ENCE 4610
Foundation Analysis and Design

Shallow Foundations Part II (Settlement)
Effects of Settlement on Structures
Schmertmann's Method
Mat Foundations and Foundation Flexibility

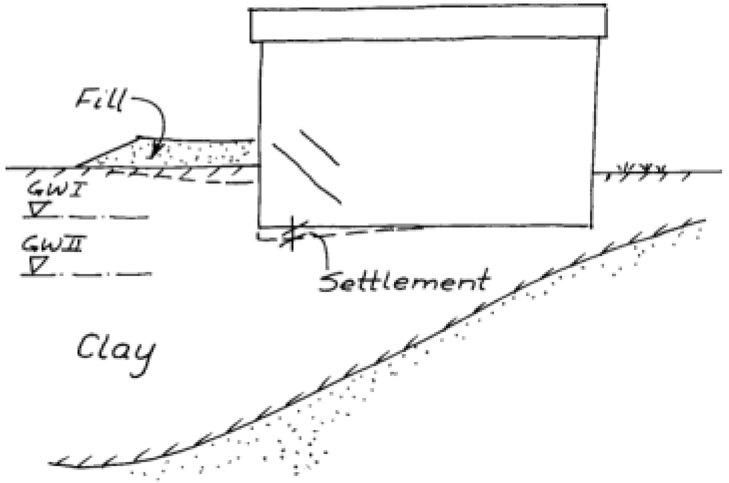
Topics for Shallow Foundations Part II (Settlement)

- Overview
  - Nature of Settlement
  - Acceptable Settlement in Structures
- Methods for Cohesive Soils
  - Primary and Secondary Consolidation, covered in Soil Mechanics
  - Elastic methods sometimes used for immediate settlements
- Methods for Cohesionless Soils
  - Hough's Method, covered in Soil Mechanics
  - Schmertmann's Method, covered here
- Mat Foundations
  - Mat foundations is a broad topic
  - Design of mat foundations in contemporary practice generally requires a computer solution
  - We will concentrate on understanding aspects of mat foundations that also apply (to some extent) to other shallow foundations
    - Limited thickness of compressible soil (vs. semi-infinite half space)
    - Floating Foundations
    - Relative flexibility of mat and soil
    - Coefficient of subgrade reaction and size effect

Two Failure Modes: Strength and Serviceability

- Strength
  - Geotechnical Strength Requirements
    - Design to prevent failure by soil plastic shear failure
    - Failure is generally catastrophic
    - Can be done either using ASD or LRFD
  - Structural Strength Requirements
    - Design to avoid structural failure of foundation components
    - Similar to other structural analyses
  - Common examples: bearing capacity failure, retaining wall failure

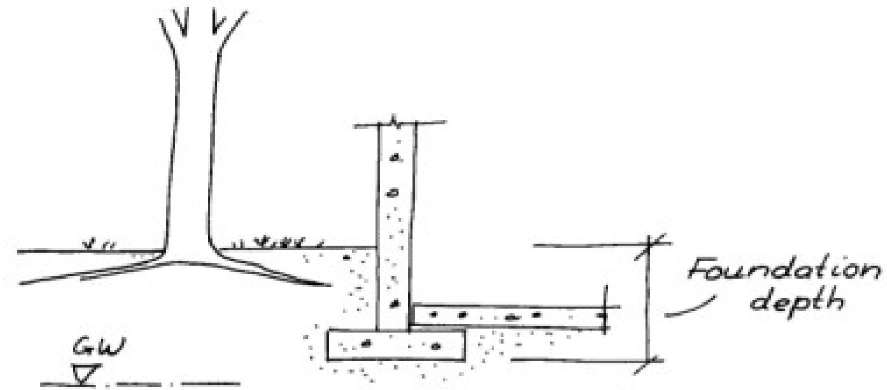
- Serviceability
  - Generally refers to failure when gradual movement of the structure makes its use difficult or impossible
  - Failure is gradual
  - Can also be analyzed using ASD or LRFD
  - Most common example in geotechnical engineering is settlement failure



Goals in Settlement Analysis

- How much will the structure settle until it's either completely or partially inoperable?
- Analysis based on:
  - Type of Structure
  - Size of Structure
  - Use of Structure

- How much will the structure settle given the geotechnical configuration?



Design considerations.

1. Seasonal changes (wet and dry periods)
2. Frost heave
3. Change of ground water level
4. Internal erosion (piping)
5. Adjecent excavations and buildings
6. Soil creep
7. Sinkholes (karst)
8. Vibrations
9. Deterioration of concrete (sulphate)

Components of Settlement

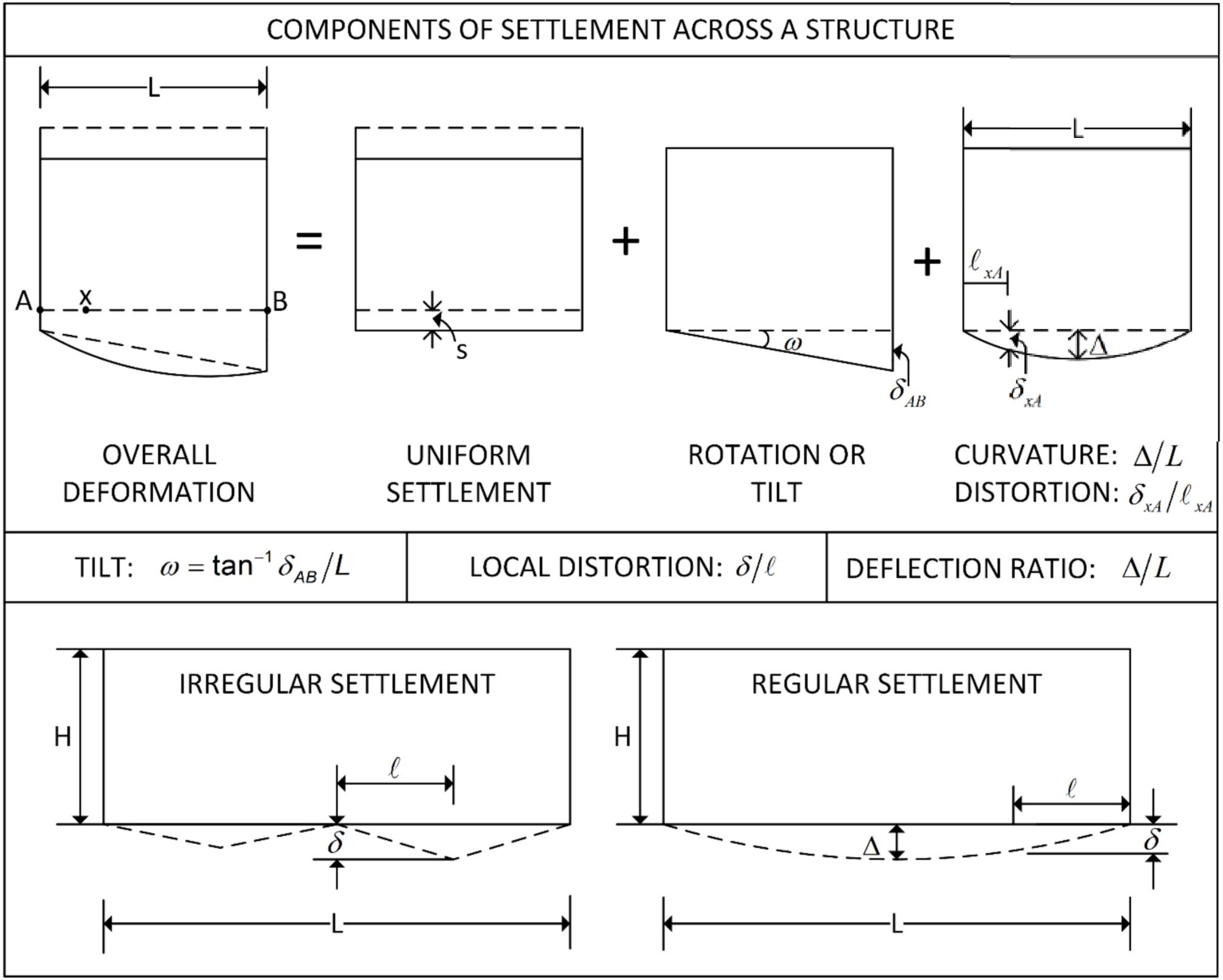


Figure 5-25 Components of Settlement
(after Duncan and Buchignani 1987, Ricceri and Soranzo 1985)

Total vs. Differential Settlement

- Total or Uniform Settlement
  - The uniform settlement of an entire structure
  - Assumes that the structure settles as a unit
  - Is expressed in units of length
  - Can be influenced by the following:
    - Neighboring structures (access, physical connection, utility connections)
    - Aesthetics
    - Drainage (a settled structure may be more vulnerable to ground water intrusion)
  - What is acceptable depends upon how the structure interacts with its neighbors

- Differential Settlement (Regular and Irregular)
  - The settlement of a structure from one column/pier/load bearing member to the next
  - Is expressed as a ratio of the distance between the relative settlement of two columns/piers and the distance between them
  - Can be influenced by the following:
    - Type of construction (steel, concrete, etc.)
    - Use of structure

Typical Values of Differential Settlement

Table 5-7 Angular Distortion Limits for Various Structures (after Skempton and MacDonald 1956, Polshin and Tokar 1957, Duncan and Buchignani 1987, and Day 1990)

Type of Structure / Condition	L/H	Tolerable δ/l Decimal	Tolerable δ/l Ratio
Small slab-on-grade structures – damage to structure	---	0.01	1/100
Steel frame with flexible siding	---	0.008	1/125
Circular steel tanks on flexible base with fixed top	---		
Considerable cracking – brick and panel	---	0.0067	1/150
Structural damage begins	---		
Safe limit – flexible brick walls (includes safety factor)	> 4		
Tilting of high buildings becomes visible	---	0.004	1/250
Slab-on-grade structures – drywall cracking	---	0.0033	1/300
Overhead cranes			
Steel or reinforced concrete frame with insensitive finish such as dry wall, glass or moveable panels	---	0.002 to 0.003	1/500 to 1/333
Circular steel tanks on flexible base with floating top	---		
Tall slender structures, such as stacks, silos, and water tanks with rigid mat foundations	---	0.002	1/500
Safe limit –cracking of buildings (includes safety factor)	---		
Steel or reinforced concrete frame with brick, block, plaster, or stucco finish	≥ 5 ≤ 3	0.002 0.001	1/500 1/1000
Frames with diagonal bracing	---	0.00167	1/600
Machinery sensitive to settlement	---	0.0013	1/750
Load-bearing brick, tile, or concrete block walls	≥ 5 ≤ 3	0.0008 0.0004	1/1250 1/2500

Example of Settlement Calculations

- Given
  - Steel framed office building with stucco, 20' column spacing, unreinforced load-bearing walls, L/H = 2 (building is relatively tall)
- Find
  - Allowable differential settlement
- Solution
  - From previous table, $\delta/L = 1/1000$
  - Typical total settlement specification = 4" (Frames structure)
  - δ_{du} = (1/1000)(20') = 0.02' = 0.25"

Schmertmann's Method

- Originally set forth by John Schmertmann, who did extensive research concerning settlement in sands (FL) and application of CPT methods to deep foundation capacity
- Several versions; modified version is presented
- Not necessary to use elastic settlement as initial settlement, as it incorporates theory of elasticity considerations

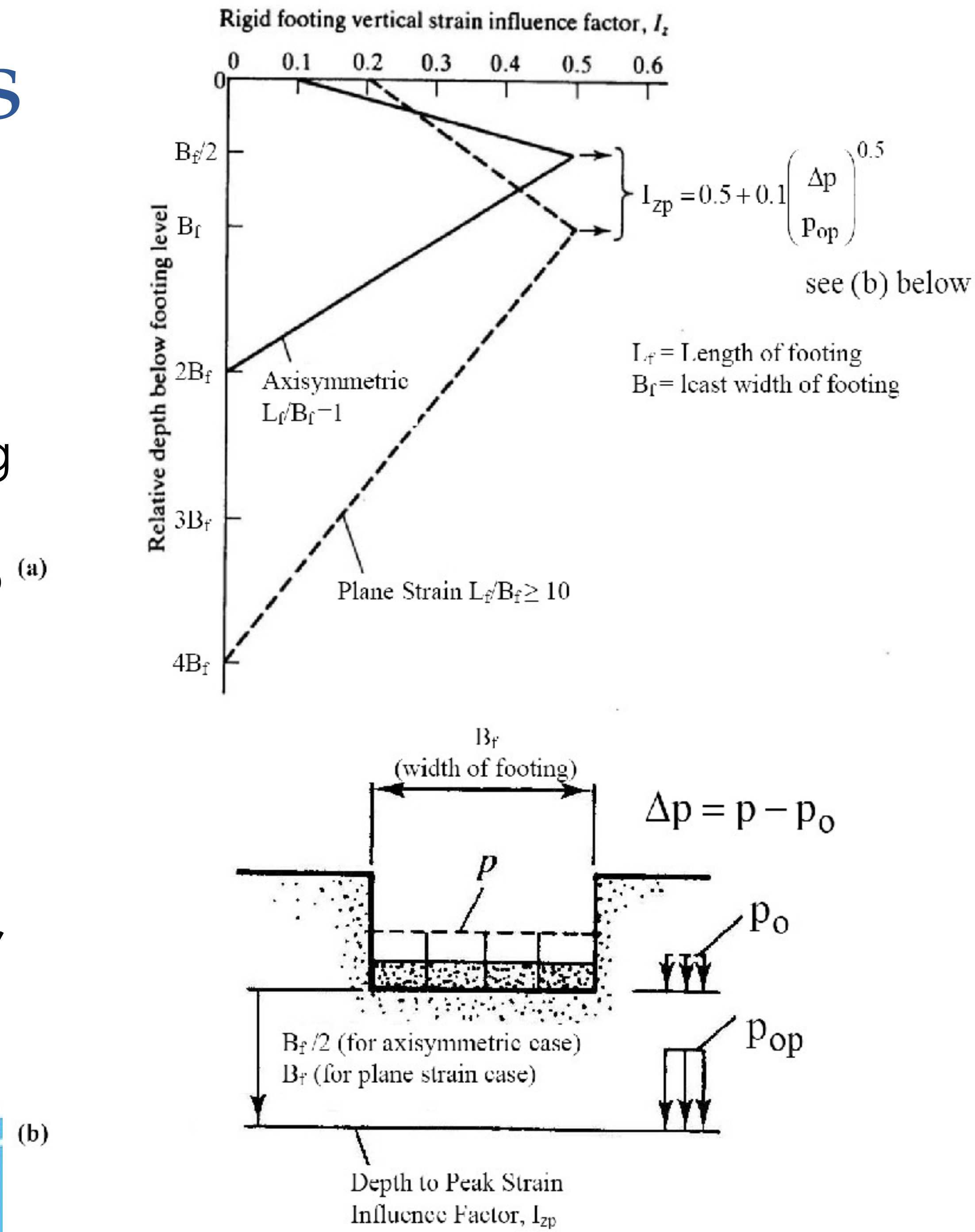


Figure 8-21. (a) Simplified vertical strain influence factor distributions, (b) Explanation of pressure terms in equation for I_{zp} (after Schmertmann, *et al.*, 1978).

Elastic Modulus of Soils for Schmertmann's Method (and other applications)

Table 5-16
Elastic constants of various soils (after AASHTO 2004 with 2006 Interims)

Soil Type	Typical Range of Young's Modulus Values, E_s (tsf)	Poisson's Ratio, ν
Clay:		
Soft sensitive	25-150	0.4-0.5 (undrained)
Medium stiff to stiff	150-500	
Very stiff	500-1,000	
Loess	150-600	0.1-0.3
Silt	20-200	0.3-0.35
Fine Sand:		
Loose	80-120	0.25
Medium dense	120-200	
Dense	200-300	
Sand:		
Loose	100-300	0.20-0.36
Medium dense	300-500	
Dense	500-800	0.30-0.40
Gravel:		
Loose	300-800	0.20-0.35
Medium dense	800-1,000	
Dense	1,000-2,000	0.30-0.40

Estimating E_s from SPT N-value

Soil Type	E_s (tsf)
Silts, sandy silts, slightly cohesive mixtures	$4\ N1_{60}$
Clean fine to medium sands and slightly silty sands	$7\ N1_{60}$
Coarse sands and sands with little gravel	$10\ N1_{60}$
Sandy gravel and gravels	$12\ N1_{60}$

Estimating E_s (tsf) from q_c static cone resistance

Sandy soils	$2q_c$ where (q_c is in tsf)

Note: 1 tsf = 95.76 kPa

8.5.1.1 Schmertmann's Modified Method for Calculation of Immediate Settlements

An estimate of the immediate settlement, S_i, of spread footings can be made by using Equation 8-16 as proposed by Schmertmann, *et al.* (1978).

$$S_i = C_1 C_2 \, \Delta p \sum_{i=1}^{n} \Delta H_i \quad \text{where} \quad \Delta H_i = H_c \left(\frac{I_z}{XE} \right) \qquad \text{8-16}$$

where: I_z = strain influence factor from Figure 8-21a. The dimension B_f represents the least lateral dimension of the footing after correction for eccentricities, i.e. use least lateral effective footing dimension. The strain influence factor is a function of depth and is obtained from the strain influence diagram. The strain influence diagram is easily constructed for the axisymmetric case ($L_f/B_f = 1$) and the plane strain case ($L_f/B_f > 10$) as shown in Figure 8-21a. The strain influence diagram for intermediate conditions can be determined by simple linear interpolation.

n = number of soil layers within the zone of strain influence (strain influence diagram).

Δp = ***net*** uniform applied stress (load intensity) at the foundation depth (see Figure 8-21b).

E = elastic modulus of layer i based on guidance provided in Table 5-16 in Chapter 5.

X = a factor used to determine the value of elastic modulus. If the value of elastic modulus is based on correlations with $N1_{60}$-values or q_c from Table 5-16 in Chapter 5, then use X as follows.

X = 1.25 for axisymmetric case ($L_f/B_f = 1$)
X = 1.75 for plane strain case ($L_f/B_f > 10$)

Use interpolation for footings with $1 < L_f/B_f < 10$

If the value of elastic modulus is estimated based on the range of elastic moduli in Table 5-16 or other sources use X = 1.0.

C_1 = a correction factor to incorporate the effect of strain relief due to embedment where:

$$C_1 = 1 - 0.5 \left(\frac{p_o}{\Delta p} \right) \geq 0.5 \qquad \text{8-17}$$

where p_o is effective in-situ overburden stress at the foundation depth and Δp is the net foundation pressure as shown in Figure 8-21b

C_2 = a correction factor to incorporate time-dependent (creep) increase in settlement for t (years) after construction where:

$$C_2 = 1 + 0.2 \log_{10} \left(\frac{t(\text{years})}{0.1} \right) \qquad \text{8-18}$$

8.5.1.2 Comments on Schmertmann's Method

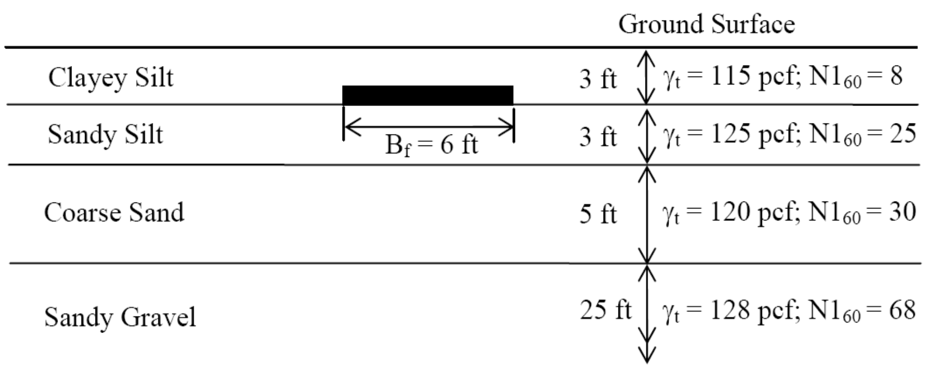
- **Effect of lateral strain**: Schmertmann and his co-workers based their method on the results of displacement measurements within sand masses loaded by model footings, as well as finite element analyses of deformations of materials with nonlinear stress-strain behavior that expressly incorporated Poisson's ratio. Therefore, the effect of the lateral strain on the vertical strain is included in the strain influence factor diagrams.

- **Effect of preloading:** The equations used in Schmertmann's method are applicable to normally loaded sands. If the sand was pre-strained by previous loading, then the actual settlements will be overpredicted. Schmertmann, *et al.* (1978) recommend a reduction in settlement after preloading or other means of compaction of half the predicted settlement. Alternatively, in case of preloaded soil deposits, the settlement can be computed by using the method proposed by D'Appolonia (1968, 1970), which includes explicit consideration of preloading.

- **C_2 correction factor**: The time duration, t, in Equation 8-18 is set to 0.1 years to evaluate the settlement immediately after construction, i.e., $C_2 = 1$. If long-term creep deformation of the soil is suspected then an appropriate time duration, t, can be used in the computation of C_2. **As explained in Sections 5.4.1 and 7.6, creep deformation is not the same as consolidation settlement.** This factor can have an important influence on the reported settlement since it is included in Equation 8-16 as a multiplier. For example, the C_2 factor for time durations of 0.1 yrs, 1 yr, 10 yrs and 50 yrs are 1.0, 1.2, 1.4 and 1.54, respectively. In cohesionless soils and unsaturated fine-grained cohesive soils with low plasticity, time durations of 0.1 yr and 1 yr, respectively, are generally appropriate and sufficient for cases of static loads. Where consolidation settlement is estimated in addition to immediate settlement, $C_2 = 1$ should be used.

The use of Schmertmann's modified method to calculate immediate settlement is illustrated numerically in Example 8-2.

Example 8-2: A 6 ft x 24 ft footing is founded at a depth of 3 ft below ground elevation with the soil profile and average $N1_{60}$ values shown. Determine the settlement in inches (a) at the end of construction and (b) 1 year after construction. There is no groundwater. The footing is subjected to an applied stress of 2,000 psf.



Solution:

Step 1: Begin by drawing the strain influence diagram. The L_f/B_f ratio for the footing is $24'/6' = 4$. From Figure 8-21(a), determine the value of the strain influence factor at the base of the footing, I_{ZB}, as follows:

$I_{ZB} = 0.1$ for axisymmetric case $(L_f/B_f = 1)$
$I_{ZB} = 0.2$ for plane strain case $(L_f/B_f \geq 10)$

Difference between axisymmetric L_f/B_f and plane strain $L_f/B_f = 9$
Difference between axisymmetric I_{ZB} and plane strain $I_{ZB} = 0.1$
Use linear interpolation for $L_f/B_f = 4$:

$\Delta(L_f/B_f)$ with respect to axisymmetric $L_f/B_f = 4\text{-}1 = 3$. Therefore

$$I_{ZB} = 0.1 + \frac{(0.2-0.1)}{9}(3) = 0.1 + \frac{0.1}{3} = 0.133$$

Step 2: Determine the maximum depth of influence, D_I, as follows:

$D_I = 2B_f$ for $L_f/B_f = 1$
$D_I = 4B_f$ for $L_f/B_f > 10$

By using linear interpolation $L_f/B_f = 4$ as before:

$\Delta(L_f/B_f)$ with respect to axisymmetric $L_f/B_f = 4\text{-}1 = 3$. Therefore

$$D_I = 2B_f + \frac{(4B_f - 2B_f)}{9}(3) = 2B_f + \frac{2B_f}{3} = \frac{6B_f + 2B_f}{3} = \frac{8B_f}{3}$$

$$D_I = \frac{8}{3}(6\,\text{ft}) = 16\,\text{ft}$$

Step 3: Determine the depth to the peak strain influence factor, D_{IP}, as follows:

From Figure 8-21(a) $D_{IP} = B_f/2$ for $L_f/B_f = 1$
$D_{IP} = B_f$ for $L_f/B_f > 10$

Use linear interpolation for $L_f/B_f = 4$:

$\Delta(L_f/B_f)$ with respect to axisymmetric $L_f/B_f = 4\text{-}1 = 3$. Therefore

$$D_{IP} = \frac{B_f}{2} + \frac{\left(B_f - {B_f}/{2}\right)}{9}(3) = \frac{B_f}{2} + \frac{B_f}{6} = \frac{3B_f + B_f}{6} = \frac{4B_f}{6}$$

$$D_{IP} = \frac{4}{6}(6\,\text{ft}) = 4\,\text{ft}$$

Step 4: Determine the value of the maximum strain influence factor, I_{ZP}, as follows:

$$I_{ZP} = 0.5 + 0.1\left(\frac{\Delta p}{p_{op}}\right)^{0.5}$$

$$\Delta p = 2{,}000\,\text{psf} - 3\,\text{ft}(115\,\text{pcf}) = 1{,}655\,\text{psf}$$

$$p_{op} = 3\,\text{ft}(115\,\text{pcf}) + 3\,\text{ft}(125\,\text{pcf}) + 1\,\text{ft}(120\,\text{pcf})$$

$$p_{op} = 345\,\text{psf} + 375\,\text{psf} + 120\,\text{psf} = 840\,\text{psf}$$

$$I_{ZP} = 0.5 + 0.1\sqrt{\frac{1{,}655\text{psf}}{840\text{psf}}} = 0.64$$

Step 5: Draw the I_Z vs. depth diagram as follows and divide it into convenient layers by using the following guidelines:

- The depth of the peak value of the strain influence is fixed. To aid in the computation, develop the layering such that one of the layer boundaries occurs at this depth even though it requires that an actual soil layer be sub-divided.
- Limit the top layer as well as the layer immediately below the peak value of influence factor, I_{zp}, to $2/3B_f$ or less to adequately represent the variation of the influence factor within D_{IP}.
- Limit maximum layer thickness to 10 ft (3 m) or less.
- Match the layer boundary with the subsurface profile layering.

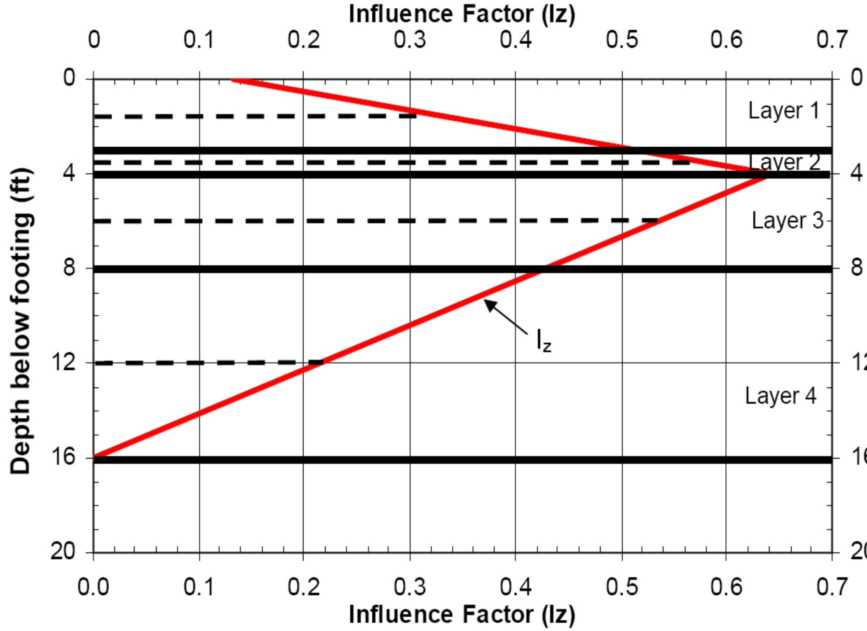
In accordance with the above guidelines, the influence depth of 16 ft is divided into 4 layers as shown below. Since the strain influence diagram starts at the base of the footing, the thickness of Layer 1 corresponds to the thickness of the sandy silt layer shown in the soil profile. Likewise, Layer 4 corresponds to the thickness of the sandy gravel layer that has been impacted by the strain influence diagram. The sum of the thicknesses of Layers 2 and 3 correspond to the thickness of the coarse sand layer shown in the soil profile. The sub-division is made to account for the strain influence diagram going though its peak value within the coarse sand layer. The minimum and maximum layer thicknesses are 1 ft (Layer 2) and 8 ft (Layer 4), respectively. The layer boundaries are shown by solid lines while the layer centers are shown by dashed lines.

Step 6: Determine value of elastic modulus E_s from Table 5-16 from Chapter 5.

Layer 1: Sandy Silt: $E = 4N1_{60}$ tsf
Layer 2: Coarse Sand: $E = 10N1_{60}$ tsf
Layer 3: Coarse Sand: $E = 10N1_{60}$ tsf
Layer 4: Sandy Gravel: $E = 12N1_{60}$ tsf

Since the elastic modulus E_s is based on correlations with $N1_{60}$-values obtained from Table 5-16, calculate the X multiplication factor as follows:

$X = 1.25$ for $L_f/B_f = 1$
$X = 1.75$ for $L_f/B_f > 10$



Use linear interpolation for $L_f/B_f = 4$

Δ (L_f/B_f) with respect to axisymmetric $L_f/B_f = 4\text{-}1 = 3$

$$X = 1.25 + \frac{(1.75 - 1.25)}{9}(3) = 1.42$$

Step 7: Using the thickness of each layer, H_c, and the relevant values for that particular layer, determine the settlement by setting up a table as follows:

Layer	H_c (inches)	$N1_{60}$	E (tsf)	XE (tsf)	Z_i (ft)	I_Z at Z_i	$\Delta H_i = \frac{I_Z}{XE} H_c$ (in/tsf)
1	36	25	100	142	1.5	0.323	0.0819
2	12	30	300	426	3.5	0.577	0.0163
3	48	30	300	426	6	0.533	0.0601
4	96	68	816	1,159	12	0.213	0.0177
						ΣH_i=	**0.1760**

Step 8: Determine embedment factor (C_1) and creep factor (C_2) as follows:

a) Embedment factor

$$C_1 = 1 - 0.5\left(\frac{p_o}{\Delta p}\right) = 1 - 0.5\left(\frac{3\,\text{ft} \times 115\,\text{pcf}}{1655\,\text{psf}}\right) = 0.896$$

b) Creep Factor

$$C_2 = 1 + 0.2\log_{10}\left(\frac{t(\text{years})}{0.1}\right)$$

- For end of construction t(yrs) = 0.1 yr (1.2 months)

$$C_2 = 1 + 0.2\log_{10}\left(\frac{0.1}{0.1}\right) = 1.0$$

- For end of 1 year:

$$C_2 = 1 + 0.2\log_{10}\left(\frac{1}{0.1}\right) = 1.2$$

Step 9: Determine the settlement at end of construction as follows:

$$S_i = C_1 C_2 \Delta p \sum H_i$$

$$S_i = (0.896)(1.0)\left(\frac{1{,}655\,\text{psf}}{2{,}000\,\text{psf}/\text{tsf}}\right)\left(0.1760\,\frac{\text{in}}{\text{tsf}}\right)$$

$$S_i = 0.130\,\text{inches}$$

Step 10: Determine the settlement after 1 year as follows:

$$S_i = 0.130\,\text{inches}\left(\frac{1.2}{1.0}\right) = 0.156\,\text{inches}$$

8.5.1.3 Tabulation of Parameters in Schmertmann's Method

To facilitate computations, Table 8-11 presents a tabulation of the various parameters involved in computation of settlement by Schmertmann's method. This table was generated by using the linear interpolation scheme demonstrated in Example 8-2. Linear interpolation may be used for L_f/B_f values between those presented in Table 8-11.

Table 8-11

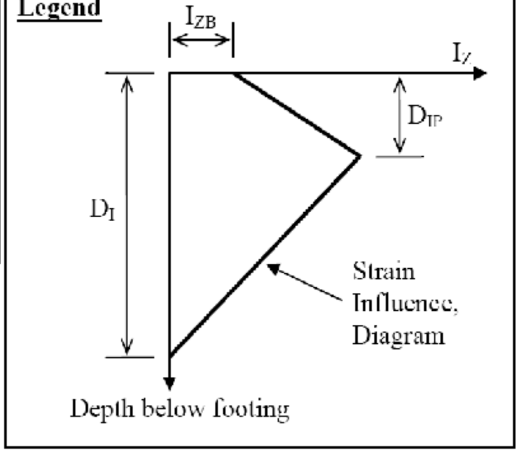
Values of parameters used in settlement analysis by Schmertmann's method

L_f/B_f	I_z at footing base, I_{ZB}	Depth to I_{zp}, D_{IP}	Depth of I_Z diagram, D_I	X factor	L_f/B_f	I_z at footing base, I_{ZB}	Depth to I_{zp}, D_{IP}	Depth of I_Z diagram, D_I	X factor
		Note 1	Note 1	Note 2			Note 1	Note 1	Note 2
1.00	**0.100**	**0.500**	**2.000**	**1.250**	**6.00**	**0.156**	**0.778**	**3.111**	**1.528**
1.25	0.103	0.514	2.056	1.264	6.25	0.158	0.792	3.167	1.542
1.50	0.106	0.528	2.111	1.278	6.50	0.161	0.806	3.222	1.556
1.75	0.108	0.542	2.167	1.292	6.75	0.164	0.819	3.278	1.569
2.00	**0.111**	**0.556**	**2.222**	**1.306**	**7.00**	**0.167**	**0.833**	**3.333**	**1.583**
2.25	0.114	0.569	2.278	1.319	7.25	0.169	0.847	3.389	1.597
2.50	0.117	0.583	2.333	1.333	7.50	0.172	0.861	3.444	1.611
2.75	0.119	0.597	2.389	1.347	7.75	0.175	0.875	3.500	1.625
3.00	**0.122**	**0.611**	**2.444**	**1.361**	**8.00**	**0.178**	**0.889**	**3.556**	**1.639**
3.25	0.125	0.625	2.500	1.375	8.25	0.181	0.903	3.611	1.653
3.50	0.128	0.639	2.556	1.389	8.50	0.183	0.917	3.667	1.667
3.75	0.131	0.653	2.611	1.403	8.75	0.186	0.931	3.722	1.681
4.00	**0.133**	**0.667**	**2.667**	**1.417**	**9.00**	**0.189**	**0.944**	**3.778**	**1.694**
4.25	0.136	0.681	2.722	1.431	9.25	0.192	0.958	3.833	1.708
4.50	0.139	0.694	2.778	1.444	9.50	0.194	0.972	3.889	1.722
4.75	0.142	0.708	2.833	1.458	9.75	0.197	0.986	3.944	1.736
5.00	**0.144**	**0.722**	**2.889**	**1.472**	**10.00**	**0.200**	**1.000**	**4.000**	**1.750**
5.25	0.147	0.736	2.944	1.486	**> 10**	**0.200**	**1.000**	**4.000**	**1.750**
5.50	0.150	0.750	3.000	1.500					
5.75	0.153	0.764	3.056	1.514					

Notes:

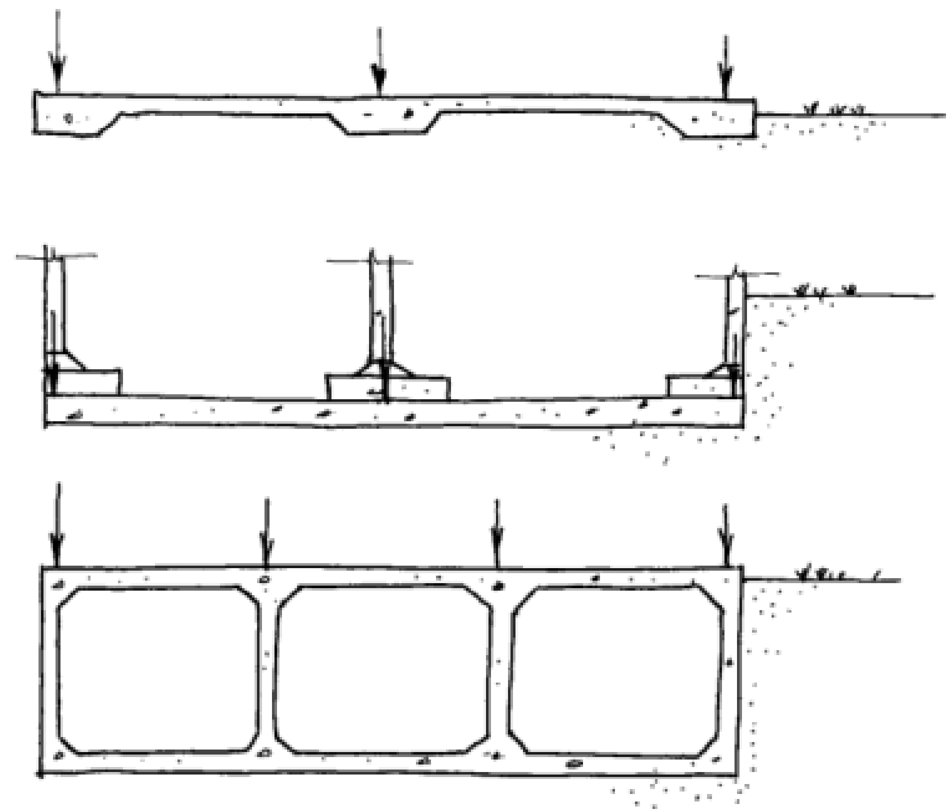
1. The depths are obtained by multiplying the value in this column by the footing width, B_f.
2. If elastic modulus is not based on SPT or CPT, then X=1.0. See Section 8.5.1.1 for a discussion on values of X factor.

Legend

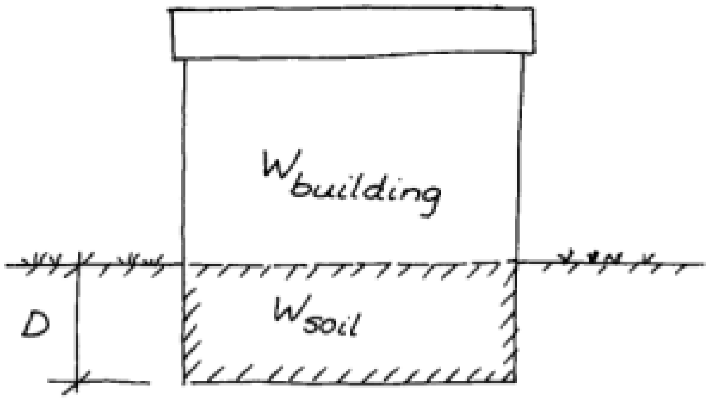


Mat Foundations

- Mat Foundations are those where the entire foundation (and not just the footing) carry the load of the structure
- The size of the mats require the consideration of factors not usually considered in shallow foundations that we have covered
- Mat foundations are complex in their implementation, we will consider four aspects of their design:
    - Limited thickness of compressible soil (vs. semi-infinite half space)
    - Floating Foundations
    - Relative flexibility of mat and soil
    - Coefficient of subgrade reaction and size effect



Floating Foundations



Floating foundation.

$W_{soil} \geq W_{Building}$

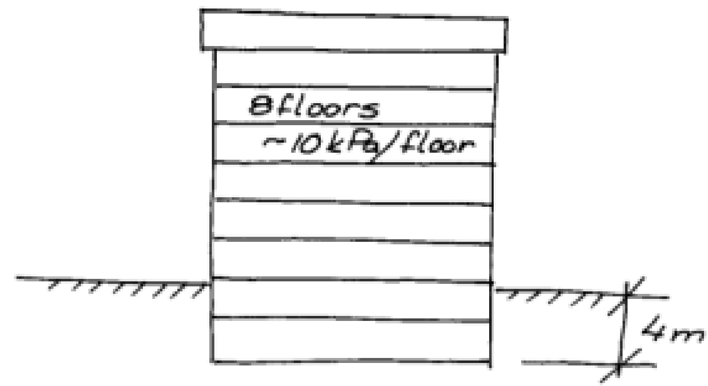
$D \rho_{soil} \geq n \cdot \Delta q$

← Number of stories

~ 10 kPa/storey

~18 kN/m³

$D = \frac{n \Delta q}{\rho_{soil}} = \frac{3 \cdot 10}{18} = 1.67\ m$



Building

DL ~ 10 kPa/floor

× 8 = 80 kPa

Excavation (4m)

$4 \cdot 1.9 \cdot 10 = \underline{76\ kPa}$

ρ g

4 kPa

Net pressure

For more information:
http://vulcanhammer.net/2023/04/05/floating-or-compensated-foundations/

Limited Elastic Soil Depth; Flexible and Rigid Foundations

- In Soil Mechanics, we looked at elastic solutions solely as semi-infinite homogeneous elastic spaces
  - Most of the foundations were flexible, i.e. they had no stiffness of their own to alter the distribution of pressure from the load to the soil
  - As we saw in the upper bound solution to bearing capacity, foundations can be considered as purely rigid as well, i.e., their displacement is uniform across the foundation
  - The space under the soil may not be semi-infinite, and at a distance *h* under the soil there be a "competent" rigid layer beyond which there is no soil deflection
- Effect of Rigidity of foundation
  - Foundation is perfectly flexible ($\Gamma = \infty$)
  - Foundation is perfectly rigid ($\Gamma = 0$)
  - Foundation is somewhere in between ($0 < \Gamma < \infty$)
- Cases to Consider
  - Flexible foundation ($\Gamma = \infty$,) limited elastic space under foundation ($h < \infty$)
  - Rigid foundation ($\Gamma = 0$,) limited elastic space under foundation ($h < \infty$)
  - Rigid foundation ($\Gamma = 0$,) infinite elastic space under foundation ($h = \infty$)
  - Varying values of Γ will produce different results
    - Flexible foundation will minimize soil stress but settle differently (sag in middle) which may affect the structure
    - Rigid foundations will settle uniformly (assuming soil is uniform) but produce higher soil stress
    - Intermediate results are…well, intermediate

Settlement of Foundations, Flexible and Rigid

$$S_e = \frac{\omega p B(1-\nu^2)}{E}$$

Where

- S_e = elastic settlement
- ω = geometry and rigidity coefficient (see table at right)
- p = uniform pressure on foundation
- B or b = width of foundation
- L or l = length of foundation
- h = distance to competent layer (for half space, h = ∞)
- h/b = ratio of foundation width to distance to completent layer
- ν = Poisson's Ratio
- E = Young's modulus of material
- Left side of table good for semi-infinite half space; right side good for a soil where a hard layer exists at a depth of h below the base of the foundation

Values of Coefficients ω

Side ratio $\alpha = l/b$	ω for half-space				ω_{mh} for finite-thickness layer at h/b				
	ω_c	ω_0	ω_m	ω_{const}	0.25	0.5	1	2	5
1 (circle)	0.64	1.00	0.85	0.79	0.22	0.38	0.58	0.70	0.78
1 (square)	$\frac{1}{2}\omega_0$	1.12	0.95	0.88	0.22	0.39	0.62	0.77	0.87
2 (rectangle)	$\frac{1}{2}\omega_0$	1.53	1.30	1.22	0.24	0.43	0.70	0.96	1.16
3 Same	$\frac{1}{2}\omega_0$	1.78	1.53	1.44	0.24	0.44	0.73	1.04	1.31
4 Same	$\frac{1}{2}\omega_0$	1.96	1.70	1.61	—	—	—	—	—
5 Same	$\frac{1}{2}\omega_0$	2.10	1.83	1.72	—	—	—	—	—
10 Same	$\frac{1}{2}\omega_0$	2.53	2.25	2.12	0.25	0.46	0.77	1.15	1.62

Note: The coefficients ω for half-space are given according to F. Schleicher with the Author's corrections, and for a finite-thickness layer, according to M. I. Gorbunov-Posadov. The table gives the following coefficients:

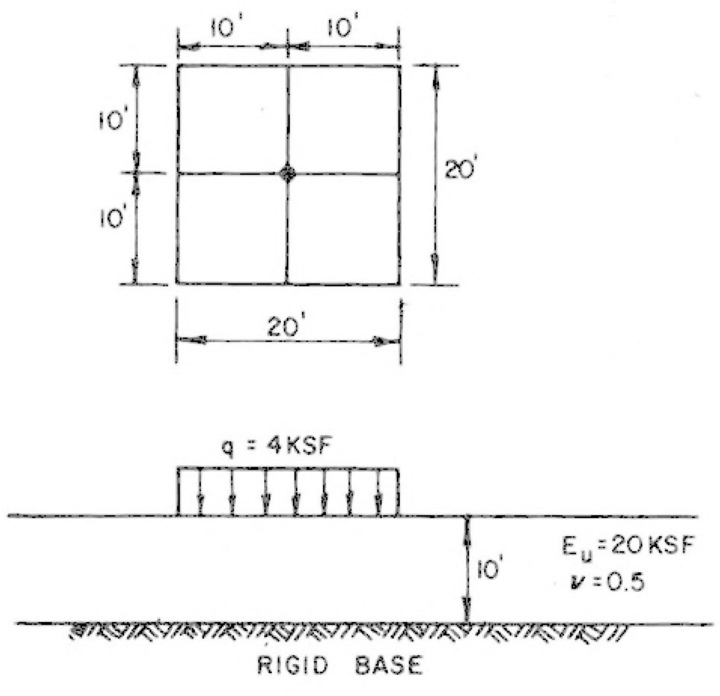
ω_c for settlements of corner points of a rectangular loading area;
ω_0 for the maximum settlement under the centre of a loaded area;
ω_m for the average settlement over the whole loaded area;
ω_{const} for settlement of absolutely rigid footings;
ω_{mh} for the average settlement of rectangular loading areas on a finite-thickness soil layer.

Example of Settlement of Flexible foundation (***Γ*** = ∞,) limited elastic space under foundation (h < ∞)

- Given
  - Problem shown at lower right
- Find
  - Average settlement assuming flexible foundation
- Solution
  - h = 10’
  - b = B = 20’
  - h/b = 0.5
  - Square Foundation
  - p = 4 ksf
  - E = 20 ksf
  - $\boldsymbol{\nu}$ = 0.5 (which, in theory, makes the soil a fluid)
  - $\boldsymbol{\omega}_{mh}$ = 0.39 (from table in previous slide)

$$S_e = \frac{\omega p B (1-\nu^2)}{E}$$

S_e = (0.39)(4)(20)(1-.5²)/20 = 1.17’ = 14.04”



Distribution of Stress and the Effect of Rigidity (Semi-Infinite Soil Mass)

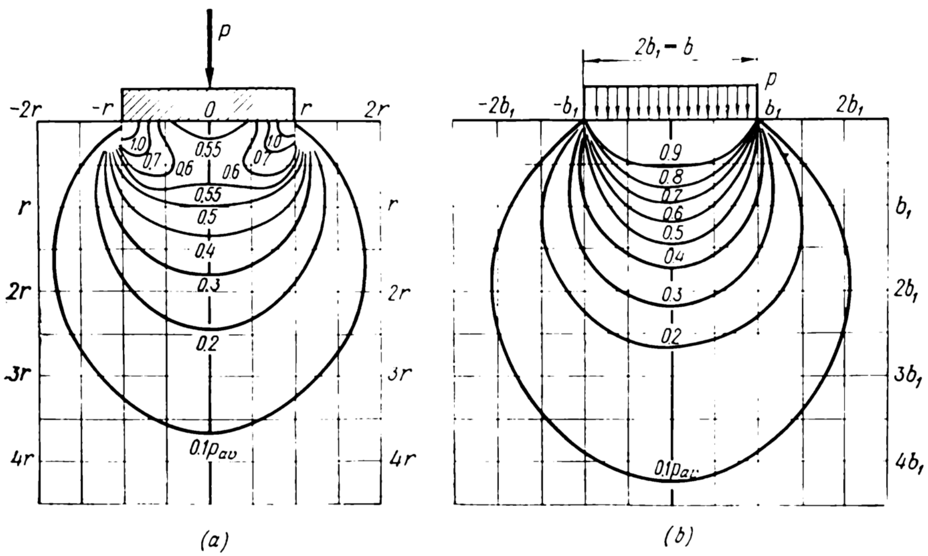


Fig. 57. Isobars in soil under foundations
(a) absolutely rigid foundation; (b) flexible foundation

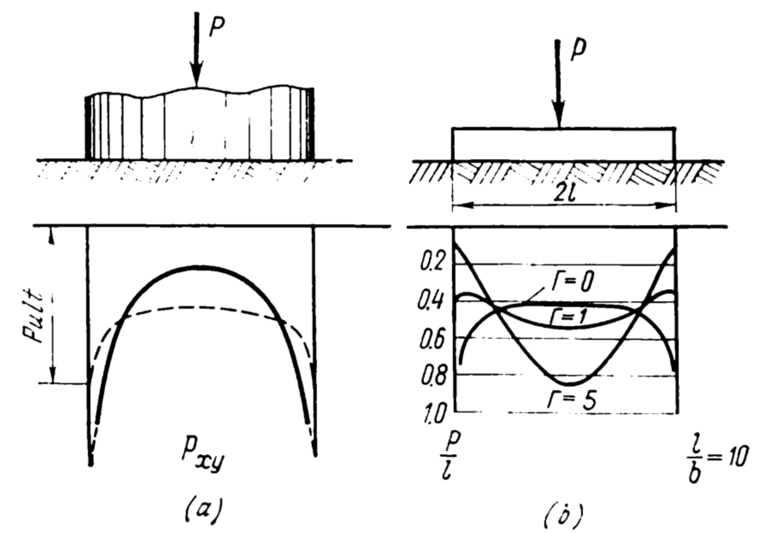


Fig. 56. Diagrams of contact pressures
(a) under an absolutely rigid foundation; (b) under foundations of various flexibilities

The solution of this integral equation for a circular base with a central load acting on an absolutely rigid foundation is of the form

$$p_{xy} = \frac{p_m}{2\sqrt{1-\left(\frac{\rho}{r}\right)^2}} \qquad (3.16)$$

where r = radius of the foundation base
ρ = distance from the centre of the base to a given point ($\rho \leq r$)
p_m = mean pressure per unit area of the base

For a planar problem

$$p_{xy} = \frac{2p_m}{\pi\sqrt{1-\left(\frac{y}{b_1}\right)^2}} \qquad (3.16')$$

where y = horizontal distance from the foundation centre to a given point
b_1 = half-width of the foundation

In both cases, the soil stress at the edge is elastically infinite

Soil Pressures Under Rigid Foundations with a Uniform Load

- Variables
  - *r* = distance from centre of circle to point of interest
  - *R* = radius of circle
  - *r/R* = ratio of point of interest distance to radius
  - *y* = distance from geometric centroid of foundation to point of interest
  - b_1 = half-width of foundation
  - y/b_1 = ratio between interest point to edge of foundation (y=0 at centre of foundation)
  - *p* = uniform pressure on foundation, stresses in soil are numbers in table times this pressure
  - *h* = distance from base of foundation to competent layer
  - *h/R* = ratio of distance to competent layer to radius of foundation
  - h/b_1 = ratio of diatance to competent layer to half-width of foundation

Table 3.8

Contact Pressures under a Rigid Foundation on a Soil Layer of Limited Thickness (Fractions of p)

r/R or y/b_1	Thickness of layer with h/R or h/b_1						
	0.25	0.5	1	2	3	5	10 and more
Circular foundations							
0.0	0.905	0.829	0.652	0.532	0.509	0.503	0.500
0.1	0.904	0.828	0.652	0.535	0.512	0.505	0.503
0.2	0.904	0.823	0.654	0.541	0.519	0.513	0.511
0.3	0.902	0.817	0.658	0.533	0.532	0.527	0.525
0.4	0.900	0.809	0.665	0.572	0.553	0.548	0.546
0.5	0.896	0.802	0.678	0.600	0.584	0.579	0.578
0.6	0.891	0.798	0.700	0.642	0.630	0.627	0.626
0.7	0.886	0.804	0.744	0.712	0.704	0.702	0.701
0.8	0.889	0.841	0.833	0.834	0.834	0.833	0.833
0.9	0.945	0.985	1.073	1.131	1.143	1.147	1.146
0.95	1.093	1.252	1.446	1.565	1.589	1.600	1.599
Strip foundations							
0.0	0.949	0.915	0.811	0.705	0.699	0.649	0.640
0.1	0.948	0.914	0.811	0.707	0.672	0.652	0.643
0.2	0.948	0.909	0.811	0.714	0.680	0.661	0.653
0.3	0.946	0.903	0.813	0.725	0.695	0.678	0.670
0.4	0.942	0.895	0.818	0.744	0.719	0.704	0.697
0.5	0.938	0.889	0.826	0.773	0.753	0.743	0.737
0.6	0.932	0.884	0.846	0.818	0.806	0.800	0.797
0.7	0.927	0.891	0.885	0.891	0.891	0.892	0.892
0.8	0.932	0.924	0.972	1.029	1.046	1.055	1.060
0.9	0.998	1.071	1.220	1.366	1.413	1.443	1.457
0.95	1.161	1.343	1.618	1.869	1.954	2.010	2.030

Relative Flexibility of Mat Foundations and Soils

$$\Gamma = 3\pi \frac{E_s\left(1-\nu_f\right)^2 l^3}{E_f\left(1-\nu_s\right)^2 t^3} \approx 10 \frac{E_s}{E_f}\left(\frac{l}{t}\right)^3$$

where

- Γ = flexibility constant, 0 for absolutely rigid foundations, infinite for absolutely flexible ones
- E_f, ν_f = Young's Modulus and Poisson's Ratio of foundation
- E_s, ν_s = Young's Modulus and Poisson's Ratio of soil
- l = half length of foundation
- t = thickness of foundation (assumed constant)

For practical purposes:

- We can use the approximate formulation
- For cases of Γ <0.16, the foundation is rigid
- For cases of Γ >31, the foundation is flexible

For more information visit:
https://vulcanhammer.net/2023/04/04/when-semi-infinite-spaces-arent-and-when-foundations-are-neither-rigid-nor-flexible/

Soil Pressures, Intermediate and Rigid Foundations ($0 \leq \Gamma < \infty$)

- Variables
  - *x* = distance from centroid of foundation to point of interest
  - *l* = half-length of foundation
  - *x/l* = ratio of distance from centroid to point of interest to distance from centroid to edge (half length)
  - ***Γ*** = flexibility variable (previously defined)
  - *H* = thickness from base of foundation to completent layer (in table, *H* is expressed in terms of the half-length *l*

Table 3.9

Reactive Pressures (Fractions of p Averaged for Sections of Length $c = \frac{1}{8} l$) for Half-Span l of Flexible Uniformly Loaded Beams on a Soil Layer of Limited Thickness H

$\pm \xi$	$\Gamma = \infty$	$\Gamma = 0$						$\Gamma = \infty$	$\Gamma = 3$					
	with H equal to													
	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞
1/16	1	1	0.970	0.924	0.828	0.718	0.639	1	0.996	0.990	0.958	0.876	0.786	0.733
3/16	1	1	0.972	0.925	0.829	0.725	0.640	1	1.002	0.985	0.956	0.874	0.789	0.738
5/16	1	1	0.976	0.929	0.836	0.741	0.668	1	1.002	0.980	0.954	0.874	0.796	0.751
7/16	1	1	0.966	0.918	0.837	0.769	0.710	1	0.998	0.975	0.934	0.866	0.808	0.772
9/16	1	1	0.956	0.910	0.857	0.816	0.770	1	1.001	0.960	0.916	0.870	0.836	0.814
11/16	1	1	0.946	0.911	0.899	0.909	0.874	1	1.001	0.960	0.904	0.893	0.899	0.898
13/16	1	1	0.960	0.927	0.987	1.059	1.070	1	1.000	0.960	0.904	0.956	1.013	1.043
15/16	1	1	1.254	1.556	1.927	2.263	2.629	1	1.000	1.190	1.474	1.791	2.073	2.251

$\pm \xi$	$\Gamma = \infty$	$\Gamma = 5$						$\Gamma = \infty$	$\Gamma = 10$					
	with H equal to													
	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞
1/16	1	1.004	0.990	0.972	0.900	0.820	0.773	1	1.030	1.018	0.994	0.938	0.875	0.889
3/16	1	1.006	0.988	0.969	0.895	0.819	0.775	1	1.032	1.010	0.989	0.931	0.871	0.858
5/16	1	1.008	0.988	0.964	0.892	0.821	0.781	1	1.036	1.008	0.981	0.923	0.866	0.821
7/16	1	0.962	0.965	0.942	0.879	0.826	0.794	1	1.027	0.920	0.953	0.902	0.859	0.742
9/16	1	1.008	0.960	0.919	0.877	0.845	0.824	1	1.037	0.980	0.925	0.889	0.862	0.842
11/16	1	1.006	0.960	0.901	0.890	0.895	0.893	1	1.023	0.980	0.898	0.887	0.889	0.870
13/16	1	1.004	0.970	0.895	0.946	0.992	1.018	1	1.016	0.970	0.881	0.915	0.956	0.980
15/16	1	1.002	1.179	1.438	1.720	1.989	2.142	1	1.004	1.114	1.380	1.615	1.822	1.988

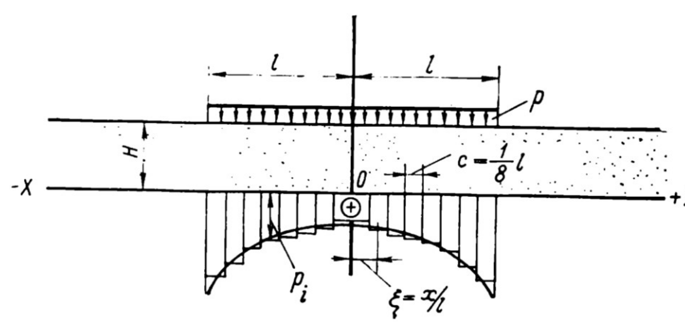
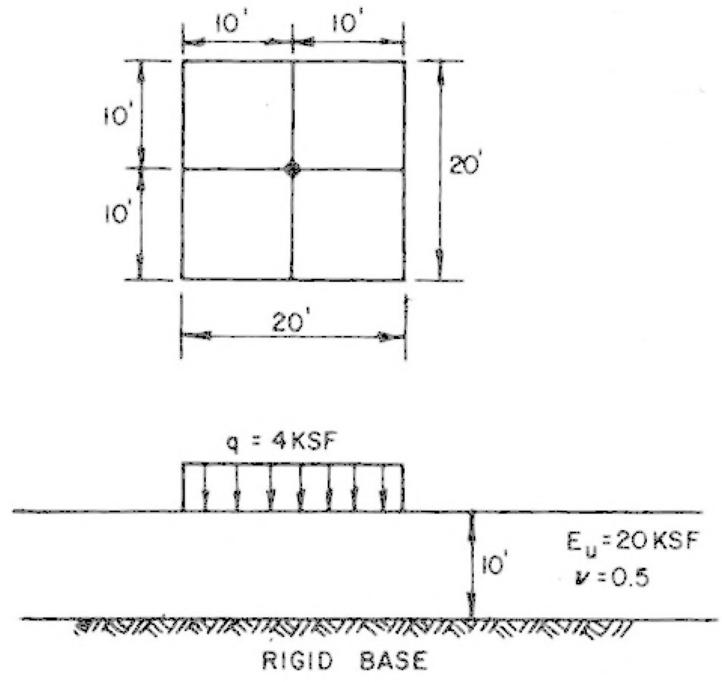


Fig. 59. Distribution of reactive pressures over the base of a flexible foundation on a limited-thickness soil layer

Example Problem

- Given
  - Problem as shown at lower right (same as previous problem)
  - Assume E_f = 720,000 ksf
  - Assume that foundation has a uniform thickness of 0.45' (5.4")
- Find
  - Soil stress under foundation considering flexibility of foundation
  - Soil stress under the foundation assuming foundation is rigid
- Solution
  - Since foundation is square, aspect ratio = 1
  - Likewise the ratio between either the half-width or the half-length of the foundation and the thickness of the layer = 1, or $H = l$
  - $\boldsymbol{\Gamma}$ = 10 (20/720000)(10/0.45)^3 = 3

$$\Gamma = 3\pi \frac{E_s\left(1-\nu_f\right)^2 l^3}{E_f\left(1-\nu_s\right)^2 t^3} \approx 10\frac{E_s}{E_f}\left(\frac{l}{t}\right)^3$$



Example Problem (Intermediate Foundation, $0 \leq \Gamma < \infty$)

Table 3.9

Reactive Pressures (Fractions of p Averaged for Sections of Length $c = \frac{1}{8}l$) for Half-Span l of Flexible Uniformly Loaded Beams on a Soil Layer of Limited Thickness H

$\pm\xi$	$\Gamma=\infty$	$\Gamma=0$						$\Gamma=\infty$	$\Gamma=3$					
	with H equal to													
	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞
1/16	1	1	0.970	0.924	0.828	0.718	0.639	1	0.996	0.990	0.958	0.876	0.786	0.733
3/16	1	1	0.972	0.925	0.829	0.725	0.640	1	1.002	0.985	0.956	0.874	0.789	0.738
5/16	1	1	0.976	0.929	0.836	0.741	0.668	1	1.002	0.980	0.954	0.874	0.796	0.751
7/16	1	1	0.966	0.918	0.837	0.769	0.710	1	0.998	0.975	0.934	0.866	0.808	0.772
9/16	1	1	0.956	0.910	0.857	0.816	0.770	1	1.001	0.960	0.916	0.870	0.836	0.814
11/16	1	1	0.946	0.911	0.899	0.909	0.874	1	1.001	0.960	0.904	0.893	0.899	0.898
13/16	1	1	0.960	0.927	0.987	1.059	1.070	1	1.000	0.960	0.904	0.956	1.013	1.043
15/16	1	1	1.254	1.556	1.927	2.263	2.629	1	1.000	1.190	1.474	1.791	2.073	2.251

$\pm\xi$	$\Gamma=\infty$	$\Gamma=5$						$\Gamma=\infty$	$\Gamma=10$					
	with H equal to													
	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞	0	$l/16$	$l/4$	$l/2$	l	$2l$	∞
1/16	1	1.004	0.990	0.972	0.900	0.820	0.773	1	1.030	1.018	0.994	0.938	0.875	0.889
3/16	1	1.006	0.988	0.969	0.895	0.819	0.775	1	1.032	1.010	0.989	0.931	0.871	0.858
5/16	1	1.008	0.988	0.964	0.892	0.821	0.781	1	1.036	1.008	0.981	0.923	0.866	0.821
7/16	1	0.962	0.965	0.942	0.879	0.826	0.794	1	1.027	0.920	0.953	0.902	0.859	0.742
9/16	1	1.008	0.960	0.919	0.877	0.845	0.824	1	1.037	0.980	0.925	0.889	0.862	0.842
11/16	1	1.006	0.960	0.901	0.890	0.895	0.893	1	1.023	0.980	0.898	0.887	0.889	0.870
13/16	1	1.004	0.970	0.895	0.946	0.992	1.018	1	1.016	0.970	0.881	0.915	0.956	0.980
15/16	1	1.002	1.179	1.438	1.720	1.989	2.142	1	1.004	1.114	1.380	1.615	1.822	1.988

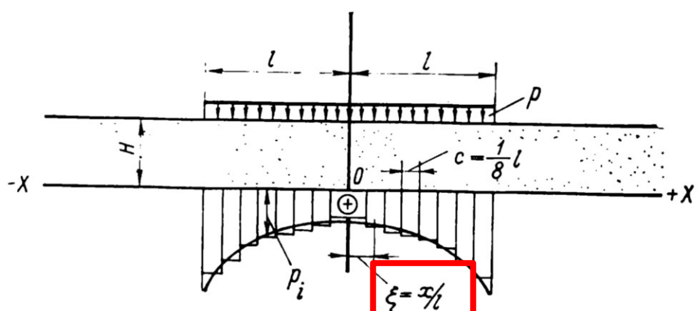
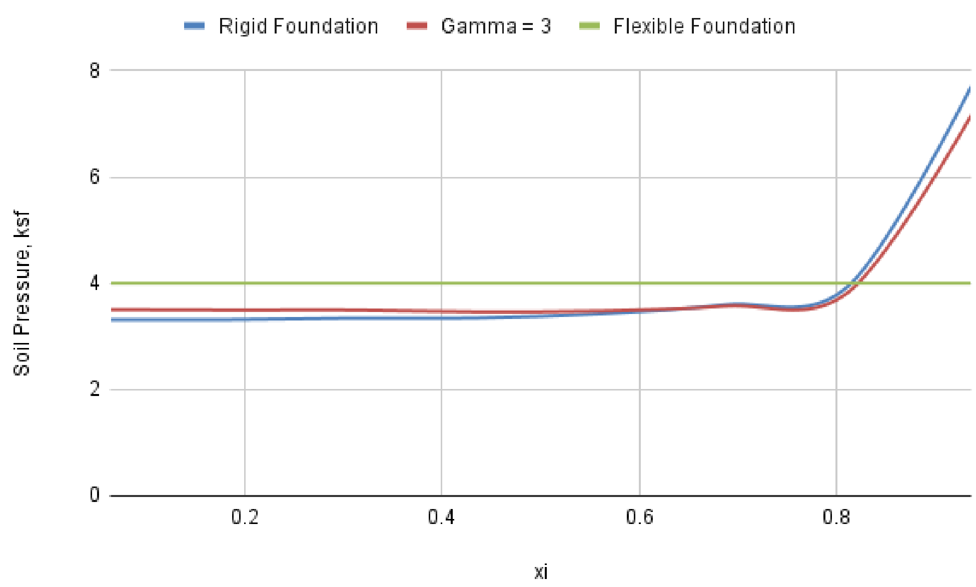


Fig. 59. Distribution of reactive pressures over the base of a flexible foundation on a limited-thickness soil layer

Example Problem (Intermediate Foundation, $0 \leq \Gamma < \infty$)

	Fraction of Pressure			Soil Vertical Reaction, ksf		
xi	Rigid Foundation	Gamma = 3	Flexible Foundation	Rigid Foundation	Gamma = 3	Flexible Foundation
0.0625	0.828	0.876	1	3.312	3.504	4
0.1875	0.829	0.874	1	3.316	3.496	4
0.3125	0.836	0.874	1	3.344	3.496	4
0.4375	0.837	0.866	1	3.348	3.464	4
0.5625	0.857	0.87	1	3.428	3.48	4
0.6875	0.899	0.893	1	3.596	3.572	4
0.8125	0.987	0.958	1	3.948	3.832	4
0.9375	1.927	1.791	1	7.708	7.164	4



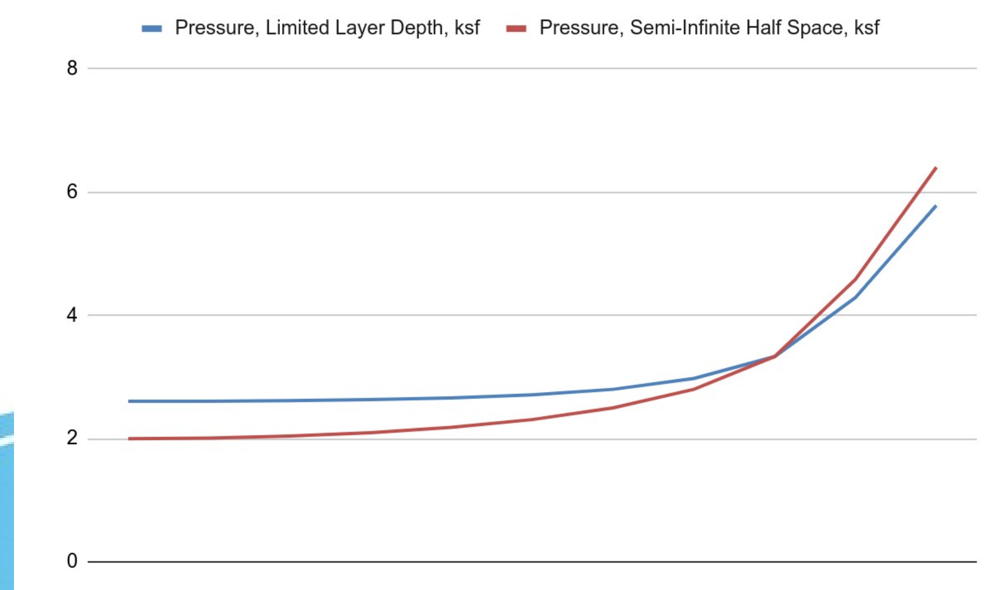
Example Problem (Rigid Foundation)

- Given
  - Same stratigraphy as before
  - Same uniform pressure as before
  - Circular foundation, 20’ in diameter
  - Rigid foundation
- Find
  - Soil stresses under foundation for stratigraphy shown
  - Soil stresses under foundation for semi-infinite soil mass
- Solution
  - h/r = 10/10 = 1
  - For semi-infinite soil mass, formula is as below:

Table 3.8

Contact Pressures under a Rigid Foundation on a Soil Layer of Limited Thickness (Fractions of p)

r/R or y/b_1	Thickness of layer with h/R or h/b_1						
	0.25	0.5	1	2	3	5	10 and more
	Circular foundations						
0.0	0.905	0.829	0.652	0.532	0.509	0.503	0.500
0.1	0.904	0.828	0.652	0.535	0.512	0.505	0.503
0.2	0.904	0.823	0.654	0.541	0.519	0.513	0.511
0.3	0.902	0.817	0.658	0.533	0.532	0.527	0.525
0.4	0.900	0.809	0.665	0.572	0.553	0.548	0.546
0.5	0.896	0.802	0.678	0.600	0.584	0.579	0.578
0.6	0.891	0.798	0.700	0.642	0.630	0.627	0.626
0.7	0.886	0.804	0.744	0.712	0.704	0.702	0.701
0.8	0.889	0.841	0.833	0.834	0.834	0.833	0.833
0.9	0.945	0.985	1.073	1.131	1.143	1.147	1.146
0.95	1.093	1.252	1.446	1.565	1.589	1.600	1.599



The solution of this integral equation for a circular base with a central load acting on an absolutely rigid foundation is of the form

$$p_{xy} = \frac{p_m}{2\sqrt{1-\left(\frac{\rho}{r}\right)^2}} \qquad (3.16)$$

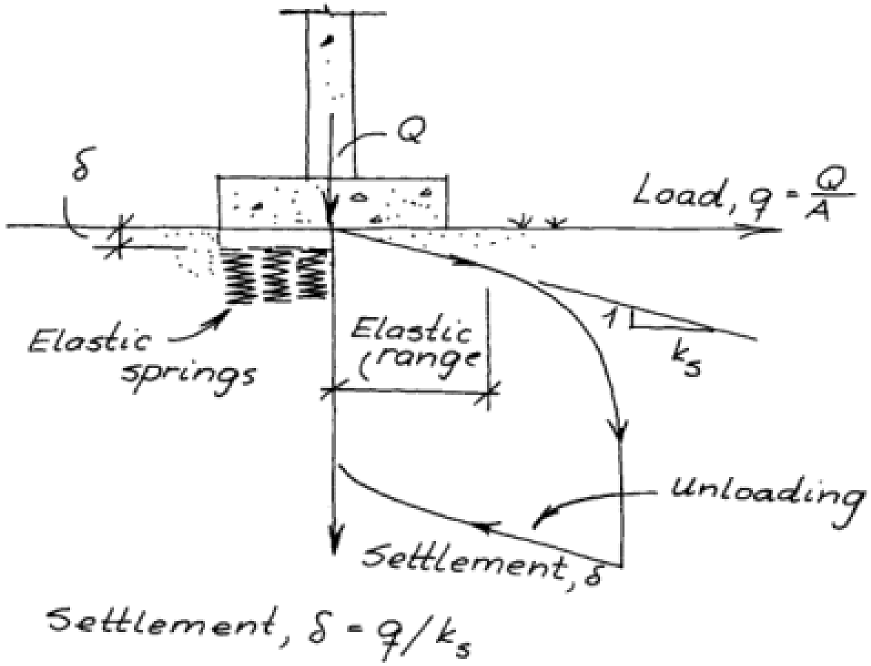
where r = radius of the foundation base
ρ = distance from the centre of the base to a given point ($\rho \leqslant r$)
p_m = mean pressure per unit area of the base

Example Problem (Rigid Foundation)

ρ/r	y, m	Pressure Ratio (from chart)	Pressure, Limited Layer Depth, ksf	Pressure Ratio, Semi-Infinite Half-Space	Pressure, Semi-Infinite Half Space, ksf
0	0	0.652	2.608	0.500	2.000
0.1	1	0.652	2.608	0.503	2.010
0.2	2	0.654	2.616	0.510	2.041
0.3	3	0.658	2.632	0.524	2.097
0.4	4	0.665	2.66	0.546	2.182
0.5	5	0.678	2.712	0.577	2.309
0.6	6	0.7	2.8	0.625	2.500
0.7	7	0.744	2.976	0.700	2.801
0.8	8	0.833	3.332	0.833	3.333
0.9	9	1.073	4.292	1.147	4.588
0.95	9.5	1.446	5.784	1.601	6.405

Coefficient of Subgrade Reaction

- Nonrigid foundations must take into account that both the soil and the foundation have deformation characteristics.
  - These deformation characteristics can be either linear or non-linear (especially in the case of the soils)
  - The best known type of foundations of this type are pavements, especially non-rigid pavements (asphalt)
- The deformation characteristics of the soil are quantified in the coefficient of subgrade reaction, or subgrade modulus



- Variables:
  - k_s = coefficient of subgrade reaction, units of force/length^3 (the units are the same as the unit weight, but not the significance!)
  - q = bearing pressure
  - δ = settlement

Plate Load Test

b. Plate Bearing Test. The plate bearing test can be used as an indicator of compressibility and as a supplement to other compressibility data.

(1) Procedure. For ordinary tests for foundation studies, use procedure of ASTM Standard D1194, Test for Bearing Capacity of Soil for Static Load on Spread Footings, except that dial gages reading to 0.001 in. should be substituted. Tests are utilized to estimate the modulus of subgrade reaction and settlements of spread foundations. Results obtained have no relation to deep seated settlement from volume change under load of entire foundation.

(2) Analysis of Test Results. (See Figure 12.) Determine yield point pressure for logarithmic plot of load versus settlement. Convert modulus of subgrade reaction determined from test K_{v1} to the property K_v for use in computing immediate settlement (Chapter 5). In general, tests should be conducted with groundwater saturation conditions simulating those anticipated under the actual structure.

Data from the plate load test is applicable to material only in the immediate zone (say to a depth of two plate diameters) of the plate and should not be extrapolated unless material at greater depth is essentially the same.

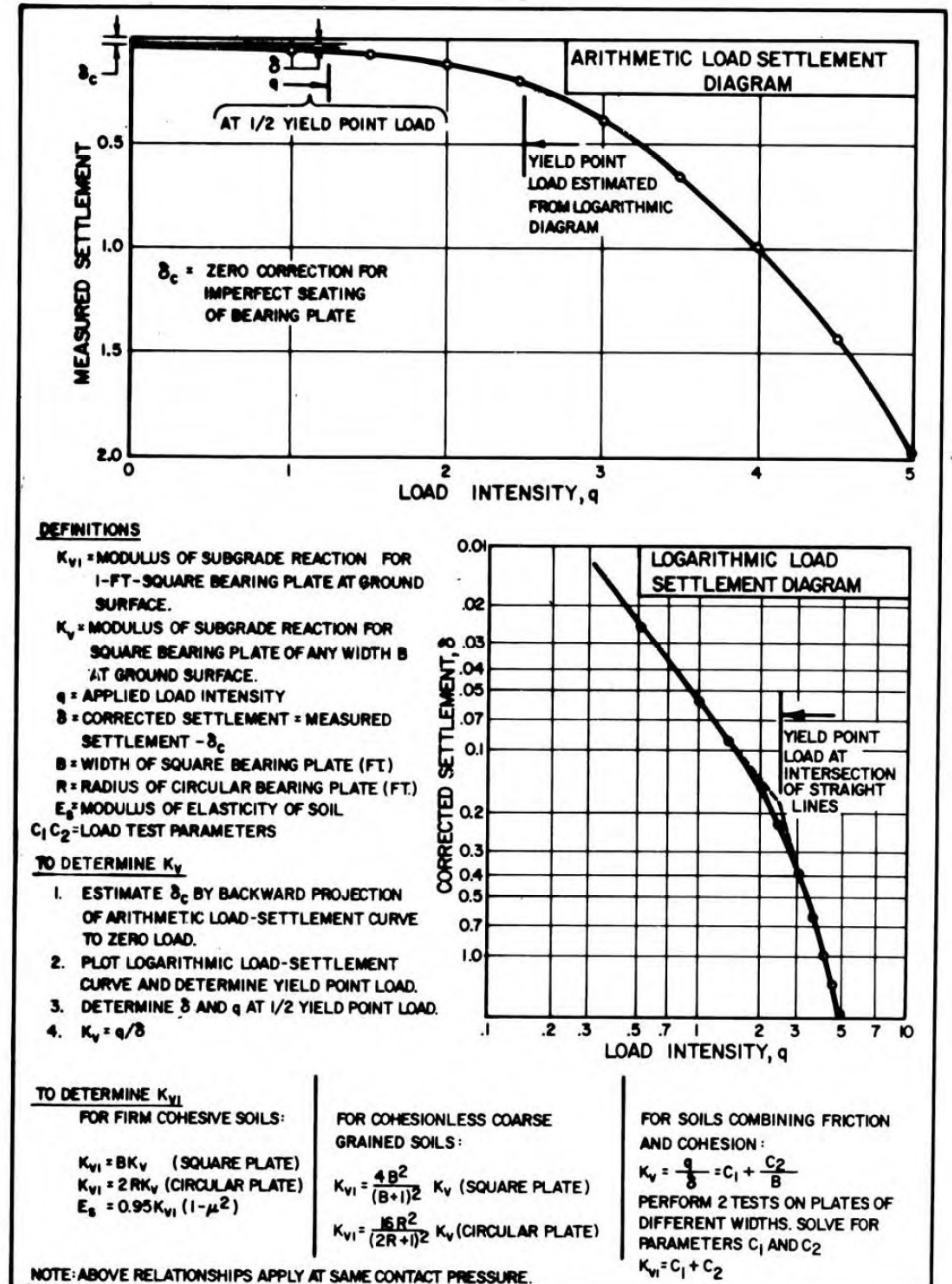


FIGURE 12

Analysis of Plate Bearing Tests

Fundamental Problems with Coefficient of Subgrade Reaction

- Example
  - Consider a foundation on an elastic medium
  - Use SFH Equation 8-19 and combine this with the basic equation for the coefficient of subgrade reaction
  - The result is this:

$$k_s = \frac{E_s}{\omega B \left(1 - \nu_s^2\right)}$$

- The coefficient of subgrade reaction is a function of Young's Modulus and Poisson's Ratio (which are mechanical properties of the soil) and foundation size, shape and rigidity (which is not a soil property)
- The coefficient of subgrade reaction is thus not a fundamental soil property

Size Effect of Foundations on Deflection Characteristics

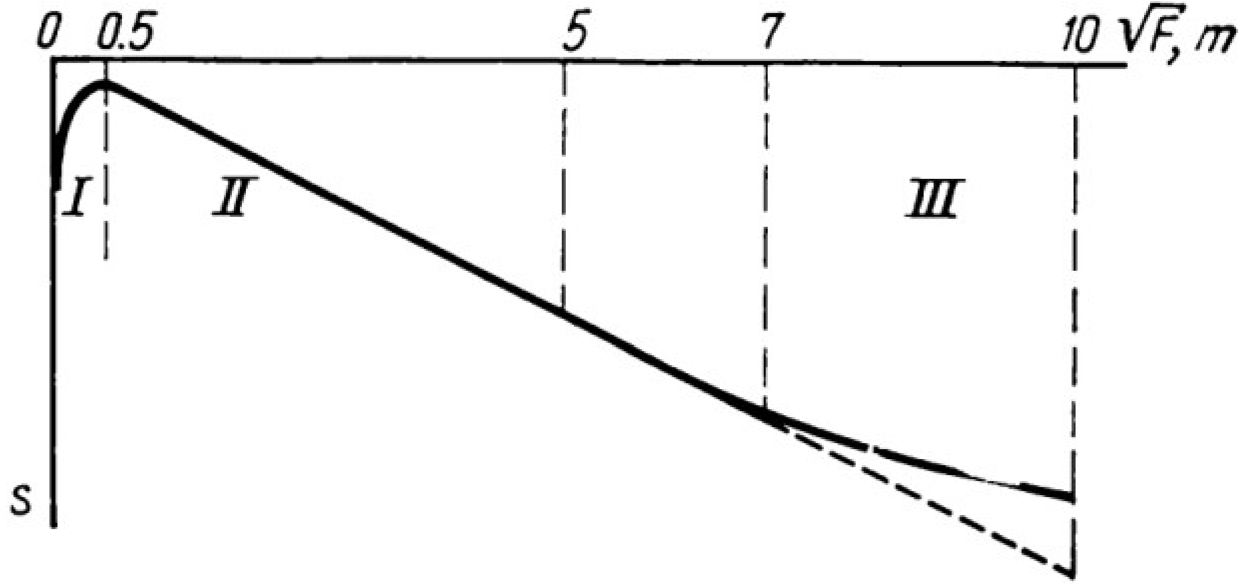


Fig. 90. Relationship between settlement of natural soils and dimensions of loading area

Relationship between settlement of natural soils and dimensions of loading area (from Tsytovich (1976).) The variable F is the area of the foundation, thus the square root of F is the basic dimension of the foundation and, in the case of square foundations, the exact dimension b or B (see below.)

Problems with Coefficient of Subgrade Reaction

- Other problems include:
  - Non-linearity of soils
  - Strain-softening of soils
  - Change in soil mechanical properties with depth
  - Change in depth affected by pressure at foundation (larger the foundation, larger the region of influence under the foundation
- Nevertheless, the coefficient of subgrade reaction is used extensively in pavement and mat foundation design
- It can be determined in a number of ways, such as plate load tests, or sometimes back calculation from elastic modulus

Questions?