

This document downloaded from
vulcanhammer.net vulcanhammer.info
Chet Aero Marine



Don't forget to visit our companion site
<http://www.vulcanhammer.org>

Use subject to the terms and conditions of the respective websites.

ENCE 3610

Soil Mechanics

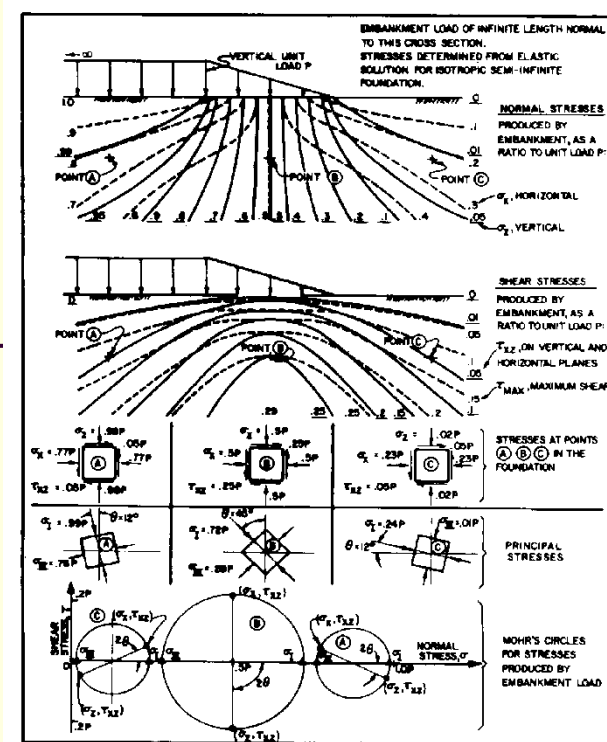


FIGURE 1
Examples of Stress Conditions at a Point

Lecture 9

Introduction to Soil Stress States

Mohr's Circle and Combined Stresses

Stresses in Soils

- Soils, like any other engineering material, experience stresses
 - We have already looked at stresses due to the “self-weight” of the soil and water combination (effective stresses)
 - We have also looked at stresses from moving water as well
- As structures are built around soils, the stress states of the soils will change
- Methods of state (stress and deflection) change
 - Elastic deformation and stress (non-path dependent,) similar to other engineering materials
 - Plastic deformation (path dependent,) which can be catastrophic under certain conditions
 - Consolidation settlement, volume change in soils due to rearrangement of the particles (unlike other engineering materials)

Stresses in Soils

- Soil stresses can and are analyzed with methods used elsewhere in mechanics of materials
- There are two important differences between soils and other materials and configurations which you may have studied
- Soils deform plastically almost exclusively
 - Our differentiation between elastic and plastic deformation and stress is, to a large extent, artificial, but necessary for analysis
- Soils act in a two- or three-dimensional continuum; for many students, it is the first introduction to continuum mechanics

Three-Dimensional Soil Stresses

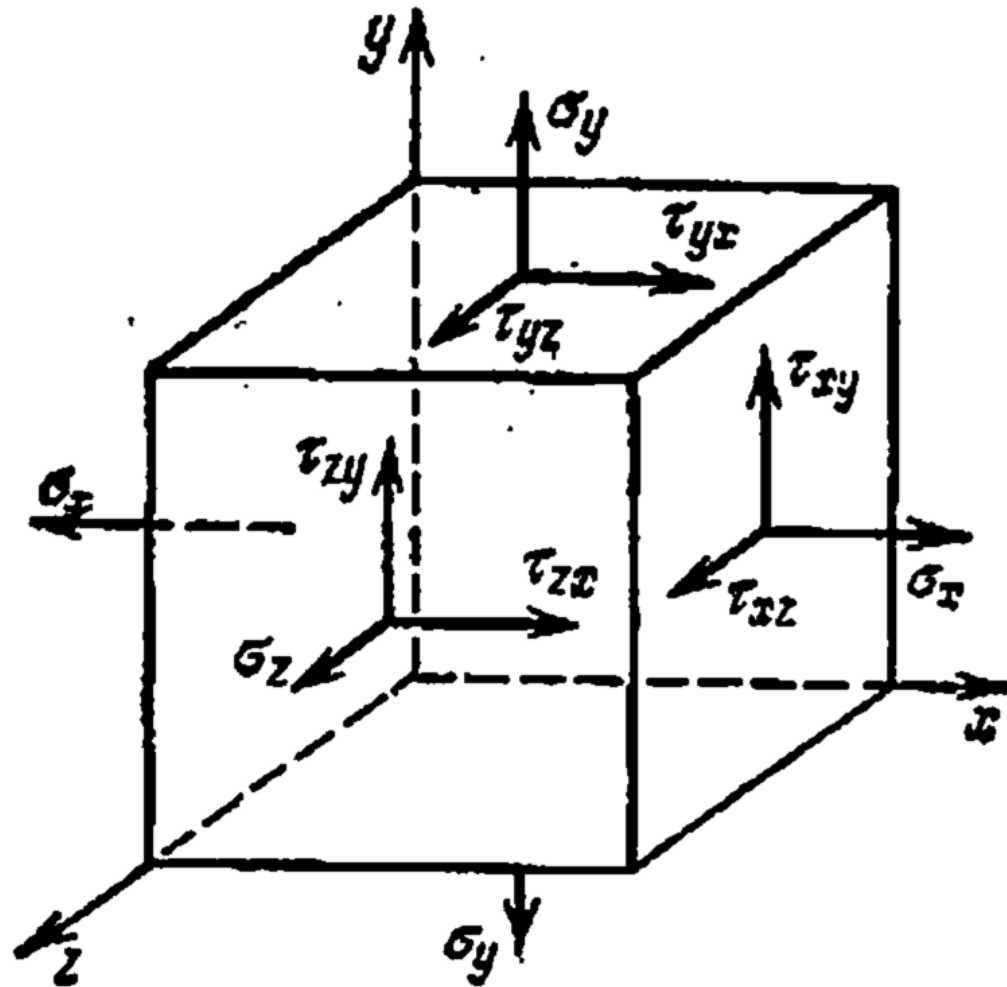
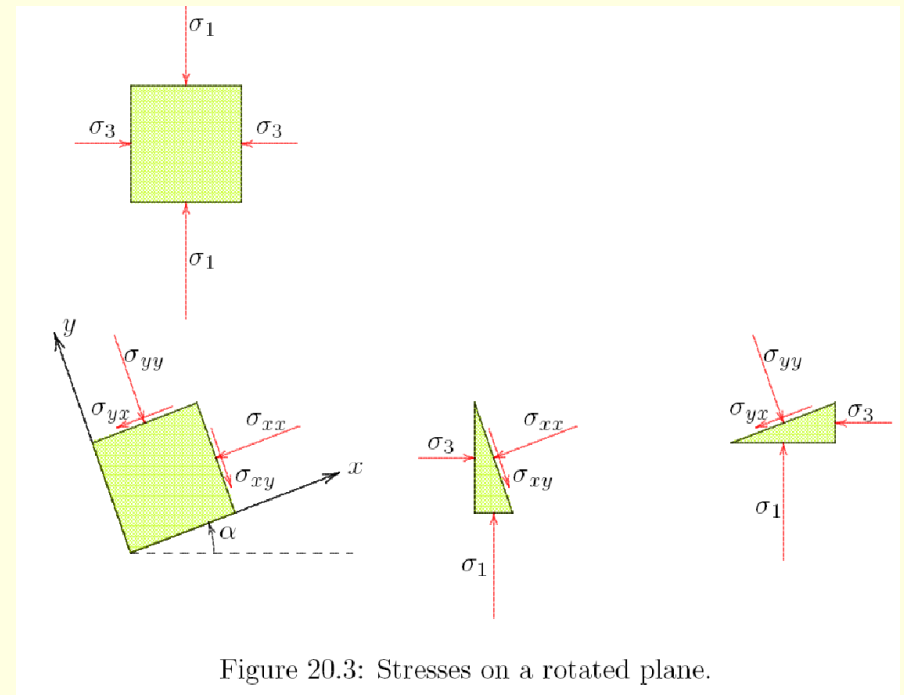


Fig. 69

Mohr's Circle for Stresses

- A convenient way to show the relationship of stresses (normal and shear) at a point
- At one time, it was the best way to handle the algebra, but this has been superseded
- It comes out of static equilibrium of stresses at a point, not theory of elasticity
- It can be done in two or three dimensions (we will restrict our studies to two)



Mohr's Circle in Two Dimensions

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left\{ \frac{\sigma_y - \sigma_x}{2} \right\}^2 + \tau_{xy}^2}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left\{ \frac{\sigma_y - \sigma_x}{2} \right\}^2 + \tau_{xy}^2}$$

$$\sigma_x = \frac{\sigma_1 + \sigma_3}{2} - \frac{(\sigma_1 - \sigma_3) \cos(2\theta)}{2}$$

$$\sigma_y = \frac{\sigma_1 + \sigma_3}{2} + \frac{(\sigma_1 - \sigma_3) \cos(2\theta)}{2}$$

$$\tau_{xy} = \tau_{yx} = \frac{(\sigma_1 - \sigma_3) \sin(2\theta)}{2}$$

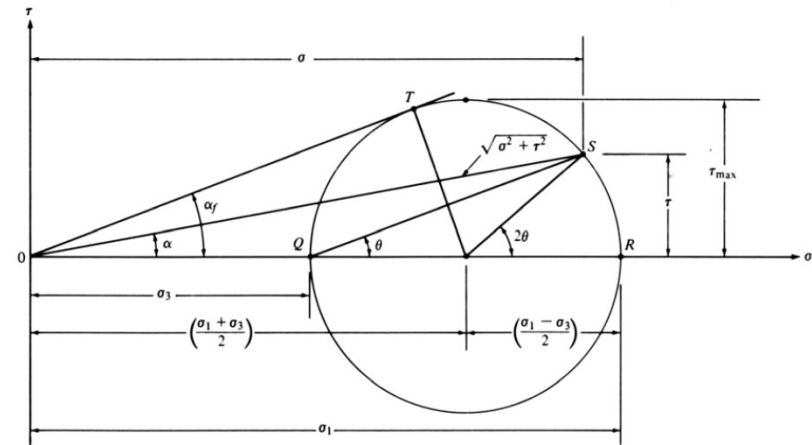


Figure B-2. Simplified version of Figure B-1(b) showing some basic relationships and characteristics of the Mohr's Circle (Cernica, 1982).

Table B-1

Summary of stresses acting on the four planes shown in Figure B-1(a)

Plane	Normal Stress	Sign	Shear Stress	Sign
P	σ_x	+	τ_{xy}	+
Q	σ_y	+	τ_{yx}	-
R	σ_x	+	τ_{yx}	+
S	σ_y	+	τ_{xy}	-

Note: In this table and Figure B-1, the sign convention is according to that commonly used in solid mechanics, i.e., tensile (normal) stresses are considered positive and are plotted to the right of the origin. In soil mechanics it is customary to indicate *compressive* stresses as positive because soil cannot sustain tensile stresses since it has virtually no strength in tension. The use of the soil mechanics convention will be illustrated in Sections B.3 and B.4.

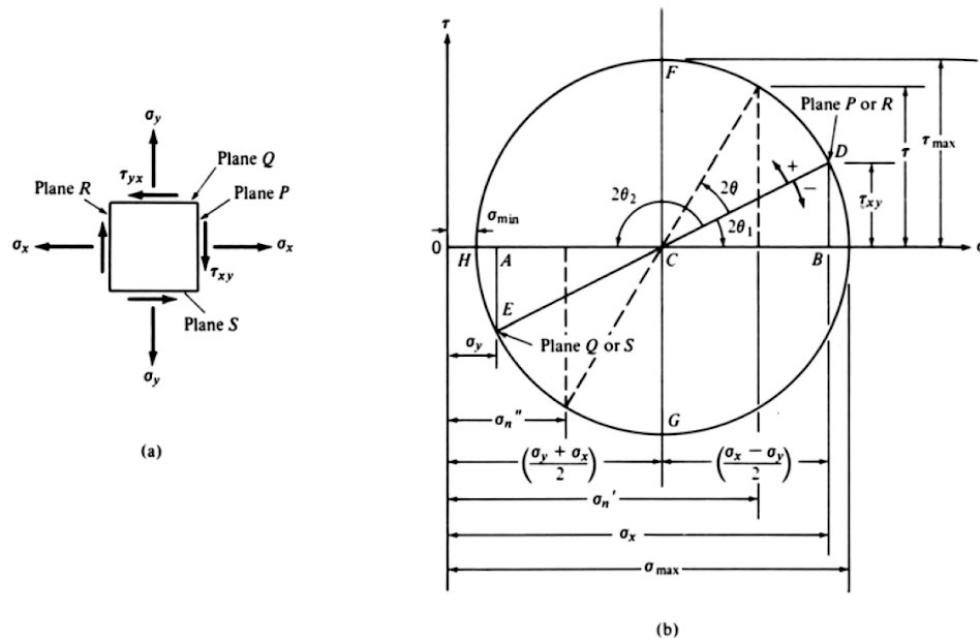
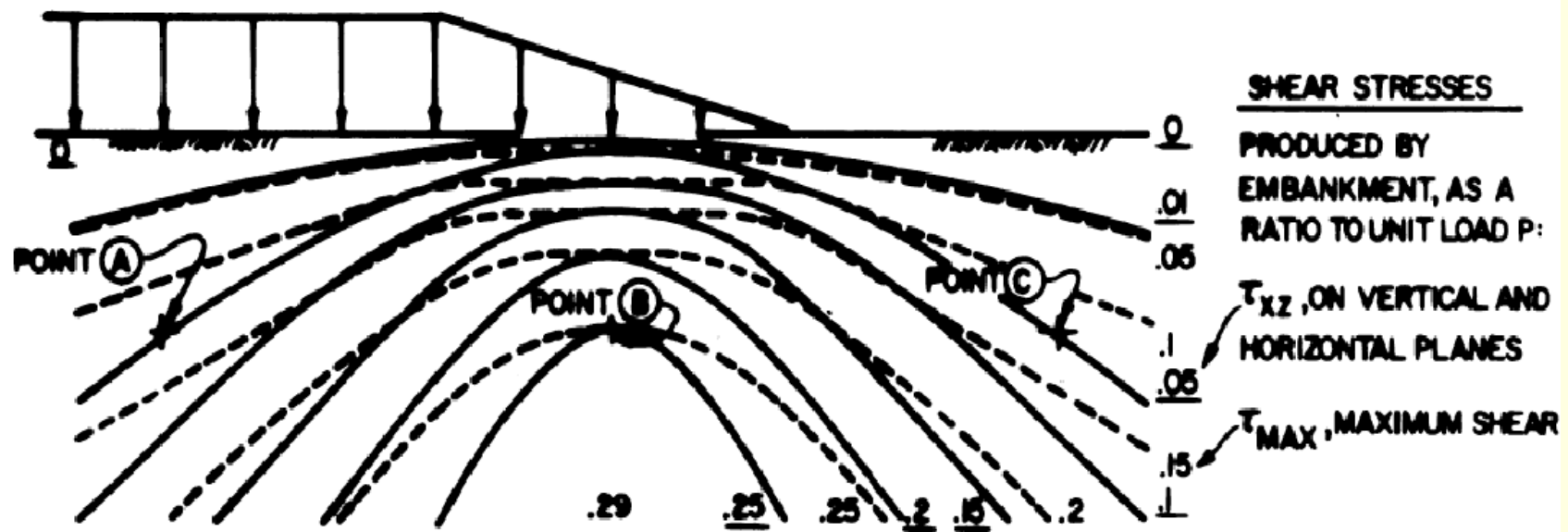
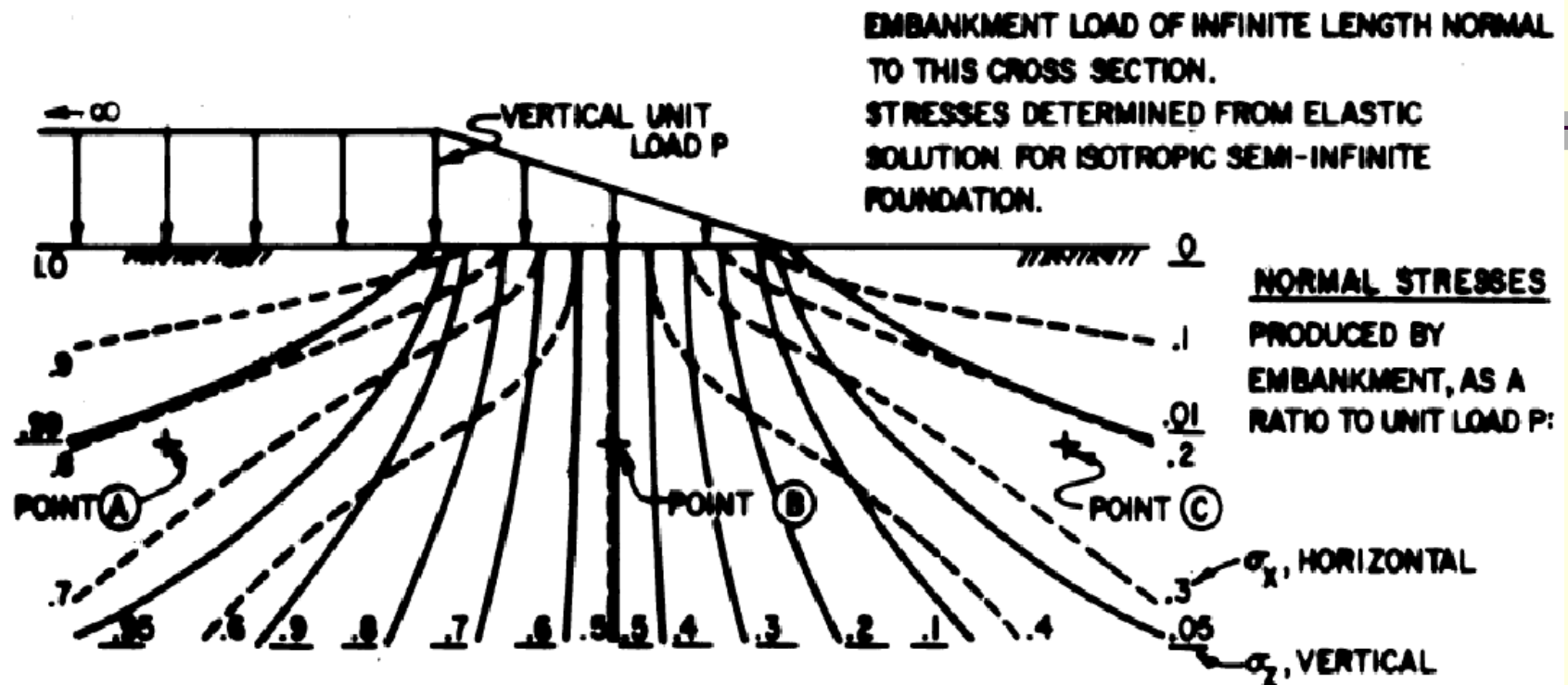
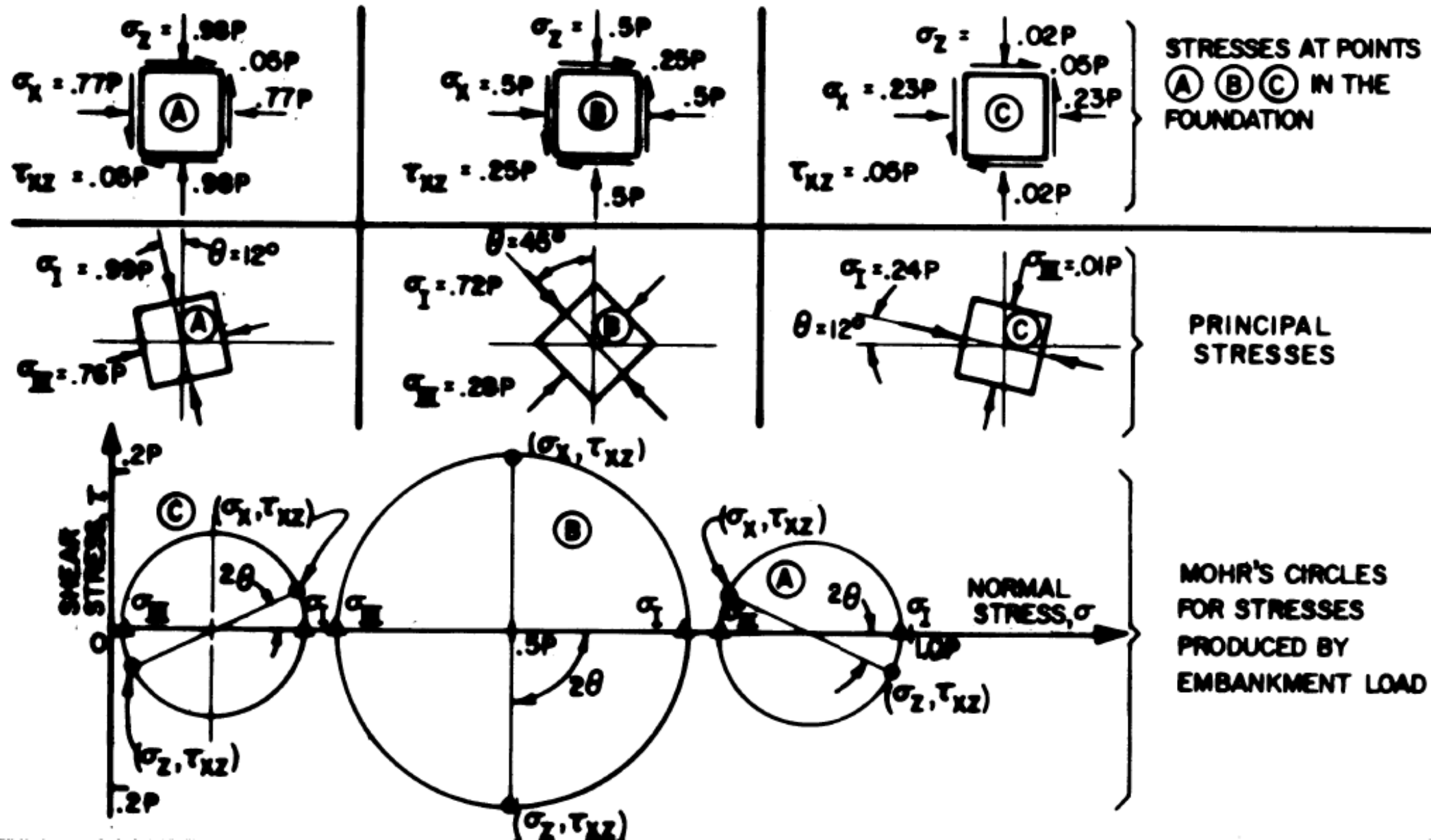


Figure B-1. Mohr's circle for general stress conditions (Cernica, 1982).

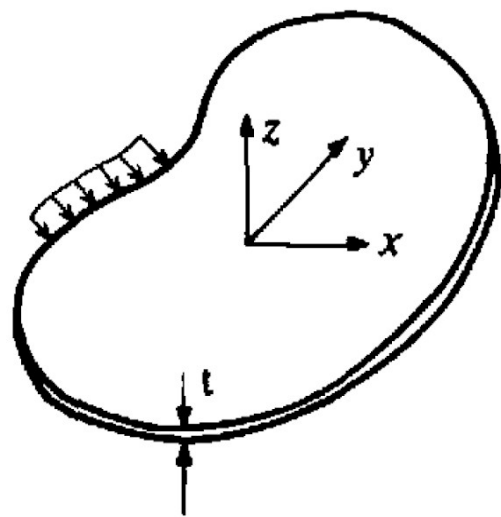
Examples Using Mohr's Circle



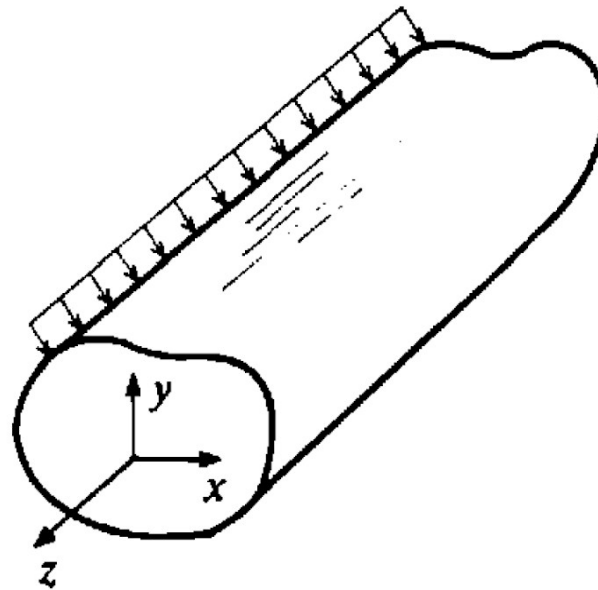
Examples Using Mohr's Circle



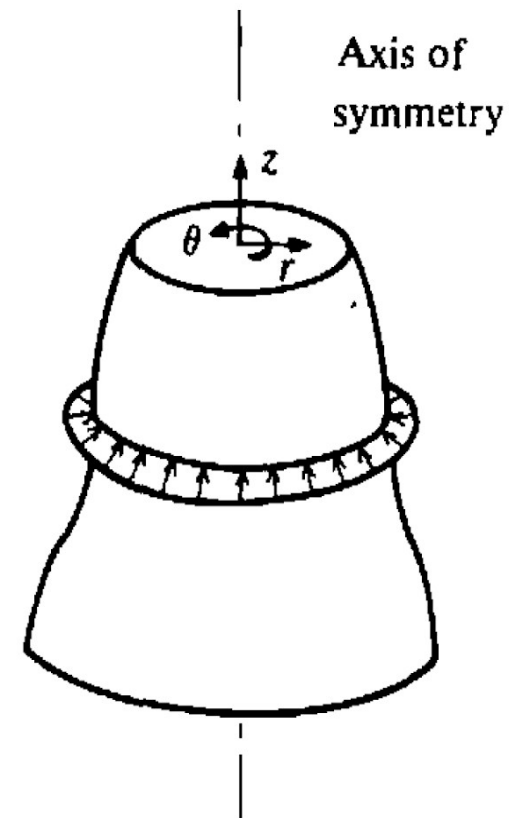
So How Do We Use Two-Dimensional Stress Concept with Three-Dimensional Soil Space?



(a) Plane stress



(b) Plane strain



(c) Axial symmetry

From Owen and Hinton (1980)

Observations on Principal and Shear Stresses in Soils

- Tension is never allowed in soils; compressive stresses are considered positive
- Although principal stresses in many cases coincide with the z-y plane used in geotechnical analysis, this is not always the case
- The understanding of principal and shear stresses is critical in determining the bearing and frictional capacities of soils

■ Stress Invariants

- The sum of the principal stresses in any stress condition is always equal to the sum of the normal (non-shear) stresses
- Thus, for two-dimensional Mohr's Circle using Mohr-Coulomb failure,
$$\sigma_1 + \sigma_3 = \sigma_x + \sigma_y$$

Frictional Forces

$$F_a = W \sin \theta_m$$

$$P_n = W \cos \theta_m$$

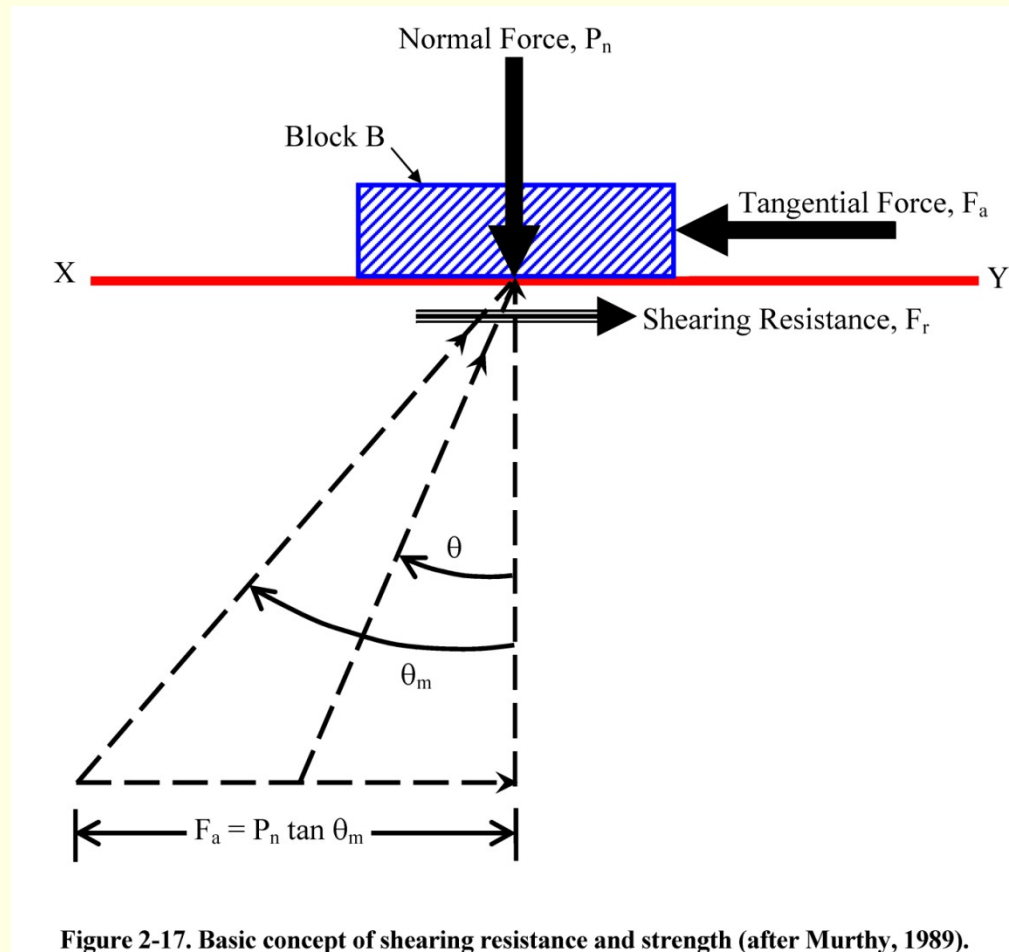
$$\frac{F_a}{P_n} = \tan \theta_m$$

By analogy:

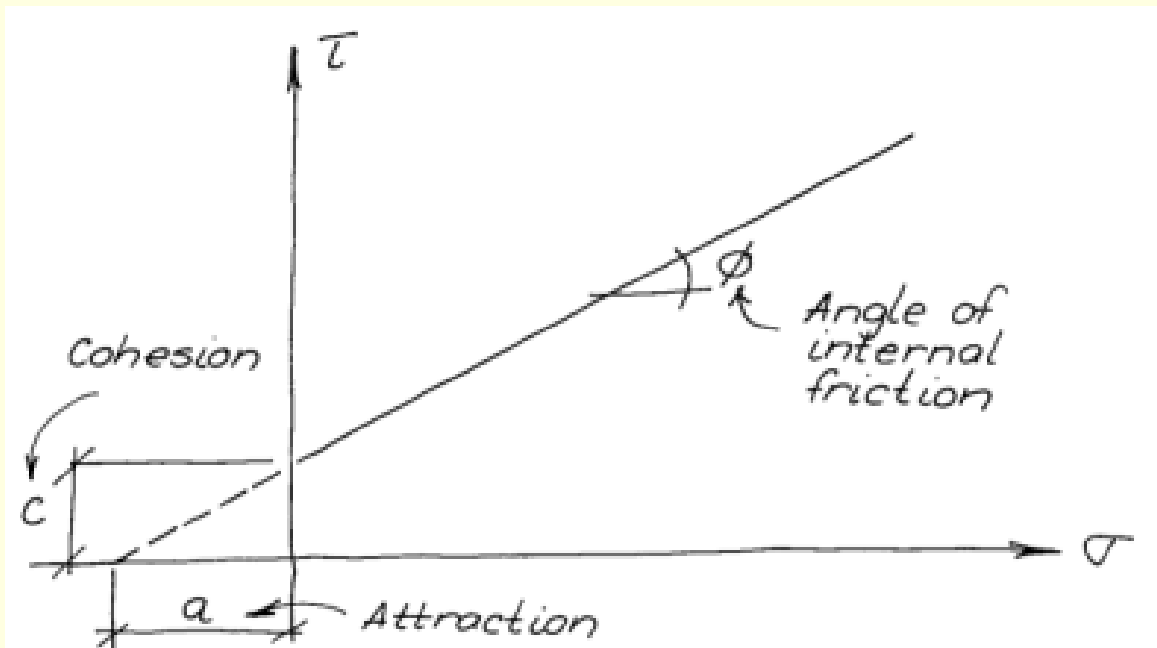
$$\frac{\tau}{\sigma} = \tan \theta \quad \text{or} \quad \tau = \sigma \tan \theta$$

More completely:

$$\tau_{crit} = c + \sigma' \tan \theta$$



Mohr-Coulomb Failure Envelope



Coulomb - Mohr

$$\tau_f = c + \sigma_f \tan \phi = (\sigma_f + a) \tan \phi$$

Terzaghi - Hvorslev

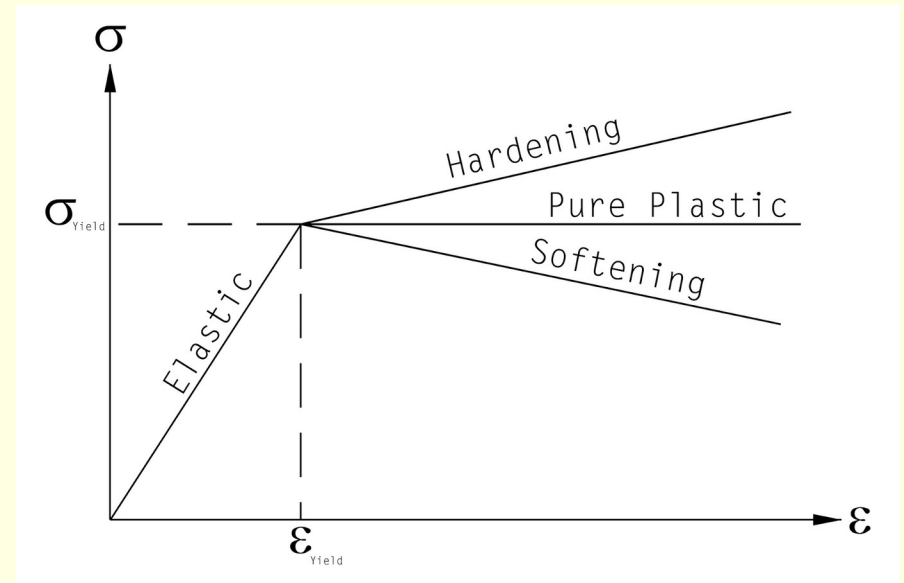
$$\tau_f = c' + \sigma_f' \tan \phi' = c' + (\sigma_f - u_f) \tan \phi'$$

$$\tau_f = (\sigma_f' + a) \tan \phi'$$

↑
Attraction

Failure: Elastic-Purely Plastic Deformation

- Suggested by the frictional model
 - The block friction is able to resist movement up to a point, after which the movement is continuous with the applied force
- Not a perfect model, but one which is adequate for much analysis in soil mechanics



Failure Criteria of Soils (incl. p-q relationships)

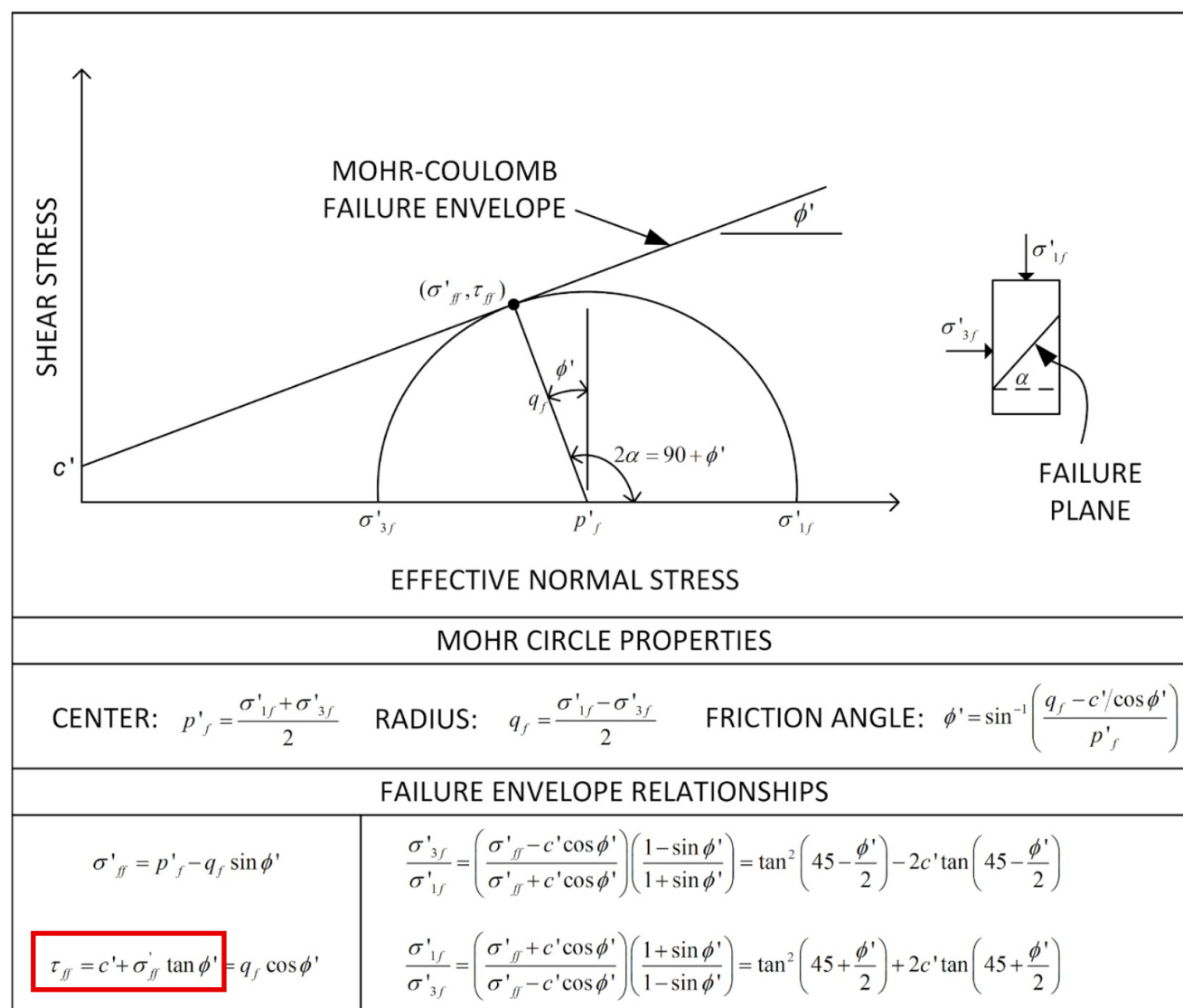


Figure 4-3 Mohr Circle Relationships

Mohr-Coulomb Failure Criterion

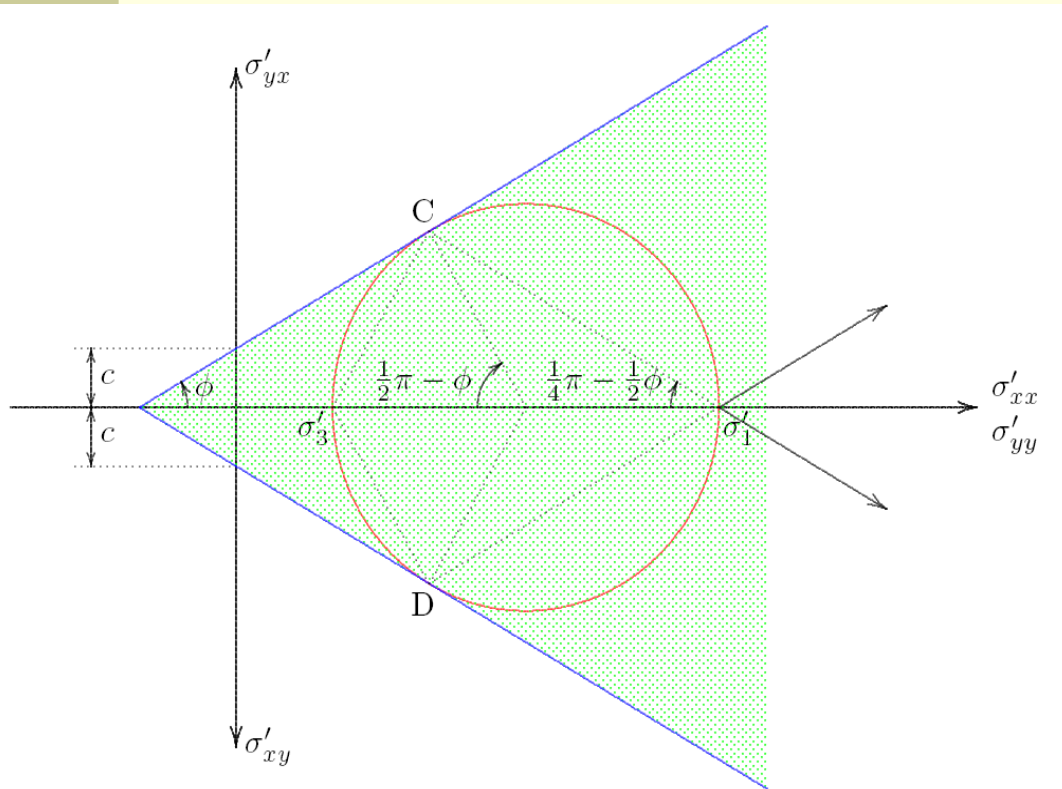
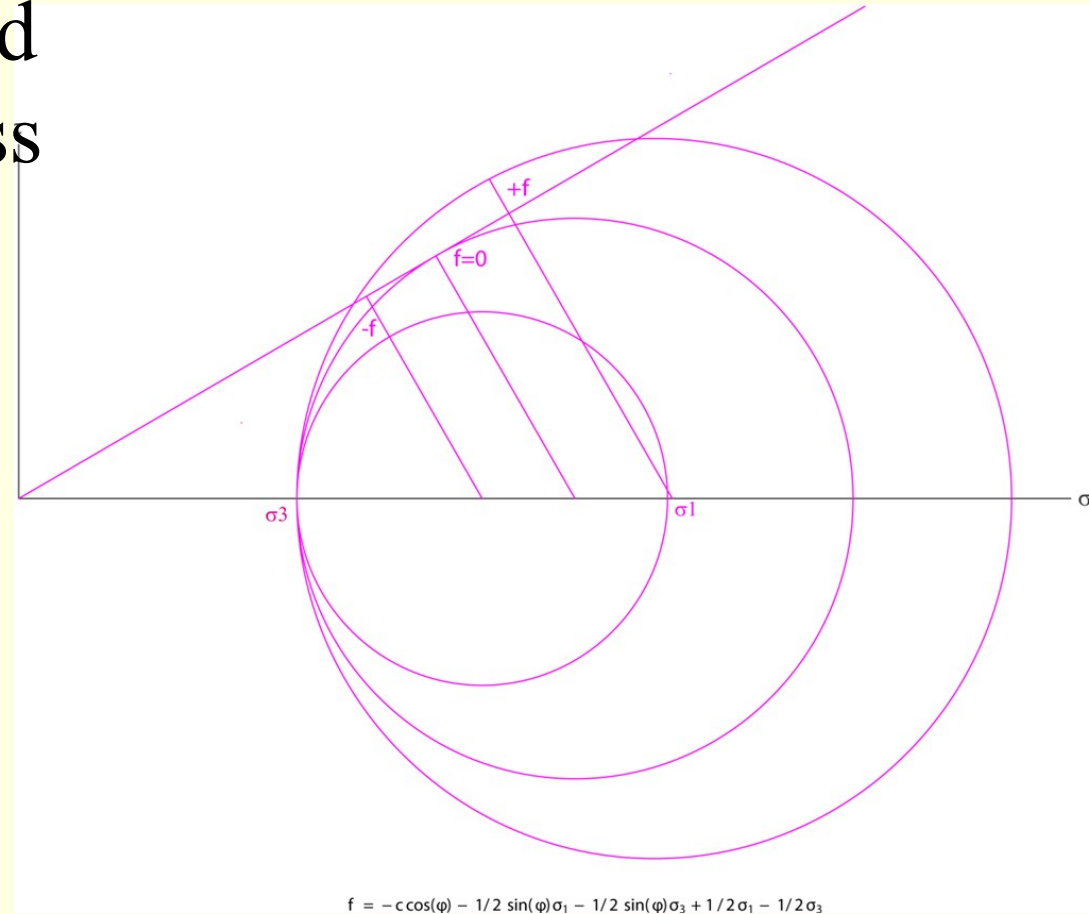


Figure 20.5: Mohr-Coulomb failure criterion.

- As long as the Mohr's circle (stress state) of a point is “within the envelope” the stress is in legal (elastic) state
- When the circle touches the envelope, failure begins, this is defined as follows:

Stress States Inside and Outside of Failure Envelope

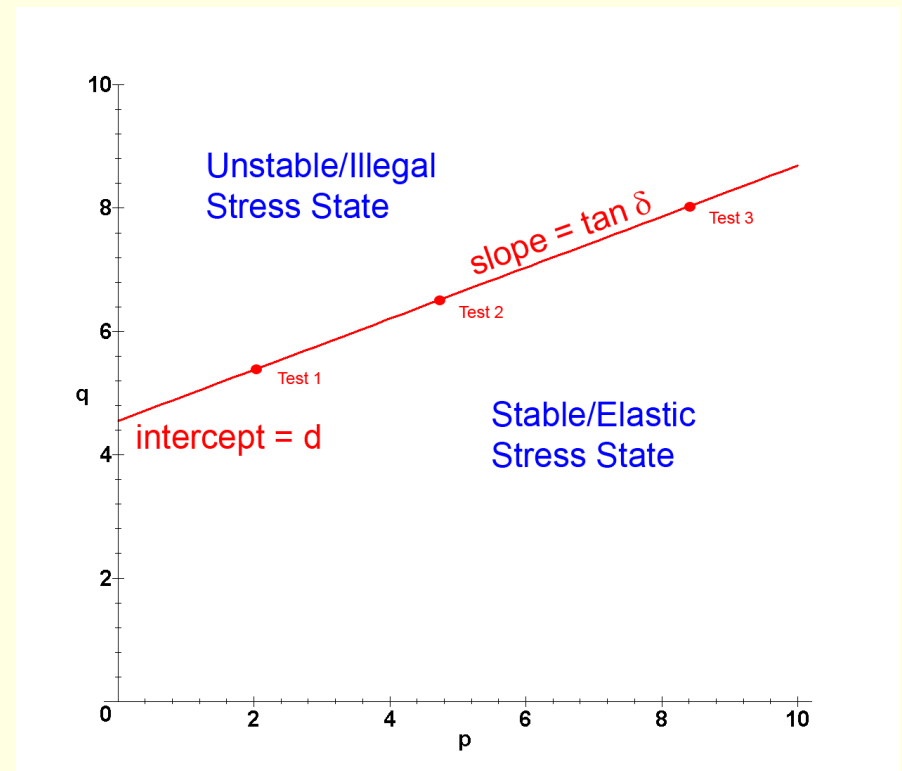
- The preceding is good to determine the stress state at failure
- But how do we evaluate stress states on either side of the envelope?
- We use the diametral failure function:



$$f = \sigma_1 - \sigma_3 - 2c \cos(\phi) - (\sigma_1 + \sigma_3) \sin(\phi)$$

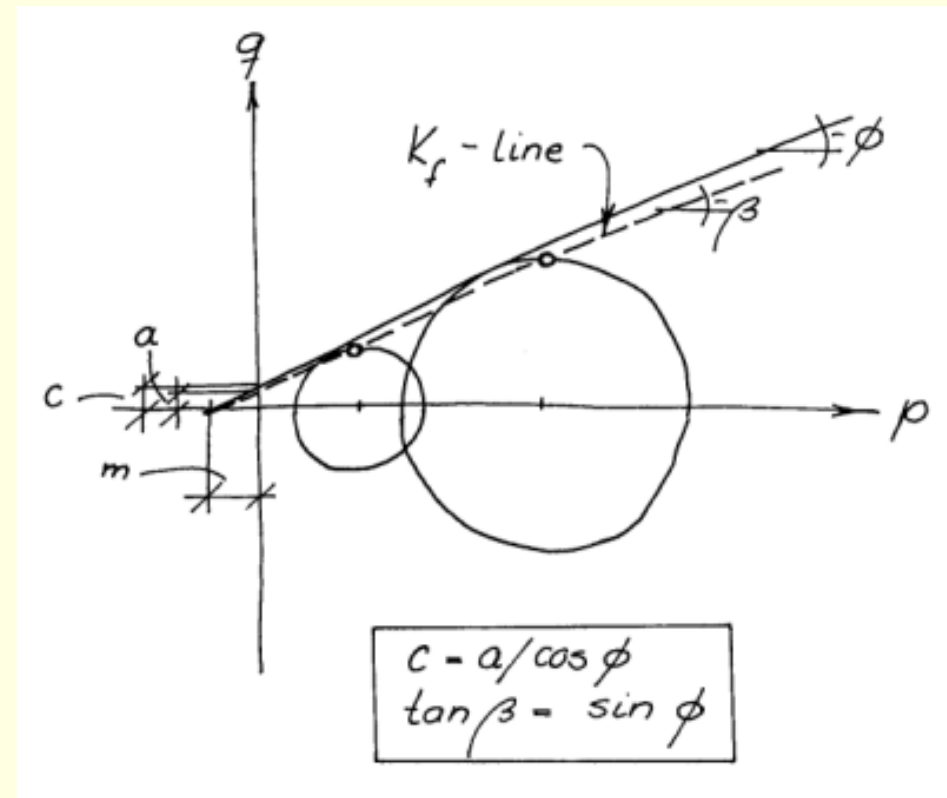
p-q Diagram

- A different method of expressing stress states than Mohr's Circle
- Makes analysis simpler in many cases
- Material on p-q diagrams elsewhere on site



p-q Diagram

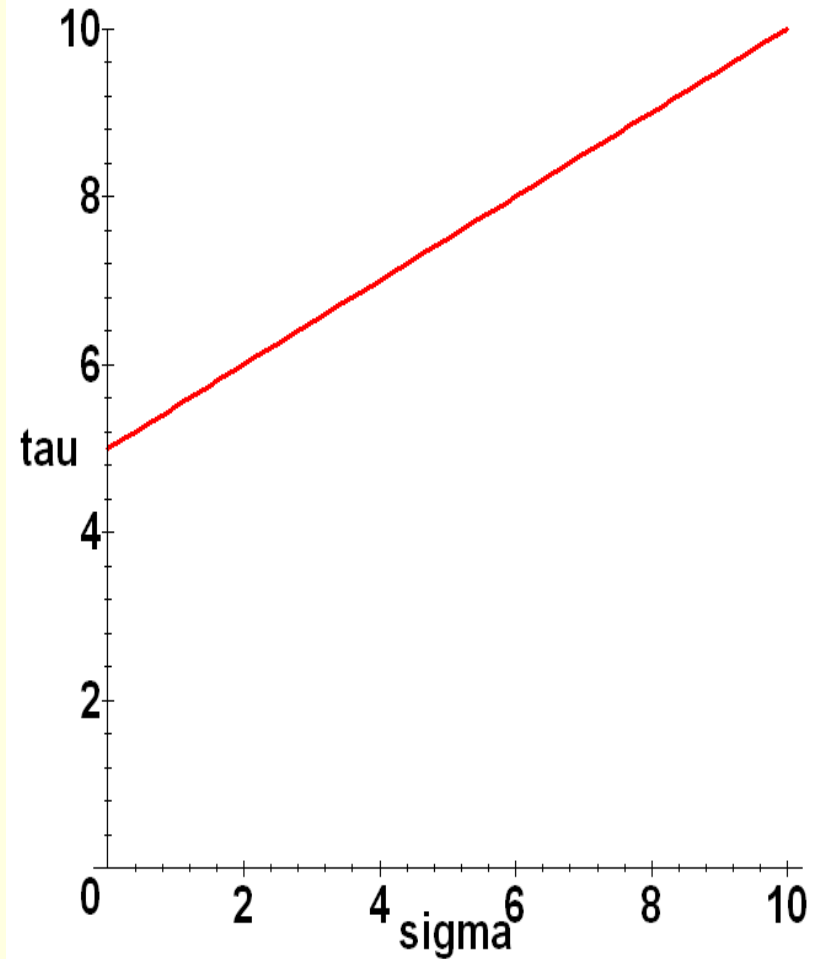
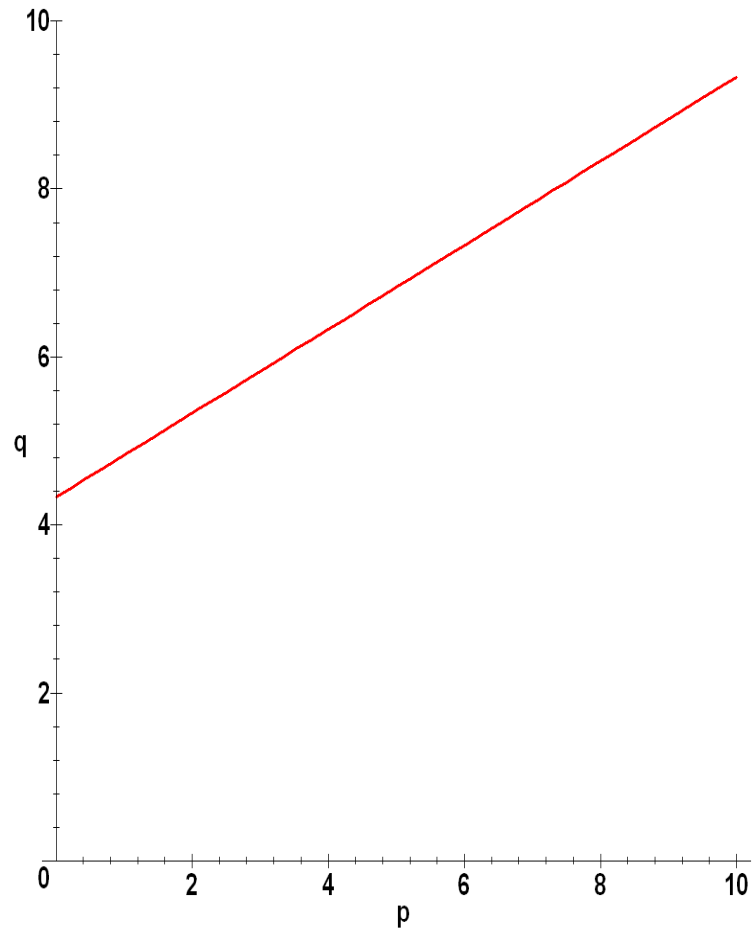
- A coordinate transformation which changes the circles in the Mohr-Coulomb τ - σ space to points in the p-q space
- We make the following substitutions:
 - $p = (\sigma_1 + \sigma_3)/2$
 - $q = (\sigma_1 - \sigma_3)/2$
 - $\tan \delta = \sin \phi$ (note notational difference from right, $\beta \rightarrow \delta$)
 - $a = c (1 - (\tan \delta)^2)^{1/2}$ (y-intercept for p-q diagram)



Example

- Given
 - Soil, $c = 5 \text{ kPa}$, $\phi = 30 \text{ deg.}$
- Find
 - Failure line plots for both sigma-tau and p-q diagrams
- Solution
 - The cohesion c is the same for both diagrams
- Solution
 - For tau-sigma diagram
$$\tau = \sigma \tan \phi + c$$
 - For p-q diagram
$$q = p \tan \delta + c (1 - (\tan \delta)^2)^{1/2}$$
 - We transform the slope by noting that $\tan \delta = \sin \phi$
 - We transform the y-intercept by multiplying the cohesion by $(1 - (\tan \delta)^2)^{1/2}$
 - From substitution, $\delta = 26.6 \text{ deg.}$, and the y-intercept for the p-q diagram = 4.33 kPa

Example: Resulting Diagrams



Questions?

