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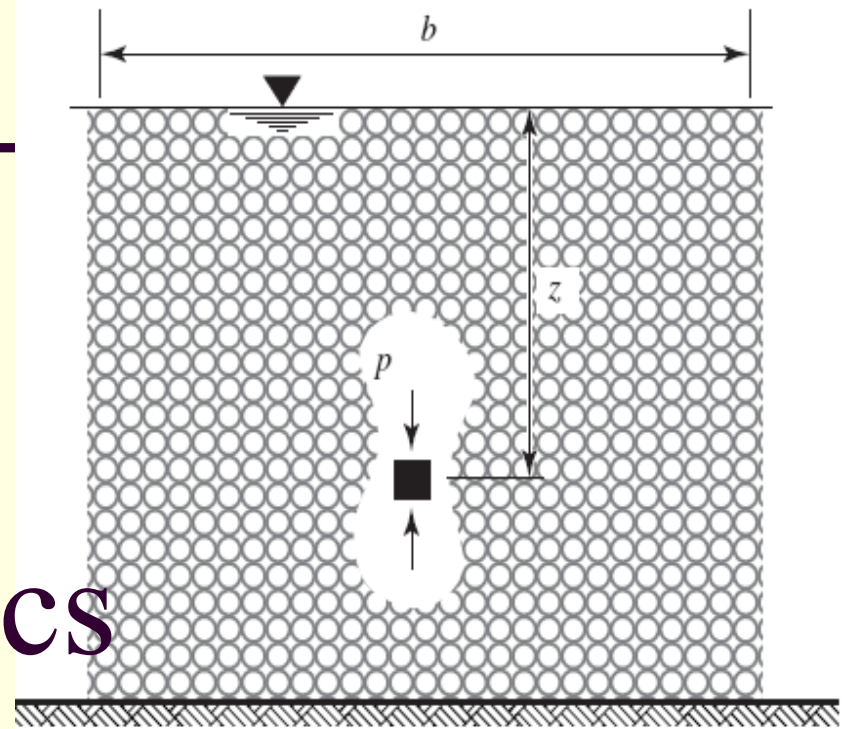


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ENCE 3610

Soil Mechanics



Lecture 5

Effective (Intergranular) Stress in Soils

Capillary Rise in Soils

Stresses in Soils

- Geostatic and Hydrostatic Stresses
 - Total Stress
 - Pore Water Pressure
 - Effective Stress and Overburden Pressure (p_o)
 - Upward and Downward Seepage Stresses
 - Lateral Earth Pressures
- External Load Stresses
 - Elastic Methods (Boussinesq, etc.)
 - Empirical Methods (2:1)
- Combined Stresses and Mohr's Circle
 - Use of Mohr's Circle (principal stresses, etc.)
 - Theory of elasticity and plasticity
- General Comments
 - The principal source of soil stress is caused by the weight of the soil (adjusted usually by the pore water pressure) above the point in the soil under examination
 - Additional stresses can and are induced by load from the soil surface, such as footings, deep foundations, embankments and other weight-bearing structures

Hydrostatic and Total Stress

- Hydrostatic Stress

- The stress induced by the weight of the water at a given depth

$$u = z\gamma_z$$

- Assumes water is at rest and not experiencing forces other than static gravity
- Also exists in the pores of the soil as it does in free water, thus the name “pore water pressure” (but pore water pressure doesn’t have to be hydrostatic)

- Total Stress

- The stress produced at any point by the overburden pressure of the soil plus any applied loads (equation does not include loads)

$$\sigma_t = \sum_{i=1}^n H_i \gamma_i$$

- Generally ignores the effect of pore water pressure
- Applies to any soil in any state of saturation

Pore Water Pressure

- Pressure of water within the pores or voids of the soil
- Can be caused by:
 - Hydrostatic pressure, either directly from above or from down flow of water
 - Capillary action
 - Seepage
 - Pressure from applied loads compressing trapped water
- Water has a buoyant effect on soils under the water table
- For hydrostatic conditions, this includes all soils under the water table
- The buoyant effect of the water equals the weight of the water the solids displace
- This reduces the effective unit weight of the soil for the purposes of calculating effective stress

Effective (Intergranular) Stress

$$\sigma = \frac{W_s + (zb^2\gamma_w - V_s\gamma_w)}{b^2}$$

$$u = z\gamma_w$$

$$\sigma = \frac{W_s + (ub^2 - V_s\gamma_w)}{b^2} = \frac{W_s - V_s\gamma_w}{b^2} + u$$

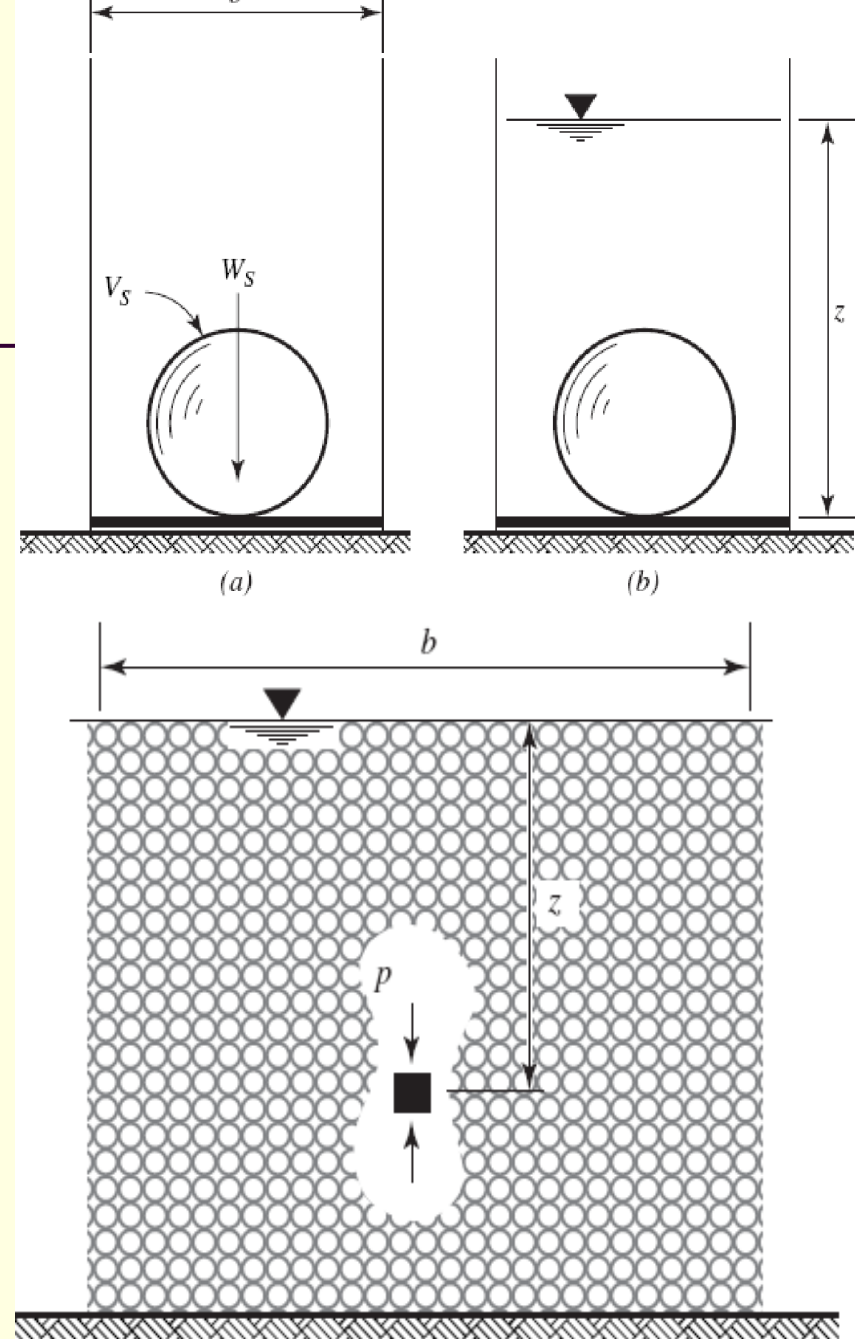
$$\sigma' = \sigma - u \quad (\text{Definition of Effective Stress})$$

$$W_s = V_s G_s \gamma_w$$

$$\sigma'_z = \frac{V_s G_s \gamma_w - V_s \gamma_w}{b^2} = \frac{V_s \gamma_w}{b^2} (G_s - 1)$$

$$V_s = \frac{V}{1+e}; z = \frac{V}{b^2}$$

$$\sigma'_z = z\gamma_w \frac{G_s - 1}{e + 1} = z\gamma_w \left[\frac{G_s + e}{e + 1} - \frac{e + 1}{e + 1} \right] = z[\gamma_{sat} - \gamma_w] = z\gamma_{sub}$$



Saturated, hydrostatic conditions

Homogeneous soil layer

Effective Stress—General Expression

- The total stress minus the pore water pressure

$$\bar{\sigma} = \sigma - u$$

- Equation only considers the load of the soil and water
- One of the most important concepts in soil mechanics

- As depth increases, soil stress generally increases because of increasing H
- Varying unit weight will vary the increase in effective stress
- Above water table, pore water pressure is generally not considered (except for capillary condition)

Methods for Computation of Effective Stress

- “Pore Water plus Total Stress” Method
 - Tabulate heights of layers and unit weights (saturated and unsaturated of each)
 - Note location of water table
 - Use this equation to compute effective stress:

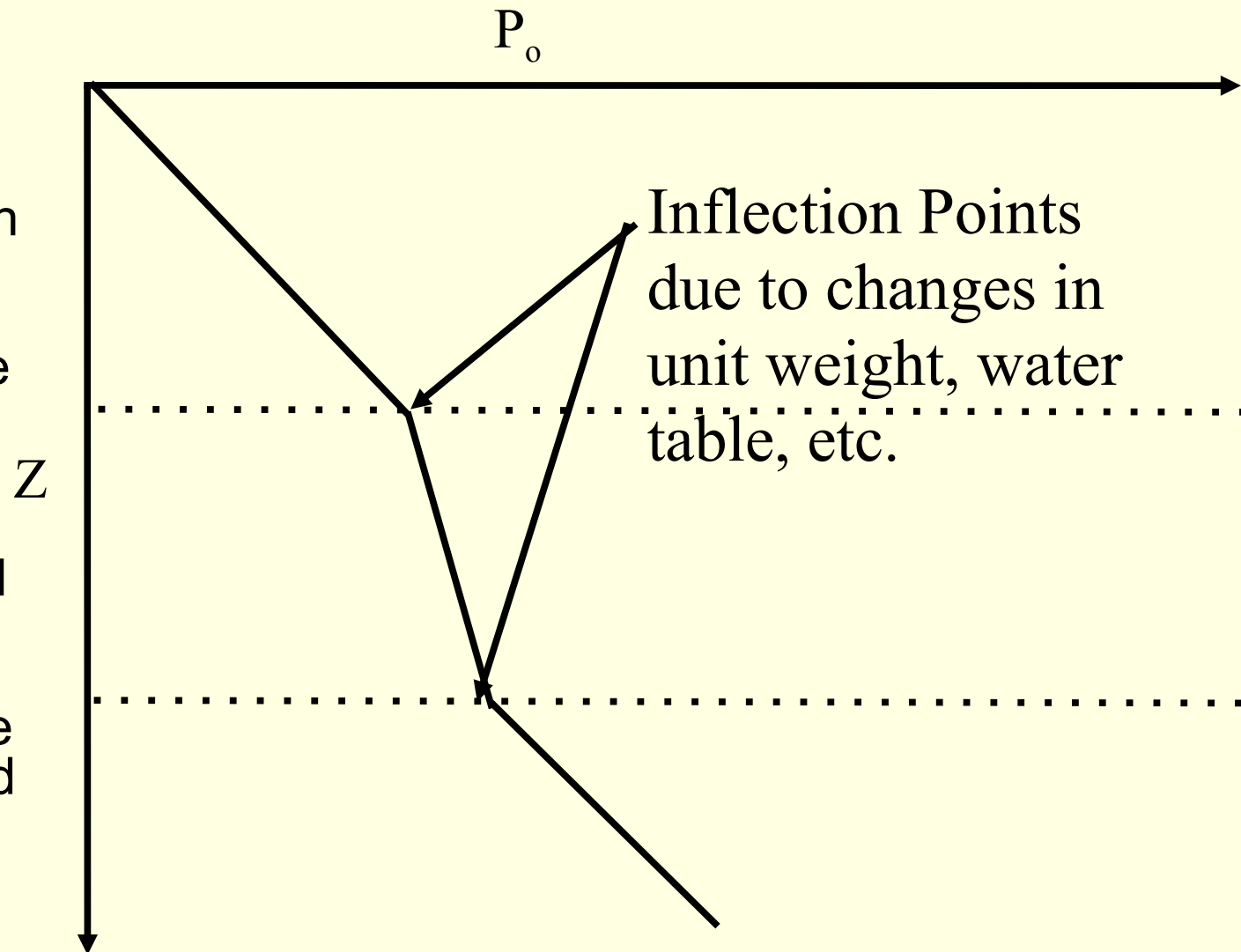
$$\sigma_o = \sigma_t - u = \sum_{i=1}^n H_i \gamma_i - u$$

- Submerged Weight Method
 - Tabulate layer heights and unit weights as with other method
 - Layers above water table use unsaturated (wet or dry) unit weight
 - Layers below water table use submerged unit weight
 - Add as with other method

$$\gamma_{sub} = \gamma' = \gamma_{sat} - \gamma_w \sim \frac{\gamma_{sat}}{2}$$

P_o Diagram

- Useful tool to visualise the increase of effective stress/overburden pressure as a function of depth and to clearly see the effect of the pore water pressure
- Can also be used in some cases to illustrate the changes that take place with applied loads and other conditions



Methods of Plotting P_o Diagrams

- Manual Plotting

- Find points of inflection (changes in unit weight, water table level, other changes in pore water pressure, etc.)
- Compute effective stress/overburden pressure at each point using standard equations
- Join points with lines
- In some cases, pore water pressure functions are not linear, so lines cannot accurately be used in P_o diagrams

- Spreadsheet Plotting

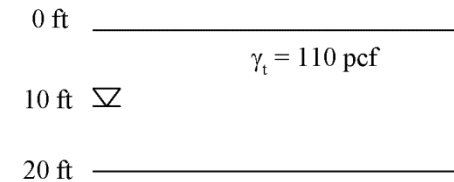
- Divide up soil into layers of constant effective stress function with depth
- Compute increase in effective stress for each layer
- Successively add the increments of increasing (usually) effective stress and create a set of data points at the bottom of each layer
- Use the data set to plot the P_o diagram

P_o Example 1

For solutions to this and the next two examples visit

https://docs.google.com/spreadsheets/d/1qHTLfyB8UIqNQD6F_LnJkgbwZuQfVJ6cp0JbPYhR3zg/edit?usp=sharing

Example 2-1: Find p_o at 20 ft below ground in a sand deposit with a total unit weight of 110 pcf and the water table 10 ft below ground. Assume $\gamma_t = \gamma_{sat}$. Plot p_t and p_o versus depth from 0 ft – 20 ft.



Solution:

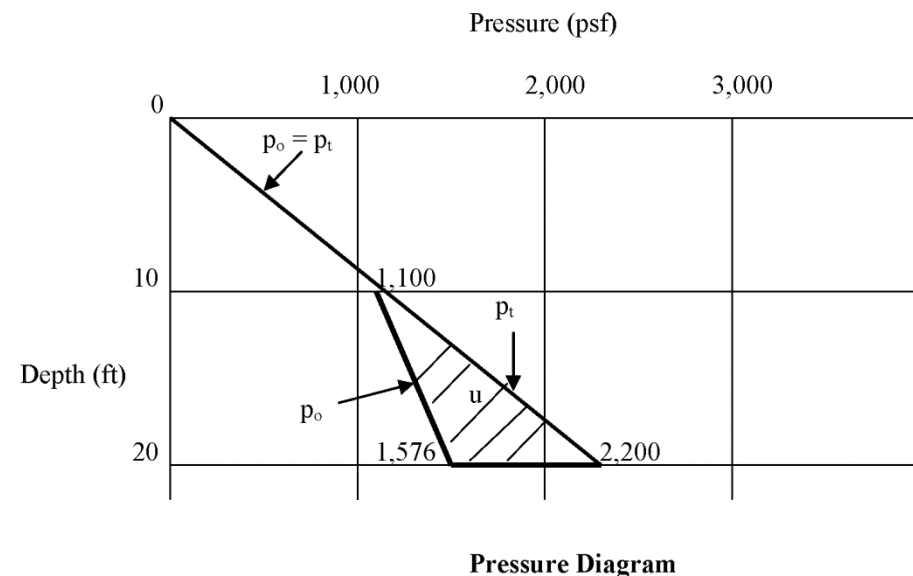
From Equation 2-13, $p_o = p_t - u$

$$p_t @ 10 \text{ ft} = p_o @ 10 \text{ ft} = 10 \text{ ft} \times 110 \text{ pcf} = 1,100 \text{ psf}$$

$$p_t @ 20 \text{ ft} = p_t @ 10 \text{ ft} + (10 \text{ ft} \times 110 \text{ pcf}) = 2,200 \text{ psf}$$

$$u @ 20 \text{ ft} = 10 \text{ ft} \times 62.4 \text{ pcf} = 624 \text{ psf}$$

$$p_o @ 20 \text{ ft} = p_t @ 20 \text{ ft} - u @ 20 \text{ ft} = 2,200 \text{ psf} - 624 \text{ psf} = 1,576 \text{ psf}$$



A plot of effective overburden pressure versus depth is called a “ p_o – diagram” and is used throughout all aspects of geotechnical testing and analysis.

Figure 2-7. Example calculation of a p_o -diagram.

P_o Example 2

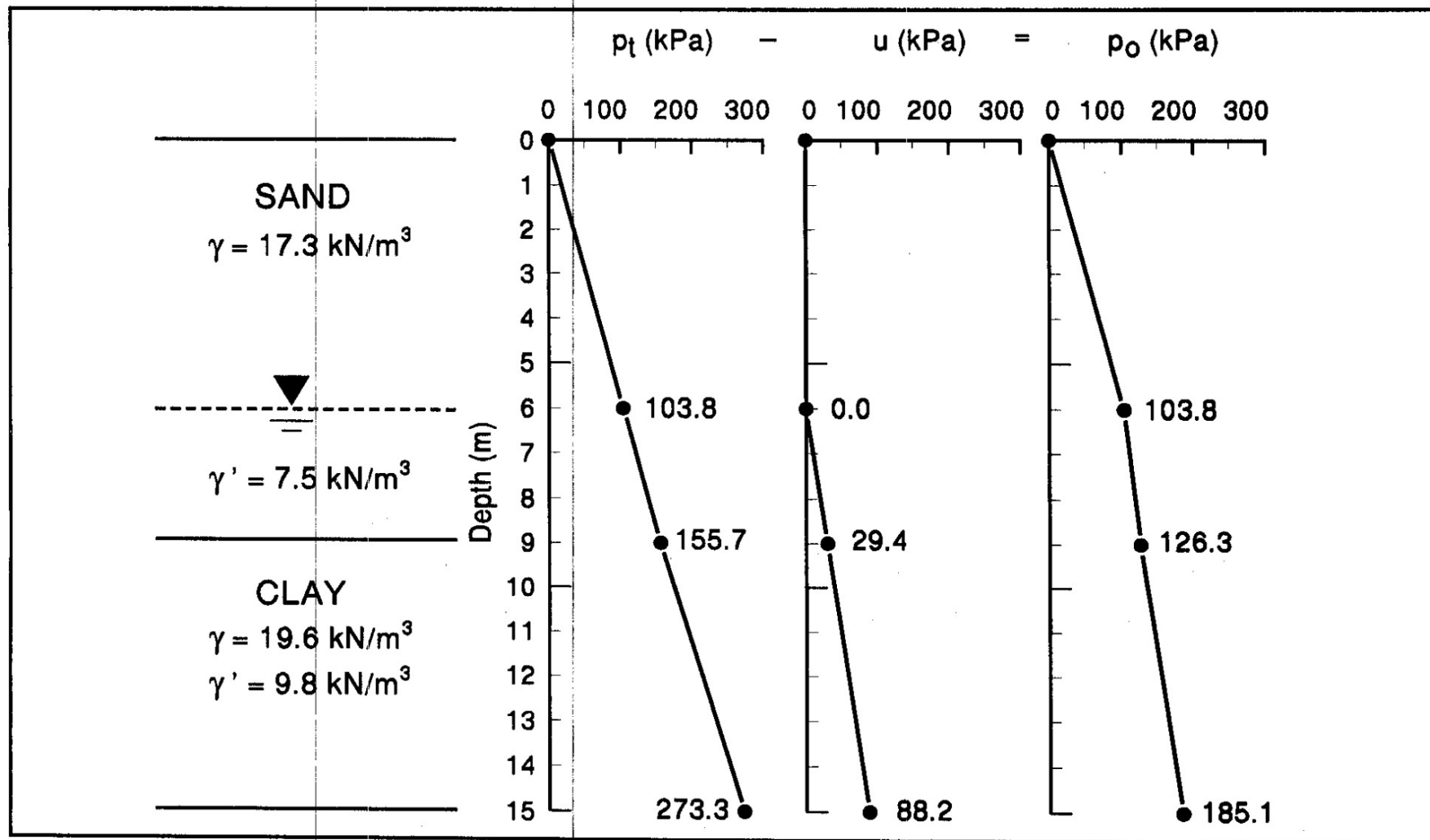
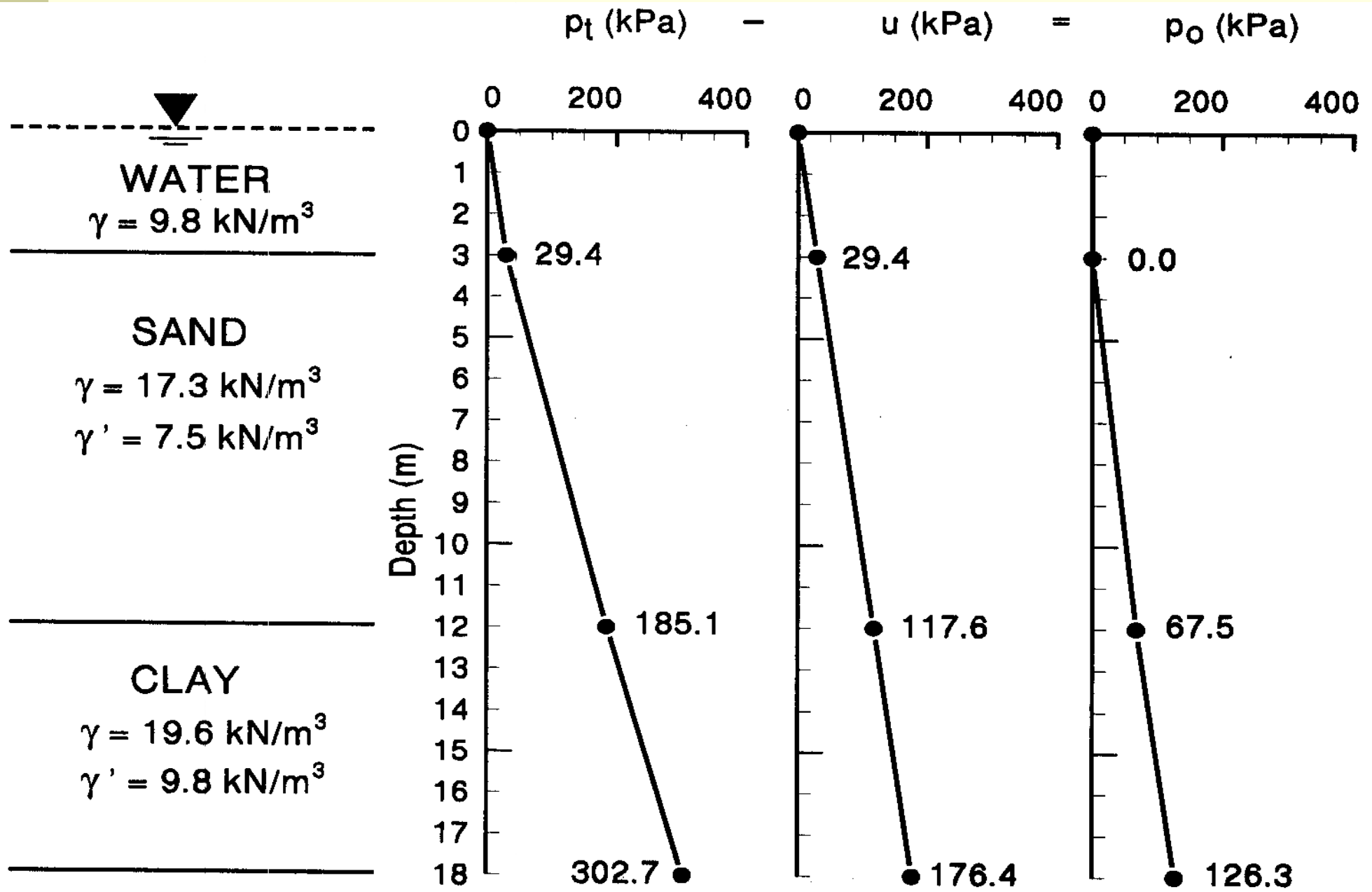
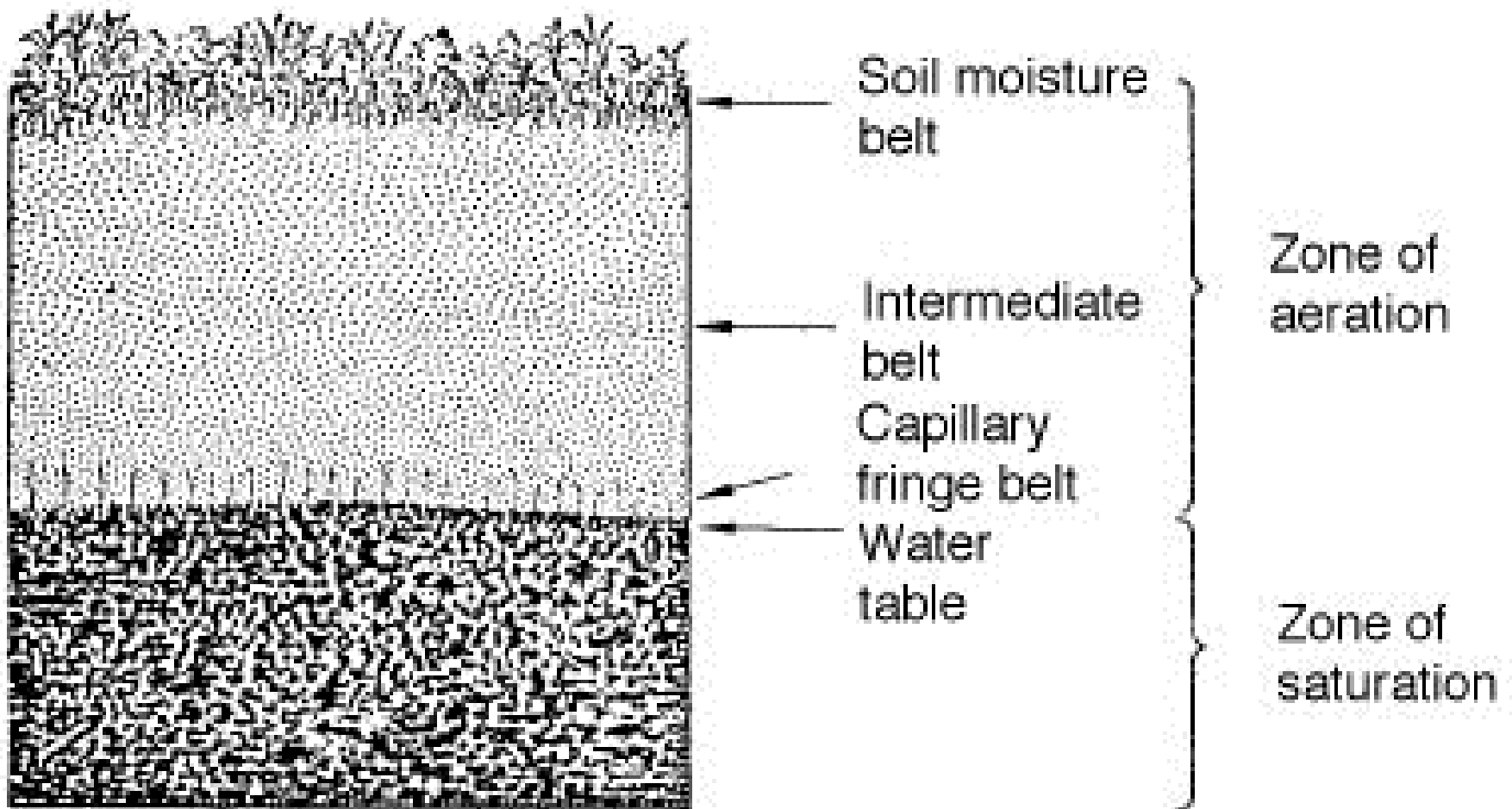


Figure 9.6 Effective Overburden Pressure Diagram - Water Table Below Ground Surface

P_o Example 3



Groundwater Zones in Soil



Capillarity and Capillary Rise in Soils

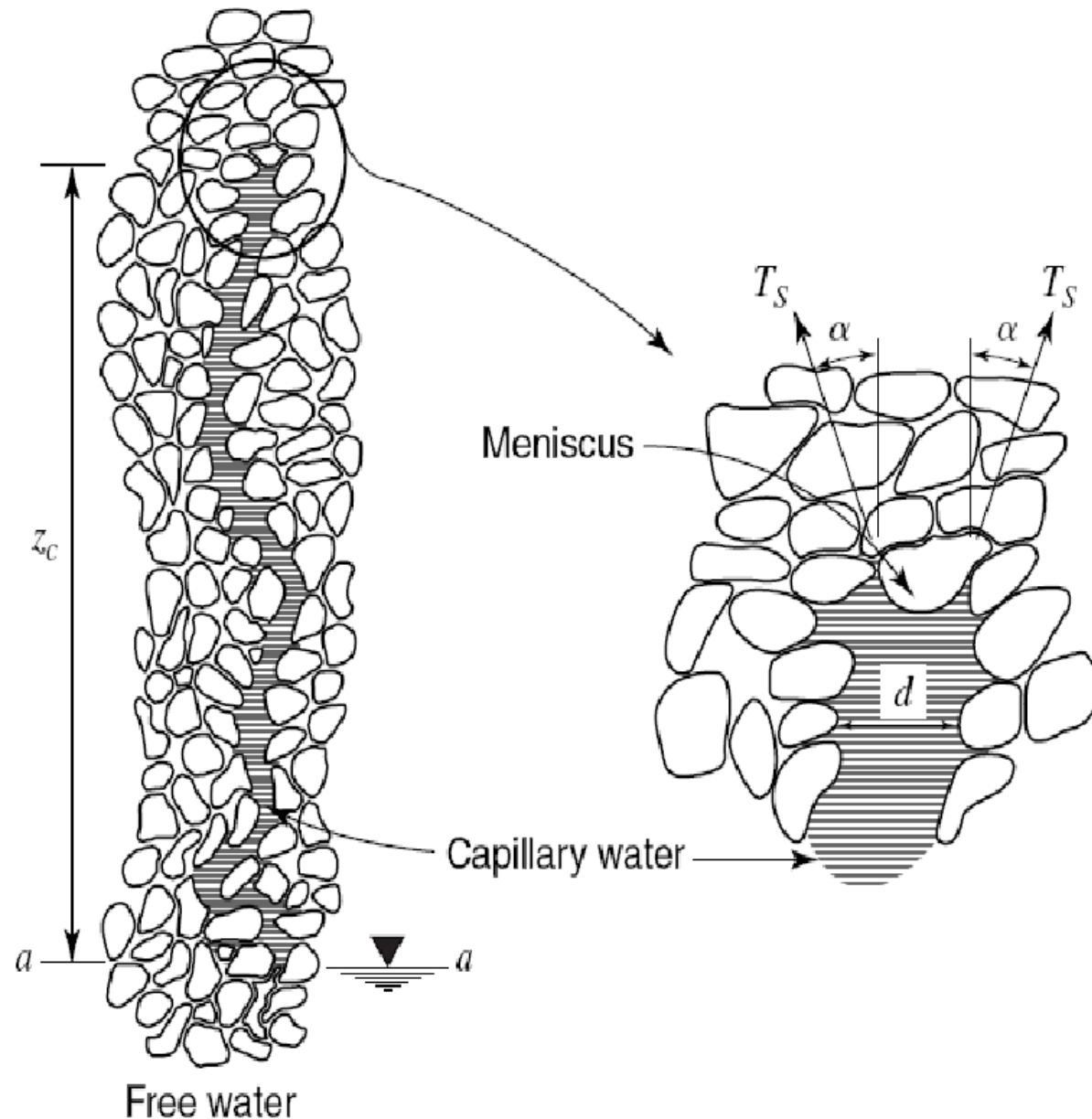


Figure 5.4 Capillary rise in soils.

- Result of surface tension in water between the water itself and a solid surface it touches (glass tube, soil particles, etc.)
- Capillary rise takes place when the surface tension in the water is greater than the force of gravity of the water under it
- Capillary rise stops when the force of the water equals the weight supporting it

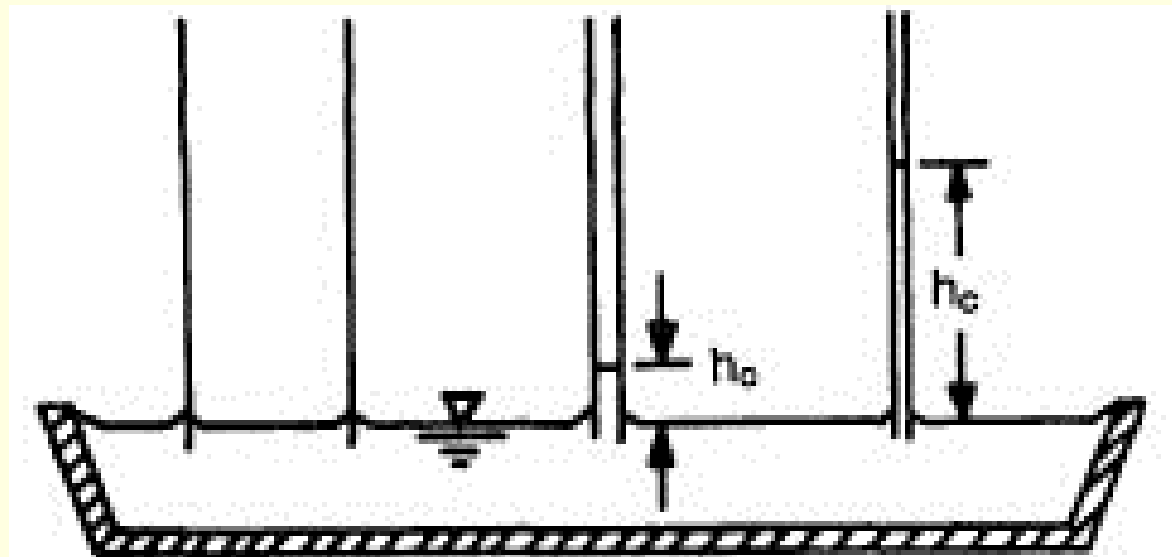
Capillarity in Soils

- A more complex phenomenon in soils due to complex geometry of particles
- More pronounced in fine grained soils
- Volume changes in fine grained soils can be in part attributed to capillarity
- Area of soil above water table where capillary rise is taking place is called the vadose zone
- Compressive stresses take place in the soil in an area with capillary rise takes place

Capillarity and Capillary Rise in Soils

- Equilibrium of capillary rise force and water head in a tube (z_c = height of water column)

$$\pi d T_s \cos \alpha = \frac{\pi z_c d^2 \gamma_w}{4}$$



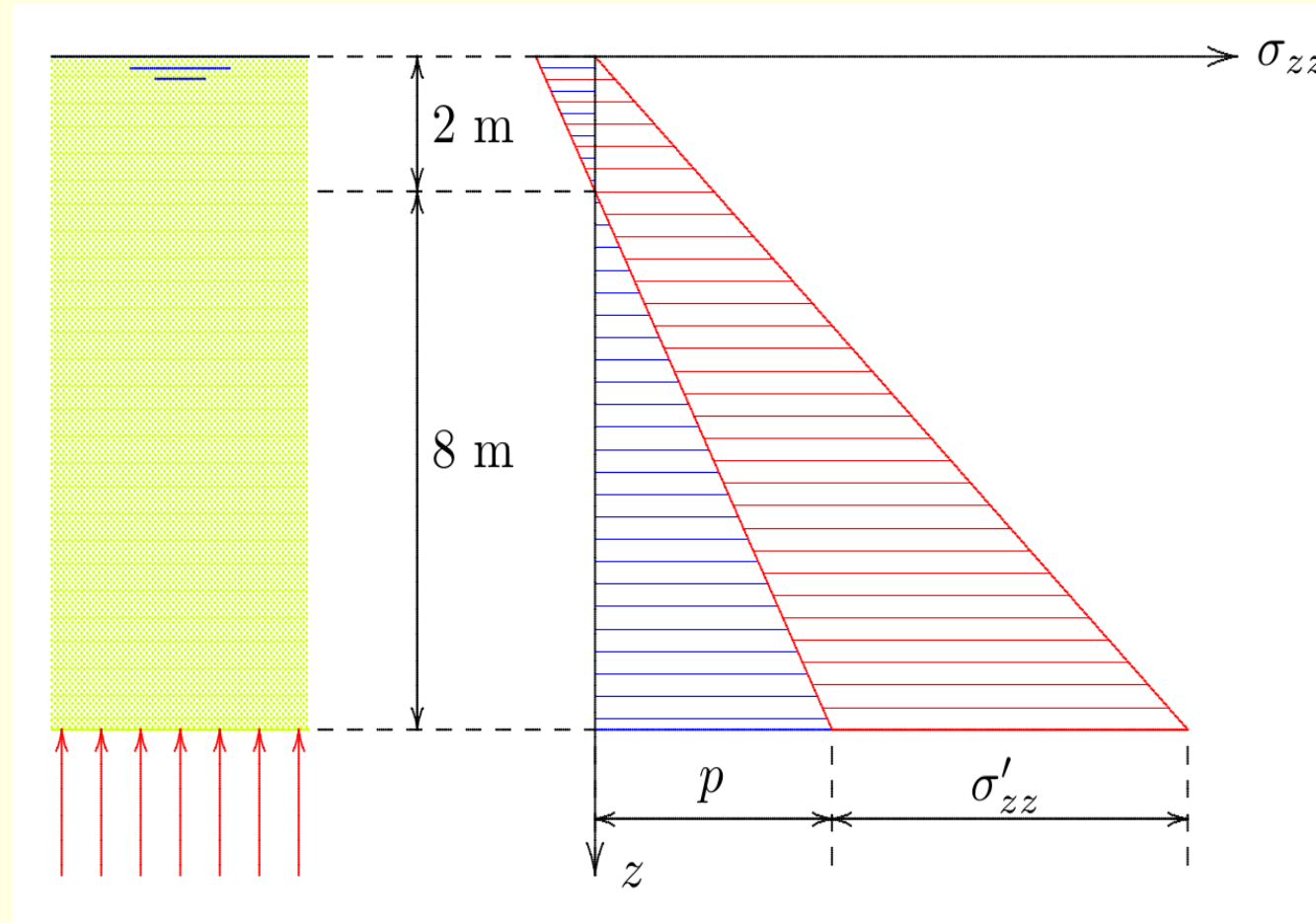
Capillarity and Capillary Rise

- Assumptions
 - $T_s = 0.0075 \text{ g/mm}$
 - $\alpha = 0$
 - $d = D_{10}/5$ (D_{10} usually expressed in millimetres)
 - $\gamma_w = .001 \text{ g/mm}^3$
- Substituting and Solving for z_c (millimetres)

$$z_c = \frac{150}{D_{10}}$$

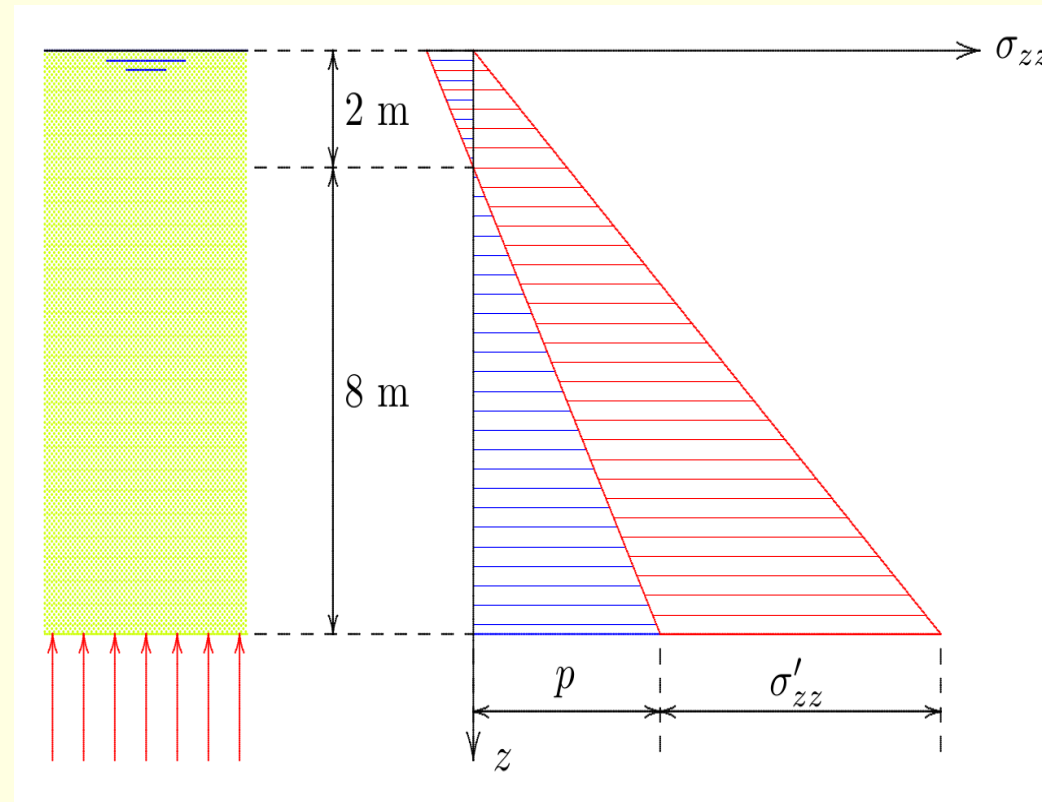
Capillarity Example

- Given
 - Soil, $D_{10} = 0.074$ mm (#200 sieve opening)
- Find
 - Capillary Rise in Soil
- Solution
 - $z_c = 150/0.074 \sim 2027$ mm ~ 2 m



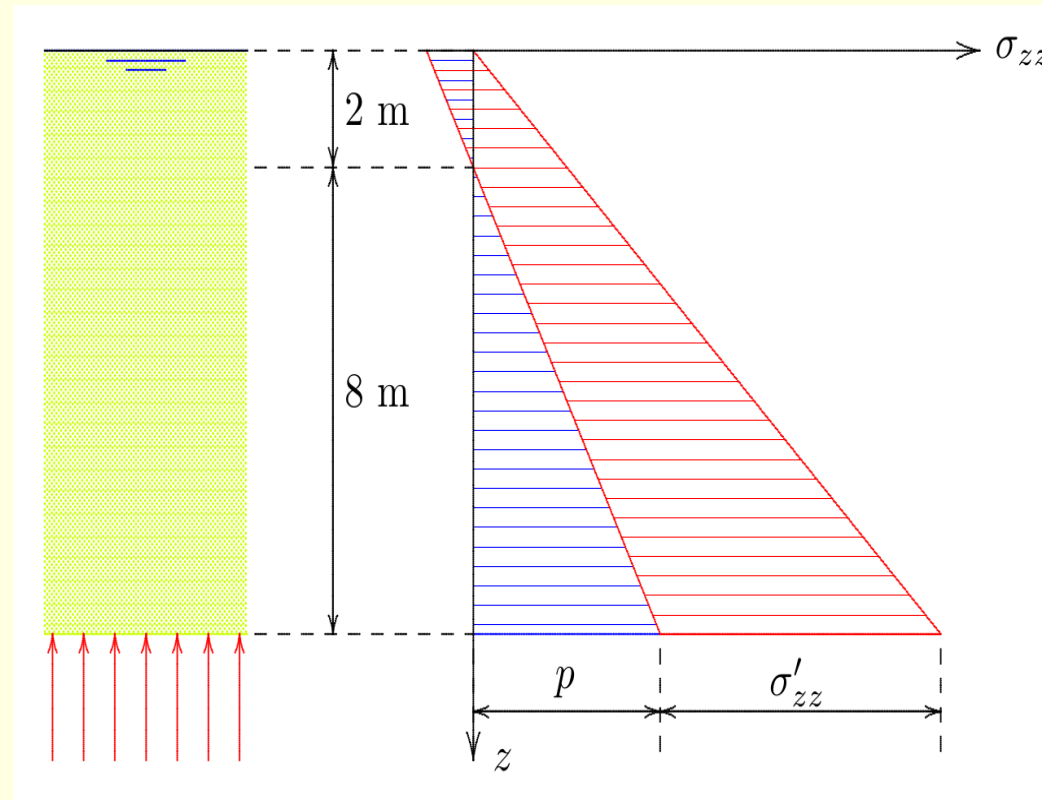
Capillary Rise and Effective Stress

- Note the following:
 - Phreatic surface is where the pore water pressure is zero
 - At the surface, since there is no surcharge, total stress is zero
 - Thus, the pore water pressure above the phreatic surface is negative with capillary rise, with the same slope as below the phreatic surface
 - There is thus a positive effective stress at the surface to balance the negative pore water pressure



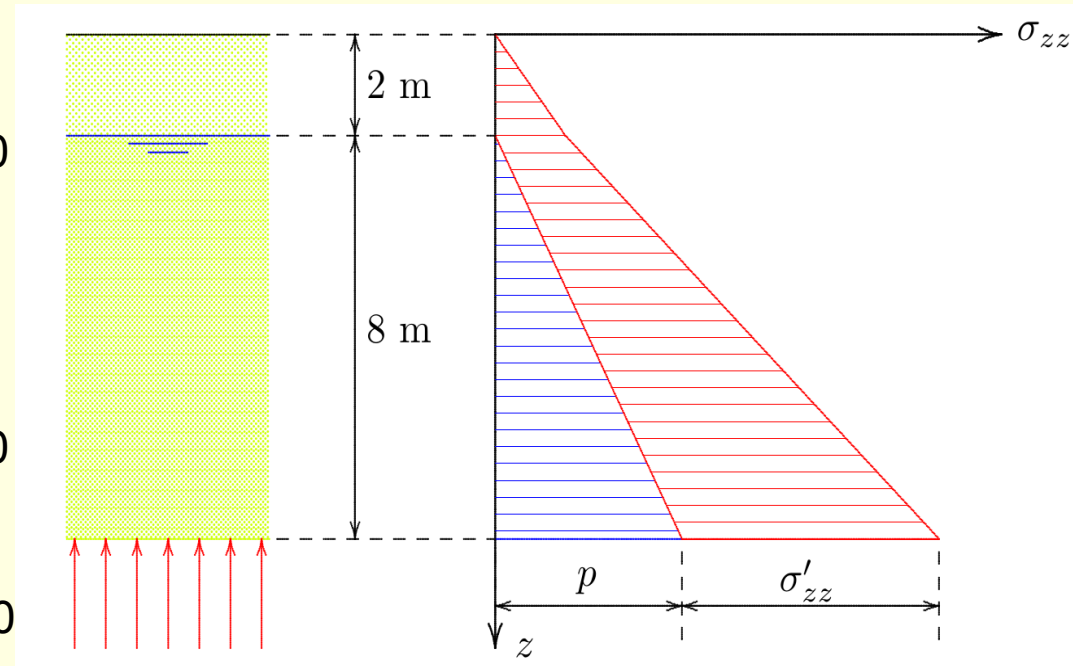
Capillary Rise and Effective Stress

- Assume that saturated unit weight = 20 kN/m³
- At surface
 - Pore water pressure = $-(2)(9.8) = -19.6$ kPa
 - Total Stress = 0 kPa
 - Effective stress = $0 + 19.6 = 19.6$ kPa
- At 2 m below surface
 - Pore water pressure = 0
 - Total Stress = $(20)(2) = 40$ kPa
 - Effective Stress = $40 - 0 = 40$ kPa
- At 10 m below surface
 - Pore water pressure = $(8)(9.8) = 78.4$ kPa
 - Total Stress = $(20)(10) = 200$ kPa
 - Effective Stress = $200 - 78.4 = 121.6$ kPa



Effective Stress without Capillary Rise

- Assume that saturated unit weight = 20 kN/m^3 and dry unit weight = 16 kN/m^3
- At surface
 - Pore water pressure = 0 kPa
 - Effective stress = 0 kPa
 - Total Stress = 0 kPa
- At 2 m below surface
 - Pore water pressure = 0
 - Total Stress = $(16)(2) = 32 \text{ kPa}$
 - Effective Stress = $32 - 0 = 32 \text{ kPa}$
- At 10 m below surface
 - Pore water pressure = $(8)(9.8) = 78.4 \text{ kPa}$
 - Total Stress = $32 + (20)(8) = 192 \text{ kPa}$
 - Effective Stress = $192 - 78.4 = 113.6 \text{ kPa}$



Questions?

