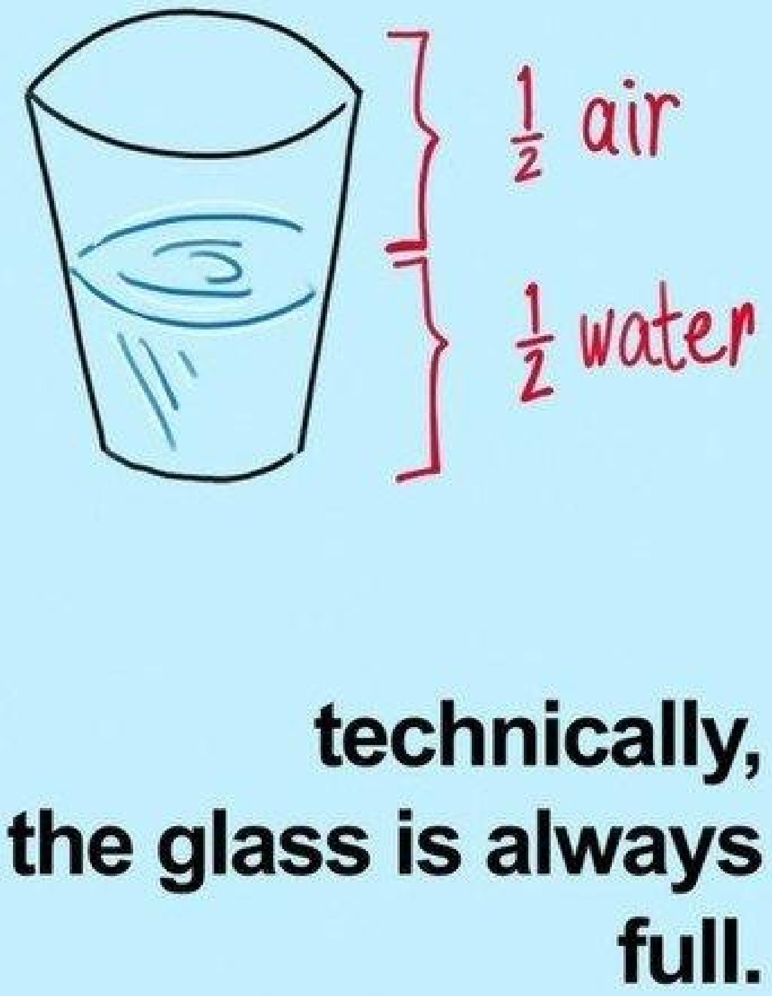
ENCE 3610
Soil Mechanics

Lecture 4
Soil Composition

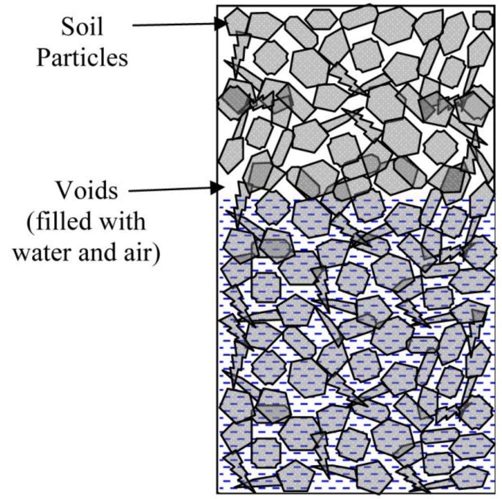
Soil Composition

- Weight-Volume Relationships
  - Water or Moisture Content
  - Unit Weight or Mass
  - Specific Gravity
- Relative Density
- Soil is a collection of particles that do not form an totally solid substance
- Soil is a combination of:
  - Soil material in particles
  - Air
  - Water
- The relationship between this combination defines much of what any particular soil can do to support foundations

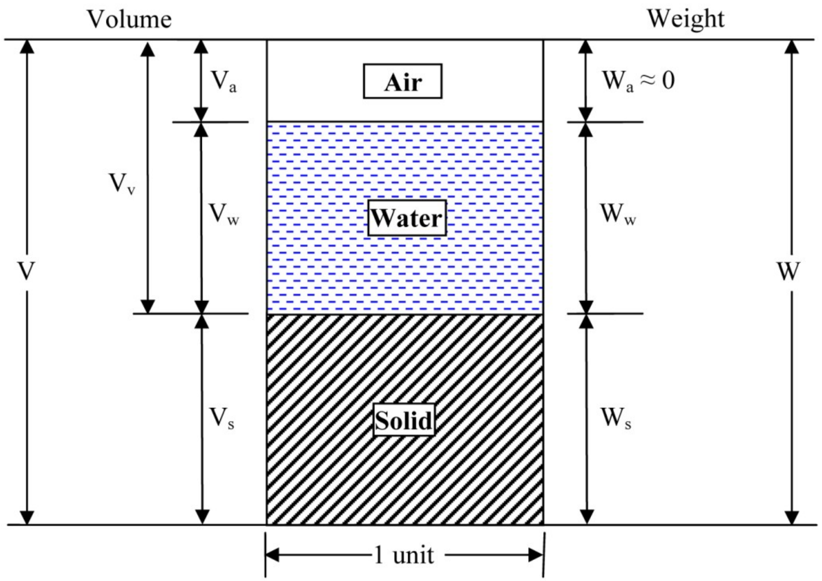


Phase Diagram

- Assumptions and Definitions:
  - Weight of air = 0
  - Dry Soil: Water weight and volume = 0, all voids filled with air
  - "Volume of voids" include all non-soil volume, both air and water
- Equalities and Summations
  - $V_{total}=V_{air}+V_{water}+V_{solids}$
  - $W_{total}=W_{water}+W_{solids}$
  - $M_{total}=M_{water}+M_{solids}$
  - $W_x=\gamma_x*V_x$ or $M_x=\rho_x*V_x$



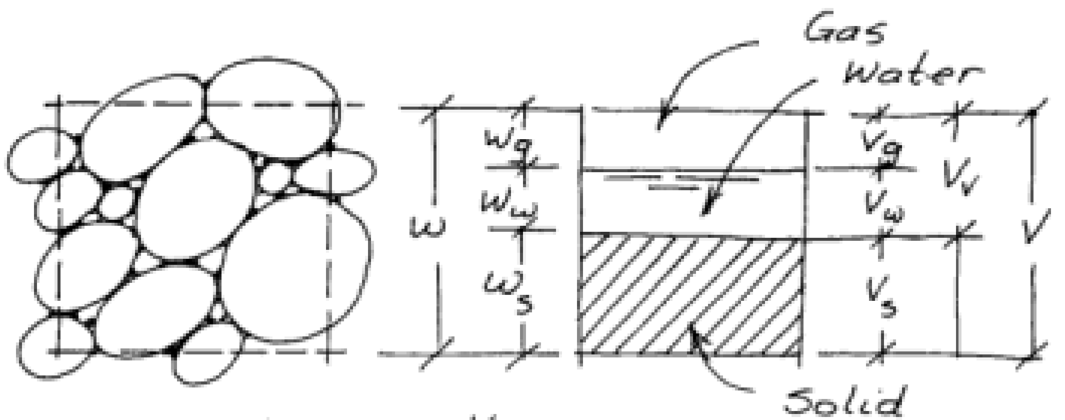
(a)



(b)

Figure 2-1. A unit of soil mass and its idealization.

Index Properties



Void ratio, $e = \frac{V_v}{V_s}$

Porosity, $n = \frac{V_v}{V} \cdot 100\ \%$

Water content, $\omega = \frac{W_w}{W_s} \cdot 100\ \%$

Unit weight, $\rho(\gamma) = \frac{W}{V}$ kN/m^3

Dry unit weight, $\rho_d(\gamma_d) = \frac{W_s}{V}$ kN/m^3

Unit weight of solids, $\rho_s(\gamma_s) = \frac{W_s}{V_s}$, kN/m^3

Specific gravity, $G = \rho_s/\rho_w$ = 2.65 (sand)
2.7 (clay)

Degree of saturation, $S_r = \frac{V_w}{V_p} \cdot 100\ \%$

Index Properties

- Water (moisture) content
  - $w = \boldsymbol{\omega} = W_w/W_s$
- Void Ratio
  - e = Volume of Voids/Volume of Solids
  - $e = V_v/V_s$
- Porosity
  - n = Volume of Voids/Total Volume
  - $n = V_v/V_t$
- $e = n/(1-n)$
- $n = e/(1+e)$
- Degree of Saturation
  - Portion of the voids which are filled with water
  - $S = V_w/V_v$
  - When a soil is said to be saturated, S = 100%
- Specific Gravity
  - Ratio of unit weight/density of solids *only* to unit weight/density of water
  - $G_s = \gamma_s/\gamma_w = \rho_s/\rho_w$

Weight-Volume Relationships and Index Properties

Table 2-1
Summary of index properties and their application

Property	Symbol	Units[1]	How Obtained (AASHTO/ASTM)	Comments and Direct Applications
Porosity	n	Dim	From weight-volume relations	Defines relative volume of voids to total volume of soil
Void Ratio	e	Dim	From weight-volume relations	Volume change computations
Moisture Content	w	Dim	By measurement (T 265/ D 4959)	Classification and in weight-volume relations
Total unit weight [2]	γ_t	FL^{-3}	By measurement or from weight-volume relations	Classification and for pressure computations
Specific Gravity	G_s	Dim	By measurement (T 100/D 854)	Volume computations

NOTES:
1 F=Force or weight; L = Length; Dim = Dimensionless. Although by definition, moisture content is a dimensionless decimal (ratio of weight of water to weight of solids) and used as such in most geotechnical computations, it is commonly reported in percent by multiplying the decimal by 100.
2 Total unit weight for the same soil can vary from "saturated" (S=100%) to "dry" (S=0%).

Table 2-2
Weight-volume relations (after Das, 1990)

Unit-Weight Relationship	Dry Unit Weight (No Water)	Saturated Unit Weight (No Air)
$\gamma_t = \frac{(1+w)G_s\gamma_w}{1+e}$ $\gamma_t = \frac{(G_s+Se)\gamma_w}{1+e}$ $\gamma_t = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$ $\gamma_t = G_s\gamma_w(1-n)(1+w)$	$\gamma_d = \frac{\gamma_t}{1+w}$ $\gamma_d = \frac{G_s\gamma_w}{1+e}$ $\gamma_d = G_s\gamma_w(1-n)$ $\gamma_d = \frac{G_s\gamma_w}{1+\frac{wG_s}{S}}$ $\gamma_d = \frac{eS\gamma_w}{(1+e)w}$ $\gamma_d = \gamma_{sat} - n\gamma_w$ $\gamma_d = \gamma_{sat} - \left(\frac{e}{1+e}\right)\gamma_w$	$\gamma_{sat} = \frac{(G_s+e)\gamma_w}{1+e}$ $\gamma_{sat} = [(1-n)G_s + n]\gamma_w$ $\gamma_{sat} = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$ $\gamma_{sat} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$ $\gamma_{sat} = \gamma_d + n\gamma_w$ $\gamma_{sat} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$

In above relations, γ_w refers to the unit weight of water, 62.4 pcf (=9.81 kN/m^3).

Unit Systems Used in Geotechnical Engineering

- Weight and Volume Relationships
  - In most cases, calculations in soil mechanics are done on a weight basis (SI or US units)
  - Exceptions include wave propagation problems (earthquakes, pile dynamics, etc.
- U.S. Units
  - Most common within the U.S., generally not used elsewhere
  - Basic units of weight: 1 pound (lb.) or kip (1000 lbs.)
  - Basic unit of length: feet
  - Will frequently write psf and pcf instead of lb/ft^2 and lb/ft^3
- SI Units
  - Technically the “official” system everywhere else
  - Basic units of weight: N or kN
  - Basic unit of length: m
- MKS Units
  - Commonly used in “metric” countries
  - Basic unit of mass: kg
  - Basic unit of length: m

Specific Gravity and Density

- Unit Weight of Water (γ_w)
  - 62.4 pcf = 0.0624 kcf
  - 9.81 kN/m^3
- Density of Water
  - 1.95 $slugs/ft^3$
  - 1000 kg/m^3
  - 1 g/cm^3 = 1 Mg/m^3 = 1 Metric Ton/m^3

- Typical Values of Specific Gravity for Soil Particles
  - Quartz Sand: 2.64 – 2.66
  - Silt: 2.67 – 2.73
  - Clay: 2.70 – 2.9
  - Chalk: 2.60 – 2.75
  - Loess: 2.65 – 2.73
  - Peat: 1.30 – 1.9
  - Except for organic soils, range is fairly narrow

Example 1

- Given
  - Atterberg limit water content calculation shown at the bottom right hand corner (see SL 2 for complete test)
- Find
  - Water content of sample
- Solution
  - In this case, the tare weight W_t (weight of the holding cup) will figure into our calculations
  - For many class problems, it will not; you can either ignore it or set it to zero in these calculations

- Weight of wet soil and tare = $W_t + W_s + W_w$
- Weight of dry soil and tare = $W_t + W_s$
- $W_w = (W_t + W_s + W_w) - (W_t + W_s) = 44.41 - 41.08 = 3.33$ g
- $W_s = (W_t + W_s) - W_t = 41.08 - 34.04 = 7.04$ g
- $w = W_w/W_s = 3.33/7.04 = .473 = 47\%$

RUN NUMBER	1
TARE NUMBER	41
A. WEIGHT OF WET SOIL + TARE	44.41
B. WEIGHT OF DRY SOIL + TARE	41.08
C. WEIGHT OF WATER, W_w (A. - B.)	3.33
D. WEIGHT OF TARE	34.04
E. WEIGHT OF DRY SOIL, W_s (B. - D.)	7.04
WATER CONTENT, $w = (\frac{W_w}{W_s} \times 100)$	47.3

Example 2

- Given:
  - Total Mass = 18.18 kg
  - Total Volume = 0.009 m^3
  - Dry Mass = 16.13 kg
  - Specific Gravity of Solids = 2.7
- Find
  - Wet Unit Weight
  - Dry Unit Weight
  - Void Ratio
  - Water Content
  - Degree of Saturation

- Compute Mass of Water
  - $M_t = M_s + M_w$
  - $18.18 = 16.13 + W_w$
  - $M_w = 2.05$ kg
- Compute Water Content
  - $w = M_w / M_s$
  - $W = 2.06/16.13 = .127 = 12.7\%$

Example 2

Compute Volumes

Volume of Water

- $V_w = M_w / \rho_w$
- $V_w = 2.05\ kg/1000\ kg/m^3 = 0.00205\ m^3$

Volume of Solids

- $V_s = M_s/ \rho_s = M_s/(G_s\ \rho_w)$
- $V_s = 16.13/((1000)(2.7)) = 0.00597\ m^3$

Volume of Air

- $V_a = V_t – V_w – V_s$
- $V_a = 0.009\text{-}0.00205\text{-}0.00597 = .00098\ m^3$

Compute Wet Density

$\gamma_{wet} = M_T/V_T = 18.18/0.009 = 2020\ kg/m^3$

Compute Dry Density

$\gamma_{dry} = M_s/V_T = 16.13/0.009 = 1792\ kg/m^3$

Void Ratio

$e = V_v/V_s = (V_w+V_a)/V_s = (.00205+0.000976)/.00597 = .507$

Compute Degree of Saturation

$S = V_w/(V_w+V_a)$

$S = .00205/(.000976+ .00205) = .677 = 67.7\%$

Example 3

- Given
  - Saturated Soil
  - Void Ratio = 0.45
  - Specific Gravity of Solids = 2.65
- Find
  - Wet Unit Weight
  - Water Content
- Assumptions
  - $V_a = 0$
  - $V_t = 1$
  - $V_s + V_w = 1$
  - $\gamma_{water} = 62.4$ lb/ft^3
- Solve for Volumes
  - $e = V_w/V_s = 0.45$
  - $V_w = 0.31$ ft^3
  - $V_s = 0.69$ ft^3
- Compute Wet Unit Weight
  - Weight of Solids = $\gamma_w V_s G_s = (62.4)(0.69)(2.7) = 114$ lb
  - Weight of Water = $\gamma_w V_w = (62.4)(0.31) = 19.4$ lb
  - Total Weight = 114 + 19.4 = 133.4 lb
  - Since volume is unity, total weight is also net unit weight = 133.4 pcf
- Compute Water Content
  - $w = W_w/W_s = 19.4/114 = 0.17 = 17\%$

Example 4

- Given
  - Well Graded Sand
  - Specific Gravity of Solids = 2.65
  - Void Ratio = 0.57
  - Porosity = 36.5%
  - Degree of Saturation = 55%
- Find
  - Wet and Dry Unit Weight of Soil
- Solution
  - Set sample volume = 1 ft^3
  - Total Volume = 1= $V_w + V_a + V_s$
  - Use porosity to compute volume of voids
    - $n = V_v/V_t$
    - $V_v = nV_t = (.365)(1) = .365\ ft^3$
- Compute volume of solids
  - $V_s = V_t - V_v = 1 - .365 = .635\ ft^3$
- $V_v = V_w + V_a$
- $S = V_w/V_v$, or $V_w = SV_v$
- $V_w = (0.55)(0.365\ ft^3) = 0.2\ ft^3$
- Dry Unit Weight
  - $Ws = \gamma_w G_s V_s = (62.4)(2.65)(.635) = 105\ lb/ft^3$
- Weight of Water
  - $W_w = \gamma_w V_w = (62.4)(0.2) = 12.5\ lb/ft^3$
- Wet Unit Weight
  - $W_t = W_w + W_v = 117.5\ lb/ft^3$

Example 4

- Can we use the "shortcut" formulae given earlier (and shown on the right) to compute the unit weight?
- The answer is "yes" *if we pick the right one*
- Since the sample is not fully saturated, we need to use the formulae in the left column
- The information in the problem (without subsequent processing) indicate the use of the underlined formula
- Substituting, $\boldsymbol{\gamma}_t$ = (2.65+(0.55)(0.57))*62.4/(1+0.57) = 117.8 pcf

Table 2-2

Weight-volume relations (after Das, 1990)

Unit-Weight Relationship	Dry Unit Weight (No Water)	Saturated Unit Weight (No Air)
$\gamma_t = \frac{(1+w)G_s\gamma_w}{1+e}$	$\gamma_d = \frac{\gamma_t}{1+w}$	$\gamma_{sat} = \frac{(G_s+e)\gamma_w}{1+e}$
$\underline{\gamma_t = \frac{(G_s+Se)\gamma_w}{1+e}}$	$\gamma_d = \frac{G_s\gamma_w}{1+e}$	$\gamma_{sat} = [(1-n)G_s+n]\gamma_w$
$\gamma_t = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	$\gamma_d = G_s\gamma_w(1-n)$	$\gamma_{sat} = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$
$\gamma_t = G_s\gamma_w(1-n)(1+w)$	$\gamma_d = \frac{G_s\gamma_w}{1+\frac{wG_s}{S}}$	$\gamma_{sat} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$
	$\gamma_d = \frac{eS\gamma_w}{(1+e)w}$	$\gamma_{sat} = \gamma_d + n\gamma_w$
	$\gamma_d = \gamma_{sat} - n\gamma_w$	$\gamma_{sat} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$
	$\gamma_d = \gamma_{sat} - \left(\frac{e}{1+e}\right)\gamma_w$	

In above relations, γ_w refers to the unit weight of water, 62.4 pcf (=9.81 kN/m^3).

Relative Density

- Density
  - Both unit weight and strength of soil can vary with particle arrangement
  - Denser soils have both higher load carrying capacity and lower settlement

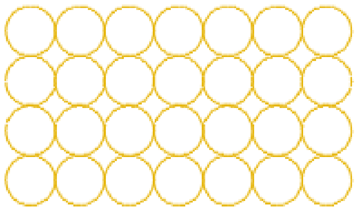


Figure 3.1: Cubic array.

$n = 0.4764$

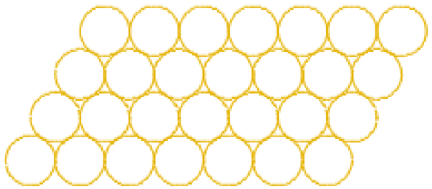


Figure 3.2: Densest array.

$n = 1 - \pi/\sqrt{18} = 0.2595$

- Definition of Relative Density (also D_r)

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

  - e_{max} = void ratio of the soil in its loosest condition
  - e_{min} = void ratio of the soil in its densest condition
  - e = void ratio in the natural or condition of interest of the soil
  - Convenient measure for the strength of a soil with less than 15% fines

Relative Density Example

- Given
  - Sand Backfill
  - Unit Weight = 109 pcf
  - Water Content = 8.6%
  - Specific Gravity of Solids = 2.6
  - e_{max} = 0.642 (loosest state)
  - e_{min} = 0.462 (densest state)
- Find
  - Void Ratio
  - Relative Density

- Assume V_t = 1 ft^3; thus, W_t = 109 lbs.
- Weight balance: 109 = $W_s + W_w$
- Water content w = W_w/W_s = 0.086
- Solving two previous equations:
  - W_s = 100.4 lbs; W_w = 8.6 lbs.
  - $V_s = W_s/\gamma_s$ = 100.4/((2.6)(62.4)) = 0.618 ft^3
  - $V_w = W_w/\gamma_w$ = 8.6/62.4 = 0.138 ft^3
  - $V_a = V_t - V_w - V_s$ = 1 – 0.138 – 0.618 = 0.243 ft^3
- e = $(V_a + V_w)/V_s$ = (0.243 + 0.138)/0.618 = 0.616

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

- D_r = (.642-.616)/(.642-.462) = 0.144 = 14.4%

Use of Relative Density for Empirical Values

TABLE 4-6 EMPIRICAL VALUES FOR ϕ, D_r, AND UNIT WEIGHT OF GRANULAR SOILS BASED ON CORRECTED N' (after Bowles, 1977)

Description	Very Loose	Loose	Medium	Dense	Very Dense
Relative density D_r	0 - 0.15	0.15 - 0.35	0.35 - 0.65	0.65 - 0.85	0.85 - 1.00
Corrected Standard Penetration N' value	0 to 4	4 to 10	10 to 30	30 to 50	50+
Approximate angle of internal friction ϕ *	25 - 30°	27 - 32°	30 – 35°	35 - 40°	38 - 43°
Range of approximate moist unit weight ,γ kN/m^3 (lb/ft^3)	11.0 - 15.7 (70 - 100)	14.1 - 18.1 (90 - 115)	17.3 - 20.4 (110 - 130)	17.3 - 22.0 (110 - 140)	20.4 - 23.6 (130 - 150)

Questions?