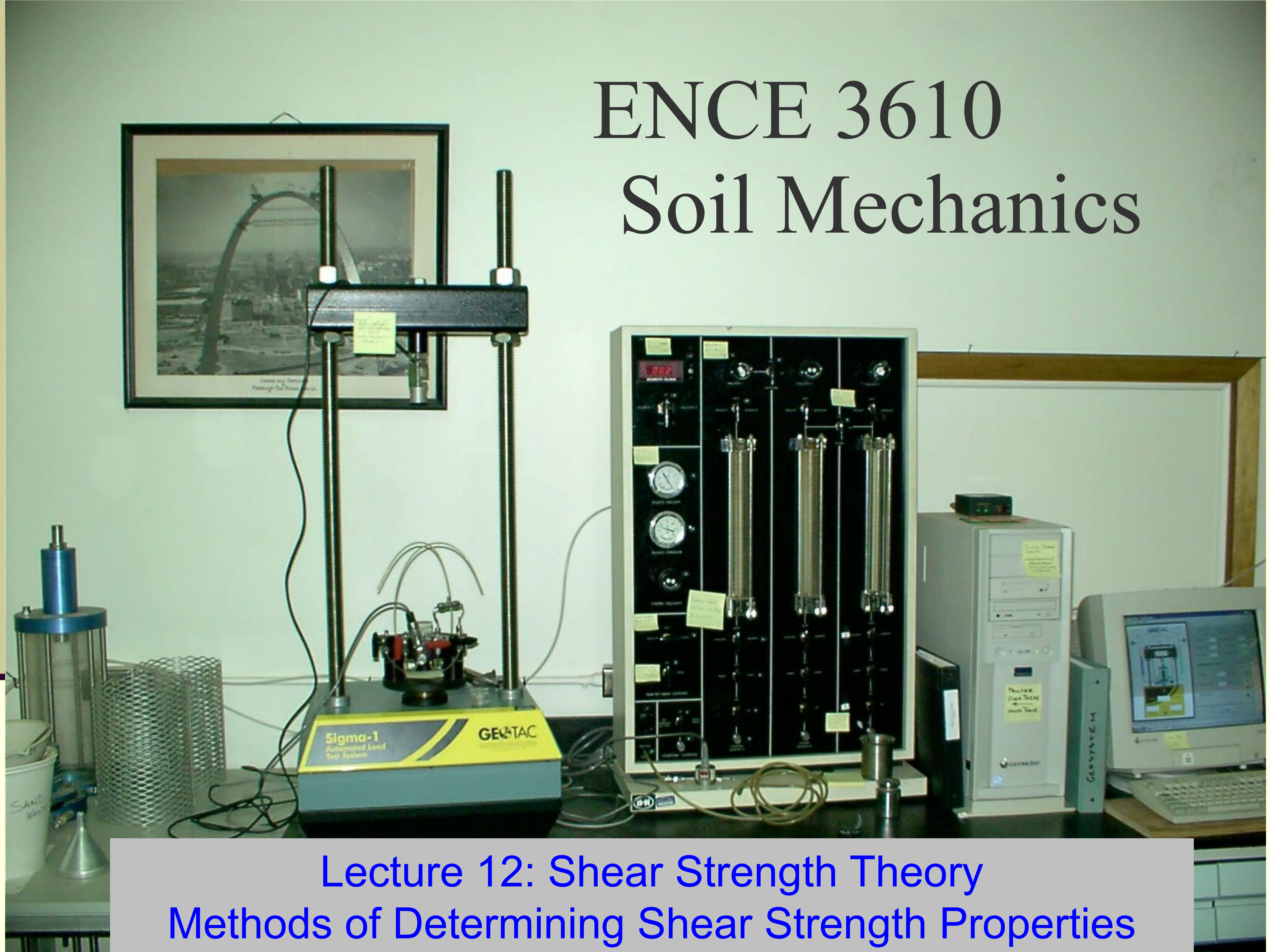
ENCE 3610
Soil Mechanics



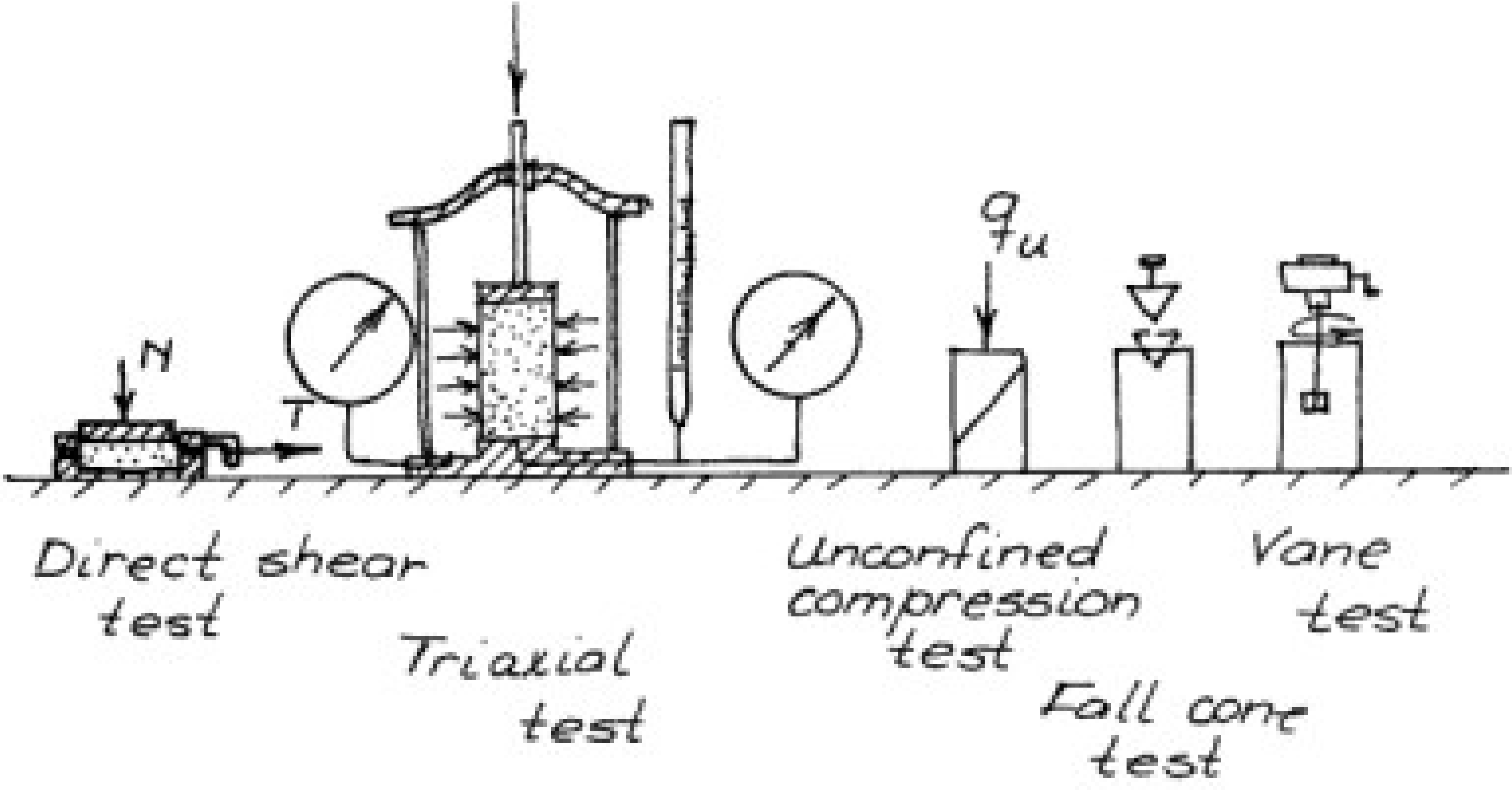
Lecture 12: Shear Strength Theory
Methods of Determining Shear Strength Properties

Review of Soil Strength

- Tension in soil is seldom (if ever) used
  - Even rock is seldom used in tension
- All applications of soil as an engineering material are in compression
- The most important failure mode to consider is shear failure
- The shear strength of the soil is the key property to determine
- The way shear strength acts in a soil depends upon the soil type

- Two questions we need to ask with soil strength
  - What constitutes soil strength?
  - How to we test for it?
- The two are not unrelated, but they are not identical either

Types of Tests for Shear Strength



Direct Shear Test

Direct Shear Test

- The most “obvious” way to test for shear strength
- Failure mechanism in direct shear test is different than in most actual soils
- Reproducibility is problematic
- Is used in some cases, but not the most common test for shear strength

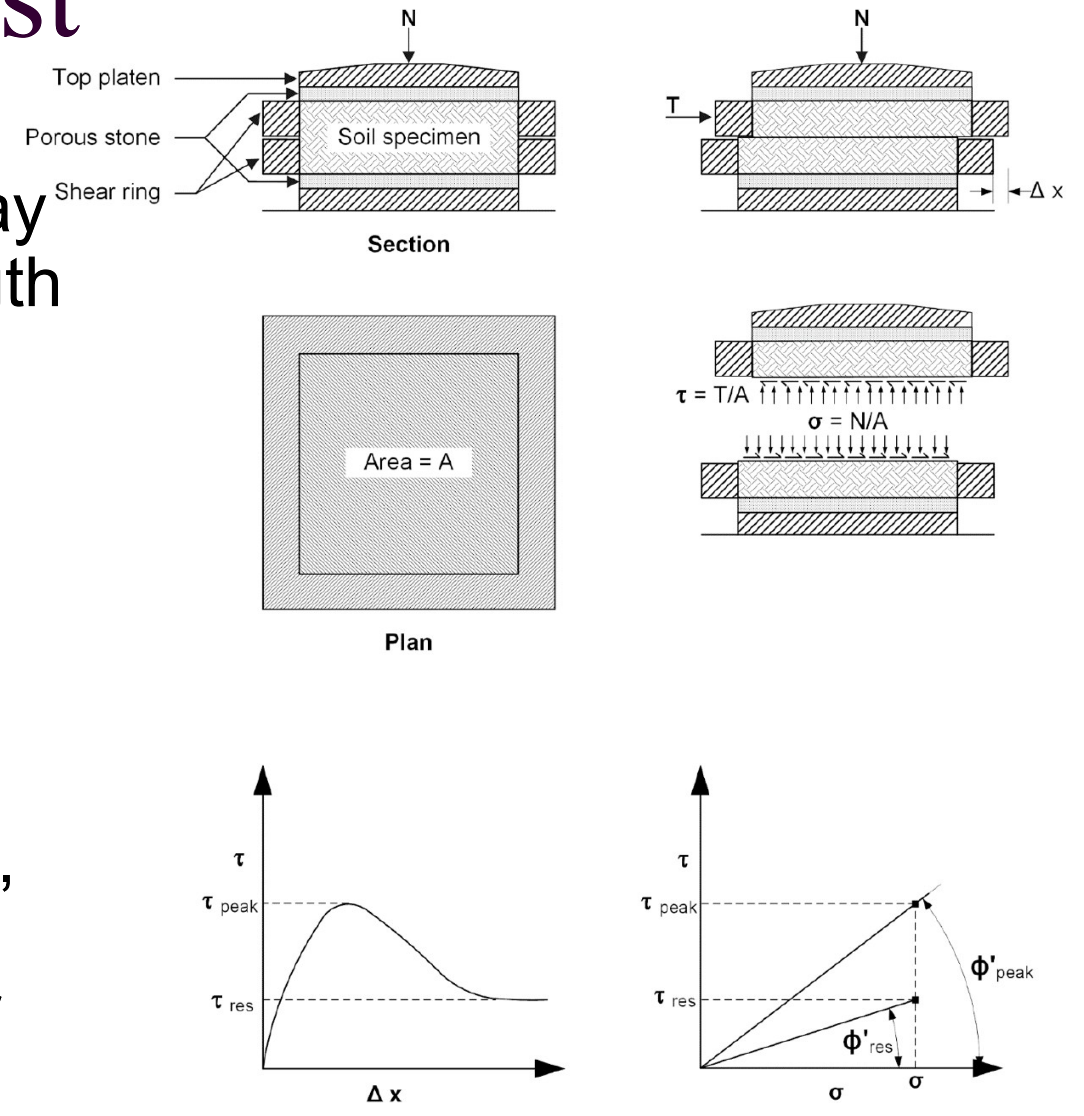


Figure 3-1 Basic Elements for a Consolidated Drained Direct Shear Test Along with Example Data Collected for One Test Specimen

Triaxial Test

- The “reference standard” for soil shear strength testing
- Elements of the test
  - Specimen, usually 1.4” diameter for cohesive soils, 2.8” for cohesionless ones
  - Wrapped in a rubber shroud to prevent water flow from load cell
  - Load cell, filled with water, pressurised separately from specimen
  - Piston which applies axial pressure to specimen
  - Provision (or not) for drainage of water at base of cell; specimen tested saturated
- Types of Triaxial Tests
  - Consolidated-Drained (CD) tests: pore water in specimen is allowed to drain during the test.
    - Usually done with cohesionless soils
    - Can be done on undisturbed, compacted or remoulded soils
  - Consolidated-Undrained (CU) tests: no drainage is allowed during the test
    - Usually done with cohesive soils
  - Unconsolidated-Undrained (UU) tests: the basis for the undrained shear strength of cohesive soils

Consolidated Drained Triaxial Test

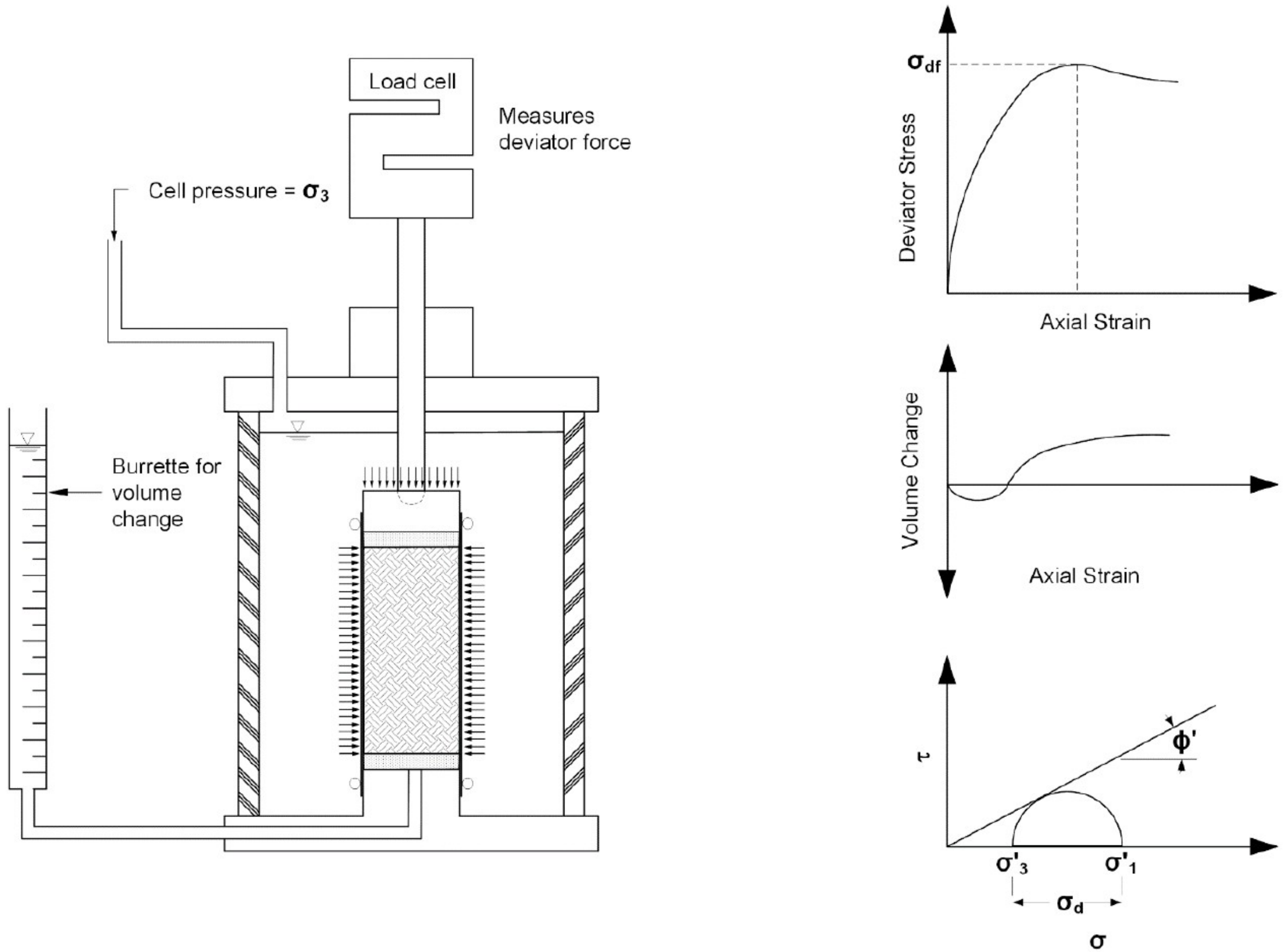


Figure 3-2 Basic Elements of a Consolidated Drained Triaxial Test Along with Example Data Collected for One Test Specimen

Consolidated Undrained Triaxial Test

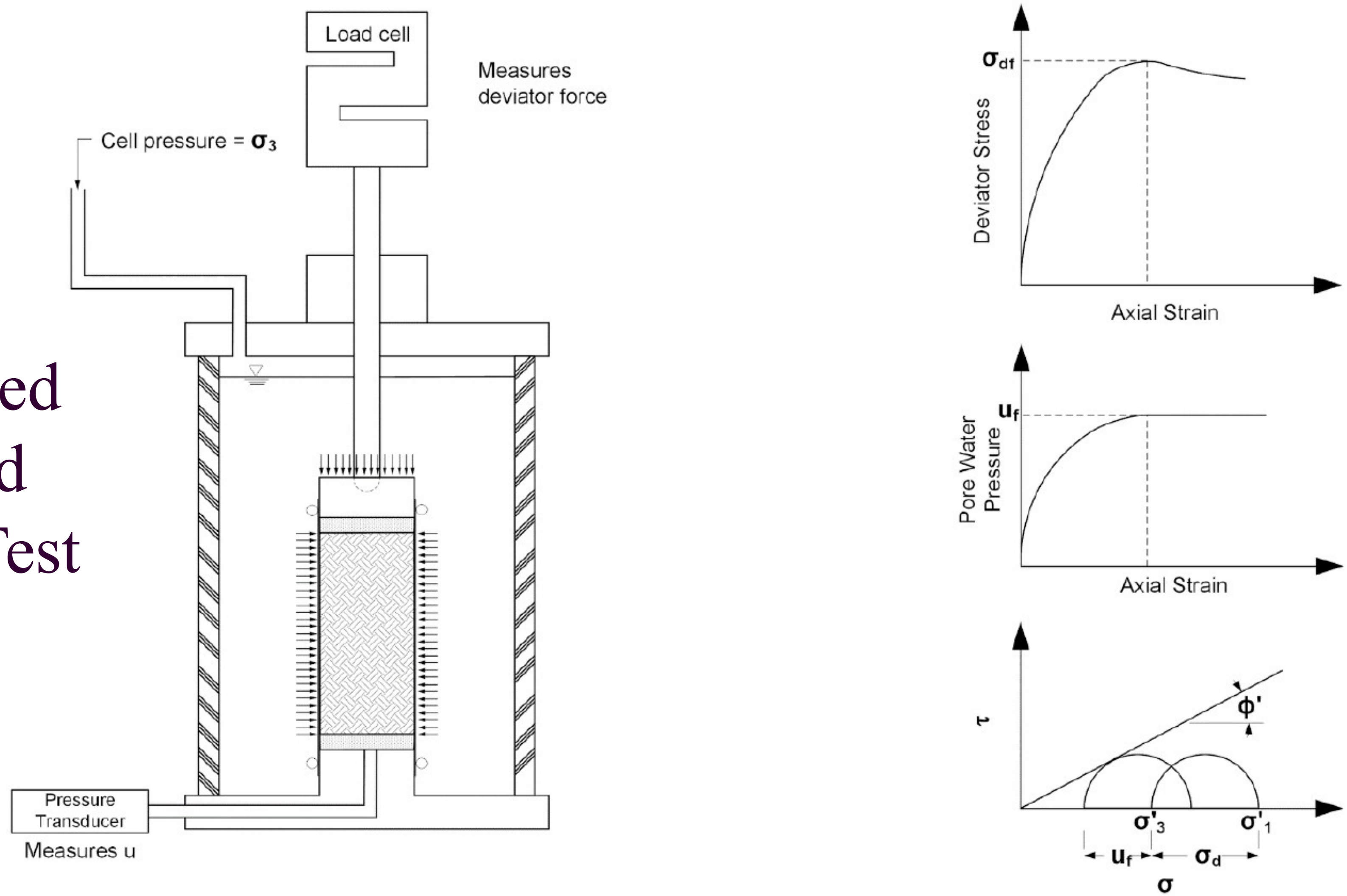


Figure 3-3 Basic Elements of a Consolidated Undrained Triaxial Test Along with Example Data Collected for One Test Specimen

Unconsolidated Undrained Triaxial Test

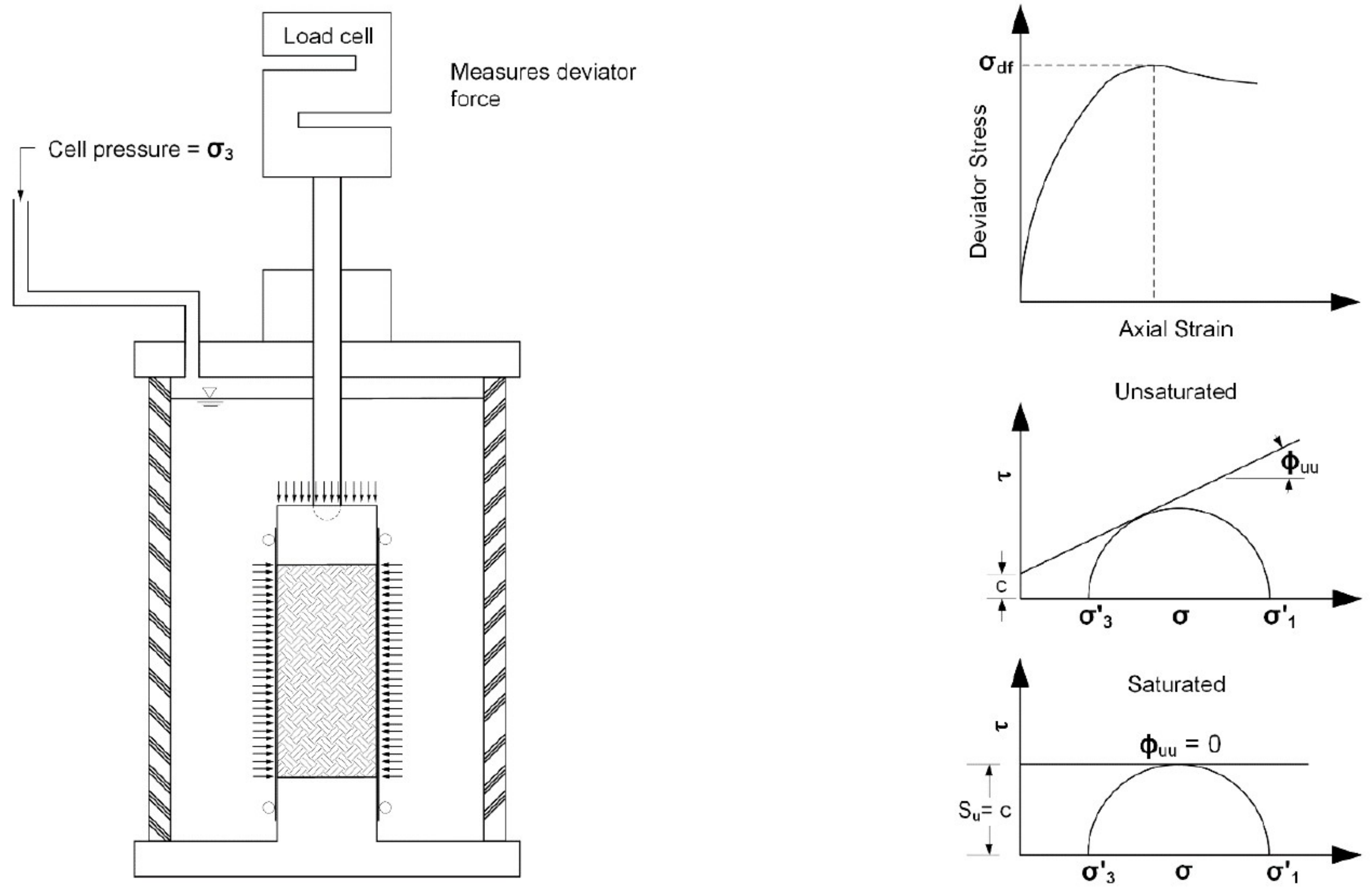
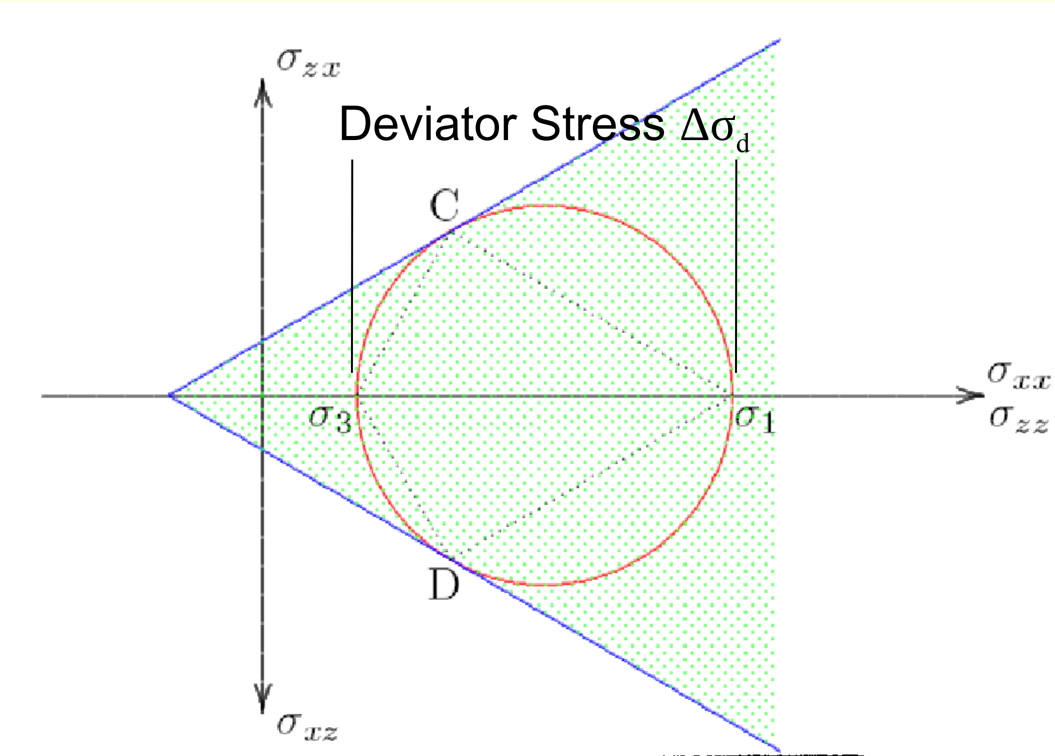
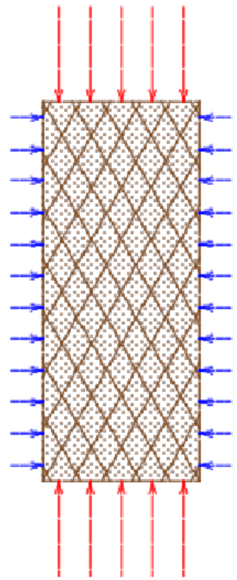
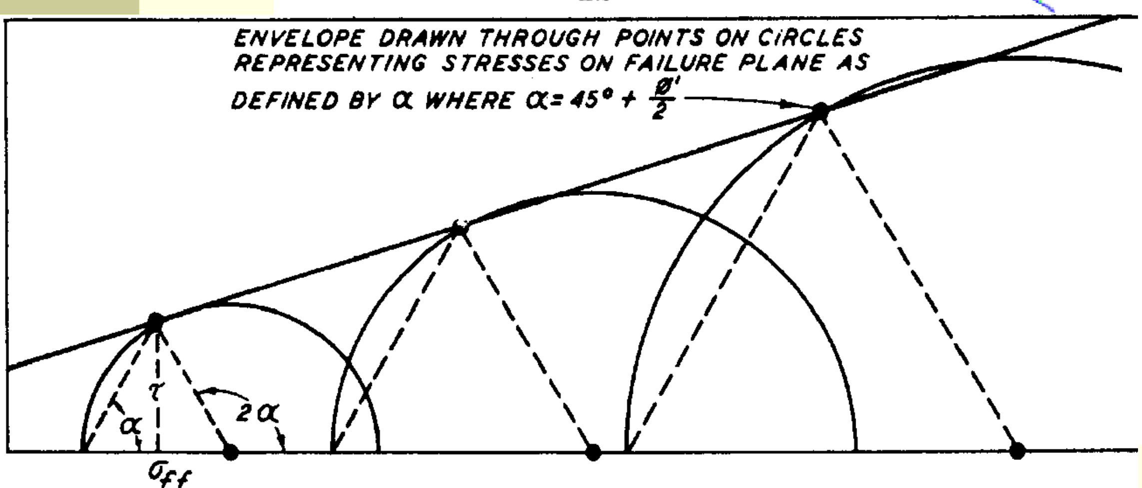


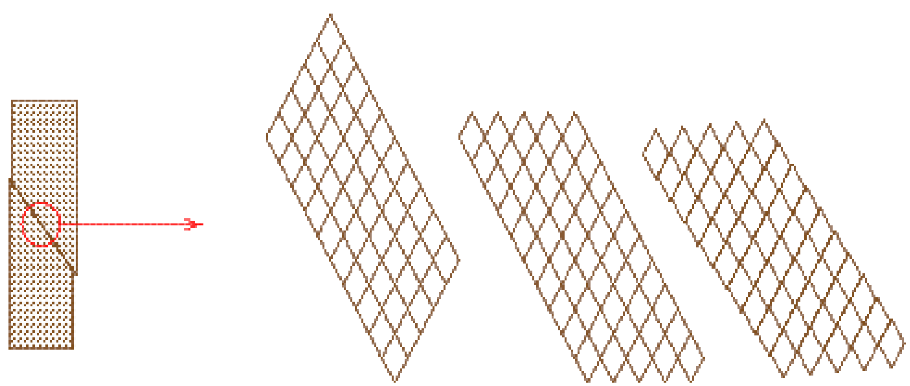
Figure 3-7 Basic elements of a UU Test Apparatus with Sample Data for a Single Test

Use of Mohr's Circle to Determine Failure Envelope



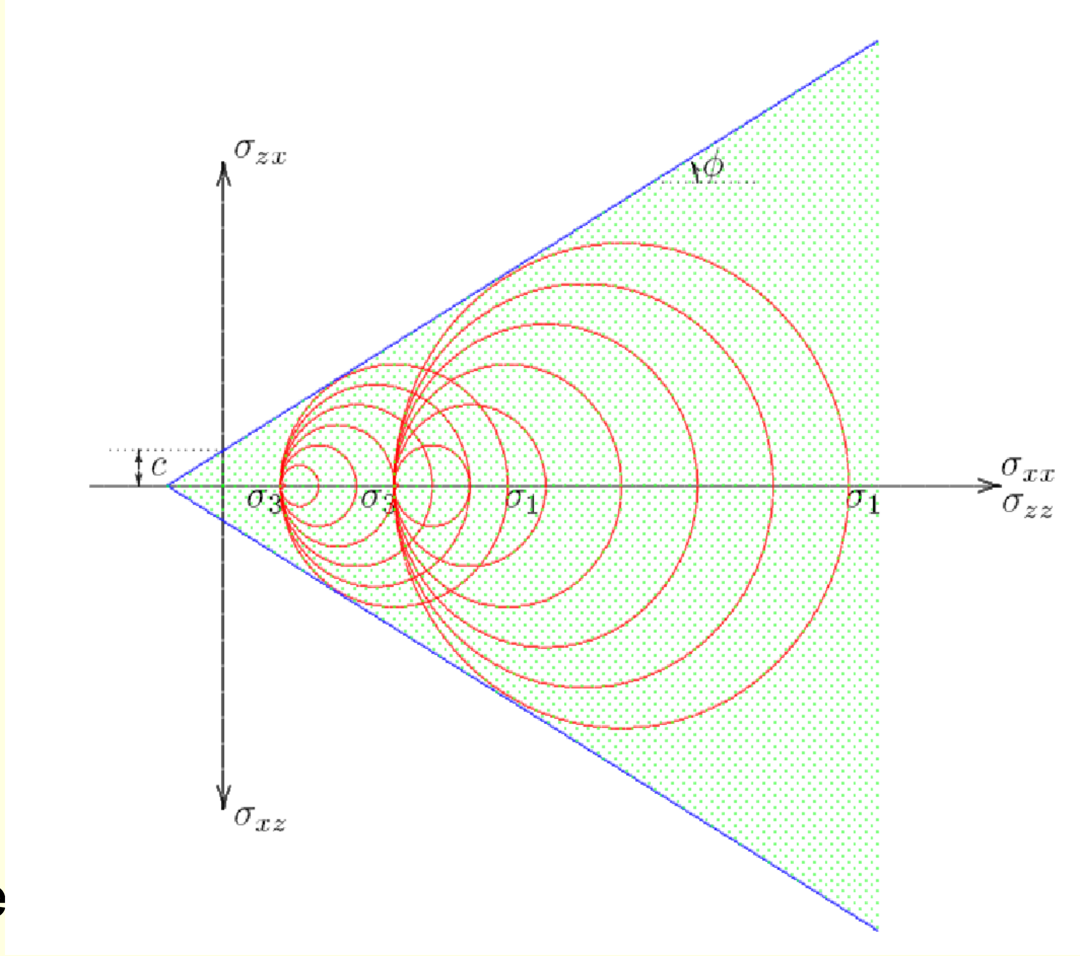






Shear Envelope for Multiple Effective Stresses (σ_3)

- Assume that the smaller principal stress σ_3 is:
  - Horizontal Stress
  - Confining Stress
- Assume that the larger principal stress σ_1 is
  - Vertical Stress
- Then the σ_3 stress is the effective stress of the soil
- Vertical stress σ_1 includes effective stress and foundation loading
- These conditions will be different in some situations, but illustrate the significance of varying σ_3



Shear Failure in Cohesionless and Cohesive Soils

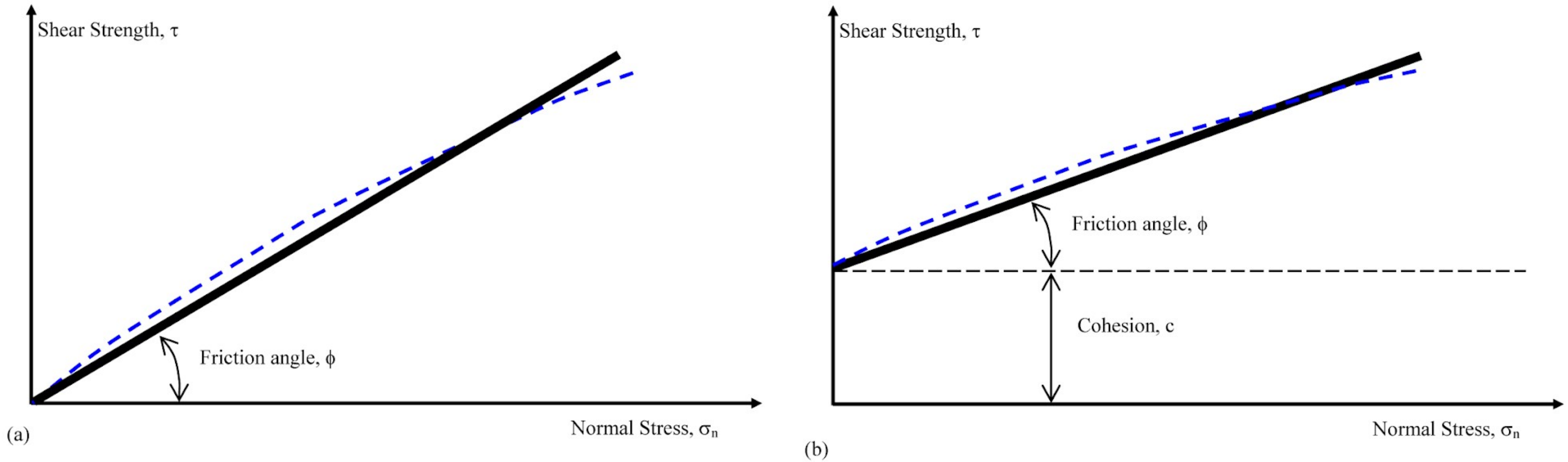


Figure 2-18. Graphical representation of shearing strength.

Typical for Cohesive Soils In Situ: φ =0, c_u > 0

Typical for Cohesionless Soils: c_u = 0, φ > 0 (but effects from overconsolidation must be considered)

Triaxial Shear Test Relationships

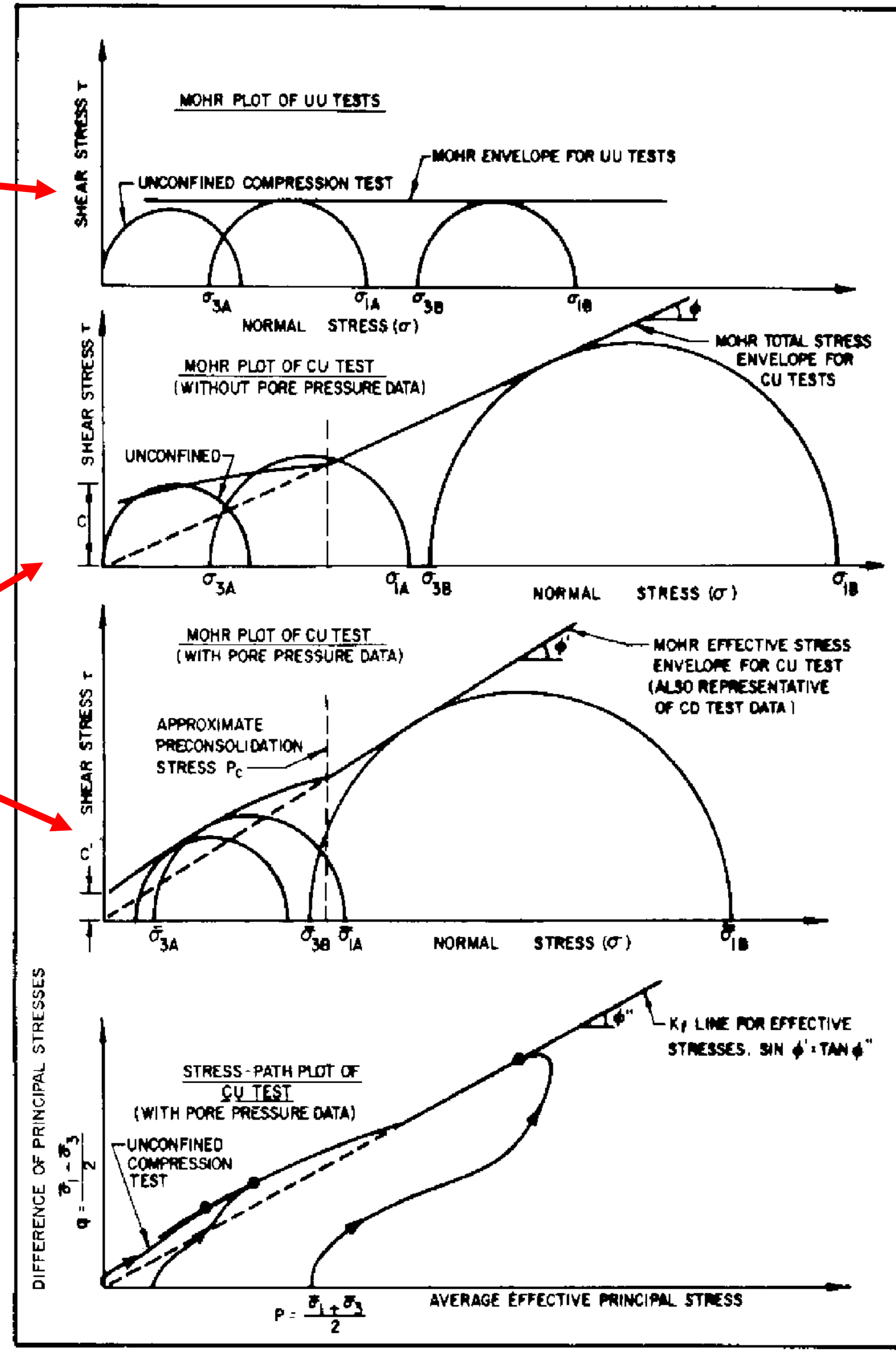


FIGURE 6
Triaxial Shear Test Relationships

Shear Strength
Purely Cohesionless Soils

- Shear strength of cohesionless soils is only developed with the presence of effective stress
- Shear strength increases with increasing effective stress
- The increase of shear strength depends upon the internal friction angle of the soil

$$s_f = c' + \sigma_f' \tan\phi'$$

$\sim c_d$ $\sim \phi_d$

(From drained tests)

$c-\phi$ analysis

For sand $c' \approx 0$

$$s_f = \sigma_f' \tan\phi'$$

ϕ- analysis

Cohesion in Soils

- True Cohesion
  - Cementation
    - Due to the presence of cementing agents such as calcium carbonate or iron oxide
  - Electrostatic and electromagnetic attractions
  - Primary valence bonding (adhesion)
    - Occurs primarily during overconsolidation

- Apparent Cohesion
  - Negative pore water pressure
  - Negative excess pore water pressures due to dilation (expansion)
  - Apparent mechanical forces
  - *Cannot be relied on for soil strength*

Purely Cohesive Soils

- All of the strength of the soil is developed from the cohesion of the soil
- The strength of the soil is essentially independent of the effective stress
  - q_u = unconfined compression strength
  - c = cohesion or shear strength
  - DO NOT CONFUSE THE TWO

$$s = c = \frac{q_u}{2}$$

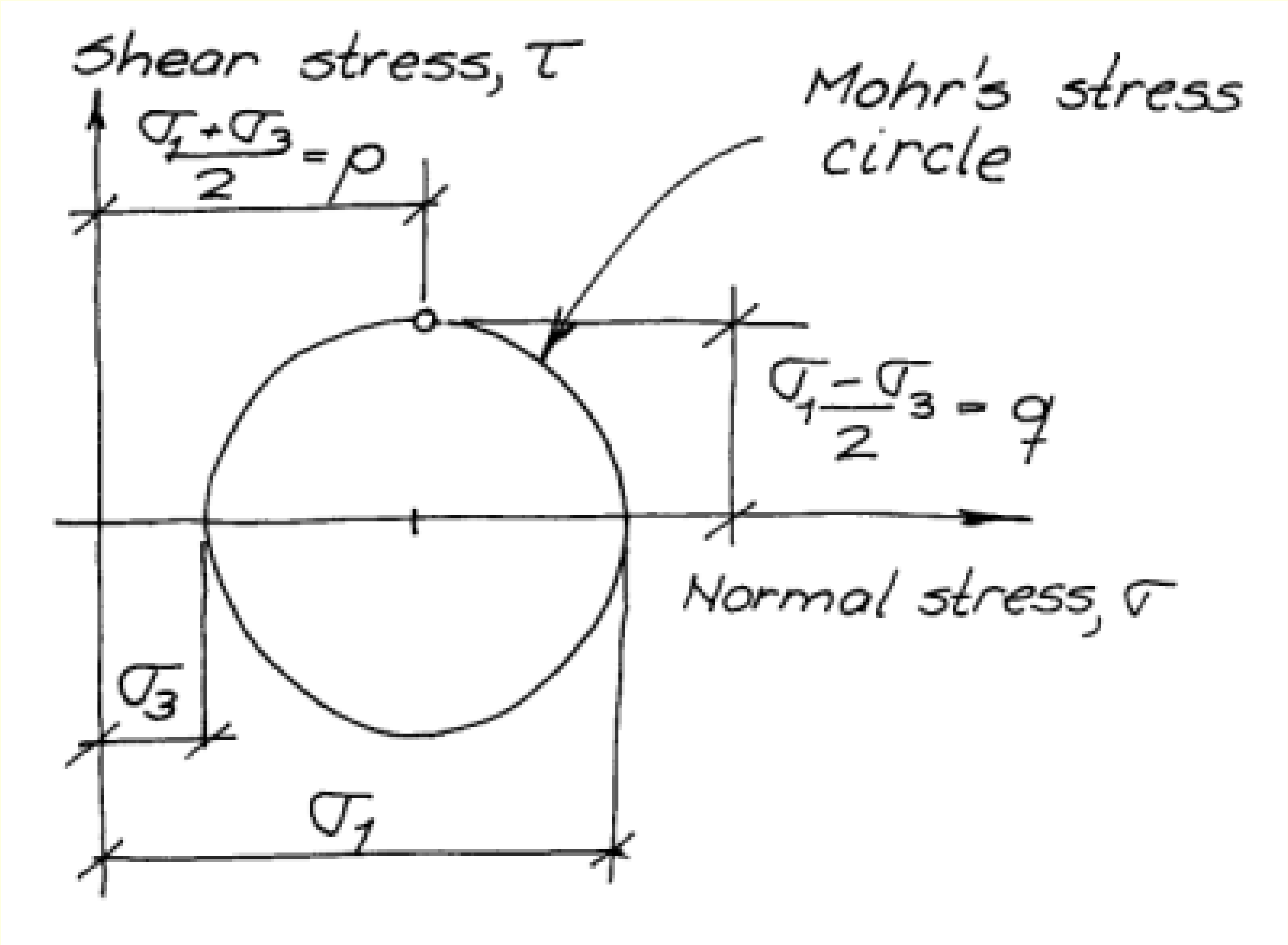
$$s_f = c_u + \sigma_f \tan \phi_u$$

For saturated soils $\phi_u = 0$

$$s_f = c_u$$

c-analysis, $\phi = 0$-analysis

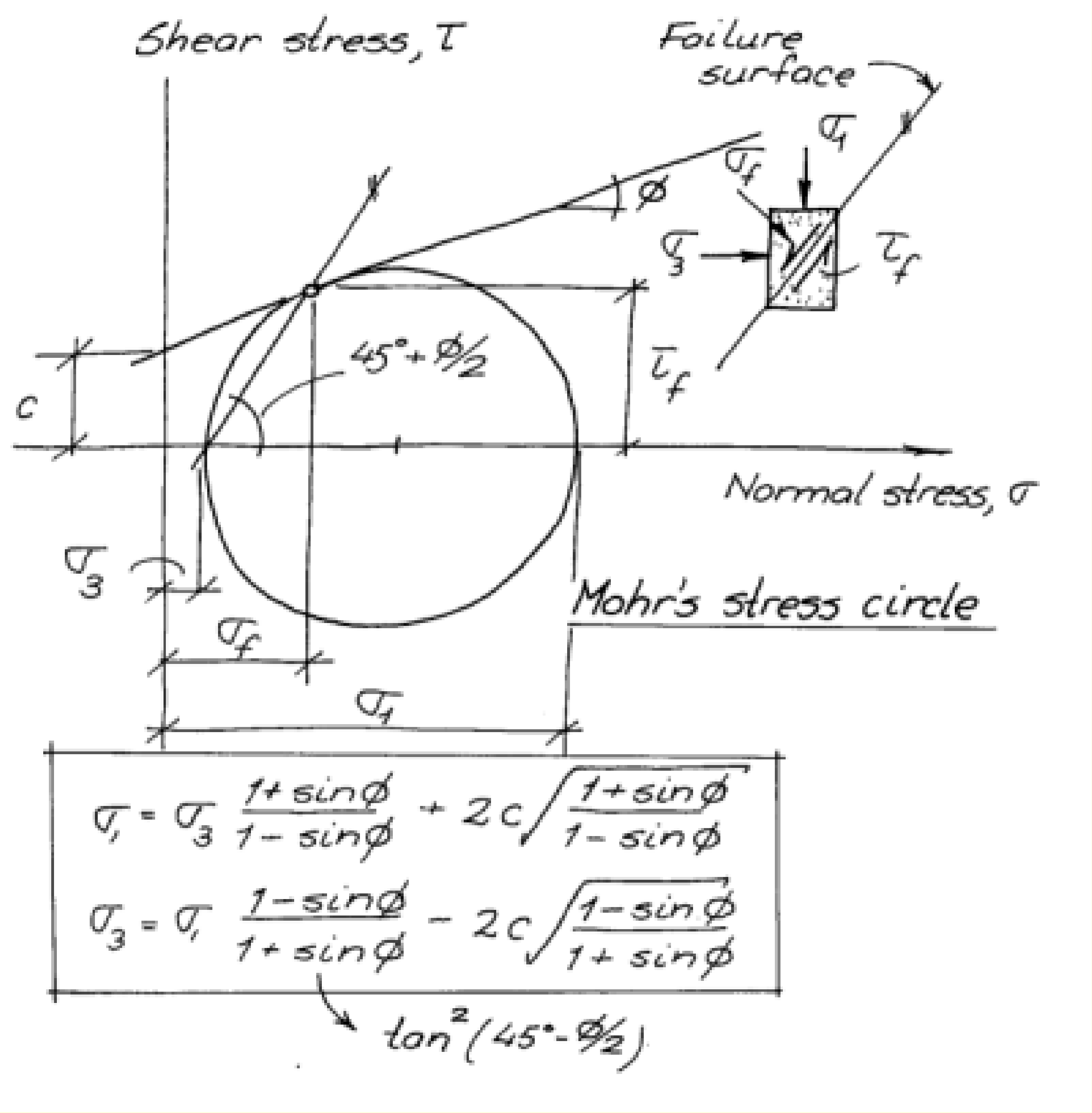
Purely Cohesive Soils



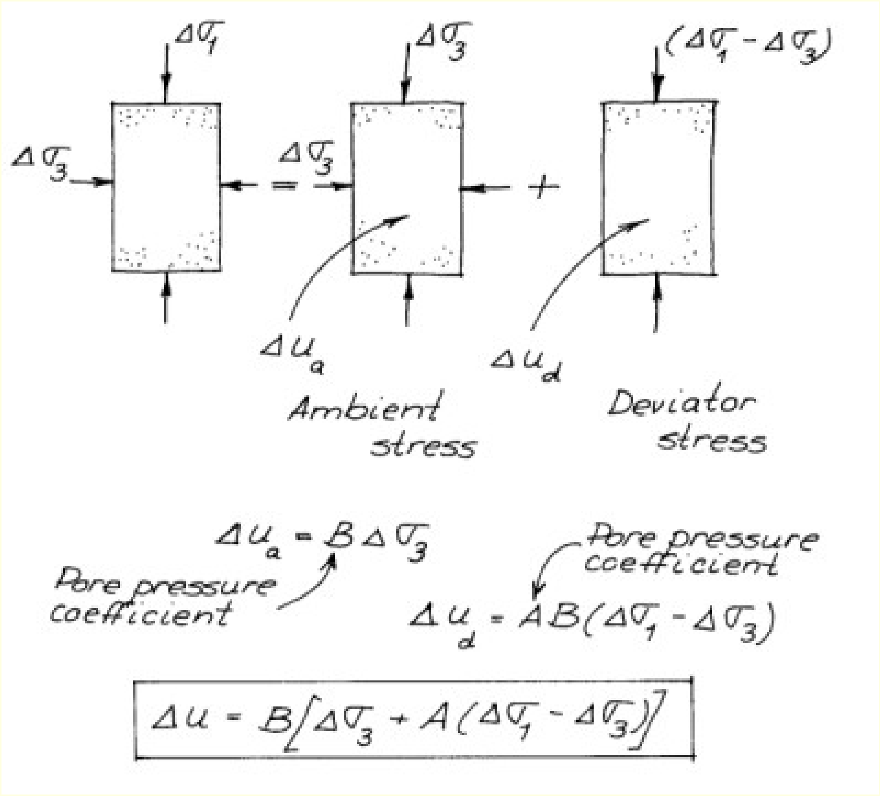
Soils with Both Cohesion and Internal Friction

- Ideally soils are either purely cohesive or cohesionless
- This is frequently not the case because:
  - Composition of soils are mixed (combinations of sands, clays and silts)
  - Drainage and/or remoulding of clays produces conditions similar to drained triaxial or direct shear conditions

Review of Mohr's Circle with Both Cohesion and Friction



Undrained Triaxial Test Example



Given

- Dry Cohesionless Soil
- Tested to determine φ
- Confining pressure = 720 psf
- Deviator stress at failure = 600 psf
- Pore water pressure at failure = 496.2 psf

Find

- Value of φ, drained and undrained

Undrained Triaxial Test Example

- Equation for failure envelope and Mohr's Circle

$$\sigma_1' = \sigma_3' \frac{1+\sin\phi}{1-\sin\phi} + 2c\,\frac{\cos\phi}{1-\sin\phi}$$

- Simplification for c = 0

$$\sigma_3' = \sigma_1' \frac{1-\sin\phi}{1+\sin\phi}$$

- Solve for sin ϕ

$$\sin\phi = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c\cot\phi + \frac{1}{2}(\sigma_1' + \sigma_3')}$$

- Simplification for c = 0

$$\varphi = \arcsin\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right)$$

Undrained Triaxial Test Example

Undrained Condition

$$\begin{aligned}
\sigma_1 &= \sigma_3 + \Delta\sigma_d \\
\sigma_1 &= 720 + 600 \\
\sigma_1 &= 1320\, psf \\
\phi &= arcsin\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) \\
\phi &= arcsin\left(\frac{1320 - 720}{1320 + 720}\right) \\
\phi &= 17.1^\circ
\end{aligned}$$

Drained Condition

$$\begin{aligned}
\sigma_3' &= \sigma_3 - \Delta u_d \\
\sigma_1' &= \sigma_1 - \Delta u_d \\
\sigma_3' &= 720 - 496.2 = 223.8\, psf \\
\sigma_1' &= 1320 - 496.2 = 823.8\, psf \\
\phi' &= arcsin\left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}\right) \\
\phi' &= arcsin\left(\frac{823.8 - 223.8}{823.8 + 223.8}\right) \\
\phi' &= 34.94^\circ
\end{aligned}$$

Example 2

- Given
  - Granular soil ($c = 0$)
  - Unit Weight = 19.6 kN/m^3
  - Internal Friction Angle = 35º
  - Proposed Structure causes vertical stress to increase 60 kPa at 4m depth
  - Also consider case where water table increases to ground surface
- Find
  - Shearing Strength 4m below surface before installation of structure
  - Whether soil will shear with additional load
  - Whether soil will shear with additional load and elevation of water table
- Solution
  - Simplest way to solve last two parts is to compute failure criterion f
    - If f is negative, failure should not occur
    - If f is positive, failure will most likely occur

Example 2

- Principal Stresses for two load cases
  - Case 1
    - Soil overburden pressure = $(19.6\ kN/m^3)(4) = 78.4\ kPa$
    - $\sigma_3 = 78.4\ kPa$
    - $\sigma_1 = 60 + 78.4 = 138.4\ kPa$
  - Case 2
    - Soil overburden pressure = $(19.6-9.81)(4) = 39.2\ kPa$
    - $\sigma_3 = 39.2\ kPa$
    - $\sigma_1 = 39.2 + 60 = 99.2\ kPa$

- Formula for failure criterion for Mohr-Coulomb failure for both cohesive and cohesionless soils (and mixed ones too) with principal stresses known

- By direct substitution
- Case 1: $f = -64.4$ kPa
- Case 2: $f = -19.4$ kPa
- In both cases $f < 0$; however, Case 2 is closer to failure and thus should be considered with greater care

Example 3:
Drained Triaxial Test on Clay

- Given
  - Consolidated-Drained (CD) Triaxial Test on Saturated Clay
  - Sample 1
    - Confining Pressure = 70 kPa
    - Failure Pressure = 200 kPa
  - Sample 2
    - Confining Pressure = 160 kPa
    - Failure Pressure = 383.5 kPa
- Find
  - Cohesion and Internal Friction Angle

Solution

See online presentation on p-q diagrams; best way to solve the problem

Unconfined Compression Test

- No confining pressure
- Only applicable to purely cohesive soils
- Simplest way to determine the “unconfined compression strength” and cohesion ($q_u/2$) of a cohesive soil

Unconfined Compression Test, Purely Cohesive Soil

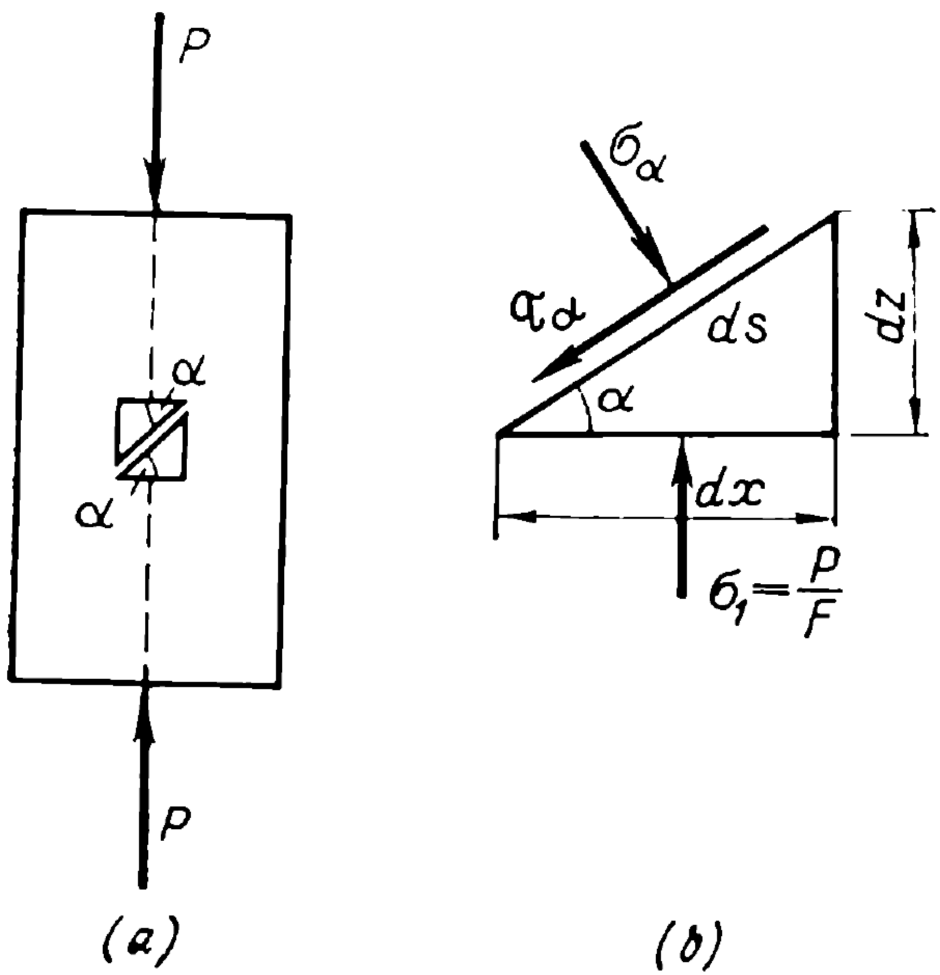


Fig. 24. Diagram of uniaxial compression test of cohesive soils

For a UC test, $\boldsymbol{\sigma}_1 = q_u = 2c$

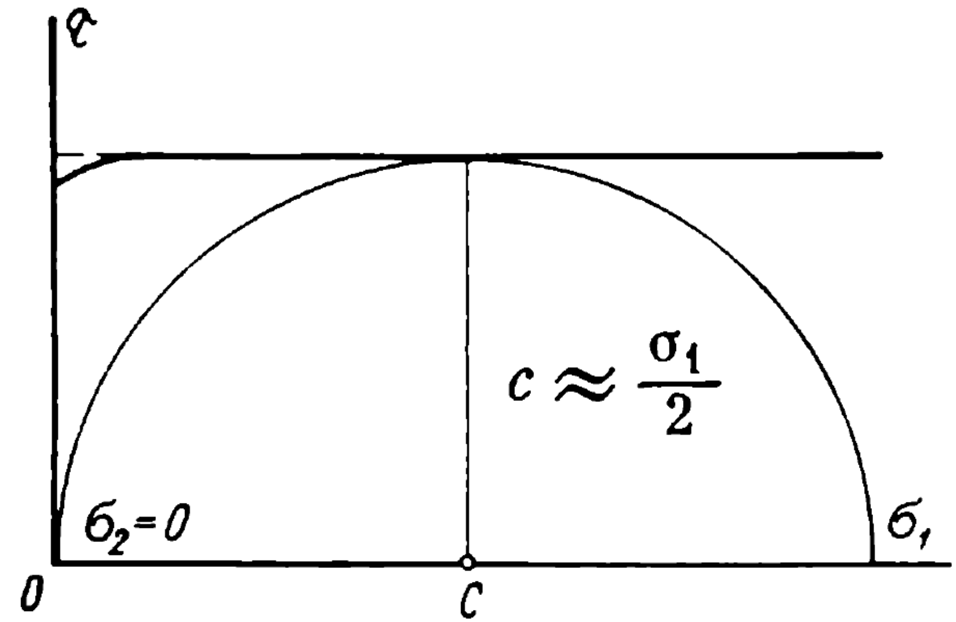


Fig. 25. Diagram of ultimat stresses at simple compression

Other Methods of Determining Shear Strength

Testing method, Field tests

Field vane test c_u

Pressuremeter test c_u

Cone penetration test $c_u = 1/10 \text{ to } 1/20\ q_c$

Standard penetration test (SPT) (c_u)

Weight sounding test (c_u)

Typical Values for Cohesionless Soils

a. Relative Density and Gradation
(Data from Schmertmann 1978)

Relative Density D_r, Percent	Fine Grained: Uniform	Fine Grained: Well-graded	Medium Grained: Uniform	Medium Grained: Well-graded	Coarse Grained: Uniform	Coarse Grained: Well-graded
40	34	36	36	38	38	41
60	36	38	38	41	41	43
80	39	41	41	43	43	44
100	42	43	43	44	44	46

b. Relative Density and In Situ Soil Tests

Soil Type	Relative Density D_r, Percent	Standard Penetration Resistance N_{60} (Terzaghi and Peck 1967)	Cone Penetration Resistance q_c, ksf (Meyerhof 1974)	Friction Angle ϕ', deg: Meyerhof (1974)	Friction Angle ϕ', deg: Peck, Hanson and Thornburn (1974)	Friction Angle ϕ', deg: Meyerhof (1974)
Very Loose	< 20	< 4	----	< 30	< 29	< 30
Loose	20 - 40	4 - 10	0 - 100	30 - 35	29 - 30	30 - 35
Medium	40 - 60	10 - 30	100 - 300	35 - 38	30 - 36	35 - 40
Dense	60 - 80	30 - 50	300 - 500	38 - 41	36 - 41	40 - 45
Very Dense	> 80	> 50	500 - 800	41 - 44	> 41	> 45

Values of Internal Friction Angle

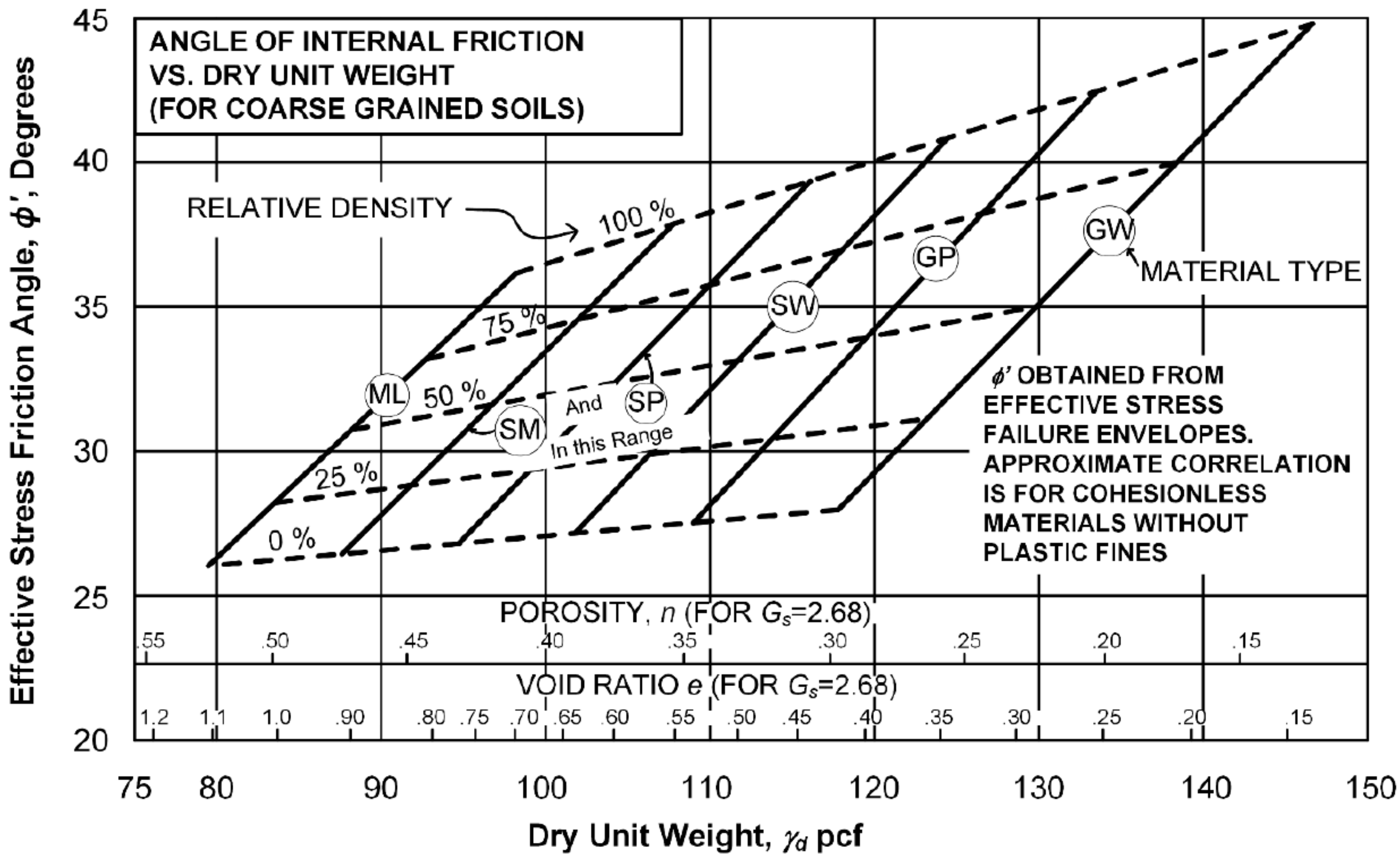


Figure 8-1 Approximate Relationship between the Effective Stress Friction Angle and Dry Unit Weight for Various Relative Densities and Types of Soil

Typical Values of Cohesion

Consistency	*Very Soft*	*Soft*	*Medium*	*Stiff*	*Very Stiff*	*Hard*
q_u = unconfined compression strength, tons per square ft	0 – 0.25	0.25 – 0.50	0.50 – 1.00	1.00 – 2.00	2.00 – 4.00	4.00 –
Standard penetration resistance, N = no. of blows per ft	0 – 2	2 – 4	4 – 8	8 – 16	16 – 32	32 –
Unit weight, pcf (saturated)	100-120		110-130	120-140		130+
Identification characteristics	Exudes from between fingers when squeezed in hand	Molded by light finger pressure	Molded by strong finger pressure	Indented by thumb	Indented by thumb nail	Difficult to indent by thumb nail

Very approximate; for preliminary purposes only

Vane Shear Test

- Useful for a quick determination of shear stress in situ
- Applicable to cohesive soils
- Good for determining undrained shear strength
  - Best if q_u < 1 ksf or 50 kPa
- Test procedures
  - ASTM D3573
  - AASHTO T 223-74

Procedure for Vane Shear Test

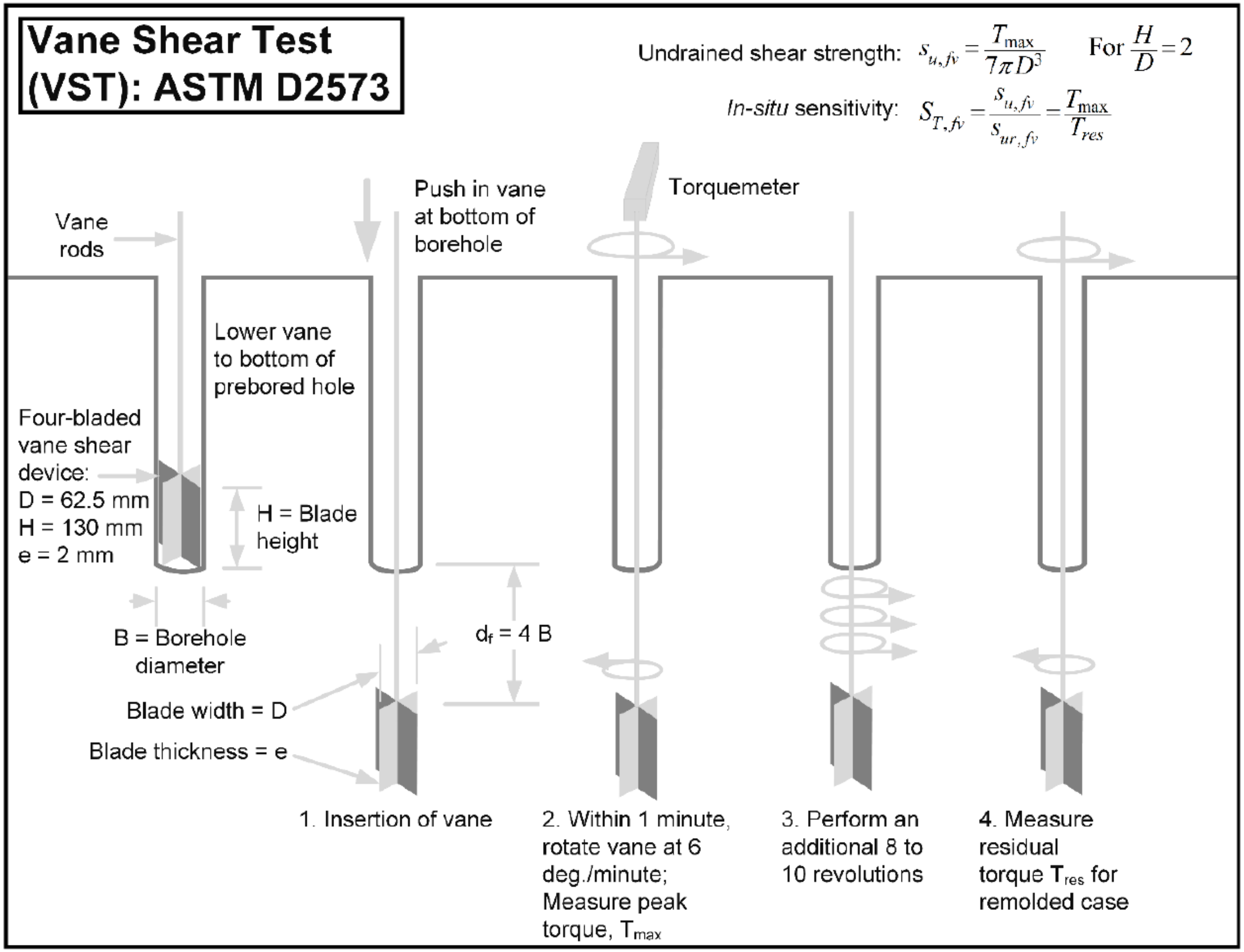
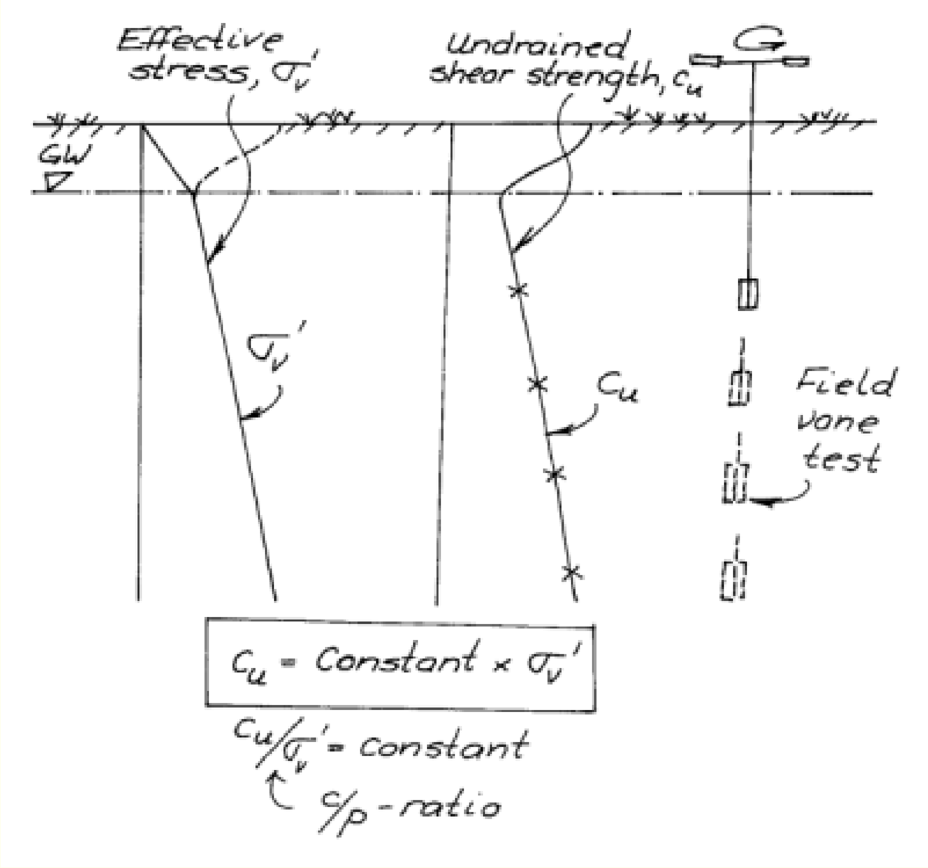


Figure 2-17 Schematic of Vane Shear Test (after Mayne 2012)

Results of Vane Shear Test

- As a general rule, undrained shear strength cu will increase with increasing effective stress.
- This is contrary to many methods in deep foundations, which assume that c_u is strictly a function of soil type and unaffected by effective stress
- We will return to this when we discuss driven piles and drilled shafts



Field Vane Shear Test Calculations

- Field Vane Shear calculation of undrained shear strength:

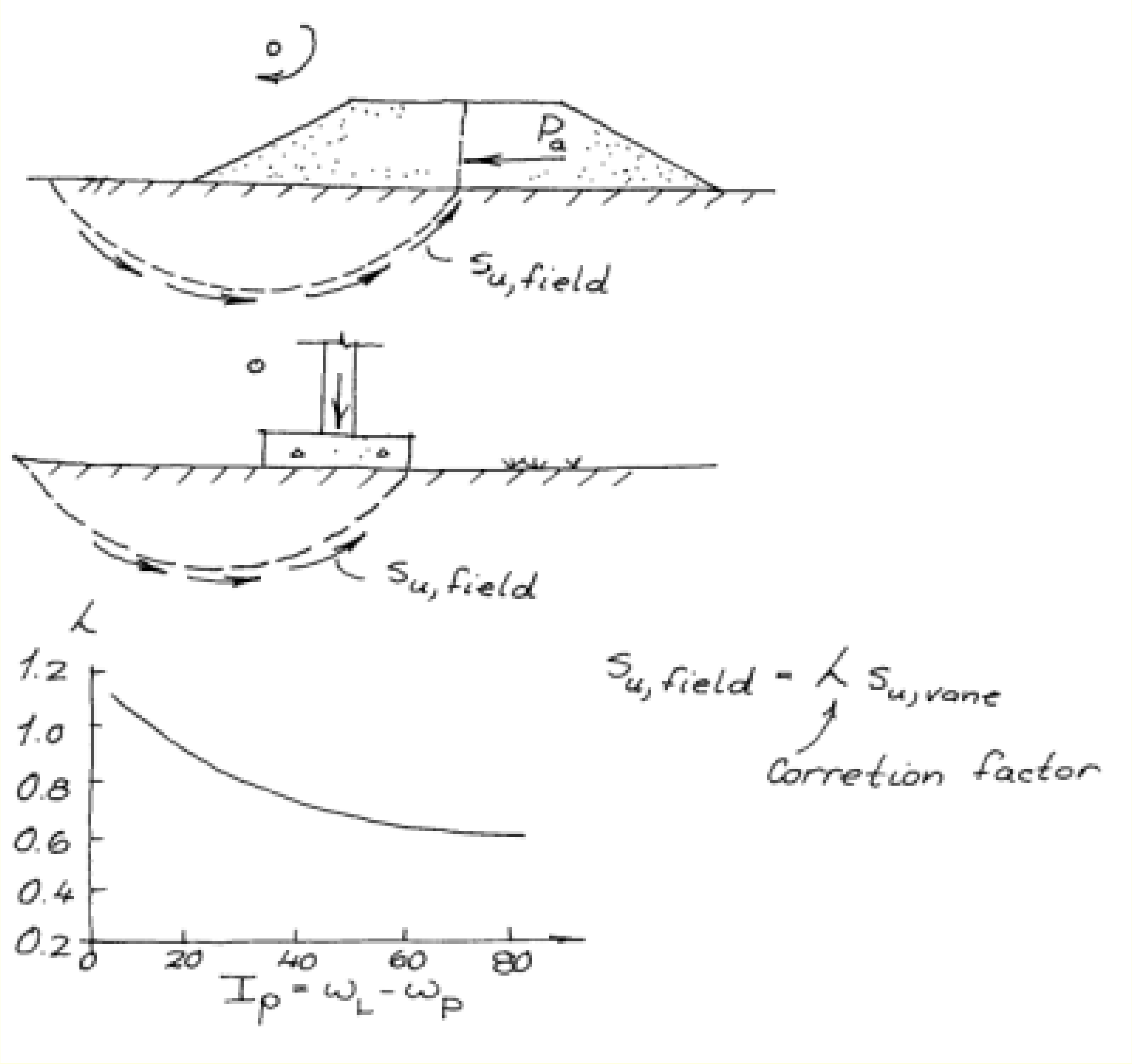
$$c_u = \frac{2T}{\pi D^2 \left(L + \frac{D}{3} \right)}$$

  - c_u = undrained shear strength
  - T = maximum torque applied
  - D = edge-to-edge diameter of vanes
  - L = length of vanes
- Assumes uniform shear strength distribution
- For most vane shear apparatuses, L/D = 2
- Corrections for vane shear test
  - Correction for plasticity index:

$$\mu = 1.7 - 0.54 \log(PI)$$

  - Should only be used for embankments
- Application of correction factors:
  - $c_{u\ corrected} = \mu\ c_{u\ uncorrected}$

Correction for Plasticity Index (Applied to Slopes and Bearing Capacity)



Sensitivity and Vane Shear

- Undrained and undisturbed strength determined by measuring maximum torque while rotating vanes at 0.1 deg./sec.
- Remoulded shear strength measured by rotating vane about ten (10) times, then recording a final torque value
- Sensitivity S_t = undisturbed shear strength/remoulded shear strength

Table 3-12

Classification of Sensitivity Values (Mitchell, 1976)

Classification	Sensitivity, S_t
Insensitive	~ 1.0
Slightly sensitive clays	1 – 2
Medium sensitive clays	2 – 4
Very sensitive clays	4 – 8
Slightly quick clays	8 – 16
Medium quick clays	16 – 32
Very quick clays	32 – 64
Extra quick clays	> 64

Questions?