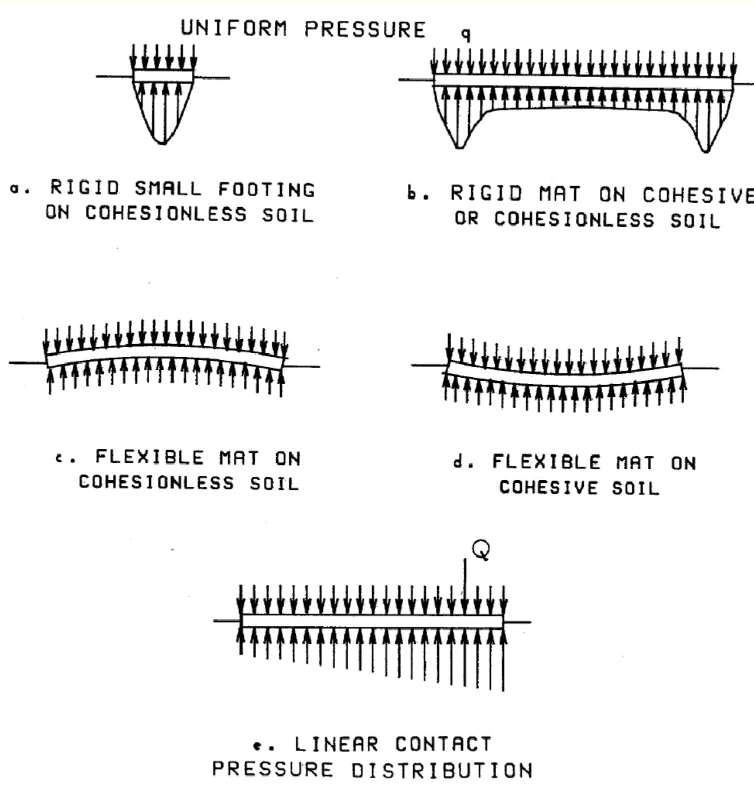


ENCE 3610
Soil Mechanics

Lecture 11

Settlement and Consolidation of Soils

Overview

- Soil is a nonhomogeneous porous material consisting of three phases
  - Solids
  - Fluid (normally water)
  - Air
- Soil deformation may occur by change in:
  - Stress
  - Water content
  - Soil mass
  - Temperature
- When soil deformation takes place below a foundation, it results in settlement
- Settlement of soils below foundations is the most common cause of foundation failure

Types of Settlement

- Stages of Settlement
  - Elastic Settlement S_e: settlement due to immediate elastic deformation before major particle rearrangement (discussed earlier)
  - Primary Consolidation Settlement S_c: settlement due to major particle rearrangement
  - Secondary Consolidation Settlement S_s: further particle rearrangement after primary settlement
- Total Settlement
  - $S_t = S_e + S_c + S_s$
- Other Types of Settlement
  - Dynamic forces
  - Expansive soil (ENCE 4610)
  - Collapsible soil (ENCE 4610)

Settlement and Consolidation

- Volume change in soils is not instantaneous: it takes place over time
- How long it takes depends upon the type of soil and its permeability
- Terms
  - Settlement refers to the distance the structure moves
  - Consolidation refers to the time it takes to move a certain distance
- For our purposes, we will assume all of this is uniaxial, i.e. one-dimensional
- Soil Type Considerations
  - Rate of consolidation depends upon permeability of the soil
  - Cohesive soils are slow, use classic consolidation theory
  - Cohesionless soils are more rapid
    - Hough's Method (ENCE 3610)
    - Schmertmann's Method (ENCE 4610, incorporates elasticity considerations)

Settlement, Starting With Elastic Theory

- We begin by assuming that the odeometer specimen/ consolidating soil is confined, either by the confining ring or the semi-infinite soil mass
- That being the case, the stress-strain relationship is given by

$$\epsilon_x = \frac{\sigma_x}{E}\beta = \frac{\sigma_x}{E}\left(1 - \frac{2\nu^2}{1-\nu}\right)$$

- where
  - ϵ_x = uniaxial strain
  - σ_x = uniaxial stress
  - E = modulus of elasticity
- Define the co[illegible] expansion:

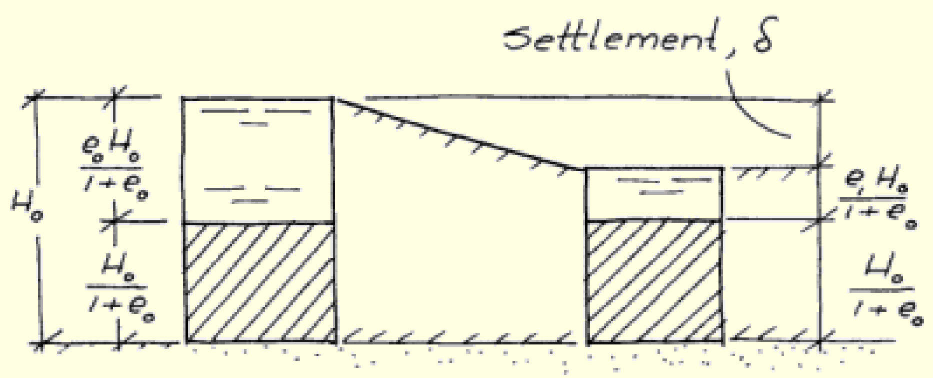
$$m_v = \frac{\epsilon_x}{\sigma_x} = \frac{\beta}{E}$$

$$\beta = 1 - \frac{2\nu^2}{1-\nu}$$

- In American (and other) practice, we normally do not use the strain, but the void ratio to determine deflection

$$\epsilon_x = \frac{\delta_p}{H_o} = \frac{e_o - e_1}{1+e_o}$$

$$\delta_p = m_v H_o \sigma_x$$



$$\delta_p = \frac{e_o H_o}{1+e_o} - \frac{e_1 H_o}{1+e_o} = \boxed{\frac{\Delta e H_o}{1+e_o}}$$

Primary settlement

Consolidation Test

- A test intended to replicate the process of primary (and secondary) settlement (distance) and time (consolidation) in the field
- Necessary to run with undisturbed specimens, as disturbance changes the soil skeleton structure and thus the results

Apparatus

- Consolidometer
  - The consolidometer has a rigid base, a consolidation ring, porous stones, a rigid loading plate, and a support for a dial indicator
  - Fixed or floating ring

Procedure

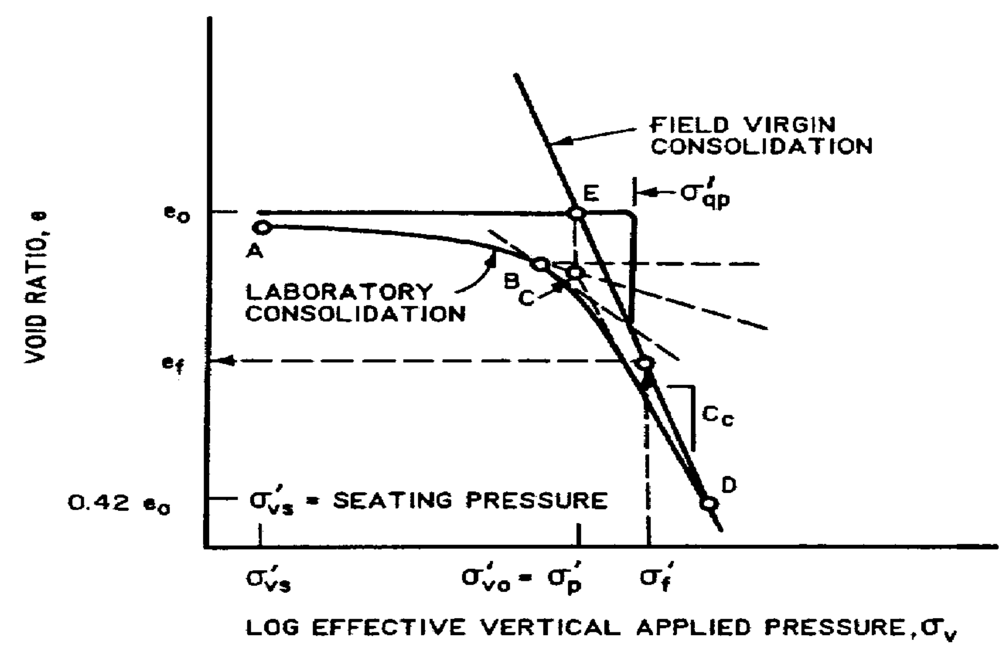
- Consecutive Loads on Specimen
  - .5 ksf
  - 1 ksf
  - 2 ksf
  - 4 ksf
  - 8 ksf
  - 16 ksf
  - 32 ksf

- Times to Record Data for Each Load
  - 0.1 min
  - 0.2 min
  - 0.5 min
  - 1.0 min
  - 2.0 min
  - 4.0 min
  - 8.0 min
  - 15 min
  - 30 min
  - 1 hr
  - 2 hr
  - 4 hr
  - 8 hr
  - 24 hr

- Record the dial reading for each load increment at completion of primary consolidation for each load (usually 24 hours)
- Remove load in decrements, recording dial indicator readings
- Conduct a water content test on specimen after unloading is complete

Actual Results of Consolidation Tests and Field Settlement

- Unfortunately, neither E nor m_v are constant as a specimen is compressed, because the material stiffens as the void ratio decreases
- The specimen/field soils tend to compressed according to the following relationship (formula for multilayer case; for single layer, n = 1):



$$S_c = \sum_{i}^{n} \frac{C_c}{1+e_o} H_o \log_{10}\left(\frac{p_f}{p_o}\right) \qquad 7\text{-}2$$

where:	C_c	=	compression index
	e_o	=	initial void ratio
	H_o	=	layer thickness
	p_o	=	initial effective vertical stress at the center of layer n
	p_f	=	$p_o + \Delta p$ = final effective vertical stress at the center of layer n.

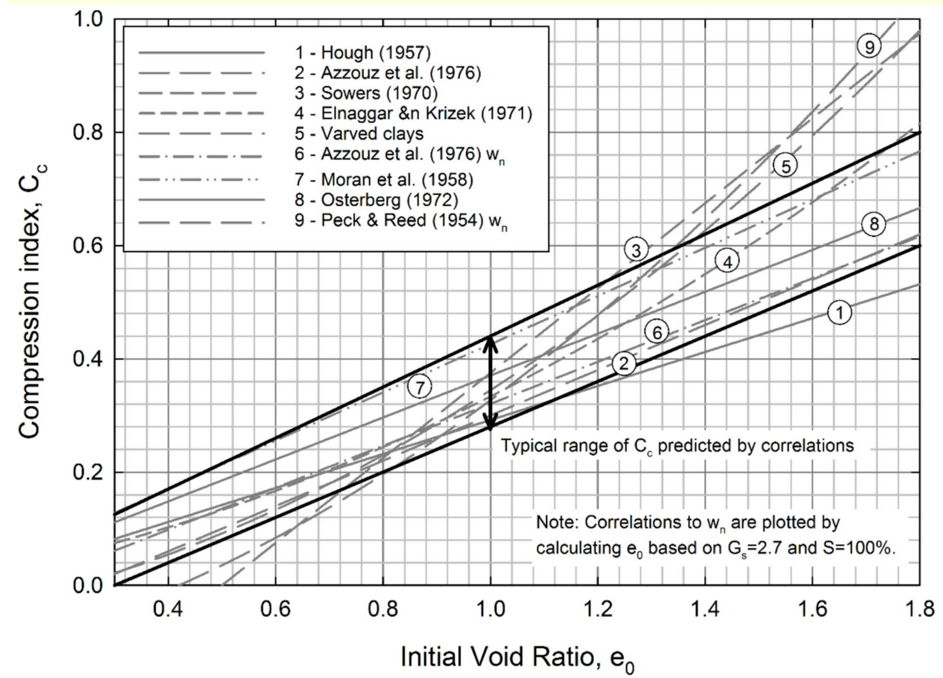
Note that "virgin" consolidation does not "kick in" until after an effective stress point is passed. The soil has been compressed to some degree by that effective stress before we add additional stress

Non-Laboratory Estimates of Compression Index

Table 5-5 Correlations for Compression Indices

Basis	Correlation	Applicable soil type	Source
LL	$C_c = 0.009(LL-10)$	Inorganic soils with sensitivity less than 4	Terzaghi and Peck (1967)
	$C_c = 0.007(LL-7)$	Remolded clays	Skempton (1944)
	$C_c = 0.006(LL-9)$	Predominantly lean to fat clay	Azzouz et al. (1976)
e_0	$C_c = 1.15(e_0-0.35)$	All clays	Nishida (1956)
	$C_c = 0.3(e_0-0.27)$	Inorganic soil; silt; silty clay; some clay	Hough (1957)
	$C_c = 0.75(e_0-0.5)$	Very low plasticity soils	Sowers (1970)
	$C_{\varepsilon c} = 0.156e_0 + 0.017$	All clays	Elnaggar and Krizek (1971)
	$C_c = 0.4(e_0-0.25)$	Predominantly lean to fat clay	Azzouz et al. (1976)
w_n	$C_c = 0.01w_n$	Chicago clays	Osterberg (1972) (in Azzouz et al. 1976)
	$C_c = 0.0115w_n$	Organic soils, peat	Moran et al. (1958)
	$C_{\varepsilon c} = \left(0.1+0.006(w_n-25)\right)$	Varved clays	Prior NAVFAC DM 7.1
	$C_c = 17.66\times10^{-5}w_n^2 + 5.93\times10^{-3}w_n - 0.135$	Chicago clays	Peck and Reed (1954)
	$C_c = 0.01w_n - 0.05$	Predominantly lean to fat clay	Azzouz et al. (1976)

Note: Liquid limit (LL) and natural water content (w_n) are in percent.

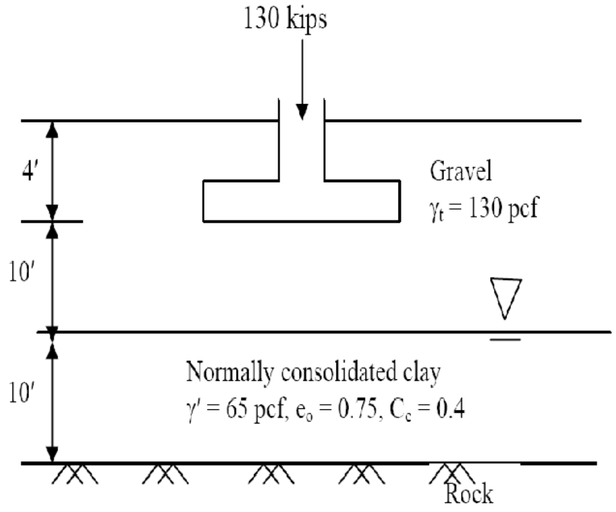


These are used if full consolidation tests are either not possible, or for preliminary analysis

Consolidation Problem (Normally Consolidated Soil)

The procedures to compute consolidation settlements discussed in Chapter 7 can be applied to spread footings also. The following example illustrates the method for determining consolidation settlement due to a load applied to a spread footing.

Example 8-3: Determine the settlement of the 10 ft × 10 ft square footing due to a 130 kip axial load. Assume the gravel layer is incompressible.



Solution:

Find overburden pressure, p_o, at center of clay layer

p_o = (14 ft × 130 pcf) + (5 ft × 65 pcf) = 2,145 psf

Find change in pressure (Δp) at center of clay layer due to applied load. Use the approximate 2:1 stress distribution method discussed in Section 2.5 of Chapter 2.

$$\Delta p = \frac{130\,\text{kips}}{(10\,\text{ft} + 15\,\text{ft})^2} = \frac{130\,\text{kips}}{625\,\text{ft}} = 0.208\,\text{ksf} = 208\,\text{psf}$$

Use Equation 7-2 to calculate the magnitude of consolidation settlement.

$$\Delta H = H \frac{C_c}{1 + e_o} \log_{10}\left(\frac{p_o + \Delta p}{p_o}\right)$$

$$\Delta H = 10\,\text{ft}\left(\frac{0.4}{1 + 0.75}\right)\log_{10}\left(\frac{2{,}145\,\text{psf} + 208\,\text{psf}}{2{,}145\,\text{psf}}\right) = 0.09\,\text{ft} = 1.1\,\text{in}$$

In reality, the magnitude of the total settlement of the foundation would be the sum of the consolidation settlement of the clay and the immediate settlement of the gravel. The gravel was assumed to be incompressible in this example. However, in practice, the component of the total settlement due to the immediate settlement of the gravel would be determined by using Schmertmann's method with only that portion of the strain influence diagram in the gravel being considered.

Overconsolidation or Preconsolidation

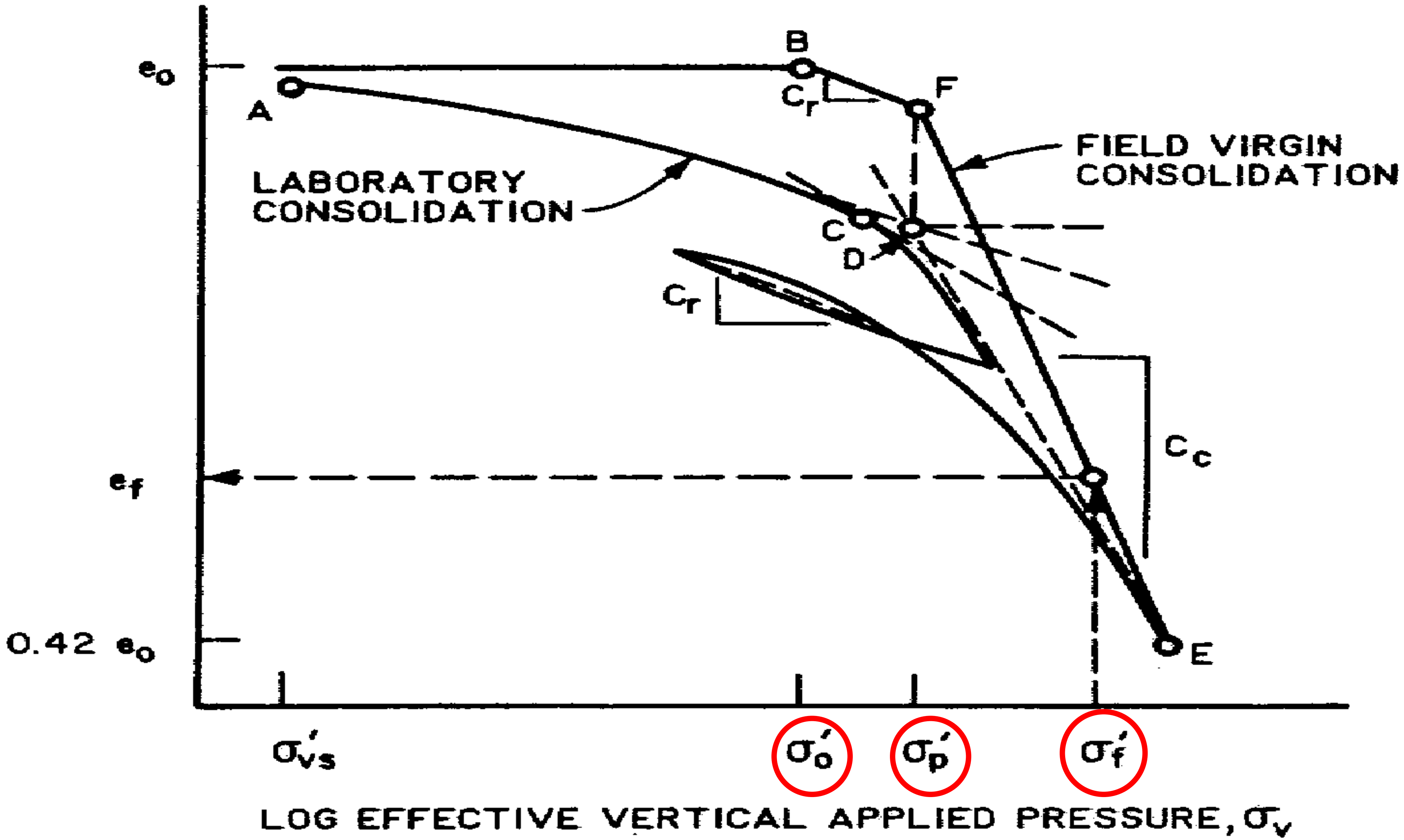
- Overconsolidation complicates the analysis of primary settlement because the ground being compressed acts as if the previous overburden pressure is still being applied, irrespective of current overburden conditions
- Requires dividing the settlement analysis into two parts
- Sometimes referred to as preconsolidation

- The ratio of the maximum overburden stress a soil has experienced to the present overburden stress

$$OCR = \frac{\sigma'_p}{\sigma'_o}$$

- If OCR >1, the soil is overconsolidated
- If OCR ~1, the soil is normally consolidated
- If OCR <1, the soil is underconsolidated

Effect on Primary Settlement



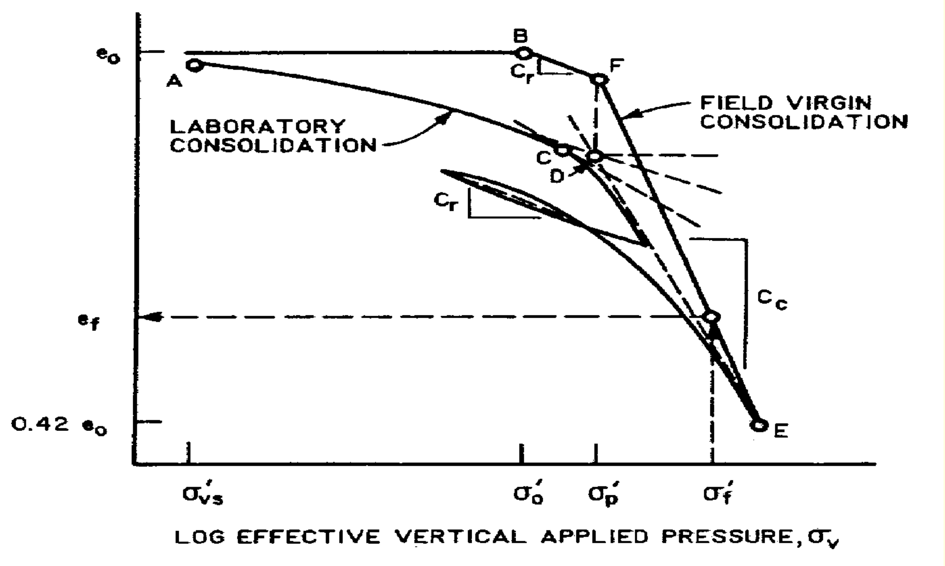
Settlement in Overconsolidated Soils (see SFH 7.5.2.2)

- Formula, $\sigma'_o + \Delta\sigma' > \sigma'_c$ (to the right of "F")

$$S = \frac{H_o}{1+e_o}\left(C_r \log_{10}(OCR) + C_c \log_{10}\left(\frac{\sigma'_f}{\sigma'_p}\right)\right)$$

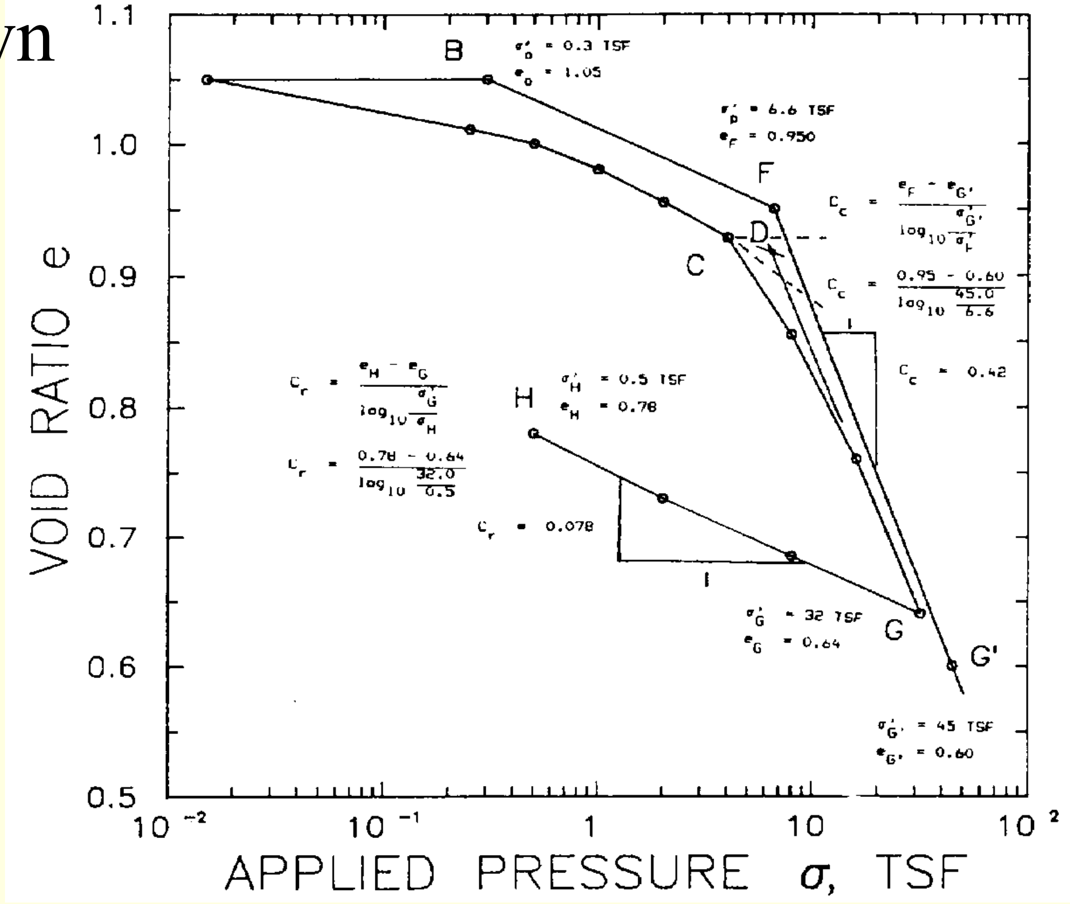
- Formula, $\sigma'_o + \Delta\sigma' < \sigma'_c$ (to the left of "F")

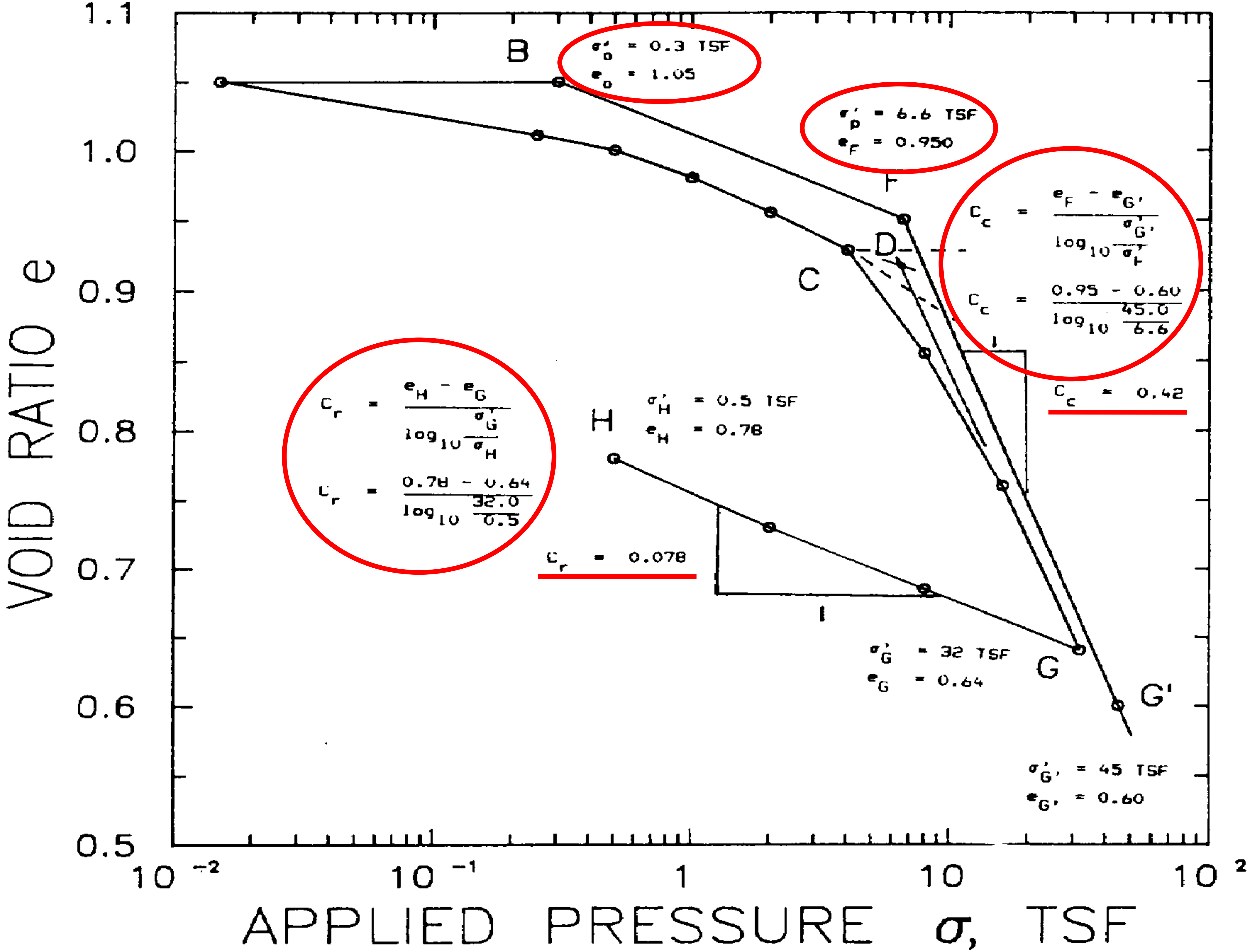
$$S = \frac{H_o}{1+e_o}\left(C_r \log_{10}\left(\frac{\sigma'_f}{\sigma'_o}\right)\right)$$



Overconsolidated Example

- Given
  - Soil with void ratio-log pressure as shown
  - 6' thick clay layer under sand layer
  - Two possible additional loadings:
    - 5 tsf
    - 7.5 tsf
- Find
  - OCR
  - Settlement for each case





VOID RATIO e
APPLIED PRESSURE σ, TSF
1.1
1.0
0.9
0.8
0.7
0.6
0.5
10 −2
10 −1
1
10
10 2
B
C
D
F
G
G'
H
σ'o = 0.3 TSF
eo = 1.05
σ'p = 6.6 TSF
eF = 0.950
Cc = (eF − eG') / log10 (σ'G' / σ'F)
Cc = (0.95 − 0.60) / log10 (45.0 / 6.6)
Cc = 0.42
Cr = (eH − eG) / log10 (σ'G / σH)
Cr = (0.78 − 0.64) / log10 (32.0 / 0.5)
Cr = 0.078
σ'H = 0.5 TSF
eH = 0.78
σ'G = 32 TSF
eG = 0.64
σ'G' = 45 TSF
eG' = 0.60
1
1

Overconsolidated Example

- OCR = 6.6/0.3 = 22
- 5 tsf additional load
  - With overburden: 5 + 0.3 = 5.3 tsf
  - 0.3 < 5.3 < 6.6, thus use

$$S = \frac{H_o}{1+e_o}\left(C_r \log_{10}\left(\frac{\sigma'_f}{\sigma'_o}\right)\right)$$

S_{pc} = (6)(12)/(1+1.05) (0.078)$\log_{10}$(5.3/0.3) =3.42”

- 7.5 tsf additional load
  - With overburden: 7.5 + 0.3 = 7.8 tsf
  - 6.6 < 7.8, thus use

$$S = \frac{H_o}{1+e_o}\left(C_r \log_{10}(OCR) + C_c \log_{10}\left(\frac{\sigma'_f}{\sigma'_p}\right)\right)$$

S_c = (6)(12)/(1+1.05) ((0.078)$\log_{10}$(22) +(0.42)$\log_{10}$(7.8/6.6)) = 4.75”

Multiple Layers

- Multiple layers can be handled by computing the single layer consolidation for each and summing the displacements
- Take care to properly compute or assign the additional vertical stresses generated by the surface load for each layer (not all of the stresses will be the same)
- Take care when dealing with overconsolidated soils to properly understand the consolidation region for each layer

Secondary Compression of Cohesive Soils

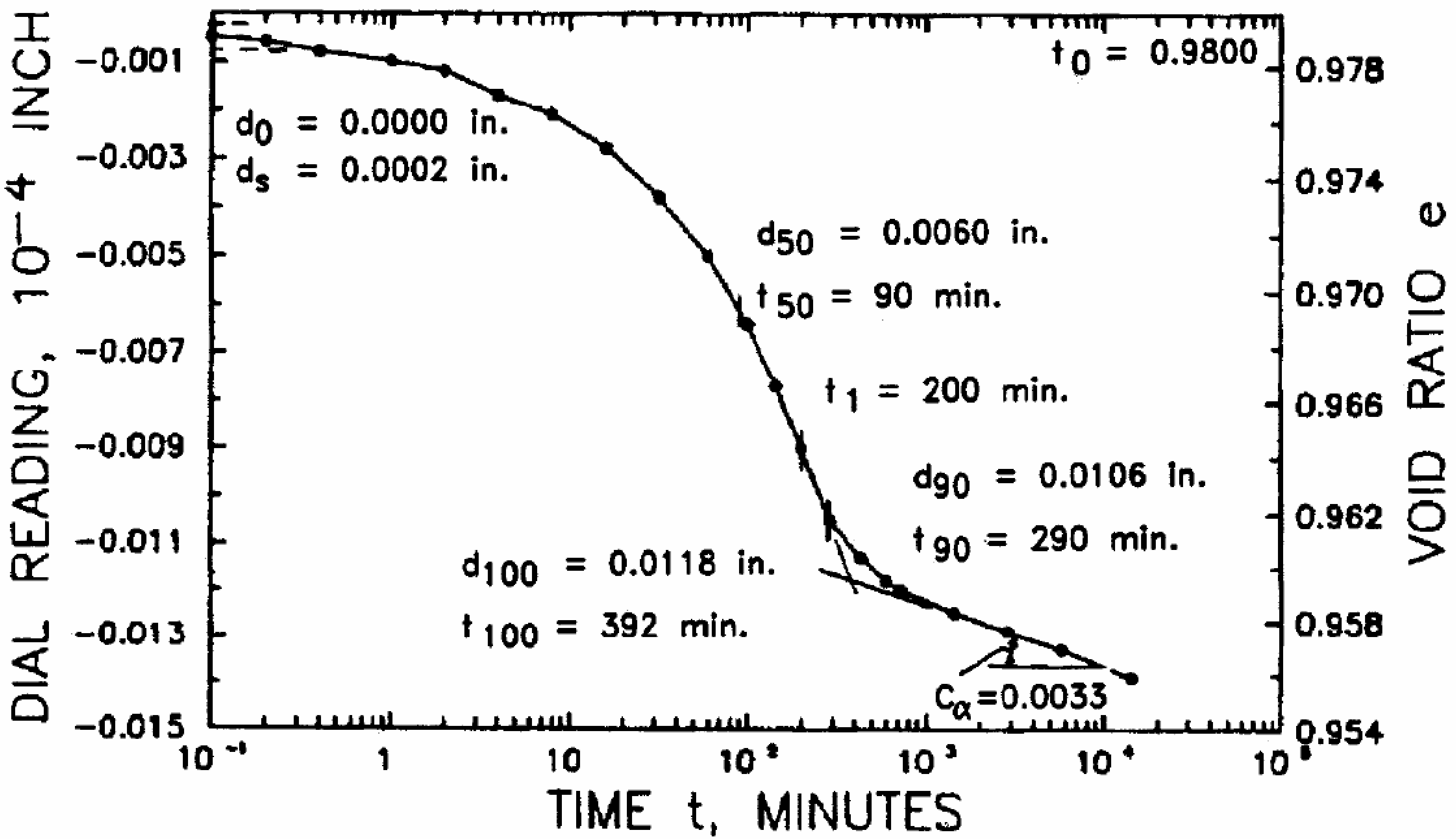


Figure 7-17: Example time plot from one-dimensional consolidometer test for determination of secondary compression (USACE, 1994). (1 in = 25.4 mm)

Computation of Secondary Compression

The settlement due to secondary compression (S_s) is then determined from Equation 7-10.

$$S_s = \frac{C_\alpha}{1+e_o} H_c \log_{10}\left(\frac{t_2}{t_1}\right) \qquad 7\text{-}10$$

where: t_1 = time when approximately 90 percent of primary compression has occurred for the actual clay layer being considered as determined from Equation 7-8.

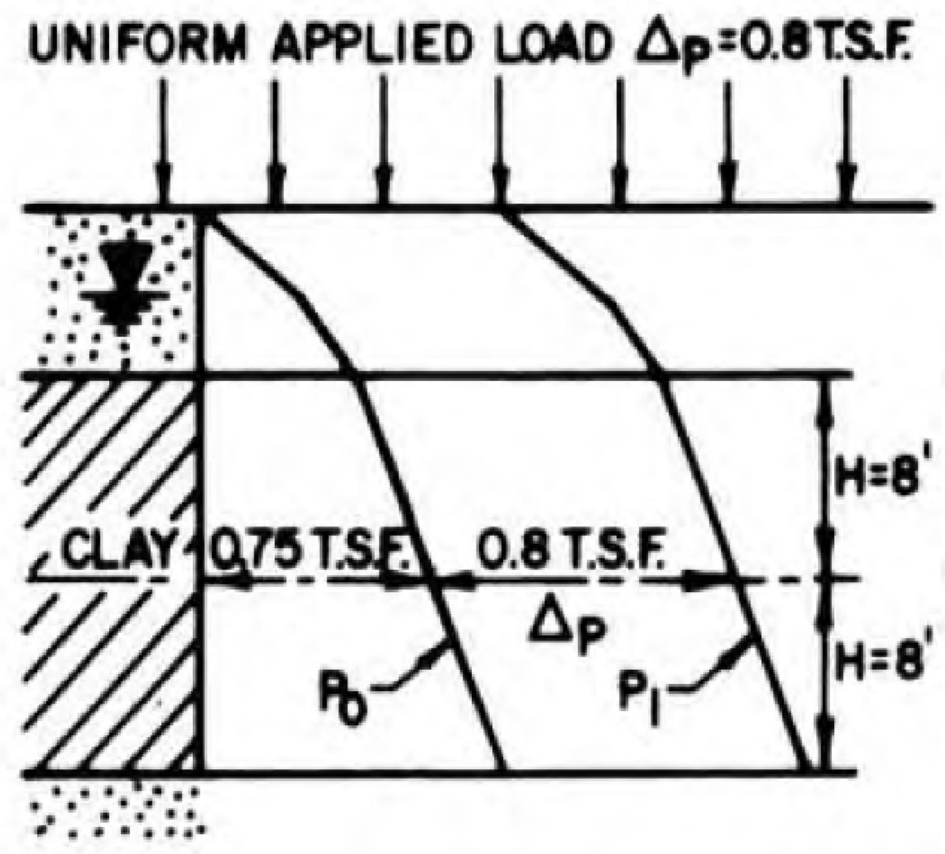
t_2 = the service life of the structure or any other time of interest.

The values of C_α can be determined from the dial reading vs. log time plots associated with the one-dimensional consolidation test as shown in Figure 7-17. Typical ranges of the ratio of C_α/C_c presented in Section 5.4.6.4 of Chapter 5 can be used to check laboratory test results.

Example of Secondary Compression

- Given
  - As shown
  - Primary Settlement Data
    - Expected time for primary consolidation ~ 13 years
    - $e_o = 1$ (at beginning of primary consolidation)
    - Normally consolidated clay, $C_c = 0.21$
  - Desired life of structure = 110 years
  - $C_\alpha = 0.02$

- Find
  - Secondary Consolidation Settlement



Secondary Compression Example

- Primary Compression Calculation

$$S_c = \frac{C_c H}{1+e_o}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) = \frac{0.21\times 16\times 12}{1+1}\log\left(\frac{.75+.8}{.75}\right) = 6.35''$$

- Compute void ratio e_p at end of primary consolidation

$$e_p = e_o - \Delta e = e_o - C_c \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) = 1 - 0.21\times\log\left(\frac{.75+.8}{.75}\right) = 0.93$$

- Secondary Compression Calculation

$$S_s = C_\alpha \frac{H}{1+e_p}\log\left(\frac{t_f}{t_p}\right) = 0.02\,\frac{16\times 12}{1+0.93}\log\left(\frac{110}{13}\right) = 1.84''$$

Total Compression = 6.35” + 1.84” = 8.19”

Hough’s Method for Settlement of Shallow Foundations in Cohesionless Soils

- Recommended by FHWA (and thus AASHTO, state DOT’s etc.) because studies show that it is consistent and conservative
- Described in Soils and Foundations Reference Manual, pp. 7-15 to 7-19
- Requires a “layer by layer” analysis and thus should be done with a spreadsheet

Hough's Method (from SFH 7.4.1)

1. Subdivide subsurface soil profile into approximately 3-m (10-ft) layers based on stratigraphy to a depth of about three times the footing width.
   - Make sure layers are approximately homogeneous with relation to SPT blow count, unit weight and type of soil.
   - Divide layers at phreatic surface (water table)
2. Correct SPT blowcounts to an $(N_1)_{60}$ value, using methods previously discussed.
3. Determine bearing capacity index (C′) using corrected SPT blowcounts, N′, determined in Step 2.
4. Calculate the effective vertical stress, σ'_o, at the midpoint of each layer and the average bearing capacity index for that layer.
5. Calculate the increase in stress at the midpoint of each layer, $\Delta\sigma$, using 2:1 method (for this course.)
6. Calculate the settlement in each layer, H_o, under the applied load using the following formula:

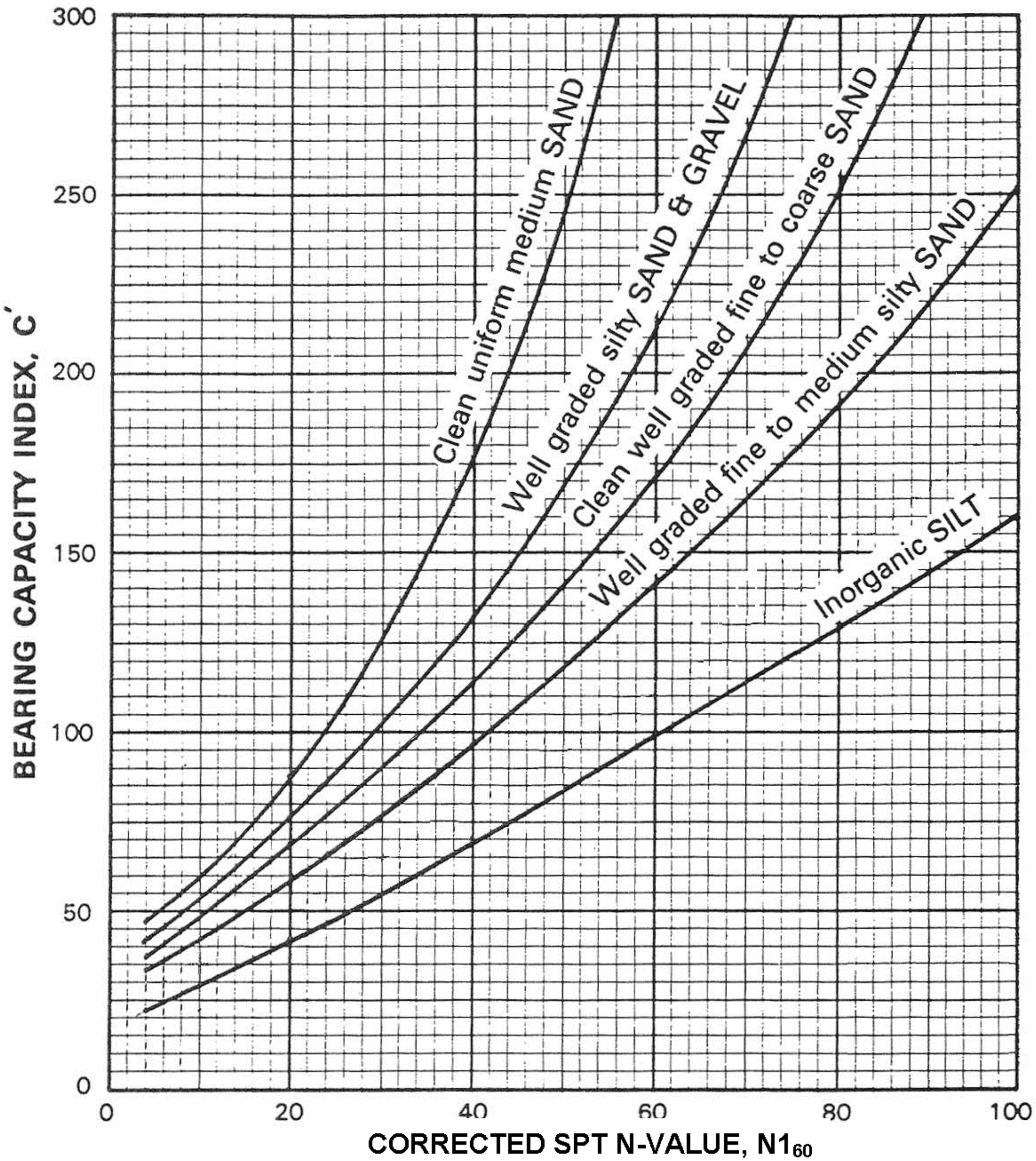
$$S_1 = \frac{H_o}{C'} \log_{10}\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

In engineering practice the formula is sometimes slightly modified by using the common logarithm (of base 10), rather than the natural logarithm (of base e), perhaps because of the easy availability of semi-logarithmic paper on the basis of the common logarithm. The formula then is

$$\varepsilon = -\frac{1}{C_{10}} \log\left(\frac{\sigma}{\sigma_0}\right). \tag{14.5}$$

Bearing Capacity Index Factors for Hough Method

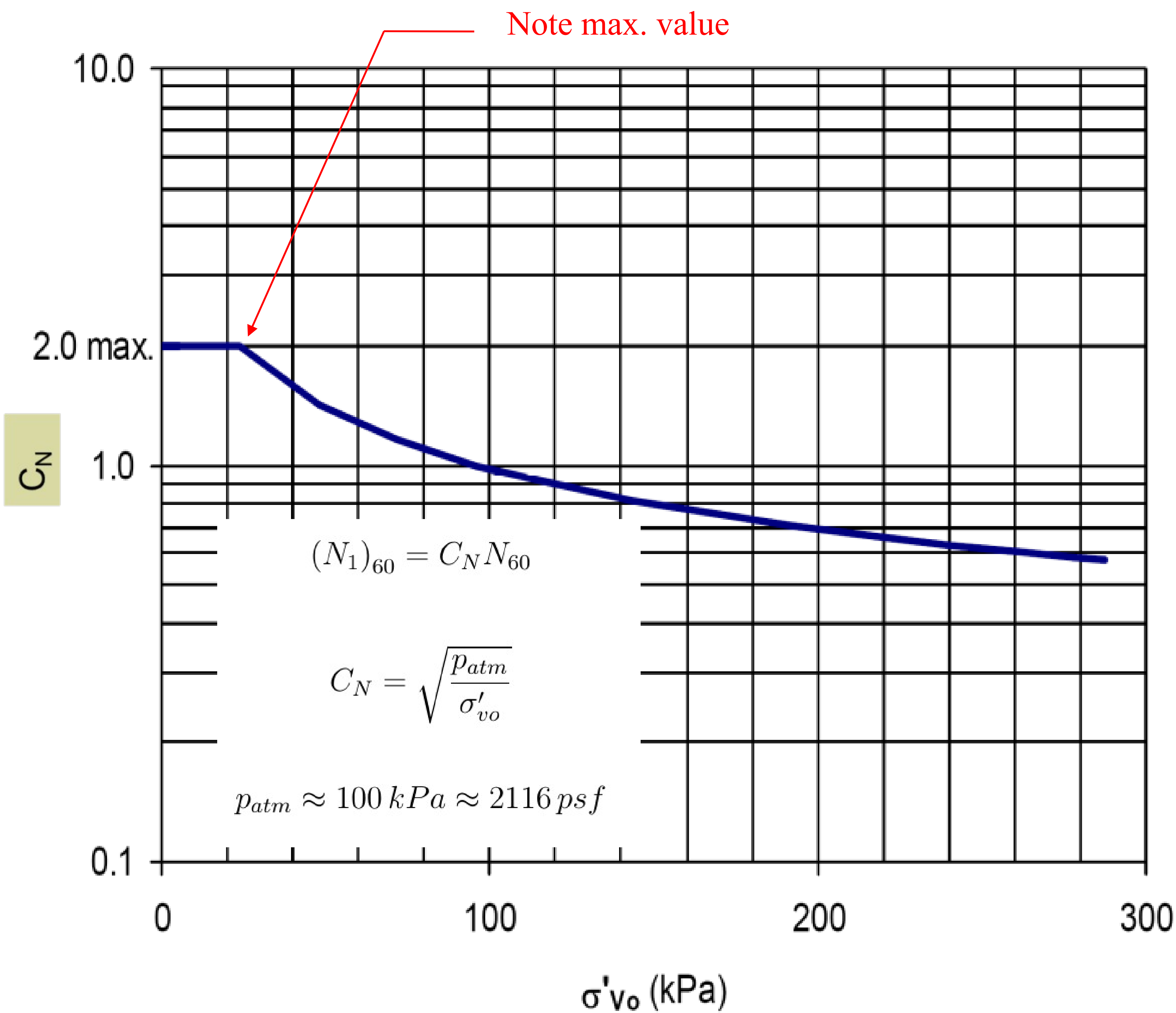
Type of soil	C	C_{10}
sand	50-500	20-200
silt	25-125	10-50
clay	10-100	4-40
peat	2-25	1-10



(Note: The "Inorganic SILT" curve should generally not be applied to soils that exhibit plasticity because N-values in such soils are unreliable)

Figure 7-7. Bearing capacity index (C') values used in Modified Hough method for computing immediate settlements of embankments (AASHTO, 2004 with 2006 Interims; modified after Hough, 1959).

Correction Factor for SPT (N) Blow Counts



$$(N_1)_{60} = C_N N_{60}$$

$$C_N = \sqrt{\frac{p_{atm}}{\sigma'_{vo}}}$$

$$p_{atm} \approx 100\,kPa \approx 2116\,psf$$

Sample Settlement Problem (Cohesionless Soils, Hough's Method)

- Given
  - Clean well-graded fine to coarse sand layer above clay layer and at surface
  - $N_{60} = 20$
  - 35' thick
  - Unit weight of soil above phreatic surface = 110 pcf
  - Submerged weight of soil below water table= 65 pcf
  - Depth of water table = 15'
- Given
  - Building placed on surface
  - Square Foundation, 30' x 30'
  - Total load on foundation = 9000 kips
- Find
  - Average settlement of building due to settlement of sand layer

Sample Settlement Problem (Cohesionless Soils, Hough's Method)

- Step 1: Compute Effective Stress Profile and Foundation Stress Profile for Bearing Strata and Foundation
  - Effective stresses using conventional methods
  - Stresses from foundation loads use 2:1 Method, thus for Layer 1, $\Delta p = (9000)(1000)/((30+7.5)(30+7.5)) = 6400$ psf

Depth at Top of Layer, ft	Layer Thickness, ft.	Depth at Bottom of Layer, ft	Depth at Center of Layer, ft	Unit Weight of Soil, pcf	Effective Stress at Top of Layer, psf	Effective Stress at Bottom of Layer, psf	Average Effective Stress in Layer, psf	Load from Foundation in the Central Axis, psf
0	15	15	7.5	110.0	0.0	1650.0	825.0	6400.0
15	20	35	25	65.0	1650.0	2950.0	2300.0	2975.2
35	25	60	47.5	52.6	2950.0	4264.8	3607.4	1498.4

Sample Settlement Problem (Cohesionless Soils, Hough's Method)

- Step 2: Correct SPT Values for Overburden Pressure
  - Layer 1:

$$\left(N_1\right)_{60} = C_N N_{60}$$

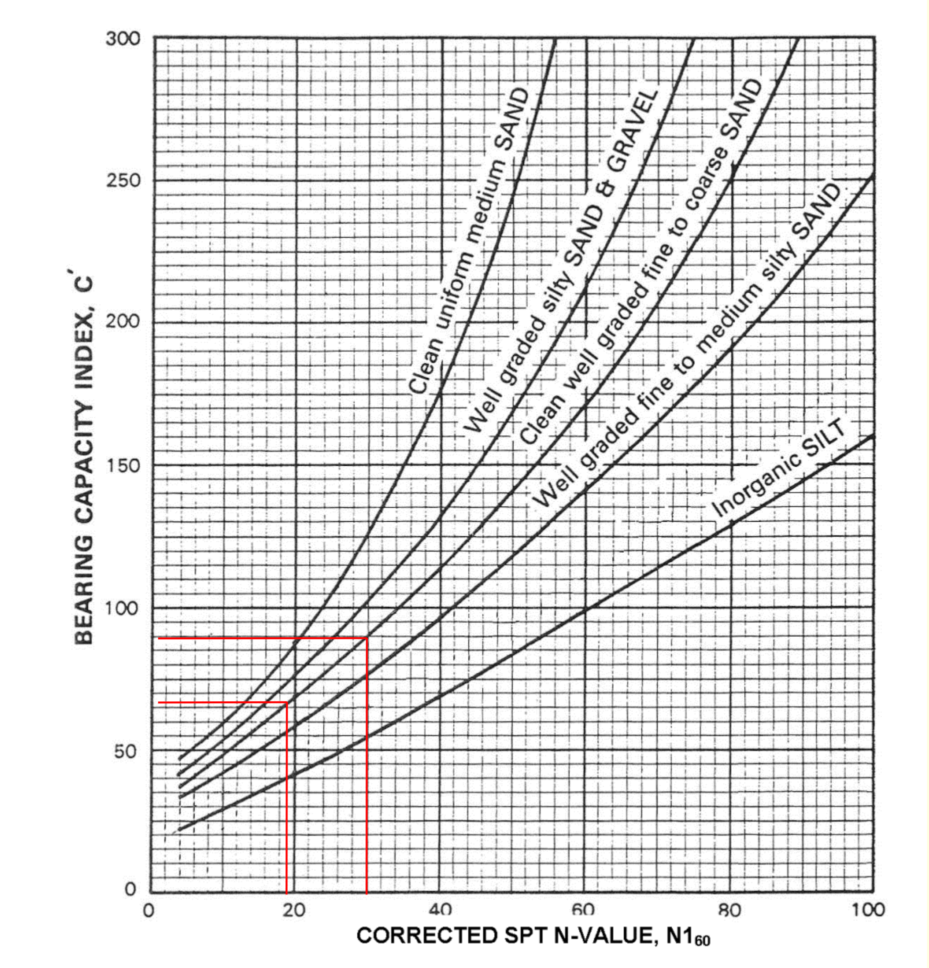
$$C_N = \sqrt{\frac{2}{\sigma'_{vo}}} \le 2 \text{ (U.S. Units, ksf)}$$

$$C_N = \sqrt{\frac{2}{0.825}} = 1.56 \le 2 \text{ (U.S. Units, ksf)}$$

$(N_1)_{60} = (1.6)(20) = 31$

  - Layer 2: $C_N = 0.9$, $(N_1)_{60} = (0.9)(20) = 19$

- Step 3: Determine Values for C'



Sample Settlement Problem (Cohesionless Soils, Hough's Method)

- Step 4: Compute Settlements of Sand Layers
  - Layer 1:

$$S_1 = \frac{H_o}{C'} \log_{10}\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'}\right) = 12 \times \frac{15'}{90} \log_{10}\left(\frac{825 + 6400}{825}\right) = 1.88''$$

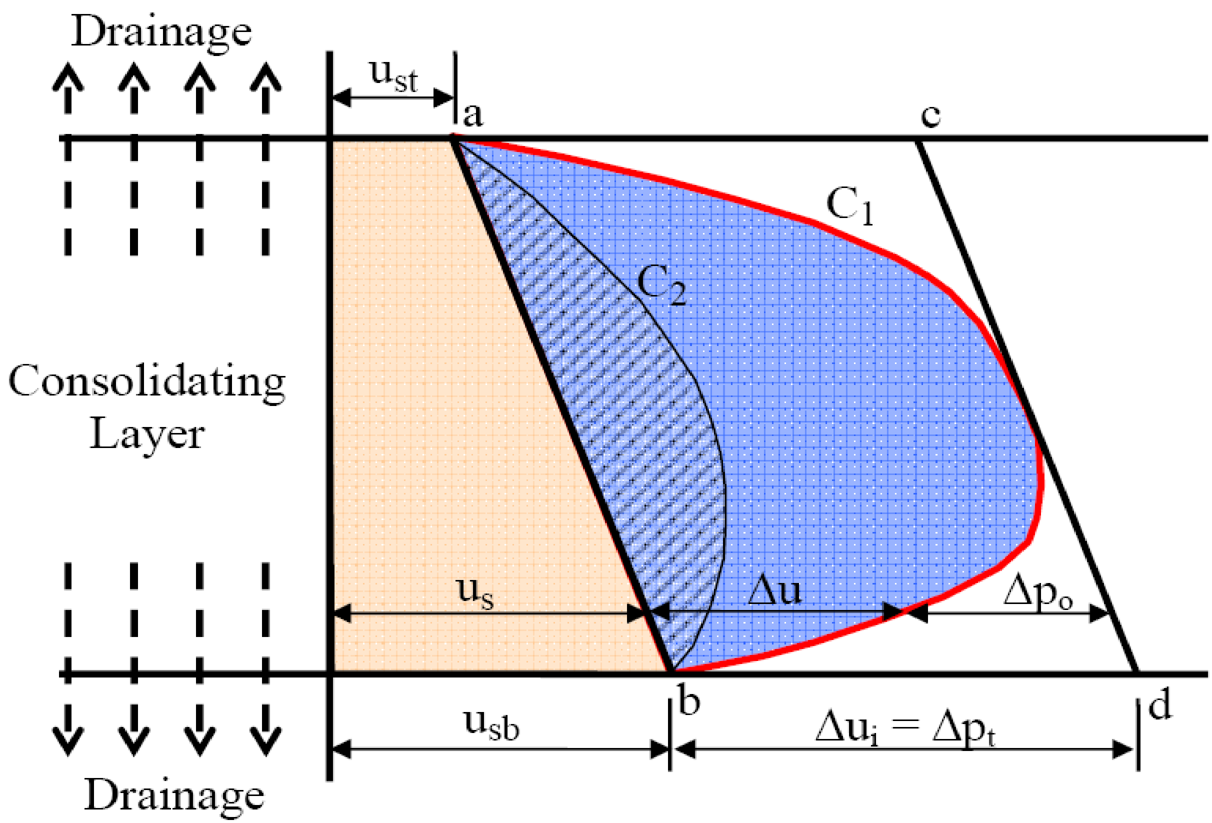
  - Layer 2:

$$S_2 = \frac{H_o}{C'} \log_{10}\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'}\right) = 12 \times \frac{20'}{65} \log_{10}\left(\frac{2300 + 2975.2}{2300}\right) = 1.33''$$

  - Total Settlement in Sand = 1.88" + 1.33" = 3.21"

[Spreadsheet Solution]

Time Rate of Consolidation



Definitions:
- u_{st} = hydrostatic pore water pressure at top of layer
- u_{sb} = hydrostatic pore water pressure at bottom of layer
- u_s = hydrostatic pore water pressure at any depth
- Δu_i = initial excess pore water pressure
- Δu = excess pore water pressure at any depth after time t
- u_t = $u_s + \Delta u$ = total pore water pressure at any depth after time t

Overview of Time Rate Computations

- Consolidation is a process that takes place over time and is the result of two processes:
  - Dissipation of excess pore water pressures created by surcharge or new foundation loads at the surface. This takes place when the excess water is drained from the soil
  - Rearrangement of the soil particles ("soil skeleton") to reflect new effective stress conditions created by foundation or surcharge

* But see Verruijt Ch. 15 online/Ch. 16 print

- Assumptions
  - Clay-water system is homogeneous
  - Saturation is complete*
  - Water is incompressible*
  - Soil particles are incompressible
  - Flow is in one direction only (in the direction of compression)
  - Darcy's Law is valid
  - Uniform Surcharge

Consolidation Model

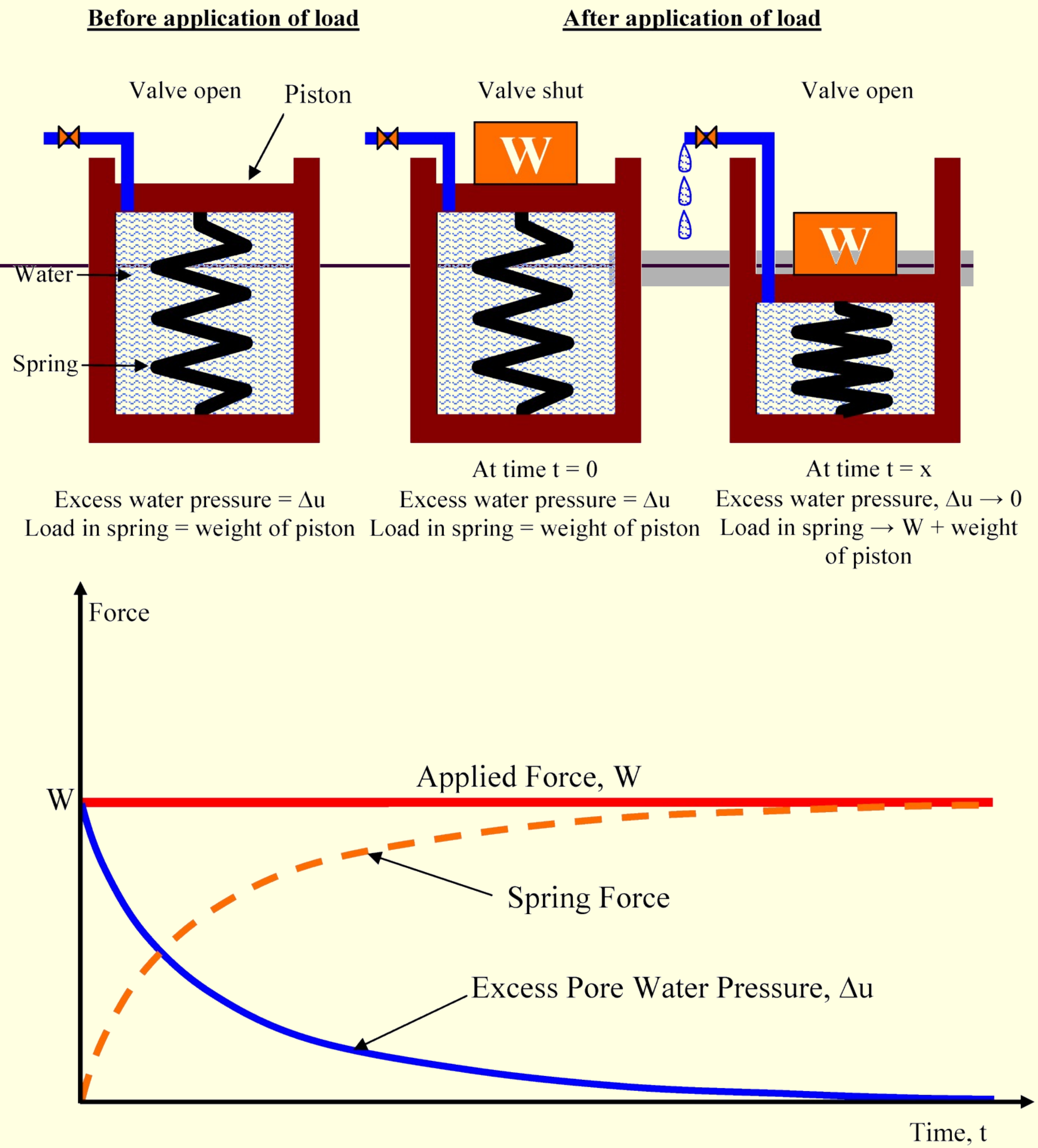


Figure 2-13. Spring-piston analogy for the consolidation process in fine-grained soils.

Deriving the Equations of Consolidation

- Basic Assumptions:
  - The relationship between the change in porosity and the time rate of flow from an element of soil:

$$\frac{\partial}{\partial x} q(x,t) = -\frac{\partial}{\partial t} n(x,t)$$

  - The practical equivalence (for small changes in strain) between changes in strain and changes in velocity:

$$\Delta n = \Delta \epsilon$$

- From this and the theory of elasticity applied to consolidation:

$$\Delta n = m_v \Delta \sigma_x$$

- Combining everything,

$$\frac{\partial}{\partial x} q(x,t) = -m_v \frac{\partial}{\partial t} \sigma_x(x,t)$$

Consolidation: Including the Effects of Permeability

- Darcy's Law:

$$q(x,t) = -k\frac{\partial}{\partial x}H(x,t)$$

- From fluid mechanics,

$$H(x,t) = \frac{u(x,t)}{\gamma_w}$$

- Defining the coefficient of consolidation,

$$c_v = \frac{k}{m_v\gamma_w}$$

- Combining everything,

$$c_v\frac{\partial^2}{\partial x^2}u(x,t) = \frac{\partial}{\partial t}\sigma_x(x,t)$$

- Notes
  - The variable c_v can be considered a constant because both k and m_v vary together with void ratio/porosity
  - The dependent variable is different from one side of the equation to another
  - This can be addressed by assuming the load initially is on the pore water and transfers itself to the soil skeleton as the water is squeezed out

Consolidation: Differential Equation, Initial and Boundary Conditions

- Basic equation (similar to heat equation):

$$c_v \frac{\partial^2}{\partial x^2} u(x,t) = \frac{\partial}{\partial t} u(x,t)$$

- Initial conditions: uniform distribution of surface pressure p throughout the thickness of the layer
- To simplify the boundary conditions, the problem at the right is assumed mirrored at the bottom layer so that the pore pressure at boundaries x = 0 and x = 2h are both zero (Dirichelet boundary conditions)
- This means that the value h is the distance of maximum drainage not the thickness of the layer

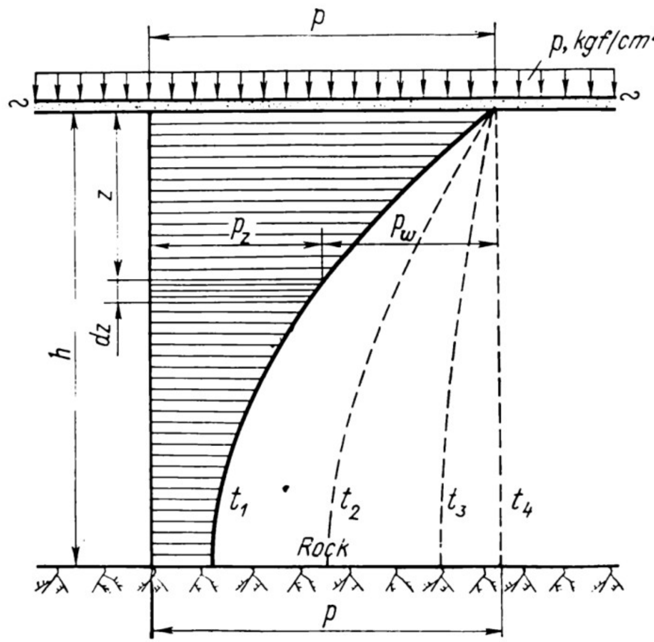


Fig. 93. Distribution of pressures in soil skeleton (p_z) and porous water (p_w) in a layer of saturated soil with a continuous load for a definite time interval

Consolidation: Solution of Equation

- Solution of pore water pressure:

$$u(x,t) = \sum_{n=1}^{\infty} 2\,p\,(1-\cos(n\pi))\sin(1/2\,\frac{n\pi\,x}{h})e^{-1/4\,\frac{cv\,n^2\pi^2 t}{h^2}}n^{-1}\pi^{-1}$$

$$u(x,t) = \frac{4p}{\pi}\left(\sin(1/2\,\frac{\pi\,x}{h})e^{-1/4\,\frac{cv\,\pi^2 t}{h^2}} + 1/3\,\sin(3/2\,\frac{\pi\,x}{h})e^{-9/4\,\frac{cv\,\pi^2 t}{h^2}} + 1/5\,\sin(5/2\,\frac{\pi\,x}{h})e^{-\frac{25}{4}\,\frac{cv\,\pi^2 t}{h^2}}\cdots\right)$$

- Invoke Effective Stress equation:

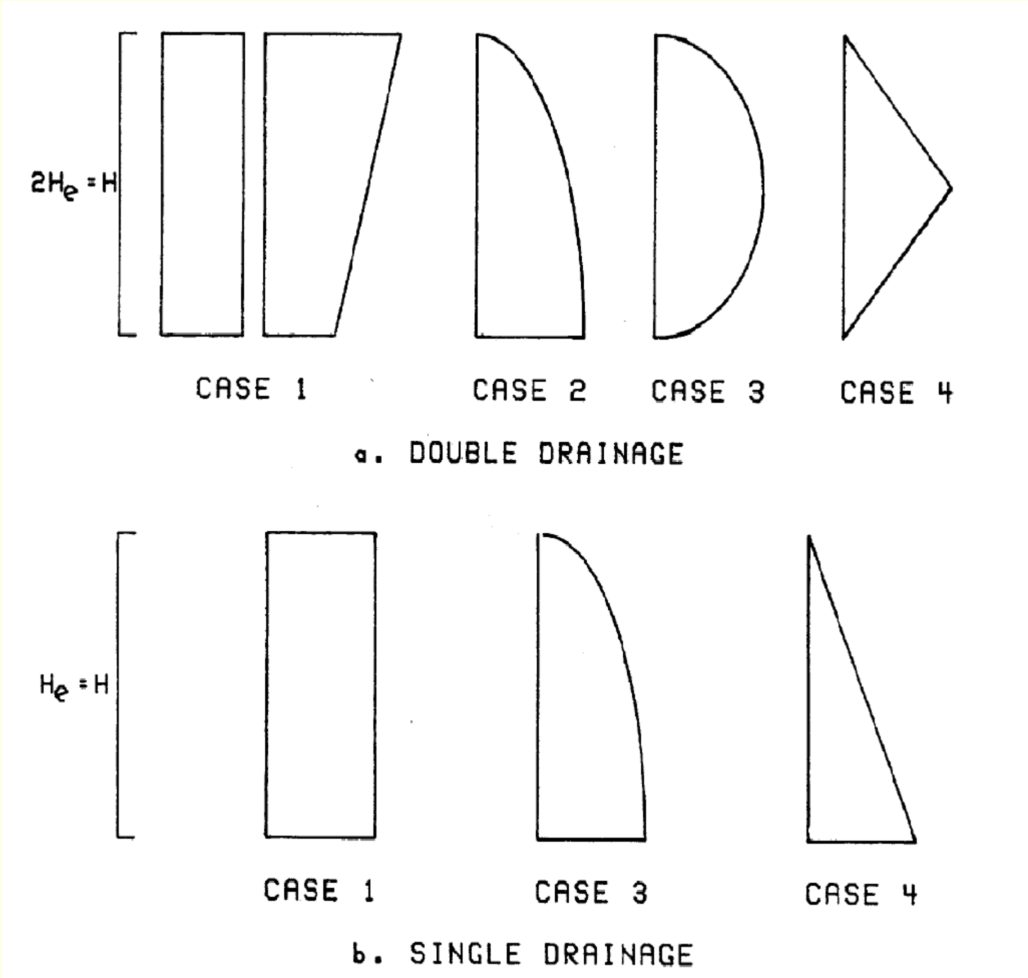
$$\sigma_x(x,t) = p - u(x,t)$$

- Solution of soil stress equation:

$$\sigma_x(x,t) = p\left(1-\frac{4}{\pi}\left(\sin(1/2\,\frac{\pi\,x}{h})e^{-1/4\,\frac{cv\,\pi^2 t}{h^2}} + 1/3\,\sin(3/2\,\frac{\pi\,x}{h})e^{-9/4\,\frac{cv\,\pi^2 t}{h^2}} + 1/5\,\sin(5/2\,\frac{\pi\,x}{h})e^{-\frac{25}{4}\,\frac{cv\,\pi^2 t}{h^2}}\cdots\right)\right)$$

Alternative Initial and Boundary (Drainage) Conditions

- Boundary Conditions
  - Speed of the drainage depends upon the conditions at the boundaries of the drainage
  - Drainage can be one or two ways
    - Theory was based on one-way drainage, but solved for two-way drainage using symmetry
- Distribution of initial pore pressure can affect time rate of consolidation (see diagram at right)
  - Solution shown is Case 1 (Uniform)



Degree of Consolidation

- Tracking pore pressures and stresses is certainly interesting, but our primary goal is to estimate the settlement at any given time after initial loading
- We first define the degree of consolidation, thus

$$U = \frac{\delta(t)}{\delta_p}$$

  - U = degree of consolidation (usually expressed as a percentage)
  - $\delta(t)$ = settlement at a given time t
  - δ_p = total consolidation settlement (from previous calculations)
- Our previous calculations showed that there is a definite relationship between the soil/pore water stress and the settlement

- Making the same assumptions (governing equation, boundary and initial conditions) as before, the degree of consolidation can be computed as follows:

$$U_o = \int_0^h \frac{\sigma_x(x,t)}{ph} dx$$

- U_o is so designated because it indicates the “standard” case (uniform initial stress distribution, boundary conditions as described)
- We should also define a time factor:

$$T_v = \frac{c_v t}{h^2}$$

Solution of Degree of Consolidation

- Solution of this equation:

$$U_o = 1 - \sum_{n=1}^{\infty} 4 \frac{e^{-1/4\,T_v\,n^2\pi^2}\left(\cos(n\pi)\cos(1/2\,n\pi) - \cos(n\pi) - \cos(1/2\,n\pi) + 1\right)}{n^2\pi^2}$$

$$U_o = 1 - 8\,\frac{e^{-1/4\,Tv\,\pi^2}}{\pi^2} - \frac{8}{9}\,\frac{e^{-9/4\,Tv\,\pi^2}}{\pi^2} - \frac{8}{25}\,e^{-\frac{25}{4}\,Tv\,\pi^2}\pi^{-2}\ldots$$

- Depending upon the value of T_v, we can use various forms of the equation to approximate the degree of consolidation
- Once again, keep in mind that T_v is based on h, not the layer thickness H. The drainage condition can significantly affect the results

Approximation for U_o in Table and Chart Form

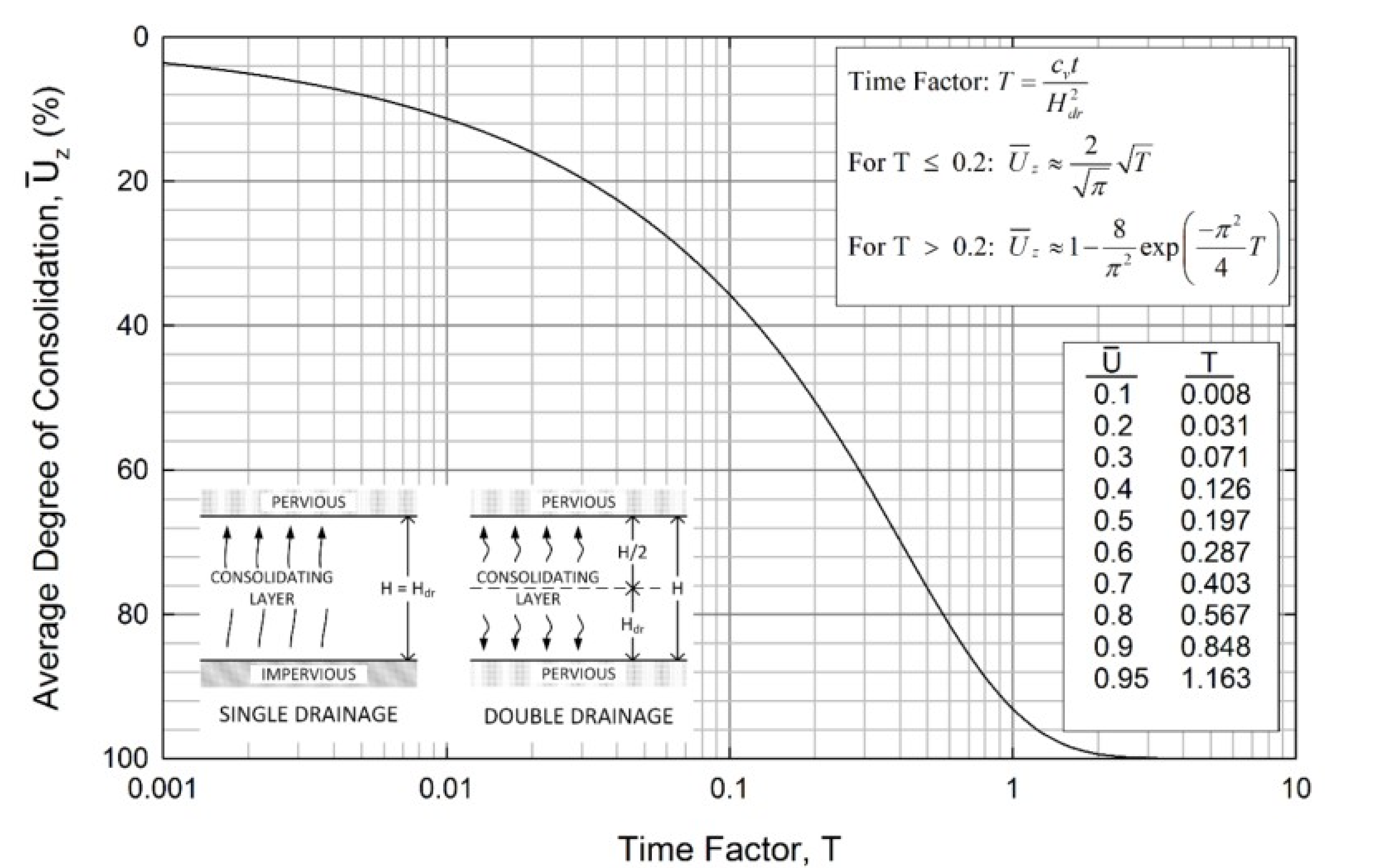
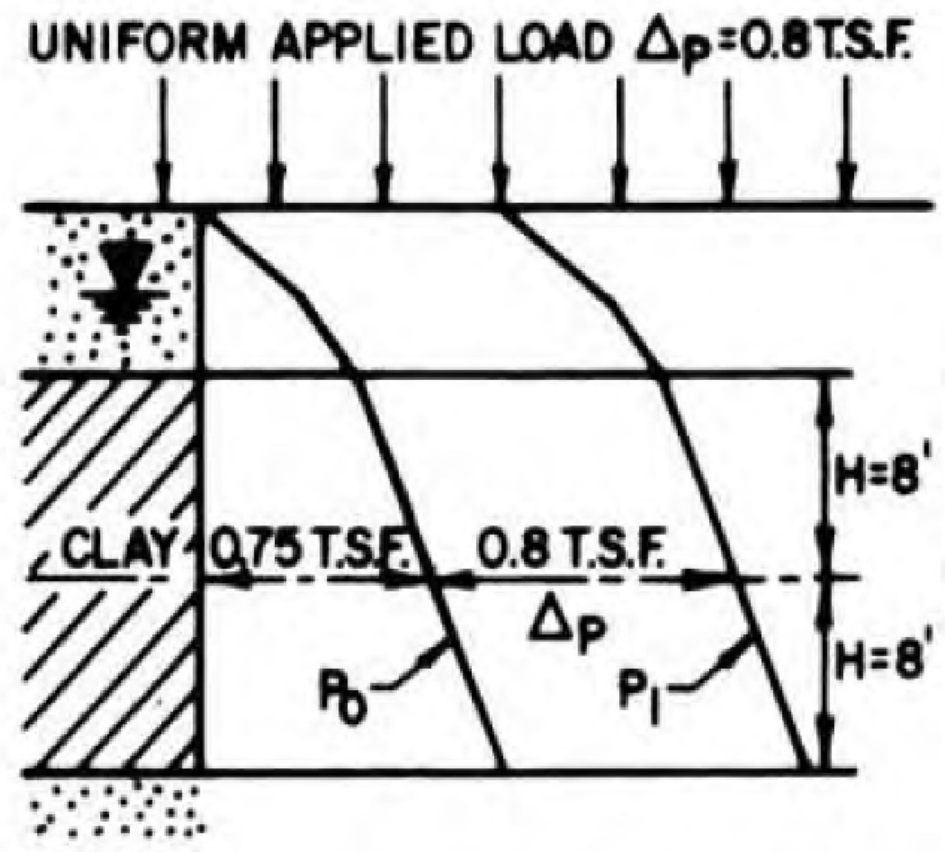


Figure 5-16 Degree of Consolidation for Instantaneous Uniform Loading and One-Dimensional Flow

Example of Time Factor Calculation

- Given
  - As shown
  - $e_o = 1.0$
  - $C_c = 0.21$
  - $c_v = 0.03$ ft^2/day
  - Primary consolidation settlement = 6.35" (from previous calculation)
- Find
  - Relationship of time of consolidation with time factor



Solution of Time Factor Calculation

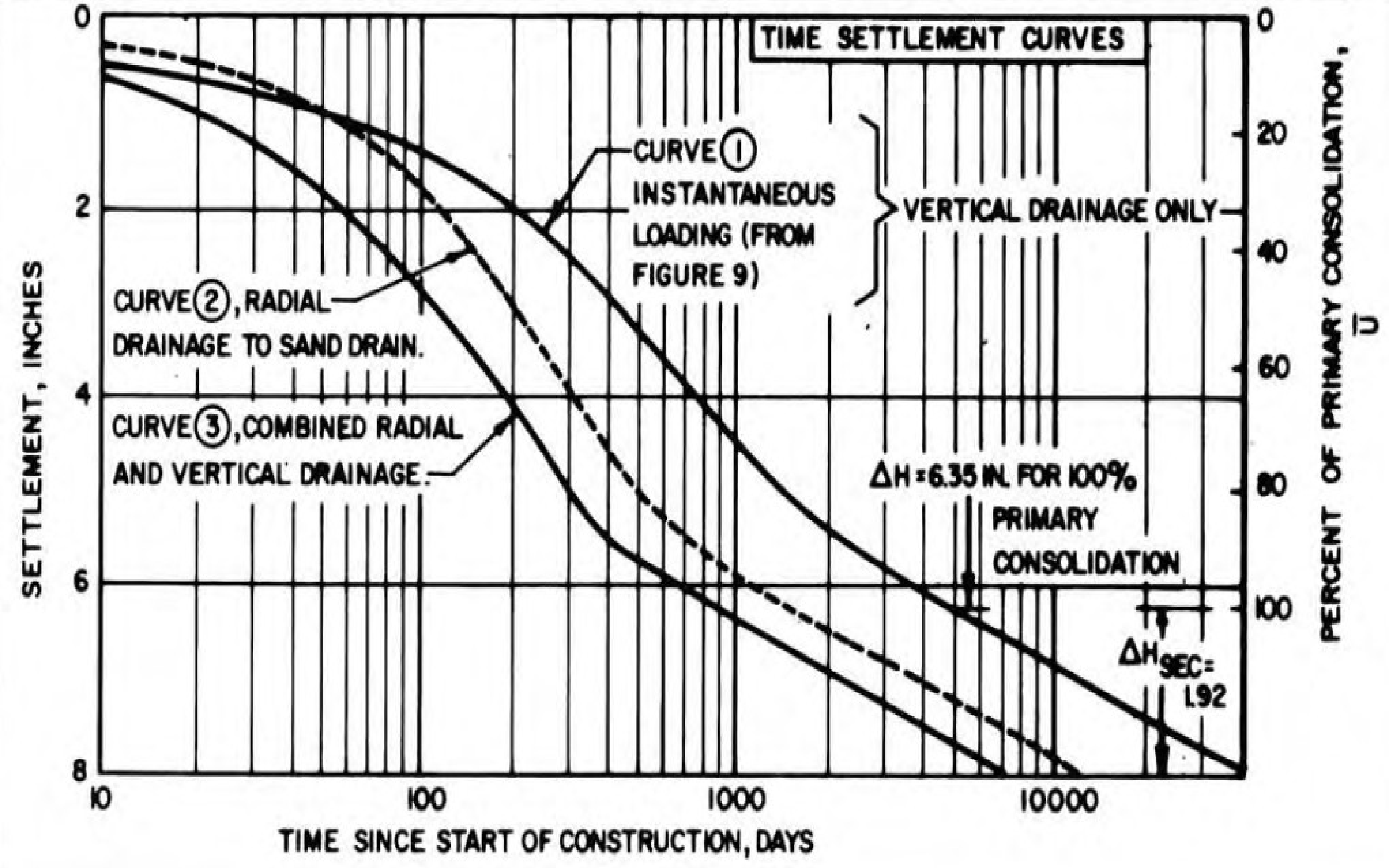
Table 7-4
Average degree of consolidation, U, versus Time Factor, T_v, for <u>uniform</u> initial increase in pore water pressure

U %	T_v
0	0.000
10	0.008
20	0.031
30	0.071
40	0.126
50	0.197
60	0.287
70	0.403
80	0.567
90	0.848
93.1	1.000
95.0	1.163
98.0	1.500
99.4	2.000
100.0	Infinity

Days	Tv	Percent Consolidation	Settlement, in.
10	0	0	0
20	0.01	10	0.64
30	0.01	10	0.64
40	0.02	15	0.95
50	0.02	15	0.95
60	0.03	20	1.27
70	0.03	20	1.27
80	0.04	25	1.59
90	0.04	25	1.59
100	0.05	28	1.78
200	0.09	32	2.03
300	0.14	40	2.54
400	0.19	48	3.05
500	0.23	53	3.37
600	0.28	60	3.81
700	0.33	63	4
800	0.38	68	4.32
900	0.42	72	4.57
1000	0.47	75	4.76

Hdr	8
Cv	0.03
Spc	6.35

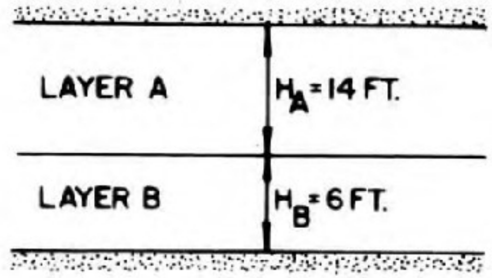
Time Factor Calculation Solution



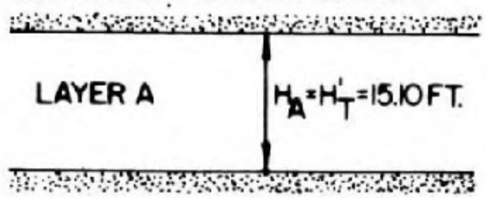
EXAMPLE OF COMPUTATION OF RATE OF CONSOLIDATION FOR A MULTI-LAYERED SYSTEM:

LAYERED SYSTEM:

ACTUAL STRATIFICATION



EQUIVALENT STRATIFICATION



KNOWN APPLICABLE SOIL PROPERTIES:

LAYER A:

$c_{vA} = 0.04$ FT.^{2}/DAY

LAYER B:

$c_{vB} = 1.20$ FT.^{2}/DAY

ASSUME: DOUBLE DRAINAGE

DETERMINATION OF EQUIVALENT LAYER THICKNESS:

1. ASSUME AN EQUIVALENT LAYER POSSESSING THE PROPERTIES OF SOIL A.
2. EQUIVALENT THICKNESS

$$
\begin{aligned}
H'_T &= H_A + H'_B \\
&= H_A + H_B \left(\frac{c_{vA}}{c_{vB}}\right)^{1/2} \\
&= 14 + 6 \left(\frac{0.04}{1.20}\right)^{1/2} \\
&= 14 + 1.10 \\
H'_T &= 15.10 \text{ FT.}
\end{aligned}
$$

3. DETERMINE $\bar{U}$ FROM FIGURE II, e.g. AT $t = 0.25$ YEARS, USING $H = (15.10)/2 = 7.55$ FT. (DRAINAGE PATH ASSUMING DOUBLE DRAINAGE) AND $c_{vA} = 0.04$ FT.^{2}/DAY, $\bar{U} = 29.1\%$

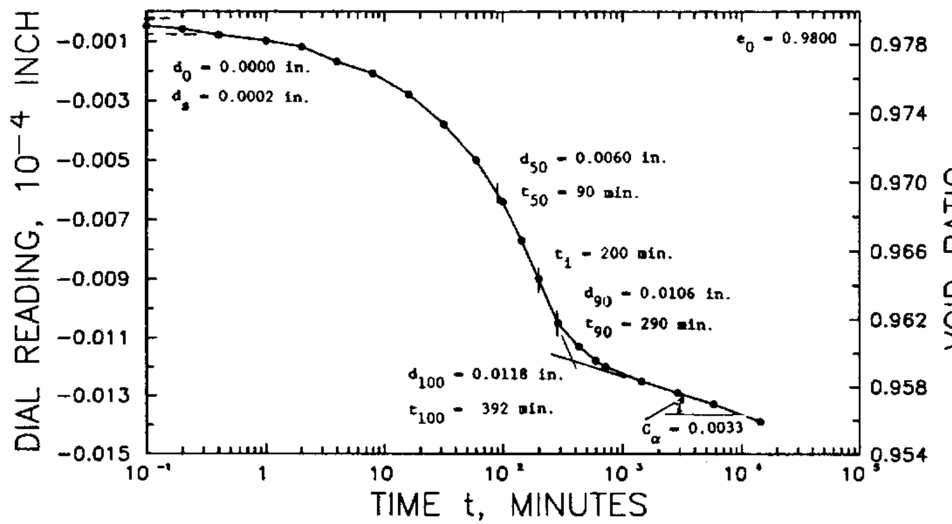
FIGURE 15 (continued)
Procedure for Determining the Rate of Consolidation
for All Soil Systems Containing "N" Layers

Coefficient of Consolidation

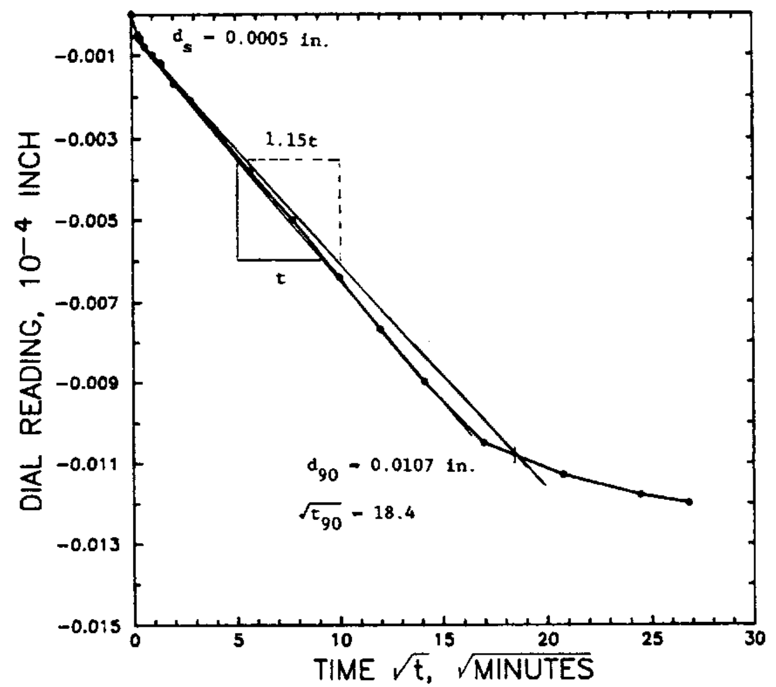
Method	Equation for c_v	Procedure	Example
Terzaghi	$\frac{0.848h_e^2}{t_{90}}$	1. Measure initial specimen height h_o and set initial dial reading d_o.	h_o = 1.148 inches d_o = 0.0000 inch h_f = 1.140 inches d_s = 0.0002 inch Refer to Figure 3-17a
		2. Measure dial reading d as a function of time t and final specimen height h_f. Plot d versus $\log_{10}t$. Determine d_s, the corrected zero point ($d_o - d_s$ = initial compression), by measuring vertical distance between time of about 0.1 min. and a time that is 4 times this.	
		3. Determine time t_{100} and compression d_{100} to 100 percent consolidation as the intersection of the tangent and asymptote of the consolidation curve. Then determine d_{90} and t_{90}.	t_{100} = 392 min. or 0.27 day d_{100} = 0.0118 in. $d_{90} = d_s + 0.9(d_{100} - d_s)$ = 0.0002 + 0.9(0.0118-0.0002) = 0.0002 + 0.0104 = 0.0106 inch. t_{90} = 290 min. or 0.20 day from Figure 3-17a
		4. Determine equivalent thickness of drainage path, $h_e = (h_o + h_f)/2$ if single drainage; $h_e = (h_o + h_f)/4$ if double drainage	$h_e = (h_o + h_f)/4$ = 2.288/4 = 0.572 inch or 0.0477 ft
		5. Calculate c_v	$c_v = \frac{0.85 \cdot 0.0477^2}{0.20}$ = 0.010 ft^2/day
Casagrande	$\frac{0.197h_e^2}{t_{50}}$	Same as Terzaghi above except determine time to reach 50 percent of consolidation t_{50}	$d_{50} = d_s + 0.5(d_{100} - d_s)$ = 0.0002 + 0.5(0.0118-0.0002) = 0.0002 + 0.0058 = 0.0060 inch t_{50} = 90 min. or 0.063 day $c_v = \frac{0.197 \cdot 0.0477}{0.063}$ = 0.007 ft^2/day
Inflection Point	$\frac{0.405h_e^2}{t_i}$	Same as above except note time to reach inflection point of curve t_i	t_i = 200 min. or 0.14 day from Figure 3-17a $c_v = \frac{0.405 \cdot 0.0477^2}{0.14}$ = 0.007 ft^2/da
Taylor	$\frac{0.848h_e^2}{t_{90}}$	1. Measure initial specimen height h_o and set initial dial reading d_o	h_o = 1.148 inches d_o = 0.0000 inch h_f = 1.140 Refer to Figure 3-17b
		2. Measure dial reading d as a function of time t and final specimen height h_f. Plot d versus $\sqrt{t}$ or square root of time.	
		3. Extend straight line portion back to $\sqrt{t}$ = 0 to obtain corrected initial reading d_o.	d_o = 0.0005 inch from Figure 3-17b
		4. Through d_o draw a straight line with inverse slope 1.15 times tangent and intersect laboratory curve to obtain t_{90}	$\sqrt{t_{90}}$ = 18.4 or t_{90} = 339 min. or 0.23 day Refer to Figure 3-17b
		5. Determine h_e as above	h_e = 0.0477 ft
		6. Calculate c_v	$c_v = \frac{0.848 \cdot 0.0477^2}{0.23}$ = 0.008 ft^2/day

Logarithm of Time Method

Square Root of Time Method



a. VOID RATIO-LOGARITHM TIME



b. DISPLACEMENT-SQUARE ROOT TIME

Accelerating Consolidation Settlement

- Rate of consolidation settlement under natural conditions is frequently unacceptable for actual construction and structure use
  - Previous Example: 75% settlement @ 1000 days = 2 years 8 months 26 days
  - Accelerating settlement if sometimes necessary to complete construction and use structure

- Concept of Consolidation Acceleration
  - Provide a high-permeability path for the water to escape, thus reducing the time for consolidation to take place
- Type of acceleration
  - Sand Drains
  - Prefabricated Vertical (Wick) Drains

Vertical Drain System

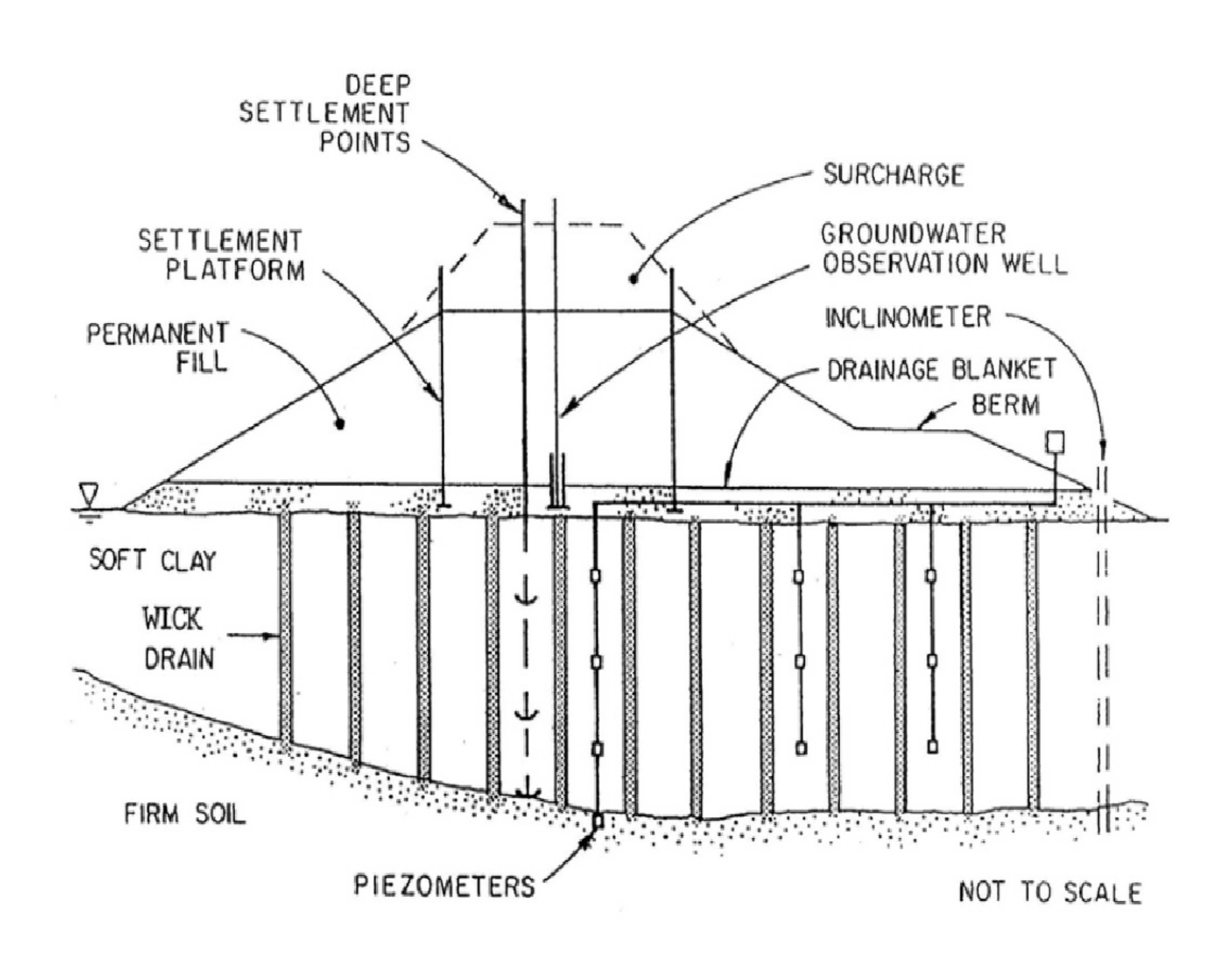
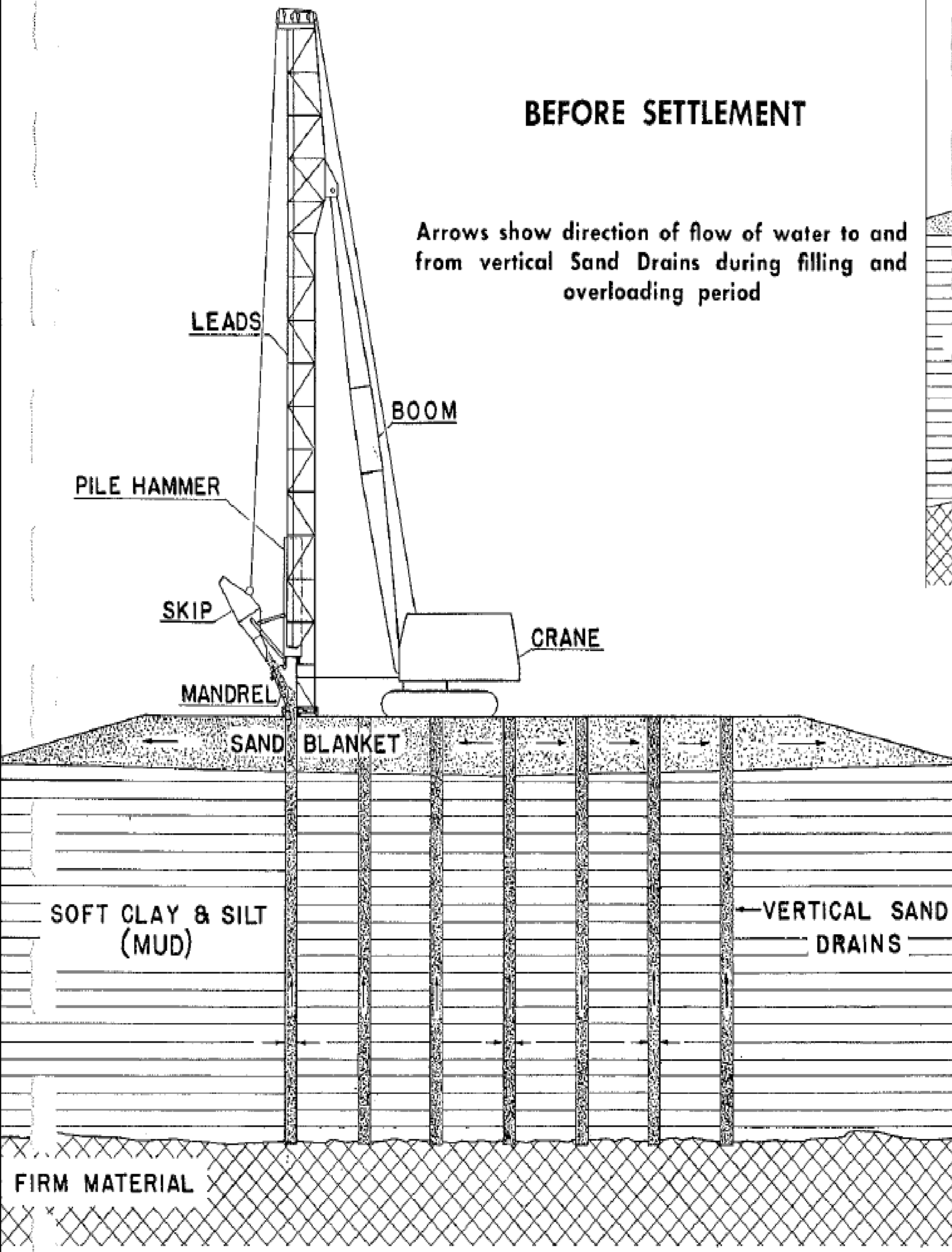
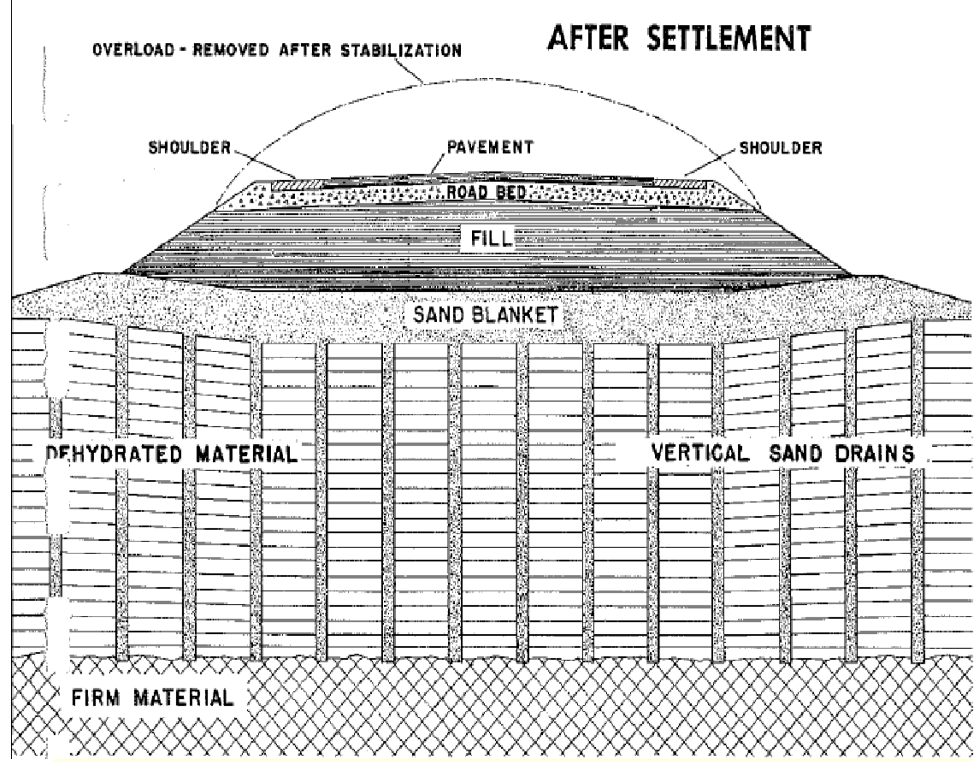


TABLE 7
Common Types of Vertical Drains

General Type	Sub-type	Typical Installation	
		d_w	s
1. Driven Sand Drain	Closed end mandrel	$18^{\pm}$ in	5 - 20 ft
2. Augered Sand Drain	(a) Screw type auger	6 - 30 in	-
	(b) Continuous flight hollow stem auger	18 in	5 - 20 ft
3. Jetted Sand Drain	(a) Internal jetting	18 in	5 - 20 ft
	(b) Rotary jet	12 - 18 in	5 - 20 ft
	(c) Dutch jet-bailer	12 in	4 - 16 ft
4. "Paper" Drain	(a) Kjellman cardboard wick	$0.1^{\pm}$ in by $4^{\pm}$ in	1.5^{+} - 4^{+} ft
	(b) Cardboard coated plastic wick	slightly thicker	-
5. Fabric Encased Sand Drain	(a) Sandwick	2.5 - 3 in	4 - 12 ft
	(b) Fabridrain	5 in	-

d_w = diameter of drain, s = drain spacing

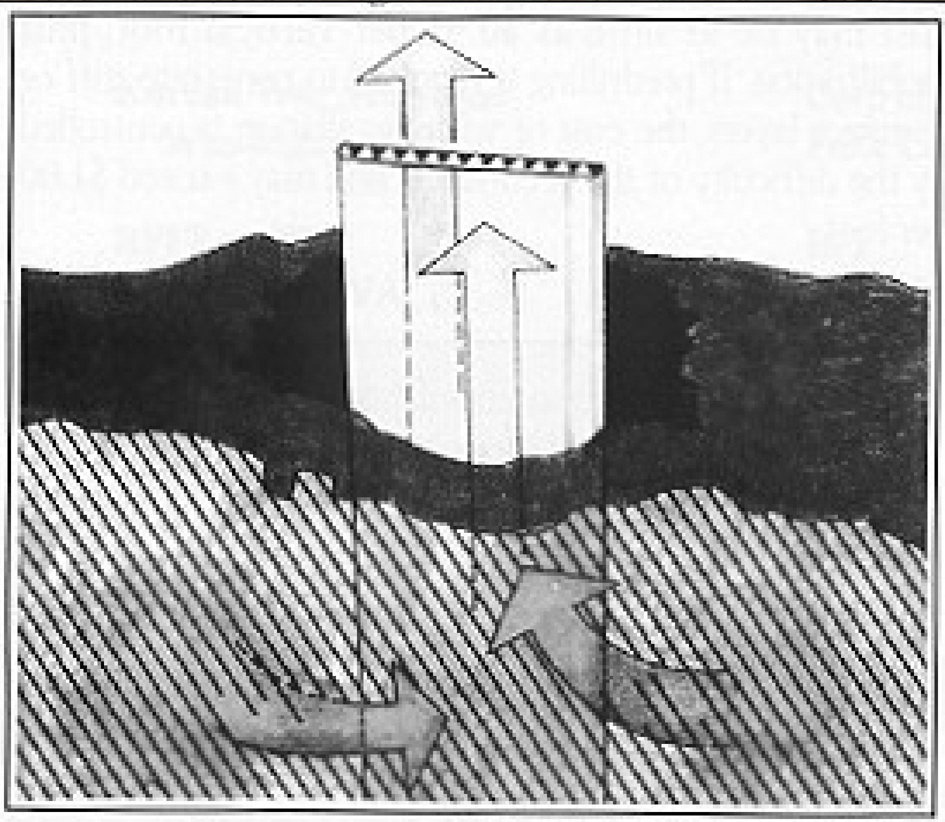




Sand Drain Installation

Wick Drains

- Geosynthetic used as a substitute to sand columns; arrayed in a similar manner to the sand drains
- Installed by being pushed or vibrated into the ground



Pore water flows laterally to the wick drains and is carried vertically up to the ground surface.

Installation of Wick Drains

Questions