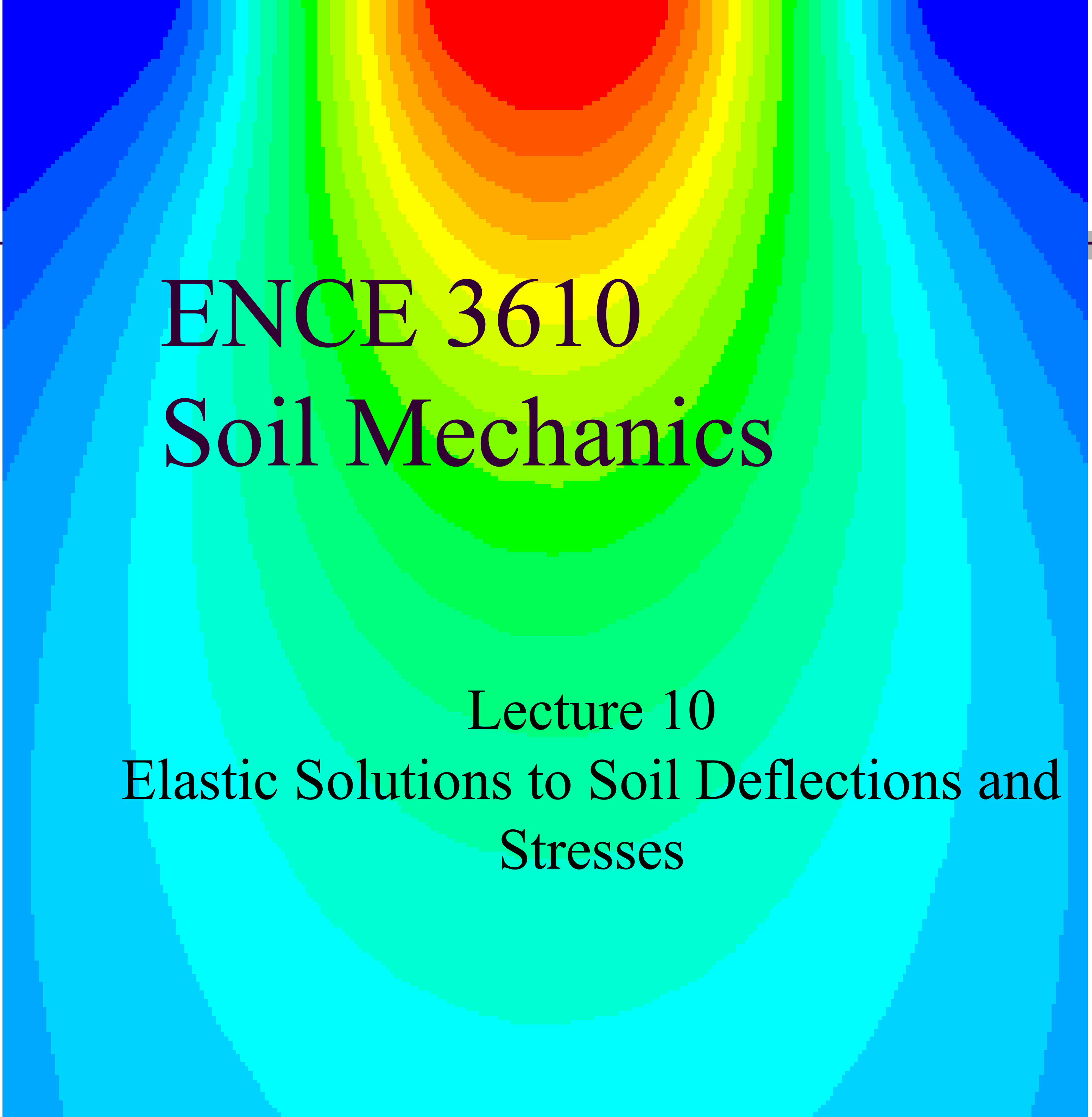


ENCE 3610
Soil Mechanics

Lecture 10

Elastic Solutions to Soil Deflections and Stresses

Using Theory of Elasticity for Estimating Stresses and Deflections

- Since it concerns both stress and strain/deflection, theory of elasticity can be used to estimate deflections in the elastic region
  - In this presentation, we will emphasize stresses, will consider deflections in more detail later
- Soils are highly nonlinear; thus, the strains must be restricted to small ones
  - Will not consider strain-softening effects, which means that the shear modulus/modulus of elasticity varies with the proximity to the load
- Theory of elasticity is applied to a semi-infinite solid (the soil) and the stresses vary (decrease) as one gets further from the load source
- For distributed loads, either a flexible or rigid foundation can be assumed, depending upon the situation at hand
  - Most solutions here—and those commonly used—assume a flexible foundation

Implementation of Theory of Elasticty

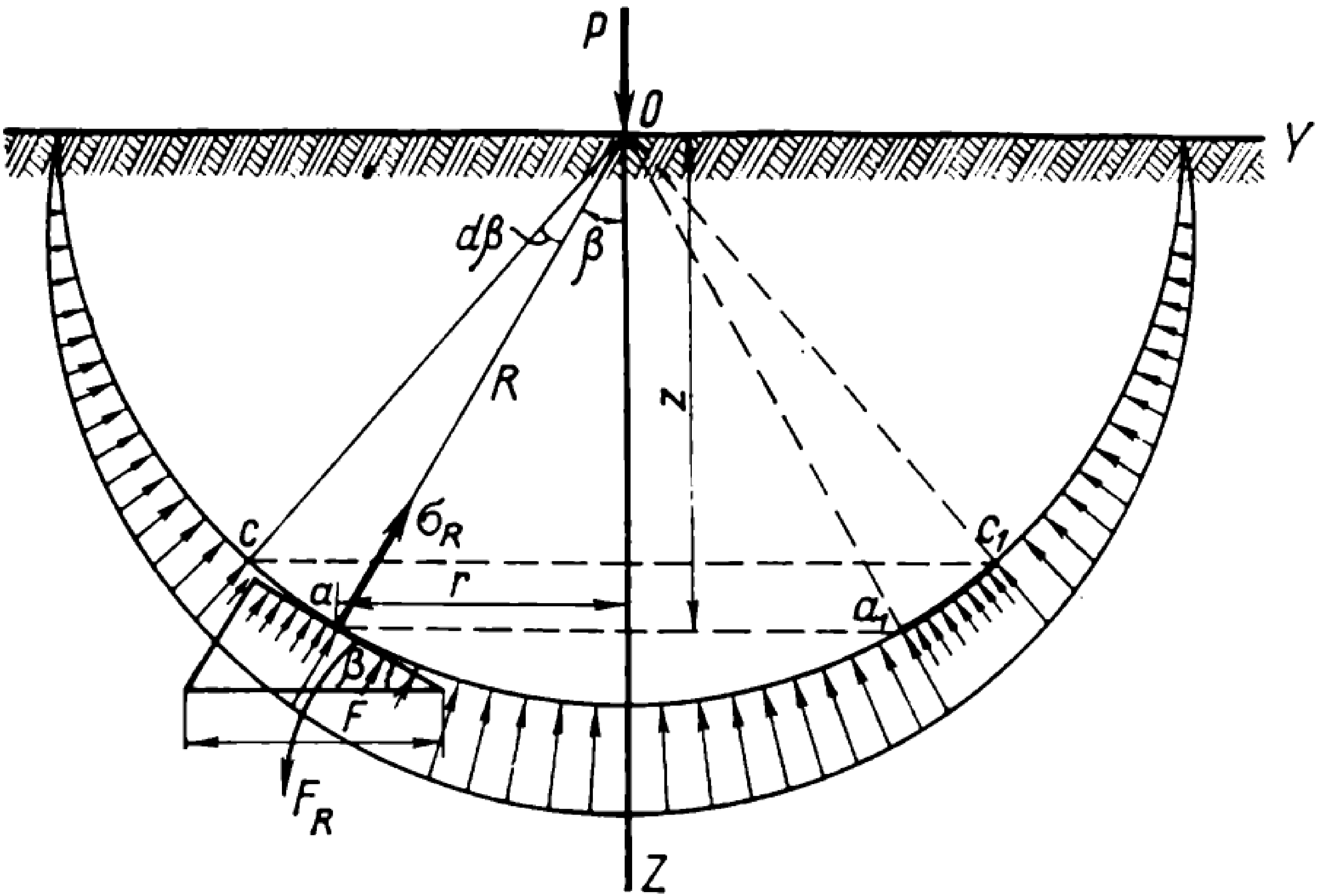
- Boussinesq Theory
  - Based on theory of elasticity
  - Homogeneous, isotropic material Semi-infinite solid
  - Original equations describe loading at a point; can be applied to various foundation shapes
  - Can be used to determine both deflections and stresses
  - Many of these solutions assume values of Poisson's Ratio for simplicity
- In all cases, our main focus will be on vertical stresses, although horizontal and shear stresses can be computed using this and other theories
- Westergaard Theory
  - Similar to Boussinesq, but no lateral deformations of the soil are assumed
  - Used with soils of alternating layers of materials
  - Used extensively in airfield pavement design
- Newmark's Method
  - Adaptation of Boussinesq Theory for structures that do not have a simple shape
- 2:1 Method
  - Empirical method commonly used to estimate structure induced stresses (FHWA)

Shapes and Solution Methods

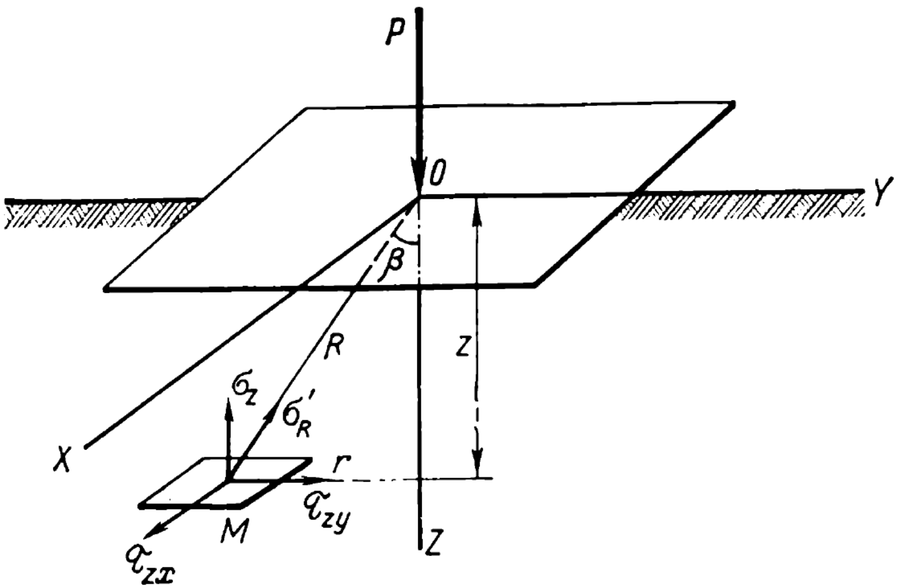
- Shapes
  - Shapes that will be considered here
    - Point Loads
    - Line Loads (Flamant)
    - Strip Loads (Flamant)
    - Rectangular Loads
    - Circular Loads
  - Chart and analytical solutions can be combined because of superposition
  - Newmark's Method can handle foundations of any shape

- Solution Methods
  - Equations
    - For simple cases, can be very useful
    - For more complex cases, generally not practical
    - Mistakes are common in literature and on the internet
  - Charts (traditionally the most extensively used, but have accuracy issues)
  - Computer program (needs check for verification)
  - Newmark's Graphical Method

Boussinesq Point Loads



Boussinesq Point Load Stresses



$$\sigma_z = \frac{3}{2} \cdot \frac{P}{\pi} \frac{z^3}{R^5}$$

$$\tau_{zy} = \frac{3}{2} \cdot \frac{P}{\pi} \cdot \frac{yz^2}{R^5}$$

$$\tau_{zx} = \frac{3}{2} \cdot \frac{P}{\pi} \cdot \frac{xz^2}{R^5}$$

The most important stresses are the vertical stresses, and be calculated thus:

$$\sigma_z = \frac{3}{2\pi\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} \cdot \frac{P}{z^2}$$

If we introduce the concept of the influence coefficient K:

$$\frac{3}{2\pi} \cdot \frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} = K$$

then we can rewrite the above equation thus:

$$\sigma_z = K \cdot \frac{P}{z^2}$$

Influence Coefficients for Point Loads

Coefficient K to Calculate Compressive Stresses from a Concentrated Force Depending on r/z Ratio

r/z	K	r/z	K	r/z	K	r/z	K
0.00	0.4775	0.50	0.2733	1.00	0.0844	1.50	0.0251
0.01	0.4773	0.51	0.2679	1.01	0.0823	1.51	0.0245
0.02	0.4770	0.52	0.2625	1.02	0.0803	1.52	0.0240
0.03	0.4764	0.53	0.2571	1.03	0.0783	1.53	0.0234
0.04	0.4756	0.54	0.2518	1.04	0.0764	1.54	0.0229
0.05	0.4745	0.55	0.2466	1.05	0.0744	1.55	0.0224
0.06	0.4732	0.56	0.2414	1.06	0.0727	1.56	0.0219
0.07	0.4717	0.57	0.2363	1.07	0.0709	1.57	0.0214
0.08	0.4699	0.58	0.2313	1.08	0.0691	1.58	0.0209
0.09	0.4679	0.59	0.2263	1.09	0.0674	1.59	0.0204
0.10	0.4657	0.60	0.2214	1.10	0.0658	1.60	0.0200
0.11	0.4633	0.61	0.2165	1.11	0.0641	1.61	0.0195
0.12	0.4607	0.62	0.2117	1.12	0.0626	1.62	0.0191
0.13	0.4579	0.63	0.2070	1.13	0.0610	1.63	0.0187
0.14	0.4548	0.64	0.2024	1.14	0.0595	1.64	0.0183
0.15	0.4516	0.65	0.1978	1.15	0.0581	1.65	0.0179
0.16	0.4482	0.66	0,1934	1.16	0.0567	1.66	0.0175
0.17	0.4446	0.67	0.1889	1.17	0.0553	1.67	0.0171
0.18	0.4409	0.68	0.1846	1.18	0.0539	1.68	0.0167
0.19	0.4370	0.69	0.1804	1.19	0.0526	1.69	0.0163
0.20	0.4329	0.70	0.1762	1.20	0.0513	1.70	0.0160
0.21	0.4286	0.71	0.1721	1.21	0.0501	1.72	0.0153
0.22	0.4242	0.72	0.1681	1.22	0.0489	1.74	0.0147
0.23	0.4197	0.73	0.1641	1.23	0.0477	1.76	0.0141
0.24	0.4151	0.74	0.1603	1.24	0.0466	1.78	0.0135
0.25	0.4103	0.75	0.1565	1.25	0.0454	1.80	0.0129
0.26	0.4054	0.76	0.1527	1.26	0.0443	1.82	0.0124
0.27	0.4004	0.77	0.1491	1.27	0.0433	1.84	0.0119
0.28	0.3954	0.78	0.1455	1.28	0.0422	1.86	0.0114
0.29	0.3902	0.79	0.1420	1.29	0.0412	1.88	0.0109
0.30	0.3849	0.80	0.1386	1.30	0.0402	1.90	0.0105
0.31	0.3796	0.81	0.1353	1.31	0.0393	1.92	0.0101
0.32	0.3742	0.82	0.1320	1.32	0.0384	1.94	0.0097
0.33	0.3687	0.83	0.1288	1.33	0.0374	1.96	0.0093
0.34	0.3632	0.84	0.1257	1.34	0.0365	1.98	0.0089
0.35	0.3577	0.85	0.1226	1.35	0.0357	2.00	0.0085
0.36	0.3521	0.86	0.1196	1.36	0.0348	2.10	0.0070
0.37	0.3465	0.87	0.1166	1.37	0.0340	2.20	0.0058
0.38	0.3408	0.88	0.1138	1.38	0.0332	2.30	0.0048
0.39	0.3351	0.89	0.1110	1.39	0.0324	2.40	0.0040
0.40	0.3294	0.90	0.1083	1.40	0.0317	2.50	0.0034
0.41	0.3238	0.91	0.1057	1.41	0.0309	2.60	0.0029
0.42	0.3181	0.92	0.1031	1.42	0.0302	2.70	0.0024
0.43	0.3124	0.93	0.1005	1.43	0.0295	2.80	0.0021
0.44	0.3068	0.94	0.0981	1.44	0.0288	2.90	0.0017
0.45	0.3011	0.95	0.0956	1.45	0.0282	3.00	0.0015
0.46	0.2955	0.96	0.0933	1.46	0.0275	3.50	0.0007
0.47	0.2899	0.97	0.0910	1.47	0.0269	4.00	0.0004
0.48	0.2843	0.98	0.0887	1.48	0.0263	4.50	0.0002
0.49	0.2788	0.99	0.0865	1.49	0.0257	5.00	0.0001

Boussinesq Point Load Displacements

The solution for the displacements is

$$u_r = \frac{P(1+\nu)}{2\pi ER}\left[\frac{r^2 z}{R^3} - (1-2\nu)(1-\frac{z}{R})\right], \tag{28.7}$$

$$u_\theta = 0, \tag{28.8}$$

$$u_z = \frac{P(1+\nu)}{2\pi ER}\left[2(1-\nu) + \frac{z^2}{R^2}\right]. \tag{28.9}$$

The vertical displacement of the surface is particularly interesting. This is

$$z = 0 : \quad u_z = \frac{P(1-\nu^2)}{\pi ER}. \tag{28.10}$$

For $R \to 0$ this tends to infinity, indicating that at the point of application of the point load the displacement is infinitely large. This singular behavior is a consequence of the singularity in the surface load, as in the origin the stress is infinitely large. That the displacement in that point is also infinitely large may not be so surprising.

We don’t use these very often because of the singularity issue.

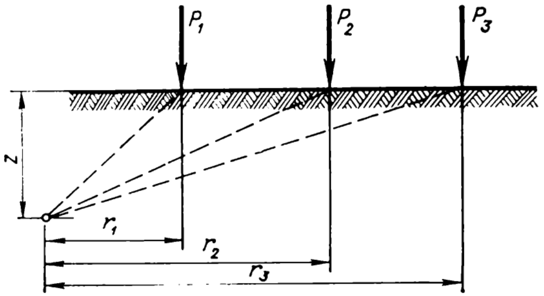
Boussinesq Point Load Illustration

- Given
  - Point Load, 45 kN
  - Point 3 m directly below the point load
- Find
  - Additional Vertical Created by Point Load (overburden/effective stresses not considered)
- Solution
  - z = 3 m, r = 0 m
  - r/z = 0
  - K = (3/((2π))/(1+(0/3)^2)^(5/2) = 0.4775 (from table, also from calculations)
  - σ_z = (0.4775)(45)/(3^2) = 2.387 kPa

$$\frac{3}{2\pi} \cdot \frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} = K$$

$$\sigma_z = K \cdot \frac{P}{z^2}$$

- In the real world point loads do not exist; however, *if the footprint of the load is "small" relative to its surroundings*, then footing loads can be analysed as "point" loads
- An example of this is given on the website, but will discuss this shortly



Flamant Line Load Stresses

In 1892 the French scientist M. Flamant obtained the solution for a vertical line load on a homogeneous isotropic linear elastic half space, see Figure 30.1. This is the two-dimensional equivalent of Boussinesq's basic problem. It can be considered as the superposition of an infinite number of point loads, uniformly distributed along the y-axis. A derivation of this solution is given in Appendix B.

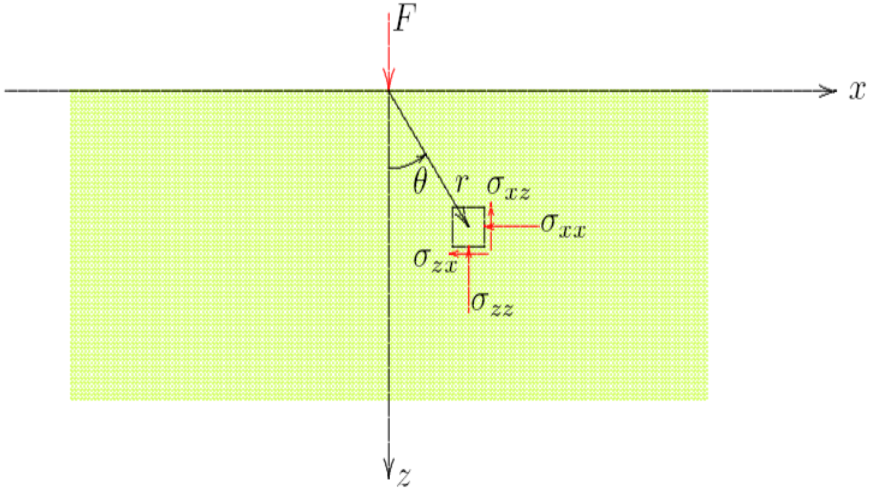


Figure 30.1: Flamant's Problem.

In this case the stresses in the x, z-plane are

$$\sigma_{zz} = \frac{2F}{\pi}\frac{z^3}{r^4} = \frac{2F}{\pi r}\cos^3\theta, \tag{30.1}$$

$$\sigma_{xx} = \frac{2F}{\pi}\frac{x^2 z}{r^4} = \frac{2F}{\pi r}\sin^2\theta\cos\theta, \tag{30.2}$$

$$\sigma_{xz} = \frac{2F}{\pi}\frac{xz^2}{r^4} = \frac{2F}{\pi r}\sin\theta\cos^2\theta. \tag{30.3}$$

In these equations $r = \sqrt{x^2 + z^2}$. The quantity F has the dimension of a force per unit length, so that F/r has the dimension of a stress.

Expressions for the displacements are also known, but these contain singular terms, with a factor $\ln r$. This factor is infinitely large in the origin and at infinity. Therefore these expressions are not so useful.

Flamant Strip Load Stresses (Verruijt)

On the basis of Flamant's solution several other solutions may be obtained using the principle of superposition. An example is the case of a uniform load of magnitude p on a strip of width $2a$, see Figure 30.2. In this case the stresses are

$$\sigma_{zz} = \frac{p}{\pi}[(\theta_1 - \theta_2) + \sin\theta_1\cos\theta_1 - \sin\theta_2\cos\theta_2], \tag{30.4}$$

$$\sigma_{xx} = \frac{p}{\pi}[(\theta_1 - \theta_2) - \sin\theta_1\cos\theta_1 + \sin\theta_2\cos\theta_2], \tag{30.5}$$

$$\sigma_{xz} = \frac{p}{\pi}[\cos^2\theta_2 - \cos^2\theta_1]. \tag{30.6}$$

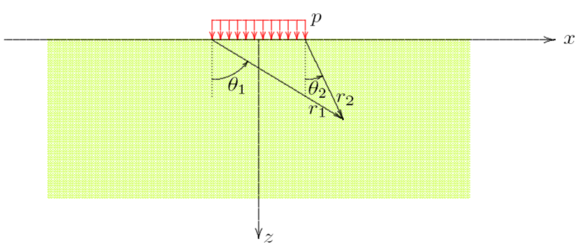


Figure 30.2: Strip load.

In the center of the plane, for $x = 0$, $\theta_2 = -\theta_1$. Then the stresses are

$$x = 0 \; : \; \sigma_{zz} = \frac{2p}{\pi}[(\theta_1 + \sin\theta_1\cos\theta_1], \tag{30.7}$$

$$x = 0 \; : \; \sigma_{xx} = \frac{2p}{\pi}[(\theta_1 - \sin\theta_1\cos\theta_1], \tag{30.8}$$

$$x = 0 \; : \; \sigma_{xz} = 0. \tag{30.9}$$

That the shear stress $\sigma_{xz} = 0$ for $x = 0$ is a consequence of the symmetry of this case. The stresses σ_{xx} and σ_{zz} are shown in Figure 30.3, as functions of the depth z. Both stresses tend towards zero for $z \to \infty$, of course, but the horizontal normal stress appears to tend towards zero much faster than the vertical normal stress. It also appears that at the surface the horizontal stress is equal to the vertical stress. At the surface this vertical stress is equal to the load p, of course, because that is a boundary condition of the problem. Actually, in every point of the surface below the strip load the normal stresses are $\sigma_{xx} = \sigma_{zz} = p$.

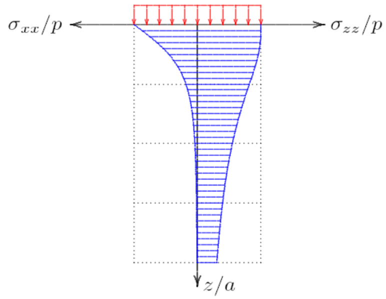


Figure 30.3: Stresses for $x = 0$.

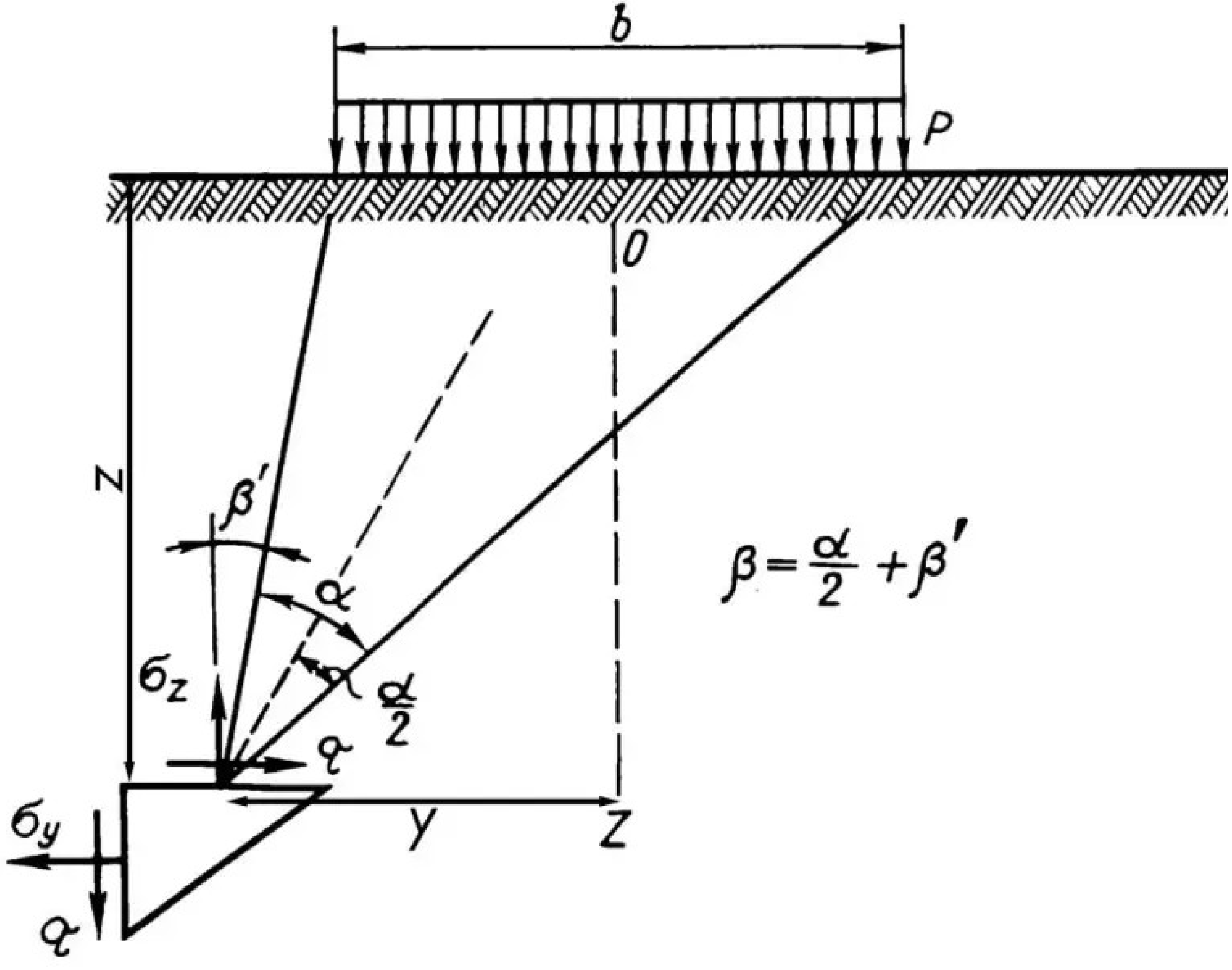
It may be interesting to further explore the result that the shear stress $\sigma_{xz} = 0$ along the axis of symmetry $x = 0$ in the case of a strip load, see Figure 30.2. It can be expected that this symmetry also holds for the horizontal displacement, so that $u_x = 0$ along the axis $x = 0$. This means that this solution can also be used as the solution of the problem that is obtained by considering the right half of the strip problem only, see Figure 30.4. In this problem the quarter plane $x > 0, z > 0$ is supposed to be loaded by a strip load of width a on the surface $z = 0$, and the boundary conditions on the boundary $x = 0$ are that the displacement $u_x = 0$ and the shear stress $\sigma_{xz} = 0$, representing a perfectly smooth and rigid vertical wall. The wall is supposed to extend to an infinite depth, which is impractical. For a smooth rigid wall of finite depth the solution may be considered as a first approximation.

The formulas (30.7) and (30.8) can also be written as

$$x = 0 \; : \; \sigma_{zz} = \frac{2p}{\pi}[\arctan(\frac{a}{z}) + \frac{az}{a^2+z^2}], \tag{30.10}$$

$$x = 0 \; : \; \sigma_{xx} = \frac{2p}{\pi}[\arctan(\frac{a}{z}) - \frac{az}{a^2+z^2}]. \tag{30.11}$$

Flamant Strip Load Stresses (Tsytovich)



$$\beta = \frac{\alpha}{2} + \beta'$$

Cartesian Axes

$$\sigma_z = \frac{p\,(\alpha + \sin(\alpha)\cos(2\,\beta))}{\pi}$$

$$\sigma_y = \frac{p\,(\alpha - \sin(\alpha)\cos(2\,\beta))}{\pi}$$

$$\tau = \frac{p\sin(\alpha)\sin(2\,\beta)}{\pi}$$

Principal Axes ($\beta = 0$)

$$\sigma_1 = \frac{p\,(\alpha + \sin(\alpha))}{\pi}$$

$$\sigma_3 = \frac{p\,(\alpha - \sin(\alpha))}{\pi}$$

Influence Coefficients

- We have seen the application of influence coefficients to point loads, where it is necessary to divide the point load by an “area” for the coefficient to be dimensionless
- Applies more easily to distributed loads such as strip, square, rectangular and circular loads where the pressure on the foundation is distributed across the area
- For distributed loads, the influence coefficient is the ratio between the uniform/reference pressure on the foundation and the stress (normal or shear) at any point, and can be defined as follows:
    - $I = K'_c = \boldsymbol{\sigma}/p = \boldsymbol{\sigma}/q = \boldsymbol{\tau}/p = \boldsymbol{\tau}/q$
- For purely flexible foundations, the influence coefficient is always less than unity

- Influence coefficients for the strip load (Tsytovich notation):

$$K_z = \frac{(\alpha + \sin(\alpha)\cos(2\beta))}{\pi}$$

$$K_y = \frac{(\alpha - \sin(\alpha)\cos(2\beta))}{\pi}$$

$$K_{yz} = \frac{\sin(\alpha)\sin(2\beta)}{\pi}$$

$$\text{Principal Axes } (\beta = 0)$$

$$K_1 = \frac{(\alpha + \sin(\alpha))}{\pi}$$

$$K_3 = \frac{(\alpha - \sin(\alpha))}{\pi}$$

Flamant Strip Load Stresses (Tsytovich)

Influence Coefficients K_z, K_y and K_{yz} for Determination of Component Stresses for a Uniformly Distributed Load under Conditions of Planar Problem

$\frac{z}{b}$	Values of y/b																	
	0			0.25			0.5			1			1.5			2		
	K_z	K_y	K_{yz}	K_z	K_y	K_{yz}	K_z	K_y	K_{yz}	K_z	K_y	K_{yz}	K_z	K_y	K_{yz}	K_z	K_y	K_{zy}
0.00	1.00	1.00	0	1.00	1.00	0	0.50	0.50	0.32	0	0	0	0	0	0	0	0	0
0.25	0.96	0.45	0	0.90	0.39	0.13	0.50	0.35	0.30	0.02	0.17	0.05	0.00	0.07	0.01	0.00	0.04	0.00
0.50	0.82	0.18	0	0.74	0.19	0.16	0.48	0.23	0.26	0.08	0.21	0.13	0.02	0.12	0.04	0.00	0.07	0.02
0.75	0.67	0.08	0	0.61	0.10	0.13	0.45	0.14	0.20	0.15	0.22	0.16	0.04	0.14	0.07	0.02	0.10	0.04
1.00	0.55	0.04	0	0.51	0.05	0.10	0.41	0.09	0.16	0.19	0.15	0.16	0.07	0.14	0.10	0.03	0.13	0.05
1.25	0.46	0.02	0	0.44	0.03	0.07	0.37	0.06	0.12	0.20	0.11	0.14	0.10	0.12	0.10	0.04	0.11	0.07
1.50	0.40	0.01	0	0.38	0.02	0.06	0.33	0.04	0.10	0.21	0.06	0.11	0.13	0.09	0.10	0.07	0.09	0.08
1.75	0.35	—	0	0.34	0.01	0.04	0.30	0.03	0.08	0.20	0.05	0.10	0.14	0.07	0.10	0.08	0.08	0.08
2.00	0.31	—	0	0.31	—	0.03	0.28	0.02	0.06	0.17	0.02	0.06	0.13	0.03	0.07	0.10	0.04	0.07
3.00	0.21	—	0	0.21	—	0.02	0.20	0.01	0.03	0.14	0.01	0.03	0.12	0.02	0.05	0.10	0.03	0.05
4.00	0.16	—	0	0.16	—	0.01	0.15	—	0.02	0.12	—	—	0,11	—	—	0.09	—	—
5.00	0.13	—	0	0.13	—	—	0.12	—	—	0.10	—	—	0,10	—	—	—	—	—
6.00	0.11	—	0	0.10	—	—	0.10	—	—	—	—	—	—	—	—	—	—	—

Strip Load Example

- **Given**
  - Strip load, b = 5 m wide, uniform pressure of p = 100 kPa
- **Find**
  - Stresses in z, y and yz directions at a point y = 2.5 m away from the centre of the strip load and z = 5 m down from the surface
- **Solution**
  - y/b = 0.5
  - z/b = 1

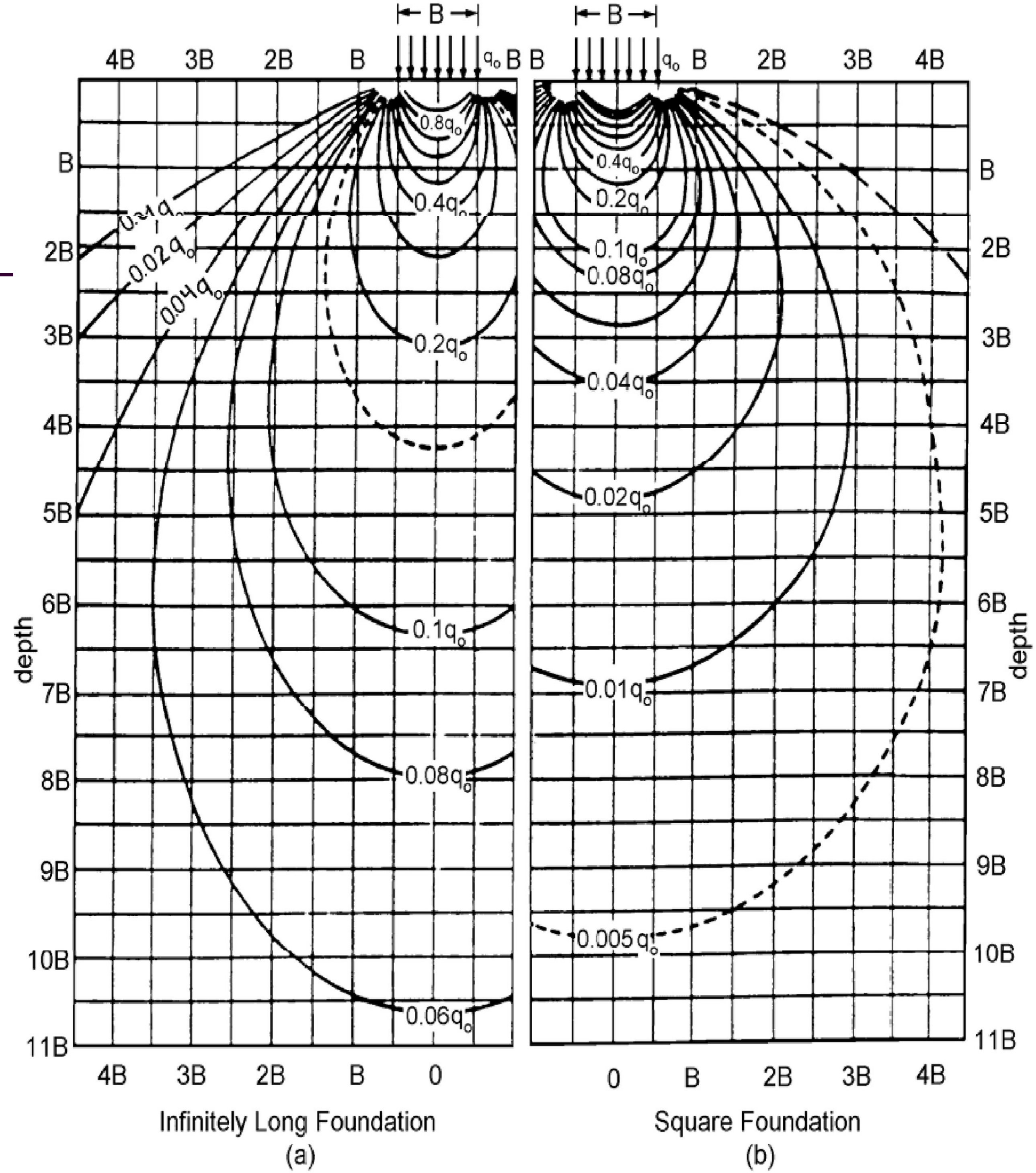
- Solution
  - Angles (from formulae)
    - $\beta' = 0$
    - $\alpha = 0.785$ radians
    - $\beta = 0.393$ radians
  - Influence Coefficients
    - $\sigma_z = K_z p$
    - $\sigma_y = K_y p$
    - $\sigma_{yz} = K_{yz} p$

Strip Load Example

- Solution
  - Influence Coefficients (from table, computed they are basically the same)
    - $K_z = 0.41$
    - $K_y = 0.09$
    - $K_{yz} = 0.16$
  - Stresses
    - $\sigma_z = 41$ kPa
    - $\sigma_y = 9$ kPa
    - $\sigma_{yz} = 16$ kPa

- Solution
  - Principal Influence Coefficients and Stresses (from formulae on slides)
    - $K_1 = 0.48$
    - $K_3 = 0.025$
    - $\sigma_1 = 48$ kPa
    - $\sigma_3 = 2.5$ kPa

Chart for Strip and Square Loads

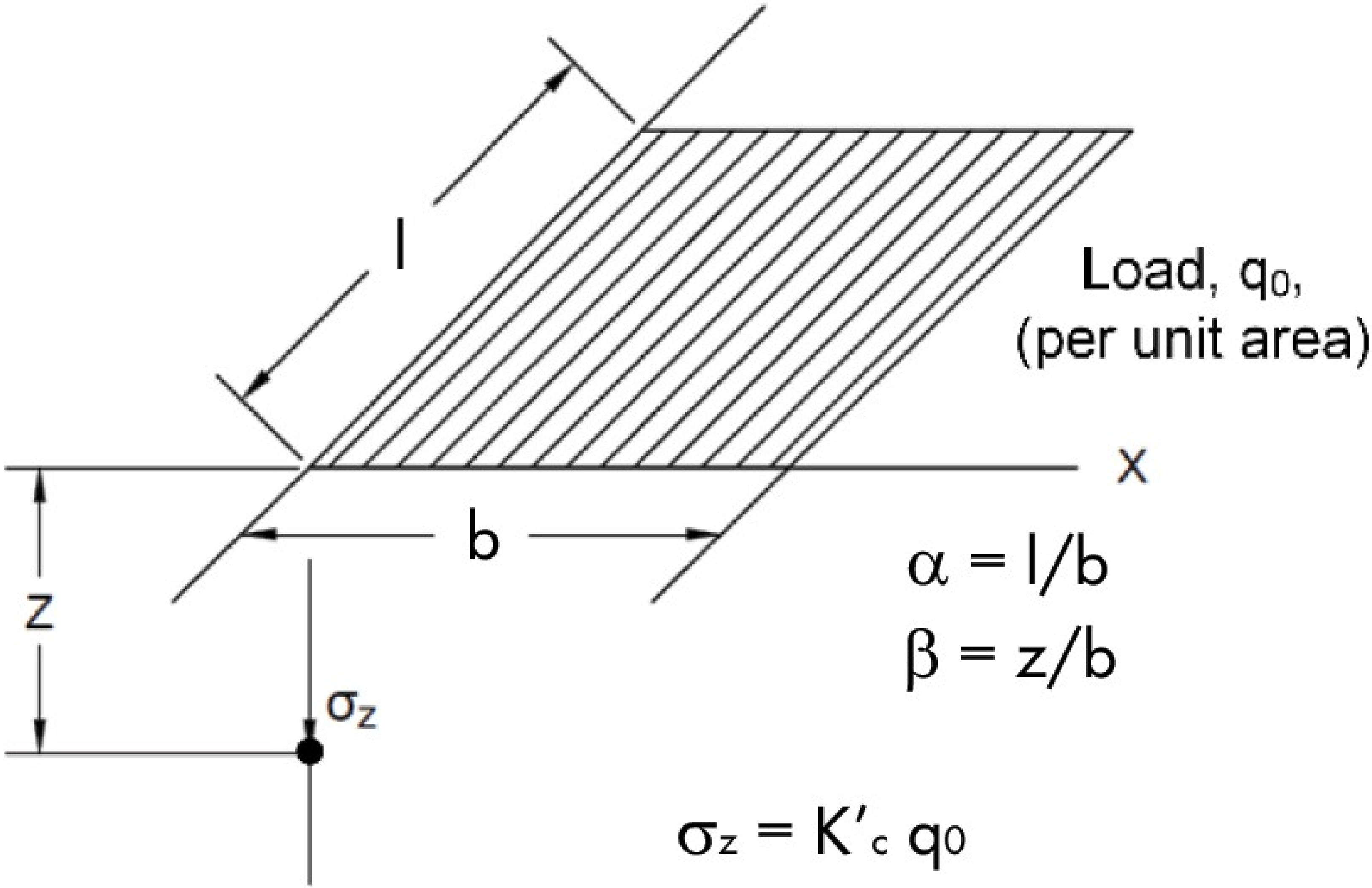


Stresses Under Squares and Rectangles

- Traditionally Engineers have used the "Fadum Charts," after the man who came up with them
- The Fadum Charts are flexible (you can interchange B and L) but are hard to read, especially as the foundation approaches a continuous one
- The following are taken from Tsytovich (1976) who constructs a table
- In both cases the stresses are under the corner of the foundation

- Definitions
  - B = shorter side of rectangle
  - L = longer side of rectangle
  - In the case of squares, $B = L$
  - K'_c = influence coefficient at at the point Z below the corner
  - $\alpha = L/B$ = aspect ratio of foundation = unity for squares
  - $\beta = Z/B$ = ratio of depth under the corner to the width of the foundation = 0 at the surface

Diagram for Corner Stresses



$$\alpha = l/b$$

$$\beta = z/b$$

$$\sigma_z = K'_c\, q_0$$

Table for Values of K'_c

Table 3.4

Coefficients K'_c

$\beta = \frac{z}{b}$	Values of $\alpha = l/b$										
	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0.0	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
0.2	0.2486	0.2489	0.2490	0.2491	0.2491	0.2491	0.2492	0.2492	0.2492	0.2492	0.2492
0.4	0.2401	0.2420	0.2429	0.2434	0.2437	0.2439	0.2440	0.2441	0.2442	0.2442	0.2442
0.6	0.2229	0.2275	0.2300	0.2315	0.2324	0.2329	0.2333	0.2335	0.2337	0.2338	0.2339
0.8	0.1999	0.2075	0.2120	0.2147	0.2165	0.2176	0.2183	0.2188	0.2192	0.2194	0.2196
1.0	0.1752	0.1851	0.1911	0.1955	0.1981	0.1999	0.2012	0.2020	0.2026	0.2031	0.2034
1.2	0.1516	0.1626	0.1705	0.1758	0.1793	0.1818	0.1836	0.1849	0.1858	0.1865	0.1870
1.4	0.1308	0.1423	0.1508	0.1569	0.1613	0.1644	0.1667	0.1685	0.1696	0.1705	0.1712
1.6	0.1123	0.1241	0.1329	0.1396	0.1445	0.1482	0.1509	0.1530	0.1545	0.1557	0.1567
1.8	0.0969	0.1083	0.1172	0.1241	0.1294	0.1334	0.1365	0.1389	0.1408	0.1423	0.1434
2.0	0.0840	0.0947	0.1034	0.1103	0.1158	0.1202	0.1236	0.1263	0.1284	0.1300	0.1314
2.2	0.0732	0.0832	0.0917	0.0984	0.1039	0.1084	0.1120	0.1149	0.1172	0.1191	0.1205
2.4	0.0642	0.0734	0.0813	0.0879	0.0934	0.0979	0.1016	0.1047	0.1071	0.1092	0.1108
2.6	0.0566	0.0651	0.0725	0.0788	0.0842	0.0887	0.0924	0.0955	0.0981	0.1003	0.1020
2.8	0.0502	0.0580	0.0649	0.0709	0.0761	0.0805	0.0842	0.0875	0.0900	0.0923	0.0942
3.0	0.0447	0.0519	0.0583	0.0640	0.0690	0.0732	0.0769	0.0801	0.0828	0.0851	0.0870
3.2	0.0401	0.0467	0.0526	0.0580	0.0627	0.0668	0.0704	0.0735	0.0762	0.0786	0.0806
3.4	0.0361	0.0421	0.0477	0.0527	0.0571	0.0611	0.0646	0.0677	0.0704	0.0727	0.0747
3.6	0.0326	0.0382	0.0433	0.0480	0.0523	0.0561	0.0594	0.0624	0.0651	0.0674	0.0694
3.8	0.0296	0.0348	0.0395	0.0439	0.0479	0.0516	0.0548	0.0577	0.0603	0.0626	0.0646
4	0.0270	0.0318	0.0362	0.0403	0.0441	0.0474	0.0507	0.0535	0.0560	0.0588	0.0603
4.2	0.0247	0.0291	0.0333	0.0371	0.0407	0.0439	0.0469	0.0496	0.0521	0.0543	0.0563
4.4	0.0227	0.0268	0.0306	0.0343	0.0376	0.0407	0.0436	0.0462	0.0485	0.0507	0.0527
4.6	0.0209	0.0247	0.0283	0.0317	0.0348	0.0378	0.0405	0.0430	0.0453	0.0474	0.0493
4.8	0.0193	0.0229	0.0262	0.0294	0.0324	0.0352	0.0378	0.0402	0.0424	0.0444	0.0463
5	0.0179	0.0212	0.0243	0.0274	0.0302	0.0328	0.0353	0.0376	0.0397	0.0417	0.0435
6	0.0127	0.0151	0.0174	0.0196	0.0218	0.0238	0.0257	0.0276	0.0293	0.0310	0.0325
7	0.0094	0.0112	0.0130	0.0147	0.0164	0.0180	0.0195	0.0210	0.0224	0.0238	0.0251
8	0.0073	0.0087	0.0101	0.0114	0.0127	0.0140	0.0153	0.0165	0.0176	0.0187	0.0198
9	0.0058	0.0069	0.0080	0.0091	0.0102	0.0112	0.0122	0.0132	0.0142	0.0152	0.0161
10	0.0047	0.0056	0.0065	0.0074	0.0083	0.0092	0.0100	0.0109	0.0117	0.0125	0.0132

Table for Values of K'_c

Table 3.4 (continued)

$\beta = \frac{z}{b}$	Values of $\alpha = l/b$										
	3.2	3.4	3.6	3.8	4	5	6	7	8	9	10
0.0	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
0.2	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492	0.2492
0.4	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443	0.2443
0.6	0.2340	0.2340	0.2341	0.2341	0.2341	0.2342	0.2342	0.2342	0.2342	0.2342	0.2342
0.8	0.2198	0.2199	0.2199	0.2200	0.2200	0.2202	0.2202	0.2202	0.2202	0.2202	0.2202
1.0	0.2037	0.2039	0.2040	0.2041	0.2042	0.2044	0.2045	0.2045	0.2046	0.2046	0.2046
1.2	0.1873	0.1876	0.1878	0.1880	0.1882	0.1885	0.1887	0.1888	0.1888	0.1888	0.1888
1.4	0.1718	0.1722	0.1725	0.1728	0.1730	0.1735	0.1738	0.1739	0.1739	0.1739	0.1740
1.6	0.1574	0.1580	0.1584	0.1587	0.1590	0.1598	0.1601	0.1602	0.1603	0.1604	0.1604
1.8	0.1443	0.1450	0.1455	0.1460	0.1463	0.1474	0.1478	0.1480	0.1481	0.1482	0.1482
2.0	0.1324	0.1332	0.1339	0.1345	0.1350	0.1363	0.1368	0.1371	0.1372	0.1373	0.1374
2.2	0.1218	0.1227	0.1235	0.1242	0.1248	0.1264	0.1271	0.1274	0.1276	0.1277	0.1277
2.4	0.1122	0.1133	0.1142	0.1150	0.1156	0.1175	0.1184	0.1188	0.1190	0.1191	0.1192
2.6	0.1035	0.1047	0.1058	0.1066	0.1073	0.1095	0.1106	0.1111	0.1113	0.1115	0.1116
2.8	0.0957	0.0970	0.0982	0.0991	0.0999	0.1024	0.1036	0.1041	0.1045	0.1047	0.1048
3.0	0.0887	0.0901	0.0913	0.0923	0.0931	0.0959	0.0973	0.0980	0.0983	0.0986	0.0987
3.2	0.0823	0.0838	0.0850	0.0861	0.0870	0.0900	0.0916	0.0923	0.0928	0.0930	0.0933
3.4	0.0765	0.0780	0.0793	0.0804	0.0814	0.0847	0.0864	0.0873	0.0877	0.0880	0.0882
3.6	0.0712	0.0728	0.0741	0.0753	0.0763	0.0799	0.0816	0.0826	0.0832	0.0835	0.0837
3.8	0.0664	0.0680	0.0694	0.0706	0.0717	0.0753	0.0773	0.0784	0.0790	0.0794	0.0796
4.0	0.0620	0.0636	0.0650	0.0663	0.0674	0.0712	0.0733	0.0745	0.0752	0.0756	0.0758
4.2	0.0581	0.0596	0.0610	0.0623	0.0634	0.0674	0.0696	0.0709	0.0716	0.0721	0.0724
4.4	0.0544	0.0560	0.0574	0.0586	0.0597	0.0639	0.0662	0.0676	0.0684	0.0689	0.0692
4.6	0.0510	0.0526	0.0540	0.0553	0.0564	0.0606	0.0630	0.0644	0.0654	0.0659	0.0663
4.8	0.0480	0.0495	0.0509	0.0522	0.0533	0.0576	0.0601	0.0616	0.0626	0.0631	0.0635
5	0.0451	0.0466	0.0480	0.0493	0.0504	0.0547	0.0573	0.0589	0.0599	0.0606	0.0610
6	0.0340	0.0353	0.0366	0.0377	0.0388	0.0431	0.0460	0.0479	0.0491	0.0500	0.0506
7	0.0263	0.0275	0.0286	0.0296	0.0306	0.0346	0.0376	0.0396	0.0411	0.0421	0.0428
8	0.0209	0.0219	0.0228	0.0237	0.0246	0.0283	0.0311	0.0332	0.0348	0.0359	0.0367
9	0.0169	0.0178	0.0186	0.0194	0.0202	0.0235	0.0262	0.0282	0.0298	0.0310	0.0319
10	0.0140	0.0147	0.0154	0.0162	0.0167	0.0198	0.0222	0.0242	0.0258	0.0270	0.0280

Equations for the Tables

Equations for square and rectangular stresses under the corners:

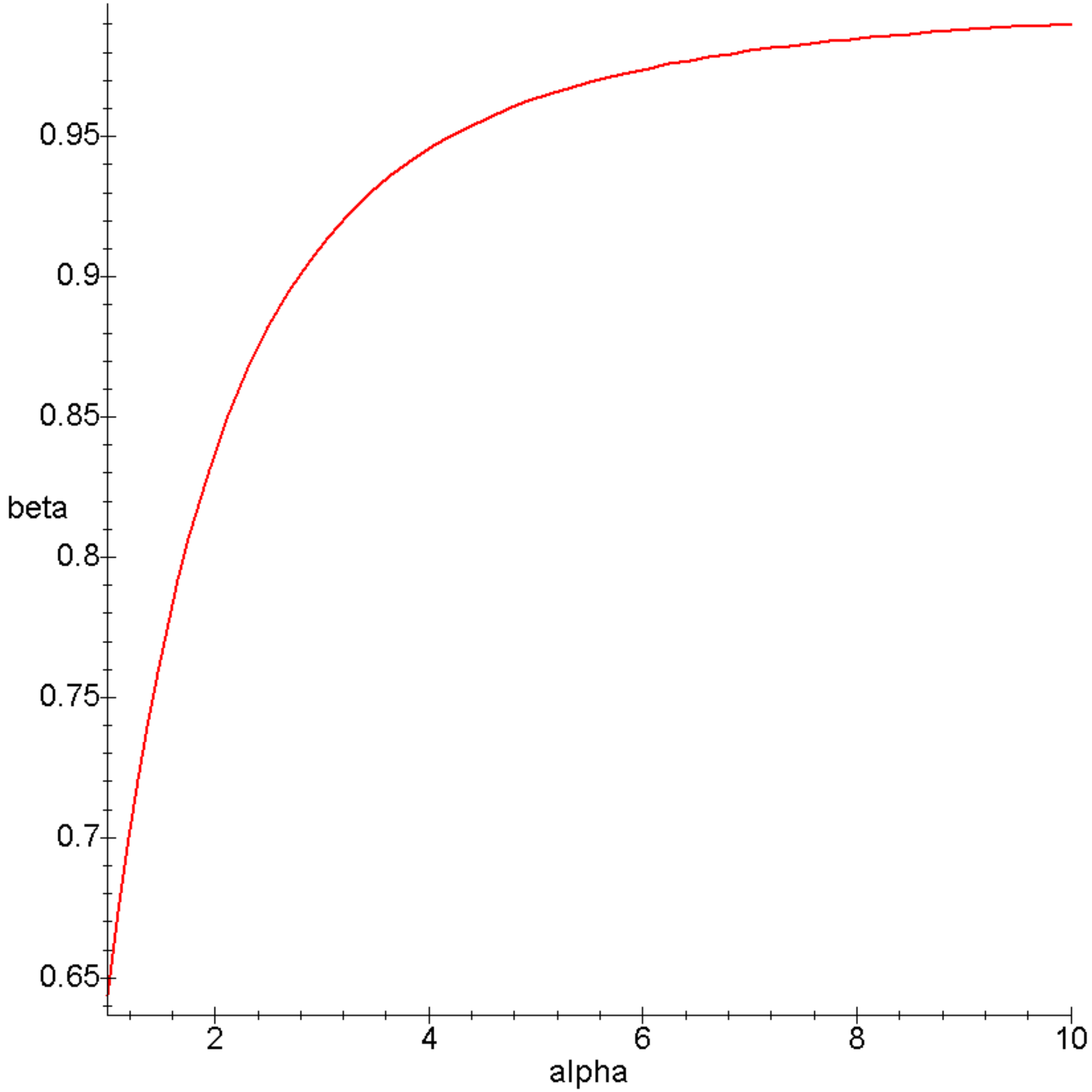
$$\beta \le 1/2\,\sqrt{2}\sqrt{\sqrt{1+6\,\alpha^2+\alpha^4}-1-\alpha^2}$$

$$K_c' = 1/4\left(2\,\frac{\alpha\,\beta\,(1+\alpha^2+2\,\beta^2)}{\sqrt{1+\alpha^2+\beta^2}\,(\beta^2\,(1+\alpha^2+\beta^2)+\alpha^2)}+\arcsin(2\,\frac{\alpha\,\beta\,\sqrt{1+\alpha^2+\beta^2}}{\beta^2\,(1+\alpha^2+\beta^2)+\alpha^2})\right)\pi^{-1}$$

$$\beta > 1/2\,\sqrt{2}\sqrt{\sqrt{1+6\,\alpha^2+\alpha^4}-1-\alpha^2}$$

$$K_c' = 1/4\left(2\,\frac{\alpha\,\beta\,(1+\alpha^2+2\,\beta^2)}{\sqrt{1+\alpha^2+\beta^2}\,(\beta^2\,(1+\alpha^2+\beta^2)+\alpha^2)}+\pi-\arcsin(2\,\frac{\alpha\,\beta\,\sqrt{1+\alpha^2+\beta^2}}{\beta^2\,(1+\alpha^2+\beta^2)+\alpha^2})\right)\pi^{-1}$$

Border Between Two Equations

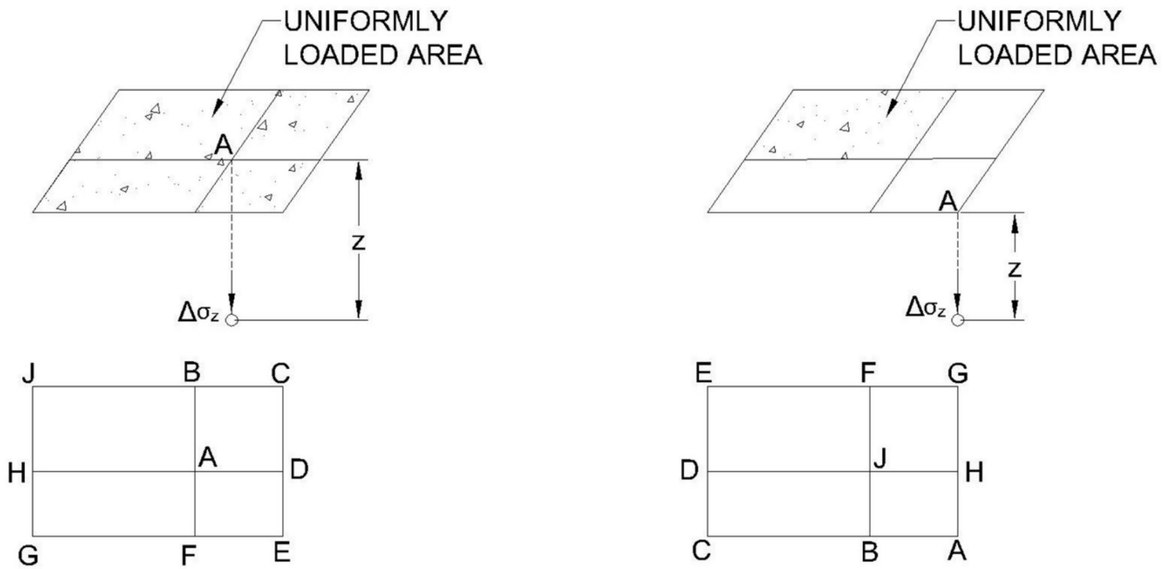


Example of Rectangular Chart Solution

- Given
  - Uniformly loaded rectangle, B = 4 m, L = 8 m, load 150 kPa
- Find
  - Soil stress 8 m below corner
- Solution
  - $\alpha = L/B = 8/4 = 2$
  - $\beta = Z/B = 8/4 = 2$
  - From charts (equations same,) $K'_c = 0.1202$, thus the stress is $(0.1202)(150) = 18.03$ kPa

- Obviously only being able to compute the stresses below the corner limits the direct application of the method
- We can, however, use superposition to obtain results for points not under a corner, even if they are not under the foundation at all
- Superposition is available because this is an elastic, path-independent method

Using Superposition with Boussinesq Charts



INFLUENCE FACTOR	RECTANGLE
I_1	ABCD
I_2	ABJH
I_3	AFGH
I_4	ADEF

$I_A = I_1 + I_2 + I_3 + I_4$

(a) Point below middle of rectangular area

INFLUENCE FACTOR	RECTANGLE
I_5	ACEG
I_6	ABFG
I_7	ACDH
I_8	ABJH

$I_A = I_5 - I_6 - I_7 + I_8$

(b) Point outside of rectangular area

Figure 4-8 Use of Superposition to Determine Change in Vertical Stress

Verrujt Newmark Example

- Given
  - Buildings as shown to the right
  - Yellow building has uniform load of 5 kPa
  - Brown building has a uniform load of 15 kPa
- Find
  - Vertical stress 8m below point A

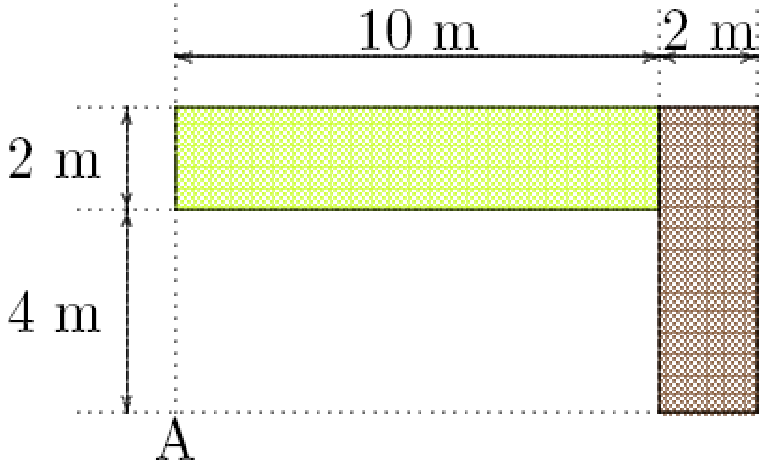


Figure 29.4: Example.

Verruijt Newmark Example

- Solution
  - Since there are two different loads, best way is to analyze two loads separately and add them together using superposition
  - Notation is per previous chart
  - In both cases, it was simpler to compute a “large” area and then subtract a void from that area
  - Influence coefficients were from the equations, can also be obtained from the tables

Yellow Foundation									
Rectangle	B, m	L, m	Z, m	alpha	beta	Iz or K'c	Pressure, kPa	Stress, kPa	
ABFG (+)	6	10	8	1.6667	1.3333	0.16	5.00	0.823	
ABJH (-)	4	10	8	2.5000	2.0000	0.13	-5.00	-0.637	
							Total	0.186	kPa
Brown Foundation									
Rectangle	B, m	L, m	Z, m	alpha	beta	Iz	Pressure, kPa	Stress, kPa	
ACEG (+)	6	12	8	2.0000	1.3333	0.17	15.00	2.551	
ABFG (-)	6	10	8	1.6667	1.3333	0.16	-15.00	-2.468	
							Total	0.083	kPa
							Complete Total	0.269	kPa

Boussinesq Circular Stresses (Analytic Solution)

As an example consider the case of a uniform load of magnitude p over a circular area, of radius a. The solution for this case can be found by integration over a circular area (S.P. Timoshenko & J.N. Goodier, *Theory of Elasticity*, paragraph 124), see Figure 28.3.

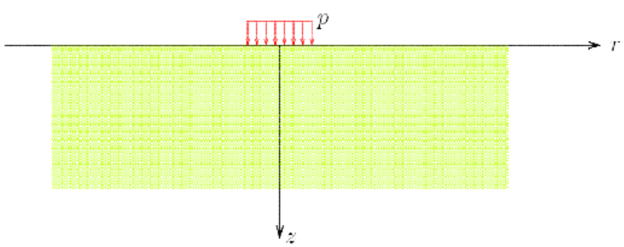


Figure 28.3: Uniform load over circular area.

The stresses along the axis $r = 0$, i.e. below the center of the load, are found to be

$$r = 0 : \quad \sigma_{zz} = p(1 - \frac{z^3}{b^3}), \tag{28.13}$$

$$r = 0 : \quad \sigma_{rr} = p[(1+\nu)\frac{z}{b} - \tfrac{1}{2}(1 - \frac{z^3}{b^3})], \tag{28.14}$$

in which $b = \sqrt{z^2 + a^2}$.

The displacement of the origin is

$$r = 0,\ z = 0 : \quad u_z = 2(1-\nu^2)\frac{pa}{E}. \tag{28.15}$$

is solution will be used as the basis of a more general case in the next chapter.

Another important problem, which was already solved by Boussinesq (see S.P. Timoshenko & J.N. Goodier, *Theory of Elasticity*, paragraph) is the problem of a half space loaded by a vertical force on a rigid plate. The force is represented by $P = \pi a^2 \overline{p}$, see Figure 28.4. The distribution of the normal stresses below the plate is found to be

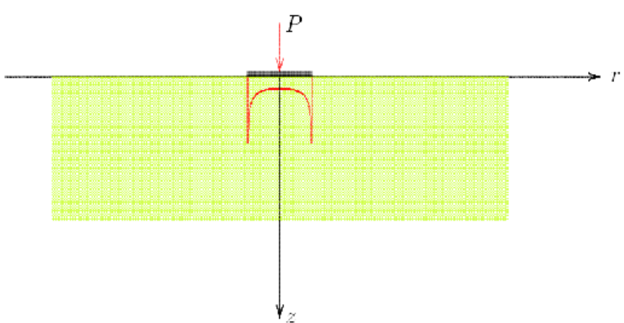


Figure 28.4: Rigid plate on half space.

$$z = 0,\ 0 < r < a : \quad \sigma_{zz} = \frac{\frac{1}{2}p}{\sqrt{1 - r^2/a^2}}. \tag{28.16}$$

This stress distribution is shown in Figure 28.4. At the edge of the plate the stresses are infinitely large, as a consequence of the constant displacement of the rigid plate. In reality the material near the edge of the plate will probably deform plastically. It can be expected, however, that the real distribution of the stresses below the plate will be of the form shown in the figure, with the largest stresses near the edge. The center of the plate will subside without much load.

The displacement of the plate is

$$z = 0,\ 0 < r < a : \quad u_z = \frac{\pi}{2}(1-\nu^2)\frac{\overline{p}a}{E}. \tag{28.17}$$

When this is compared with the displacement below a uniform load, see (28.15), it appears that the displacement of the rigid plate is somewhat smaller, as could be expected.

Circular Tank Example

- Given
  - Circular Tank, 25 metres diameter
  - Soil, 18 kN/m^3 unit weight, water table very deep
  - Weight of tank 6100 metric tons = 59,800 kN
- Find
  - Vertical stress induced by tank 10 metres below the centre of the tank
  - Effective stress induced by the overburden
  - Combined stress
- Solution
  - Bearing Pressure = 59,800 kN/491 m^2 = 122 kPa
  - Effective stress due to overburden = (10 m)(18 kN/m3) = 180 kPa
  - Stress induced at 10 m = 92.3 kPa (see below)
  - Combined stress with effective stress = 180+92.3 = 272.3 kPa

$$\sigma_z = p\left(1-\left(\frac{z}{b}\right)^3\right) = p\left(1-\left(\frac{z}{\sqrt{z^2+a^2}}\right)^3\right) = 122\left(1-\left(\frac{10}{\sqrt{10^2+12.5^2}}\right)^3\right) = 92.3kPa$$

Circular Stresses Other Than Directly Under the Centre

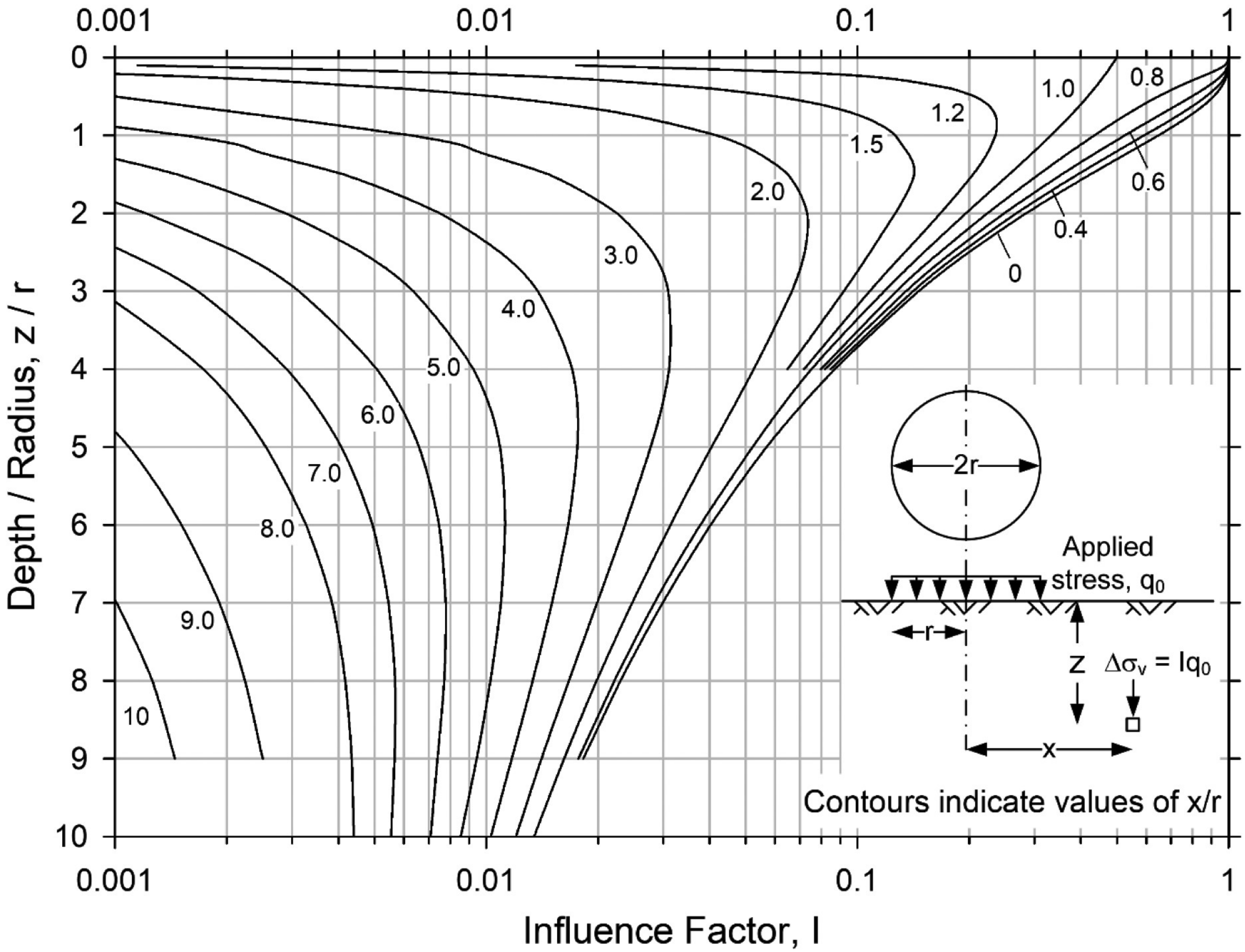


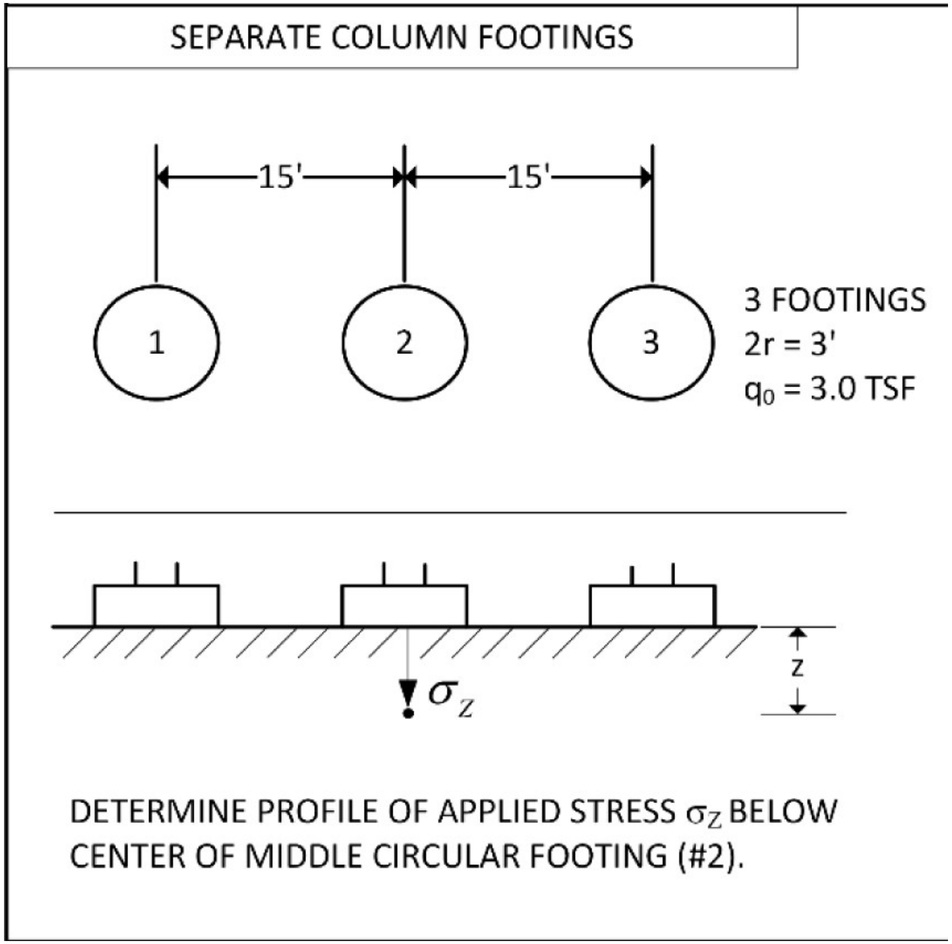
Figure 4-6 Influence Factors for a Circular Loaded Area – Boussinesq (after Ahlvin and Ulery 1962, Poulos and Davis 1974)

$I = K'_c$

$\Delta\sigma_v = Iq_0 = K'_c\, q_0$

Example of Circular Loads and Substitution of Point Loads

- Given
  - Problem shown at right
- Find
  - Vertical stress from load at a point 10' below the centre of the middle load 2 using three methods:
    - Assuming all three loads are point loads
    - Assuming all three loads are circular loads
    - Assuming the centre load 2 is a circle load (which allows us to use the equation of Verruijt) and loads 1 and 3 point loads (which does the same thing)

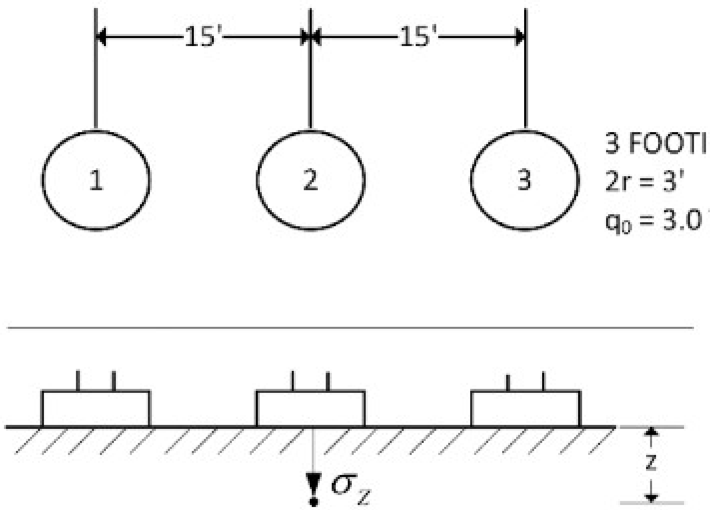


Example of Circular Loads and Substitution of Point Loads

- Point Load Method
  - $\sigma_z = 3Pz^3/(2\pi R^5)$ (Verruijt)
  - P = (3)(7.069) = 21.206 tons (for all columns)
  - $R = (4^2 + 15^2)^{0.5}$ = 15.524' (for columns 1 and 3)
  - R = 4' (for column 2)
  - σ_z = 0.001 tsf (columns 1 and 3)
  - σ_z = 0.633 tsf (column 2)
  - σ_z = 0.634 tsf (summation by superposition)
- Point Loads for Columns 1 and 3, Circle Load for Column 2
  - Stresses the same for Columns 1 and 3 = 0.001 tsf
- Circle Load for Column 2
  - $\sigma_z = p(1 - (z/b)^3) = q_0(1 - (z/b)^3)$ (Verruijt, directly under column)
  - q_0 = 3 tsf
  - z = 4'
  - a = 3/2 = 1.5'
  - $b = (4^2 + 1.5^2)$ = 4.272'
  - $\sigma_z = (3)(1 - (4/4.272)^3)$ = 0.537 tsf
  - σ_z = 0.539 tsf (by superposition)

Example of Circular Loads and Substitution of Point Loads

SEPARATE COLUMN FOOTINGS



3 FOOTINGS
$2r = 3'$
$q_0 = 3.0$ TSF

DETERMINE PROFILE OF APPLIED STRESS σ_Z BELOW CENTER OF MIDDLE CIRCULAR FOOTING (#2).

REFER TO FIGURE 4-6 (VERTICAL STRESS UNDER CIRCULAR LOADED AREA) TO DETERMINE INFLUENCE VALUE, I, FOR EACH FOUNDATION.

z (FT)	$\frac{z}{r}$	I_1 $x/r = 10$	I_2 $x/r = 0$	I_3 $x/r = 10$	$\sum I$	$\sigma_z = \Sigma I q_0$ (TSF)
2	1.33	0.000	0.646	0.000	0.646	1.939
4	2.67	0.000	0.200	0.000	0.200	0.600
6	4.00	0.001	0.087	0.001	0.088	0.265
10	6.67	0.002	0.035	0.002	0.039	0.117
15	10.00	0.003	0.015	0.003	0.021	0.062
20	13.33	0.003	0.009	0.003	0.015	0.045
25	16.67	0.003	0.007	0.003	0.013	0.039

Point Load Method, $\boldsymbol{\sigma}_z$ = 0.634 tsf

Point Loads for Columns 1 and 3, Circle Load for Column 2, $\boldsymbol{\sigma}_z$ = 0.539 tsf

Elastic Settlement

- Based on theory of elasticity
- Load applied at a point or over an area on a semi-infinite half space
- Can estimate both deflections and stresses
- Theory of Boussinesq most commonly used; will discuss stresses later
- Especially useful in computing the settlement of structures on firmer material, such as intermediate geomaterials and rock, but can also be used for soils for preliminary and initial settlements

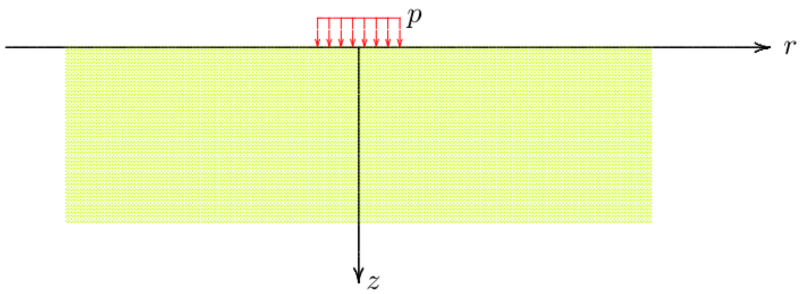


Figure 28.3: Uniform load over circular area.

As an example consider the case of a uniform load of magnitude p over a circular area, of radius a.

The displacement of the origin is

$$r = 0,\ z = 0\ :\quad u_z = 2(1-\nu^2)\frac{pa}{E}$$

Definitions of Soil, Rock and Intermediate Geomaterials (IGM)

The geotechnical specialist is usually concerned with the design and construction of some type of geotechnical feature constructed on or out of a geomaterial. For engineering purposes, in the context of this manual, the geomaterial is considered to be primarily rock and soil. A geomaterial intermediate between soil and rock is labeled as an intermediate geomaterial (IGM). These three classes of geomaterials are described as follows:

- **Rock** is a relatively hard, naturally formed solid mass consisting of various minerals and whose formation is due to any number of physical and chemical processes. The rock mass is generally so large and so hard that relatively great effort (e.g., blasting or heavy crushing forces) is required to break it down into smaller particles.

- **Soil** is defined as a conglomeration consisting of a wide range of relatively smaller particles derived from a parent rock through mechanical weathering processes that include air and/or water abrasion, freeze-thaw cycles, temperature changes, plant and animal activity and by chemical weathering processes that include oxidation and carbonation. The soil mass may contain air, water, and/or organic materials derived from decay of vegetation, etc. The density or consistency of the soil mass can range from very dense or hard to loose or very soft.

- **Intermediate geomaterials** (IGMs) are transition materials between soils and rocks. The distinction of IGMs from soils or rocks for geotechnical engineering purposes is made purely on the basis of strength of the geomaterials. Discussions and special design considerations of IGMs are beyond the scope of this document.

Elastic Settlements

$$S_e = \frac{\omega p B(1-\nu^2)}{E}$$

Where

- S_e = elastic settlement
- ω = geometry and rigidity coefficient (see table at right)
- p = uniform pressure on foundation
- B or b = width of foundation
- L or l = length of foundation
- ν = Poisson's Ratio
- E = Young's modulus of material
- Left side of table good for semi-infinite half space; right side good for a soil where a hard layer exists at a depth of h below the base of the foundation

Table 5.2

Values of Coefficients ω

Side ratio $\alpha = l/b$	ω for half-space				ω_{mh} for finite-thickness layer at h/b				
	ω_c	ω_0	ω_m	ω_{const}	0.25	0.5	1	2	5
1 (circle)	0.64	1.00	0.85	0.79	0.22	0.38	0.58	0.70	0.78
1 (square)	$\frac{1}{2}\omega_0$	1.12	0.95	0.88	0.22	0.39	0.62	0.77	0.87
2 (rectangle)	$\frac{1}{2}\omega_0$	1.53	1.30	1.22	0.24	0.43	0.70	0.96	1.16
3 Same	$\frac{1}{2}\omega_0$	1.78	1.53	1.44	0.24	0.44	0.73	1.04	1.31
4 Same	$\frac{1}{2}\omega_0$	1.96	1.70	1.61	—	—	—	—	—
5 Same	$\frac{1}{2}\omega_0$	2.10	1.83	1.72	—	—	—	—	—
10 Same	$\frac{1}{2}\omega_0$	2.53	2.25	2.12	0.25	0.46	0.77	1.15	1.62

Note: The coefficients ω for half-space are given according to F. Schleicher with the Author's corrections, and for a finite-thickness layer, according to M. I. Gorbunov-Posadov. The table gives the following coefficients:

ω_c for settlements of corner points of a rectangular loading area;
ω_0 for the maximum settlement under the centre of a loaded area;
ω_m for the average settlement over the whole loaded area;
ω_{const} for settlement of absolutely rigid footings;
ω_{mh} for the average settlement of rectangular loading areas on a finite-thickness soil layer.

Example of Elastic Settlement in Rock

- Given
  - Same circular foundation as before, only now seated on rock, RMR = 50 (see SFH Eq. 5-29)
  - E_s = 145,000 psi = 1,000,000 kPa
  - Poisson's Ratio = 0.33
- Find
  - Deflection at centre
- Solution
  - From previous table, ω = 1.0 (Same result can be found in SFH Table 8-13, C_d = 1.0) for flexible foundation
  - Result is computed below
  - Using Verruijt Equation 28.15 (online) will yield the same result

$$\delta_v = \frac{\omega p B \left(1 - v^2\right)}{E} = \frac{1 \times 122 \times 25 \times \left(1 - 0.33^2\right)}{1000000} = .0027\, m = 2.7\, mm$$

Equivalent Thickness Method to Estimate Elastic Soil Deflections

- It is possible to use elastic theory to estimate the deflections of soil for complex foundations shapes
- Some definitions:
  - Coefficient of Volume Compressibility $m_v = \boldsymbol{\beta}/E$
  - E = modulus of Elasticity
  - $\boldsymbol{\beta}=1-(2\boldsymbol{\nu}^2/(1-\boldsymbol{\nu}))$
  - $\boldsymbol{\nu}$ = Poisson's Ratio
  - We will explain these in more detail when we get to consolidation
  - h_{eq} = equivalent height of soil being compressed under a point
  - s = settlement at a point on the surface
  - p = uniform pressure on a given foundation
  - b = width (small dimension) of foundation

- The settlement of the soil at a point can be estimated by the equation

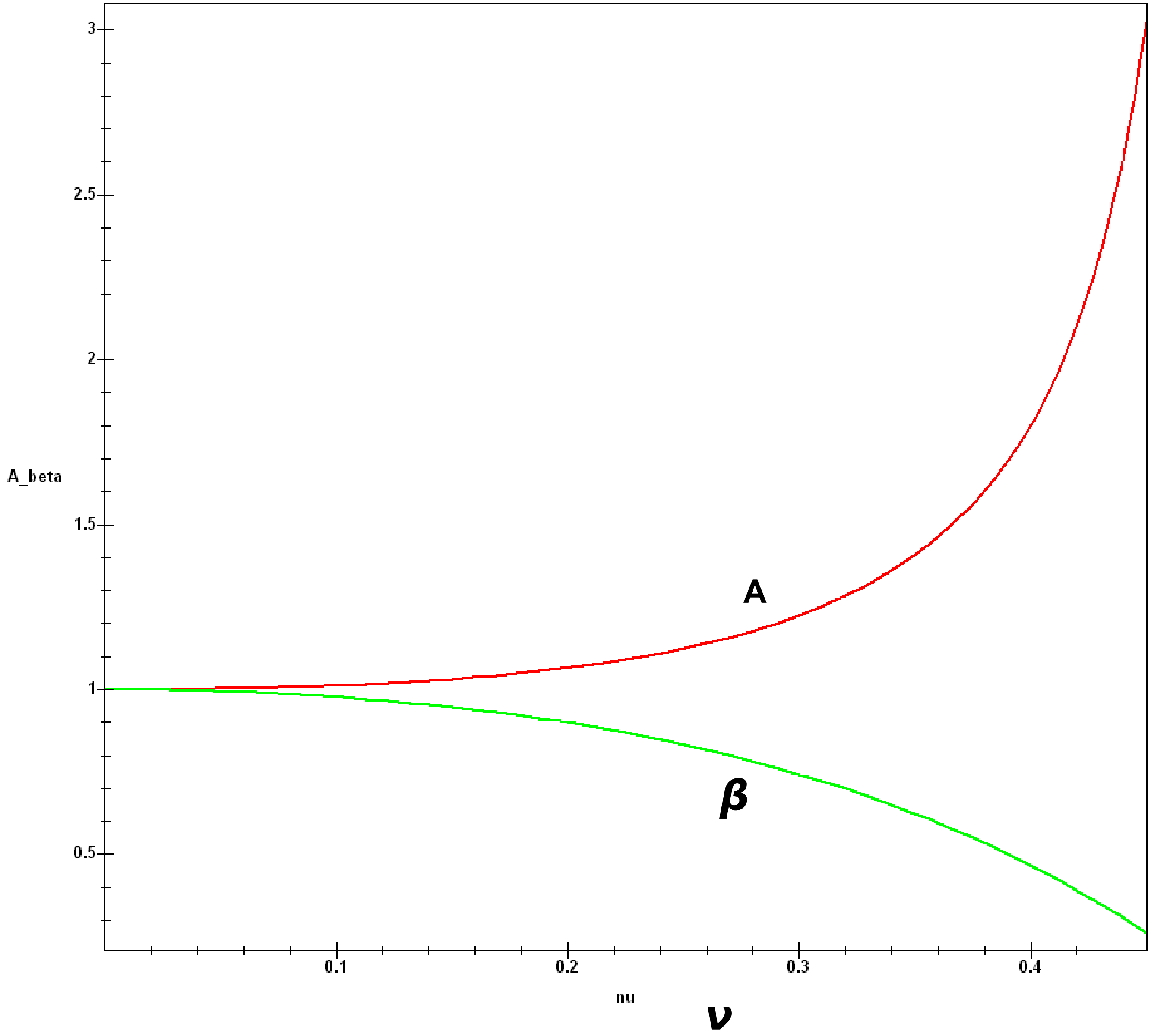
$$s=h_{eq}pm_v$$

- This leaves us to determine the equivalent height
- Equating the general equation for elastic settlements and solving for the equivalent height yields

$$h_{eq} = A\boldsymbol{\omega}b$$

where $A = (1-\boldsymbol{\nu})^2/(1-2\boldsymbol{\nu})$ and $\boldsymbol{\omega}$ is given in the earlier table

Values of A and $\boldsymbol{\beta}$ for Equivalent Thickness Method



Example of Elastic Thickness Method

- We will use the same example we used earlier from Verruijt for the stresses. The complete spreadsheet solution is given with the same spreadsheet
- The soil properties are as follows: E = 10,000 kPa, ν = 0.25
- The four regions (see earlier chart) are as follows:
  - Yellow Foundation Positive, corners ABFG, pressure +5 kPa
  - Yellow Foundation Negative, corners ABJH, pressure – 5kPa
  - Brown Foundation Positive, corners ACEG, pressure +15 kPa
  - Brown Foundation Negative, corners ABFG, pressure – 15 kPa
- The "hand solution" for the first region is as follows:
  - We compute A and β as follows:
    - $A = (1-0.25)^2/(1-(2)(0.25)) = 1.125$
    - $\beta = 1-(2)(0.25)^2/(1-0.25) = 0.833$
    - You can verify these using the plot of these parameters.
  - We compute the coefficient of volume compressibility m_v = 0.833/10000 = 0.0000833 1/kPa
  - We compute the value of $\alpha = l/b = 10/6 = 1.6667$
  - We compute the corner value for ω (since we are dealing with corners as was the case with stresses.) We can use the table for ω or we can compute it using the formulae from "Analytical Boussinesq Solutions for Strip, Square and Rectangular Loads," but it is ω = 0.71.
  - We compute the equivalent height h_{eq} = (1.125)(0.71)(6) = 4.8 m. *This value will be different for each corner point considered.*
  - The settlement for this portion of the analysis is s = (4.8)(5)(0.000083333) = 0.002 m = 1.998 mm.

Example of Elastic Thickness Method

Yellow Foundation							
Rectangle	B, m	L, m	alpha	omega (corner)	heq, m	Pressure p, kPa	Deflection, mm
ABFG (+)	6	10	1.6667	0.71	4.80	5.00	1.998
ABJH (-)	4	10	2.5000	0.83	3.76	-5.00	-1.565
						Total	0.433
Brown Foundation							
Rectangle	B, m	L, m	alpha	omega (corner)	heq, m	Pressure p, kPa	Deflection, mm
ACEG (+)	6	12	2.0000	0.77	5.17	15.00	6.462
ABFG (-)	6	10	1.6667	0.71	4.80	-15.00	-5.994
						Total	0.468
						Complete Total	0.901

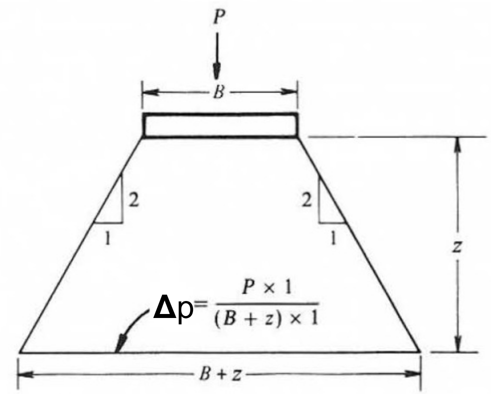
60 Degree Loading 2:1 Method

- For continuous foundations, formula in diagram at top
- For rectangular loads, formula in center diagram at right
  - Note the use of total load in equation
- For circular structures, it’s as easy to use the Boussinesq solution at the foundation centre:

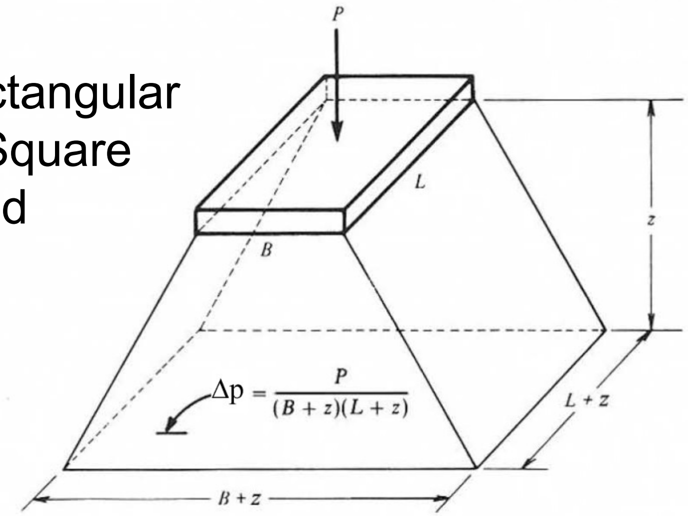
  $\Delta\sigma_z = q^*(1-(z/b)^3)$, $b = (r^2 + z^2)^{1/2}$

  - Note the use of the unit load in this form, not the total load
- Should not be used anywhere other than the center of the foundation

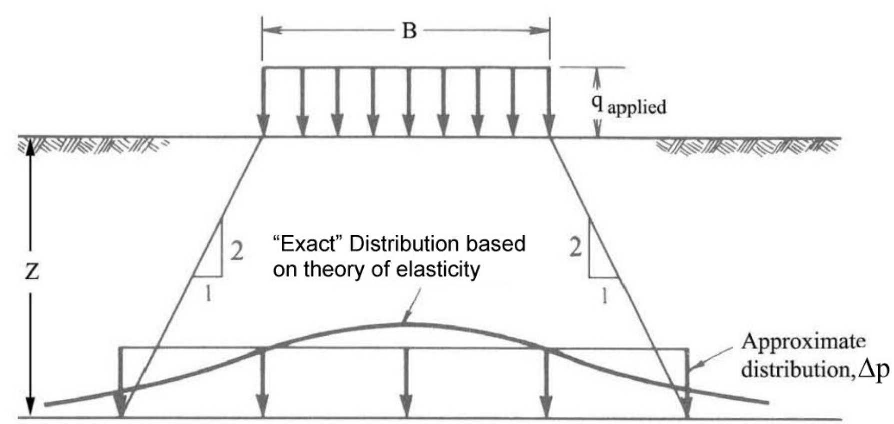
Strip Load



Rectangular or Square Load



(a)



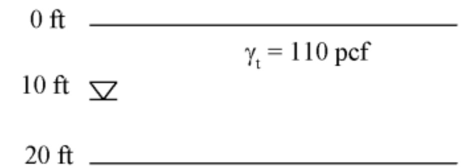
(b)

Figure 2-10. Distribution of vertical stress by the 2:1 method (after Perloff and Baron, 1976).

Combined Effective Stress and Applied Stress Example

The **Δp** is determined here using Boussinesq theory, 2:1 follows

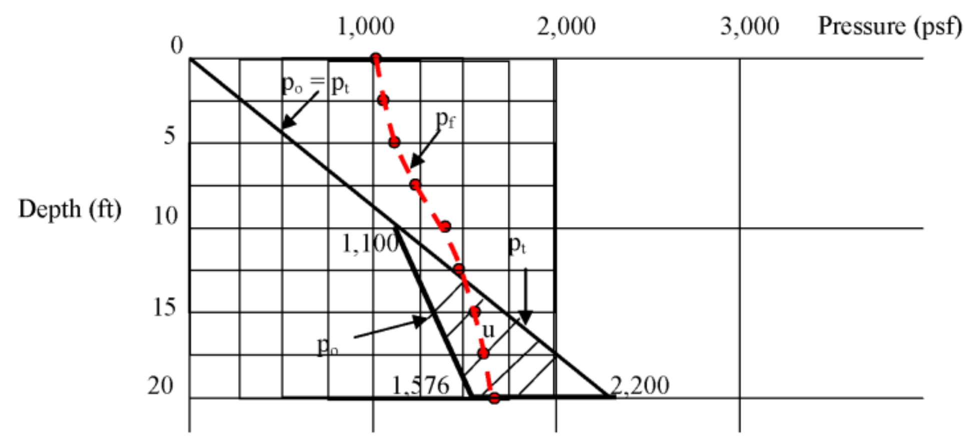
Example 2-2: For the Example 2-1 shown in Figure 2-7, assume that a 5 ft wide strip footing with a loading intensity of 1,000 psf is located on the ground surface. Compute the stress increments, Δp, under the centerline of the footing and plot them on the p_o diagram shown in Figure 2-7 down to a depth of 20 ft.



Solution:

For the strip footing, use the left chart in Figure 2-9. As per the terminology of the chart in Figure 2-9, B = 5 ft and q_o= 1,000 psf. Compile a table of stresses for various depths and plot as follows:

Depth z, ft	z/B	Isobar Value, x	Stress, Δp = x(q_o), psf	p_o, psf	p_f= p_o+Δp psf
2.5	0.5	0.80	800	(110)(2.5)=275	1,075
5.0	1.0	0.55	550	(110)(5.0)=550	1,100
7.5	1.5	0.40	400	(110)(7.5)=825	1,225
10.0	2.0	0.32	320	(110)(10.0)=1,100	1,420
12.5	2.5	0.25	250	1,100+(12.5-10.0)(110-62.4)=1,219	1,469
15.0	3.0	0.20	200	1,100+(15.0-10.0)(110-62.4)=1,338	1,538
17.5	3.5	0.18	180	1,100+(17.5-10.0)(110-62.4)=1,457	1,637
20.0	4.0	0.16	160	1,100+(20.0-10.0)(110-62.4)=1,576	1,736



Pressure Diagram

Figure 2-12. Example calculation of p_f with stress increments from strip load on p_o-diagram.

Stresses from Strip Loads using 2:1 Method

- Given
  - Problem as shown in previous slide
- Find
  - Additional Stresses due to strip load
- Solution
  - Formula for strip load stresses $\Delta p = P/(B+z)$, where P is the load per unit length of strip load
  - Can also be written in terms of unit load, $\Delta p = q_0/(1+z/B)$
  - For z=10', substitute as follows: $\Delta p = 1000/(1+10/5) = 333$ psf
  - Values for all depths at the right
  - These results are added to the final stress as the Boussinesq results are

Depth, ft.	Boussinesq Δp, psf	2:1 Δp, psf
2.5	800	667
5	550	500
7.5	400	400
10	320	333
12.5	256	286
15	200	250
17.5	180	222
20	160	200

Questions?