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Forks in the road: decisions that have shaped and will shape the teaching and practice of geotechnical engineering

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Article

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Abstract

Geotechnical engineering spans a wide range of applications, including tunnels, foundations, dams, and retaining structures. It deals with a material known to be difficult to model: a particulate material whose mechanical response is affected by all three invariants of the stress tensor, by loading rate, by density and by fabric. New problems and greater complexity in old problems have come about with the effects of climate change. Progress in certain technologies—notably artificial intelligence—also defines the new landscape in which geotechnical engineers must operate. This paper focuses on mechanics-based geotechnical engineering applications. The paper reviews some of the major decisions that were made by the engineers and researchers who developed geotechnical engineering to the point at which it was an identifiable separate discipline and the consequences that these decisions have had on the development of the discipline and on its teaching. The paper identifies some key modelling choices that were made that have had an undeservedly disproportionate impact on the teaching and practice of geotechnical engineering. The focus of the paper is therefore on these decisions and choices, and what should be taught in their place today. Challenges that future geotechnical engineers may face, as well tools that will be available to them, are also discussed in the context of what should be taught in undergraduate and graduate courses.

1. Empiricism, science and geotechnical design

1.1 The pre-science days

Construction in, on or with soil is nothing new: we have been building structures of the most varied types for millennia. One might infer from this that geotechnical engineering, which is the engineering of structures or systems of which soil is an integral part, would be a settled subject. However, the fact that we can design and construct does not mean that we do these things as well as we could, and it does not mean that the models that we use in analysis and design are correct.

In any type of activity, improved processes and products result from trial and error, but only up to a point. This attempt to arrive at better ways of doing things without a full understanding of the factors at play and their interrelationships is known to us as empiricism, and progress can at times be painful. An interesting twist in how both individuals and populations learn and add to knowledge in an empirical manner resulted from the development of the World Wide Web, the internet, and smart search engines. The combination of these three technologies, and the access by a large fraction of the Earth's population to them has given people much more access to knowledge and the possibility of experimenting with knowledge they find online, keeping what works, and discarding what does not. Whereas individuals in their daily lives and people working in the trades have benefited from the rapidly accumulating body of easily accessible specialized knowledge, it is possible to argue that the same is not true of a profession, which geotechnical engineering is. There are two reasons for this. One, common to all professions, is that new knowledge and its transmission are curated with a higher level of formality in professions. For a profession like geotechnical engineering, another reason for this is that, at least for the more challenging projects, the engineering profession today must rely on science, and science cannot be found or taught or developed so easily and so loosely. There is a method to science, and not to rely on science would take us back a hundred years, when results in terms of economy and safety were far from satisfactory.

There is a common misconception that all engineering done before the advent of science was conservatively done. The inference seems to be common-sensical, because it would be natural to proceed cautiously when one does not know very well what one is doing, i.e., when one is proceeding by trial and error. Ancient structures, with their robust pillars and

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arches, also convey this impression that we have always built conservatively. However, that has not necessarily been so. While cases of serious engineering failures would not have appeared in geotechnical scientific journals – because they, as such, did not exist before the first half of the 20th century – we can still learn about how things could go wrong in the pre-science days of geotechnical engineering by referring, for example, to court decisions. An interesting case is that of *Stees v. Leonard*. Here is an excerpt of a pertinent part:

> The action was brought to recover damages for a failure of defendants to erect and complete a building on a lot of plaintiffs, on Minnesota street, between Third and Fourth streets, in the city of St. Paul, which, by an agreement under seal between them and plaintiffs, the defendants had agreed to build, erect, and complete, according to plans and specifications annexed to and made part of the agreement. The defendants commenced the construction of the building, and had carried it to the height of three stories, when it *fell to the ground*. The next year, 1869, they began again and *carried it to the same height as before, when it again fell to the ground*, whereupon defendants refused to perform the contract. Stees v. Leonard, 20 Minn. 494, 449 (Minnesota Supreme Court, 1874) [emphasis added].

There are other cases like this recorded in court proceedings that show the inadequacy of a trial-and-error approach, which lacks a basis on the underlying science. The number of events is most certainly a multiple of those we can learn about from consulting such records. Starting a geotechnical engineering course with a case history like this and following that with a discussion of the scientific method gives students an appreciation for what the subject is about, its importance, and why science matters.

1.2 The development of the science

The scientific method is the formulation of a hypothesis about some question or problem and then the idealization and execution of experiments to validate the hypothesis. If the hypothesis is properly validated, we have a model, which we can then use to guide further scientific inquiry or the development of engineering design methods.

Until the early 20th century, all that anyone working with soil and rock could count on was empirical knowledge. It was not until scientists like Philipp Forchheimer (whom his student, Karl Terzaghi, later emulated in many respects) started seeking to frame some flow problems as boundary-value problems (Goodman, 1999) that the science of soil mechanics started coming into form. It was a natural step to go from flow problems to consolidation theory, in which flow is coupled with deformation, and that development is credited as the birth of soil mechanics. Although Terzaghi's one-dimensional consolidation theory was imperfect (see Goodman (1999), Salgado (2008), or Salgado (2022b) for an account of why that is so and of the sad events involving Terzaghi and Forchheimer) its flaws were not fatal to its application to a range of practical problems, and it was by no means a misstep. It will not be discussed further in the present paper.

Once consolidation theory was in place, the same general approach—looking for the science to underpin design methods in the incipient engineering discipline that we now call geotechnical engineering—was followed for other problems. Bearing capacity theory, as an example, follows from work done during the industrial revolution on metal indentation (Prandtl, 1920, 1921; Reissner, 1924). This path was by no means easy. Faced with hurdles, the pioneers took detours and made decisions that have had significant implications for how geotechnical engineering is practiced and how it is taught at universities even today.

1.3 Structure of the paper

This paper examines how these difficulties and resulting decisions, many related to how to model the mechanical response of soil, have shaped the development of the discipline and its teaching. Understanding of soil mechanics is vastly superior today. The paper puts forth some ideas regarding key content that should be taught at the undergraduate and graduate level that is consistent with current understanding and that—contrary to opinions sometimes expressed—is easily learned by students. Due to space limitations, the paper covers only three of the fundamental model choices that shaped soil mechanics and geotechnical engineering, but there are more.

The three topics addressed are the use of the Mohr-Coulomb and Tresca yield criteria to model soil shear strength, the use of an associated flow rule with these models, and the neglect of shear strain localization. These choices have guided the development of the discipline and have led to a significant body of work. Among topics not covered are the reliance of analyses on infinitesimal strains, the neglect of fabric effects on material response, and the use of total-stress undrained analyses in clays.

A final topic is the decisions and choices that we, as subject matter experts, are making now and the possible impact that teaching decisions relating to these choices may have on the future of the discipline. This relates, in particular, to the themes that are currently being identified by many researchers as the future of geotechnical engineering: artificial intelligence and analysis of soil as a particulate medium (i.e., a "micro" as opposed to a "macro" approach). But I also briefly discuss three additional timely topics as they relate to geotechnical engineering: mining tailings, climate change and offshore wind energy.

2. The original sin: soil as a Mohr-Coulomb material and clay as a "cohesive" material

2.1 Background

Traditionally, soil, although a particulate material, has been treated as a continuum—a solid, to be more precise.

Solids—when plastic—may experience plastic deformation when their shear strength is exceeded. This strength may depend on the normal effective stress on the eventual plane of shearing, or it may be independent of it. The first type of shear strength is known as frictional; the second type, as cohesive.

To understand why, today, students learn that there are two types of soils—"cohesionless soils" and "cohesive soils"—we must travel back to the 1950s, when the science of soil mechanics was in development. After a relatively successful study of 1D consolidation using the coupling of deformation with flow, Terzaghi and co-workers set about dealing with problems involving shear strength, such as the calculation of the bearing capacity of foundations.

The state of the mechanics of foundations at the time was fundamentally this: little progress had been made in the practice of foundation engineering in the preceding century. We discussed earlier the case of *Stees v. Leonard*, in which a contractor tried, not once, but twice, to erect a building on soil that could not support it. In the lawsuit that followed, they misidentified the cause of the problem, which was a bearing capacity problem, as the existence of "quick sand" at the site. But, even as the understanding that one must design against bearing capacity "failures"—i.e., bearing capacity ultimate limit states—started forming, the means to calculate this bearing capacity lagged behind.

The practice of foundation engineering was to try to build based on prior experience, an experience that was often not applicable to the conditions at hand. In this environment, in which scientific knowledge hardly existed, it is not surprising that Terzaghi believed that "[…] [b]ecause of the unavoidable uncertainties involved in the fundamental assumptions of the theories and in the numerical values of the soil constants, simplicity is of much greater importance than accuracy" (Terzaghi & Peck, 1967, p. 153). This thinking permeated much of Terzaghi's work at the time, and it is therefore no surprise that he also believed that "[…] [i]n spite of the apparent simplicity of their general characteristics, the mechanical properties of real sands and clays are so complex that a rigorous mathematical analysis of their behavior is impossible" (Terzaghi, 1943, p. 5).

We now know that there are three things that are incorrect in Terzaghi's two statements. First, simplicity and accuracy are not necessarily antithetical. Something can be both simple and inaccurate, and *vice versa*. To state that something simple but inaccurate is superior to something not simple but accurate does not appear sensible. Second, the mechanical properties of sand and clay are not even apparently simple. Refer to Figure 1 and Figure 2 for stress-strain plots for sand and clay sheared under drained and undrained conditions in triaxial compression. The stress q in the figures is the Mises shear stress (a multiple of the octahedral shear stress). Without an understanding of the mechanics of these soils, it is impossible to make sense of transitions and reversals between contractive and dilative response, of the existence of a peak shear stress to normal effective stress ratio, of the existence of a critical state, or of the transition to a residual strength at large shear strains and sufficiently large normal effective stresses for clays. Lastly, the final part of Terzaghi's second statement is also (today) incorrect, because researchers are developing fairly rigorous relationships for modeling soil behavior. Monotonic mechanical response is not considered today a challenge to model (see e.g., Chakraborty et al., 2013b; Dafalias & Herrmann, 1986; Li & Dafalias, 2000; Loukidis & Salgado, 2009b; Manzari & Dafalias, 1997; Woo & Salgado, 2015). Figure 1 and Figure 2 show simulations done using an advanced constitutive model that clearly match the experimental response quite well.

Faced with what he deemed an impossibility, it is not surprising that Terzaghi proposed the concepts of an "ideal sand"—a linear elastic, perfectly plastic Mohr-Coulomb type of material with non-zero friction angle and $c = 0$—and an "ideal clay" —a linear elastic, perfectly plastic material following a Tresca yield criterion (Terzaghi, 1943). Terzaghi referred to this material as a "cohesive" material, a term that survives to this day. As to sand, engineers soon started assuming non-zero cohesion also for sand, deviating from the original "ideal sand" concept that Terzaghi had advanced.

So the Mohr-Coulomb yield criterion (Figure 3a) would be used for sand, and the Tresca yield criterion (Figure 3b) would be used for clay. The only way to understand this postulation is to assume that Terzaghi observed increasing strengths for sand tested at increasing confining stresses under drained conditions, but constant strength for clay with increasing *total stresses* when samples were tested under undrained conditions. Based on this limited set of observations, Terzaghi postulated behaviors for soil that are not real. To this, Schofield (1988) later referred as "Terzaghi's error." This criticism is tempered by the recognition that the "ideal clay" model turned out to be an effective basis to build a body of analysis for problems involving saturated clay, and that even erroneous models of soil behavior were better than the crude form of knowledge available in those days. Additionally, the greater harm concerning sands was the subsequent use of a Mohr-Coulomb material with nonzero cohesion for sand, rather than the original ideal sand concept. Consequently, some viable theories have evolved from these simple "ideal" soil models, but the failure to accurately describe the sources of shear strength in soils remained.

2.2 The error

We have argued that Terzaghi's "ideal sand" and "ideal clay" models led to an erroneous description of soil behavior. This is true even if one is simply interested in calculating shear strengths and has no interest in realistically simulating any other aspects of behavior. But why is it so? For the answer, we look to plasticity theory.

Perhaps nothing has been as damaging to the teaching of soil mechanics than the notion that soil can generally be considered

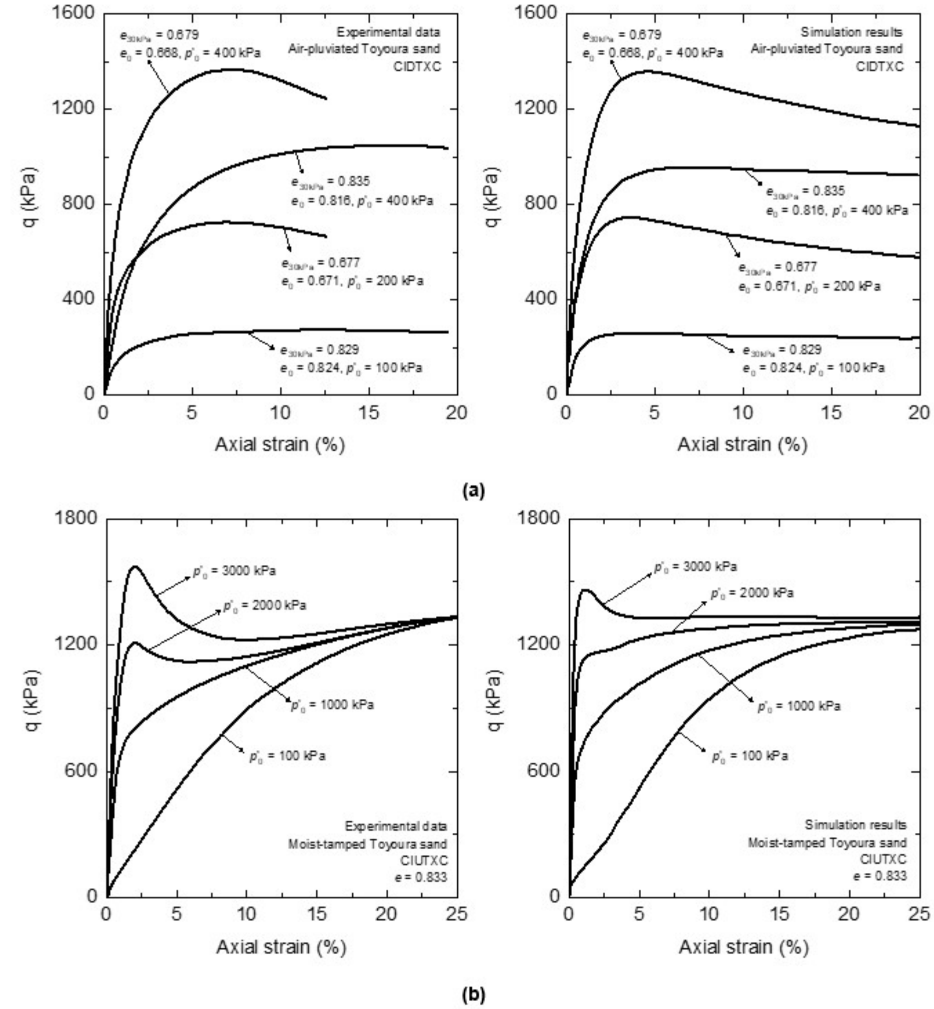

Figure 1. Results of triaxial compression tests performed on sands (left) and respective simulations (right): (a) drained; (b) undrained (Woo & Salgado, 2015).

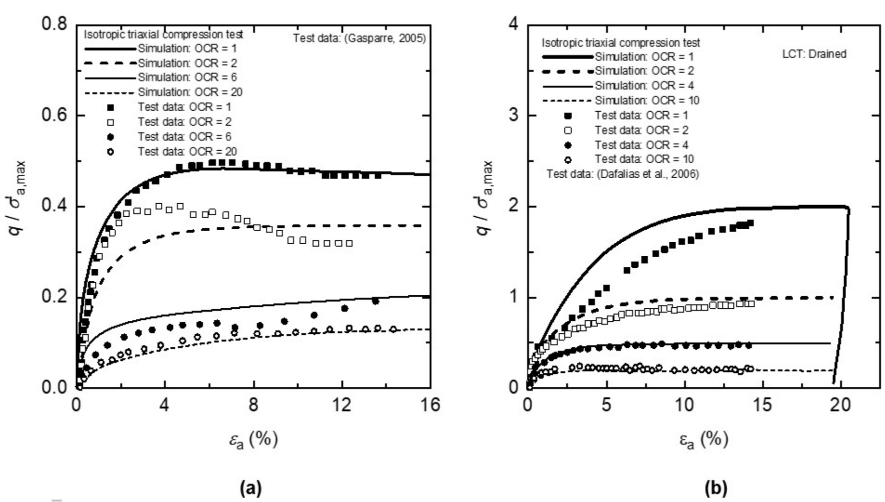

Figure 2. Results of triaxial compression tests performed on clays: (a) undrained (b) drained (Chakraborty et al., 2013b; Dafalias et al., 2006; Gasparre, 2005).

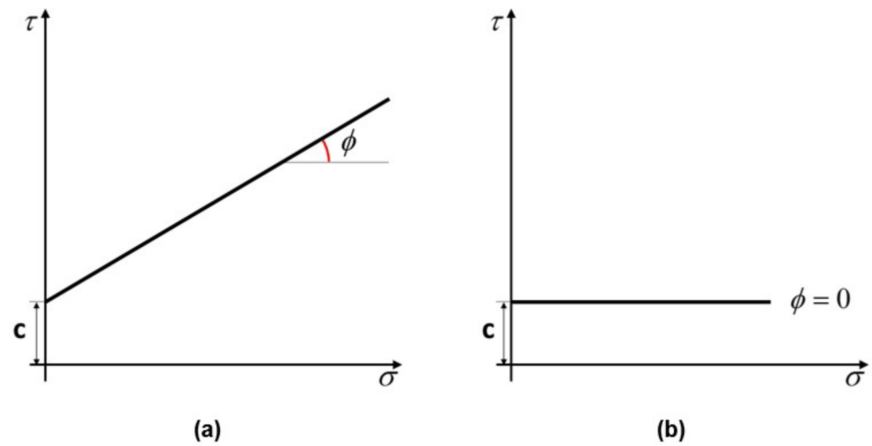

Figure 3. Relationship between normal and shear stresses for (a) a Mohr-Coulomb material, idealized in the 1950s as an "ideal sand" if *c*= 0 and (b) a Tresca material, idealized in the 1950s as an "ideal clay".

to follow the Mohr-Coulomb yield criterion. A material that follows the Mohr-Coulomb yield criterion experiences plastic strains only when the stress state satisfies the relationship:

$$F = \left(\sigma_1 - \sigma_3\right) - \left(\sigma_1 + \sigma_3\right)\sin\phi - 2\mathrm{c}\cos\phi = 0 \qquad (1)$$

where σ_1 and σ_3 are the maximum and minimum principal stresses, respectively. The function F of stresses is referred to as the yield function, and $F = 0$ is referred to as the yield criterion. The parameters ϕ and c are the friction angle and the cohesion, respectively, of the material. Terzaghi's ideal sand has non-zero ϕ and $c = 0$, and the ideal clay has zero ϕ. As discussed earlier, in later work, engineers abandoned the original concept of zero c in sand and started using nonzero c and ϕ to describe sand. No explanation was provided for the source of what should amount to a frictional strength component and a stress-independent (frictionless or cohesive) strength component. What this step left both educators and practitioners with was a model that was not based on an understanding of soil behaviour, since ϕ and c were the starting point of the analysis, that is, the model fundamental parameters.

Unfortunate implications of this paradigm were the misunderstanding that clean, uncemented sands could have non-zero c, and that clays had a constant c, a result directly implied by the "ideal clay" model. Initially, educators taught students that a set of tests had to be done at more or less the "appropriate" level of effective stresses, and straight-line fits to the corresponding data points would yield the correct values of ϕ and c. This presented a variety of questions, one of which regarded the applicable level of effective stress for a problem in which the soil experiences a wide range of stress levels, as in the bearing capacity problem. In some of these problems, stresses can be as high as several or even tens of megapascals. Clearly, performing shear strength tests at these elevated stress levels was not realistic.

Fortunately, even as Terzaghi made these influential choices, others (e.g., Taylor, 1948) were attempting to understand what the real sources of shear strength were. Taylor laid the foundation for what would later be known as critical-state soil mechanics. In this framework for the mechanics of soil, soil is a frictional material capable of volume change; a second source of shear strength results from this dilative response.

2.3 What should be taught instead

What emanated from the studies of Roscoe et al. (1958), Schofield (2006), Taylor (1948) and others was the understanding that soil is always a frictional material. In the absence of cementation, a fully saturated or completely dry soil derives its strength exclusively from friction if under sufficiently high confining stress and/or sufficiently low density (see, e.g., Salgado 2022b). Although beyond the scope of this paper, current understanding of the mechanical response of unsaturated soil also points to suction being translated into a greater effective stress, with correspondingly greater frictional strength. If either density is sufficiently high or effective stress is sufficiently low, soil also derives its strength from dilatancy.

It follows that, whether teaching at the graduate or undergraduate level, we should teach our students that soil takes its strength from two sources: friction and dilatancy. It is essential to stress that unstructured soil (soil without cementation or any source of extraneous cohesion) is frictional, lacking cohesion. A good starting point for this discussion is plastic deformation in the absence of any tendency to change volume: the so-called critical state. Surprisingly, based on anecdotal evidence, this is a concept to which undergraduate students are often not exposed. The concept is, however, easy to teach. The easiest way to teach it is to show students that the critical state is simply a purely frictional state. At critical state, the soil derives its strength from the frictional strength between soil particles, there being no other source of shear strength. And frictional strength only exists in the presence of non-zero effective normal stress.

It is sometimes surprising to students who have somehow learned otherwise that even clays are purely frictional materials. An example that can be used to get this last point across is that of a clay deposit forming at the bottom of a lake (Salgado, 2022b). It is easy for students to understand that the soil right at the surface of the bottom of the lake, composed of particles that have recently deposited out of water, lacks shear strength. The reason for that is that the clay there is essentially a slurry: it is under zero effective stress and has a very high void ratio. In the absence of nonzero normal effective stress, that clay has zero shear strength because it is a frictional material. An example for sand that can be given, to which undergraduate students can easily relate, is that someone picking up some sand on the beach can easily manipulate the soil, for it lacks strength, and it lacks strength because it is under nearly zero normal effective stress.

The other component of shear strength is due to dilatancy, which can best be explained by referring to a figure such as Figure 4, which shows that spherical particles that are closely packed must separate in the direction normal to that of shearing. This separation must occur against an existing normal effective stress, which requires work to be done. Where does the work come from? From the applied shear stress. So, the applied shear stress must overcome not only frictional strength to cause the material to deform plastically, but also this confining stress opposing the required soil dilation.

These two concepts are easy for students to understand. This basic understanding of the physical processes underlying shear strength development in soil can then be used throughout their course of study of geotechnical engineering applications (retaining structures, foundations, slopes and other structures), and should effectively inoculate them against the flawed concepts of "cohesive" or "cohesive-frictional" soils. From that point on, students will understand that soils are truly potentially dilative, frictional materials.

At the undergraduate level, one of the easiest ways to teach how dilatancy works is to use the Bolton (1986) friction angle calculation framework for sands. This work has been extensively referred to and has been extended to apply to sands with fines (see, e.g., Carraro et al., 2009; Salgado et al., 2000) and sands at low confining stresses (Chakraborty & Salgado, 2010). Concisely, for a sand, the peak friction angle ϕ_p is written as the summation of a critical-state friction angle ϕ_c and an angle due to dilatancy:

$$\phi_p = \phi_c + A_\psi I_R \tag{2}$$

where A_Ψ is a parameter in Bolton's equation having value of 3 for triaxial conditions and 5 for plain-strain conditions, and I_R is the relative dilatancy index given by:

$$I_R = I_D(Q - \ln p') - R \tag{3}$$

where I_D is relative density, p' is the mean effective stress and *Q and R* are fitting parameters.

This is essentially the approach that I follow in my geotechnical engineering text (Salgado, 2022b). In the introductory soil mechanics course, I follow a Socratic approach in in-person sessions combined with a variety of content delivery methods (such as reading assignments, video lectures and problem-solving sessions) and assessment methods (including quizzes; laboratory reports and laboratory quizzes; exams; and a term project). The use of the Socratic sessions in which everyone participates during the semester allows verification that the students have been able to learn these concepts quite well.

At the Ph.D. level, one must go much beyond this. It is important then to cover constitutive modeling (mainly the most recent models, such as bounding-surface or two-surface models) and particle-based methods.

3. Compounding the original sin: reliance on the associated flow rule

3.1 Background

The teaching of geotechnical engineering tends to emphasize stresses, but strains are just as much a part of the solution to any boundary-value problem in geomechanics. The only exposure that students seem to get to strains is through stress-strain plots typically shown or obtained in the laboratory and through the coverage of consolidation. A standard discussion surrounds the facts that loose sands

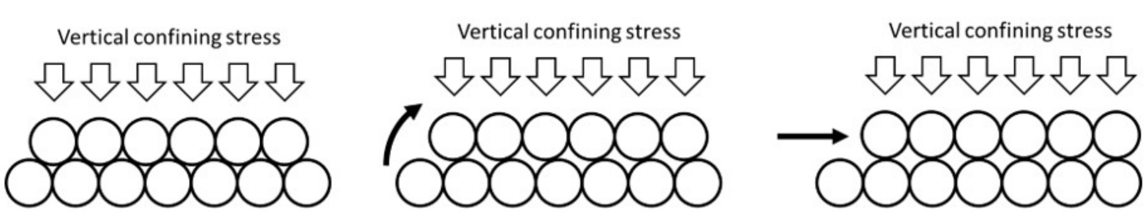

Figure 4. Particle climbing action for densely arranged particles (Salgado, 2022b).

contract or dilate less than dense sands and that dense sands may contract initially, but then end up being ultimately dilative. Strains are typically not linked back to stresses with any rigor, and that is sometimes true even at the graduate level. Yet, this link is crucial to the modeling of the mechanical response of soil.

The relationship is rather obvious to students in the context of elasticity. There is a general sense that application of a stress increment leads to a strain increment, and that its removal returns the body to its original configuration. When it comes to plasticity, matters turn more complex.

The rate of the plastic strain tensor in classical plasticity models is obtained from the plastic flow rule:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial G}{\partial \sigma_{ij}} \tag{4}$$

where i and j are indices taking values 1, 2 or 3; σ_{ij} are the six components of the (symmetric) stress tensor; $\dot{\lambda}$ is the plastic multiplier; and G is the plastic potential, a function of the stress tensor:

$$G = G(\sigma) \tag{5}$$

Given that there are six independent stress components, Equation 4 states that the plastic strain increments or rates are determined by a six-dimensional surface defined by Equation 5. The meaning of the term $\partial G/\partial\sigma_{ij}$ is that of a gradient in that space. This can best be visualized if we represent the stress tensor using its three principal stresses, in which case we are able to represent these equations in 3-dimensional space (see Figure 5). The gradient can then be visualized as being normal to the 3-dimensional surface defined by Equation 5. This visualization of a 6-dimensional process in 3-dimensional space can only be taken so far, as discussed by Woo & Salgado (2014).

If the gradient is aligned with the σ_1 axis, for example, that means that only the ε_1 strain component will change, with $\dot{\varepsilon}_2 = \dot{\varepsilon}_3 = 0$. So $\partial G/\partial\sigma_{ij}$ determines the proportion or ratio between each pair of strain rate components.

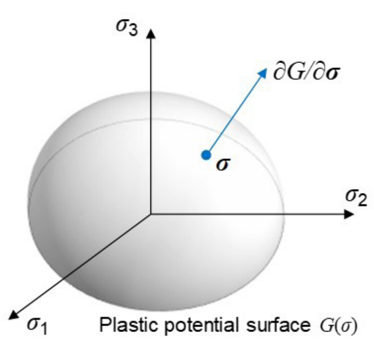

Figure 5. Plastic potential surface represented in principal stress space and its stress gradient, which enters the formulations of the flow rule.

In metal plasticity, which developed considerably during the industrial revolution, it was observed that there was no plastic volume change during plastic deformation. Although we don't show this here, this leads to the result that plastic strain rate is normal to the yield surface given by Equation 1 if plastic strain rates are plotted in the same space (with a separate scale) as stresses. This led to the adoption of what we now call an associated flow rule for the plastic strain rate, where F is used as the plastic potential:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial F}{\partial \sigma_{ij}} \tag{6}$$

If we are working with clays using total stresses in undrained loading simulations, we are in effect using Terzaghi's "ideal clay" model. There is then no volumetric strain, and Equation 6 is applicable. In drained simulations or effective-stress simulations, an associated flow rule does not apply. This can be observed by performing experiments and observing the lack of normality between the plastic strain rate and the yield surface. However, it is important to understand what the fundamental error of use of an associated flow rule is in those cases.

3.2 The error

A material undergoing plastic deformation (yielding), in contrast with only elastic deformation, dissipates energy. We can think of energy dissipation as the energy that has to be expended to change the material internally (i.e., to permanently deform it in some manner). The rate of plastic energy dissipation D_p per unit volume for infinitesimal-strain plasticity is given by:

$$D_p = \sigma_{ij}\dot{\varepsilon}_{ij}^{p} \tag{7}$$

where σ_{ij} is the stress, and $\dot{\varepsilon}_{ij}^{p}$ is the time rate of plastic strain.

Taking Equation 1 and Equation 6 into Equation 7, we obtain the following for the rate of plastic dissipation:

$$D_p = \dot{\lambda}[2c\cos\phi] \tag{8}$$

What Equation 8 tells us is that the rate of plastic energy dissipation is entirely due to the existence of a cohesion c. If $c = 0$, then no energy is dissipated during plastic flow. If we think of sand in realistic terms, it has no cohesion. So Equation 8 is telling us that the shearing of sand does not require energy dissipation, which we know to be incorrect. This result is also baffling to the typical graduate student. How can a cohesive-frictional material—that is what a Mohr-Coulomb material is supposed to be—dissipate no energy upon plastic deformation when $c = 0$? Is friction not intricately linked to energy dissipation?

The inescapable conclusion is that the use of the Mohr-Coulomb yield criterion with an associated flow rule to model real soils in effective-stress analysis is simply wrong. Sand, loaded under drained conditions, which corresponds to the vast majority of applications involving sands, cannot be modeled with a Mohr-Coulomb model even as an approximation, unless a flow rule that is not associated is used. Unfortunately, drained analysis with a Mohr-Coulomb material and an associated flow rule is what a large body of work in geotechnical engineering is based on. This is the content that many geotechnical engineering students get in the classroom, likely without elaboration about the limitations of the concepts.

3.3 What must be taught instead

If one must use the Mohr-Coulomb model, it is important not to teach any of the theories in which an associated flow rule was assumed and, where needed, stress that the flow rule for a Mohr-Coulomb material cannot be associated if realism is to be achieved. This difference is far from just conceptual, with important numerical consequences.

Consider, for example, the bearing capacity problem in sand. The unit bearing capacity q_{bL} in sand can be seen as the summation of two terms:

$$q_{bL} = q_0 N_q + \frac{1}{2}\gamma B N_\gamma \tag{9}$$

where q_0 = overburden stress, γ = unit weight, and N_q and N_γ are bearing capacity factors. We ignore any depth correction factor that might be incorporated into Equation 9 for the purposes of the discussion that follows. The classical equations for the two bearing capacity factors are:

$$N_\gamma = 1.5\left(N_q - 1\right) \tan\,\phi \tag{10}$$

and

$$N_q = \frac{1+\sin\phi}{1-\sin\phi} e^{\pi\tan\phi} \tag{11}$$

Equation 10 is due to Brinch Hansen (1970), who proposed it based on results from the method of characteristics. The method of characteristics assumes an associated flow rule, as does most of the work published using limit analysis. We now know that these two equations cannot be correct, for sand does not follow an associated flow rule. How innacurate are the results? We can answer this by referring to the equations proposed by Loukidis & Salgado (2009a) for a sand with a non-associated flow rule:

$$N_q = \frac{1+\sin\phi}{1-\sin\phi} e^{J(\phi,\psi)\pi\tan\phi} \tag{12}$$

and

$$N_\gamma = \left(N_q - 1\right)\tan(1.34\phi) \tag{13}$$

where J is a function given by

$$J(\phi,\psi) = 1 - \tan\phi\left[\tan(0.8(\phi-\psi))\right]^{2.5} \tag{14}$$

and ψ is the dilatancy angle.

The dilatancy angle in simple shear loading is defined as:

$$\sin\psi = -\frac{\dot{\varepsilon}_{\mathrm{v}}}{\dot{\gamma}_{\max}} \tag{15}$$

where $\dot{\varepsilon}_v$ is the time rate of volumetric strain and $\dot{\gamma}_{\max}$ is the rate of the maximum shear strain.

The dilatancy angle is a measure of how much volumetric strain results from shearing of the material. A flow rule associated with the Mohr-Coulomb yield function leads to $\psi = \phi$. It is more realistic for sands to assume $\psi < \phi$. This would correspond to a non-associated flow rule. Figure 6 illustrates the impact that the choice of an associated instead of a non-associated flow rule has on engineering computations related to the bearing capacity problem. The figure shows value of N_γ resulting from realistic pairings of ψ and ϕ and from $\psi = \phi$. Values for $\psi = \phi$ significantly exceed values for $\psi < \phi$.

How much difference does the choice of flow rule make in the calculation of the bearing capacity of a footing? Let us consider the bearing capacity factors and the limit bearing capacity q_{bL} of strip footings calculated using the two sets of equations. As an example, we take a friction angle $\phi = 45°$; dilatancy angle $\psi = 45°$ and 18°; and unit weight of sand = 19 kN/m^3. Table 1 presents the computed bearing capacity factors—N_γ and N_q—and the bearing capacity q_{bL} of two strip

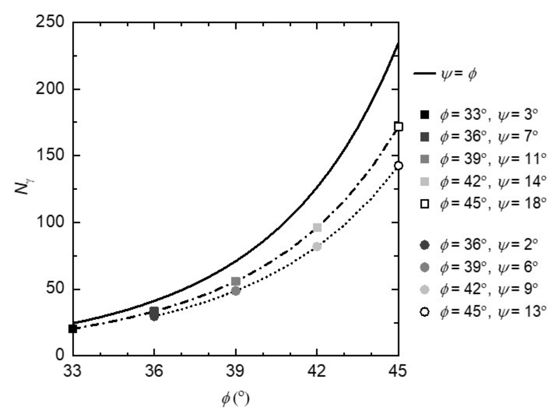

Figure 6. Comparison of values of bearing capacity factor N_γ calculated based on the assumption of associated flow ($\psi = \phi$) with values calculated based on non-associated flow ($\psi < \phi$).

Table 1. Effect of flow rule non-associativity on bearing capacity of strip footings: results of calculations using Equations 12 and 13.

Flow rule	ϕ (°)	ψ (°)	N_q	N_γ	q_{bL} (kN/m^2)			
					embedment = 0 m		embedment = 1 m	
					B = 1 m	B = 2 m	B = 1 m	B = 2 m
Associated	45	45	135	235	2230	4459	4792	7022
Non-associated		18	99	172	1631	3262	3511	5142

footings with width B = 1 m and 2 m, with an embedment of 0 m and 1 m, with the depth factor on the overburden term of the bearing capacity equation neglected.

The resulting bearing capacity for footing on the surface of a deposit of the material following the associated flow rule is 37% greater than that calculated for a material following the non-associated flow rule. This very significant overestimation of the bearing capacity of a strip footing resulting from use of the associated flow rule is an error that is unconservative. Given the nature of shallow foundation design, with serviceability controlling in the majority of design cases, this error is not as consequential to final design as it otherwise would be.

This simple example, for one of the classical problems of soil mechanics, illustrates the level of error resulting from use of theories based on a Mohr-Coulomb material following an associated flow rule. Ideally, these would not be taught but for providing historical perspective. The teaching of methods of analysis and design that rely on realistic soil models would be the best approach, and it is possible in many instances. Failing that, whenever the Mohr-Coulomb yield criterion is used, it must be used with a non-associated flow rule.

Lastly, use of a non-associated flow rule does not heal the defects of a model relying on the Mohr-Coulomb yield criterion. The model is still exceedingly simple—having constant ϕ and ψ—and will not be realistic for calculations requiring a higher degree of realism. In such cases, use of a more sophisticated constitutive model is required.

4. Shear strain localization and its implications

4.1 Background

In undergraduate laboratory classes, students typically see or perform triaxial tests on dense sand specimens; they observe the resulting “failure plane” that eventually develops through the specimen. In most classrooms, that observation leads to nothing more, but it should. That is the best time to make a number of crucial points that are today essential for a well-rounded geotechnical engineer to understand.

The first important point regarding that “failure plane” is that it is not a plane at all. The second is that “failure” is too vague a term, and it confuses students to use it. It is better to speak of what has happened as the shearing of the sand or, if one is especially attached to the word, as a shear “failure” of the sand specimen. Back to the first point, today it is possible to show to students videos taken of the shearing of sand. In videos of the shearing of sands, we can clearly see that a band of particles, with thickness of the order of 5 to as many as 10 particle diameters, is what constitutes that “plane.” The “plane” is what we know today as a shear band.

Shear bands in soil have been studied as early as the 1970s (Vardoulakis et al., 1978). It is however very important to teach students this for the following reason: a plane is an abstraction from which no pattern of soil behavior can be inferred, but a band, containing a number of soil particles, has a behavior that results from the interactions of the particles in it. This interaction of particles in the band directly produces the constitutive behavior of the soil. Once students understand this, it is much easier for them to understand how shaft resistance develops along a pile or why the pressure on a retaining wall is what it is.

The localization of shearing in a band results from the mechanical behavior of soil: from the softening, i.e., loss of shear strength that occurs with the progression of shearing. With continuing shearing, the soil will tend to weaken at the location where this process first starts, shear strain then localizes there, sparing regions surrounding the band of further deformation. It is vital to understand this process because any simulations that we attempt of boundary-value problems involving such materials depend on correctly capturing the width of the shear bands. Mechanicians speak of the “length scale” of the material as determinative or intrinsically linked to the material behavior.

Shear bands are also seen in soils following a Mohr-Coulomb yield criterion with c = 0 if they also follow a non-associated flow rule. This is closely linked to the fact that, in these materials, plastic energy does dissipate—due to friction—once plastic shearing starts. It is then natural for shearing to continue where it started instead of diffusing to surrounding regions, because that would require greater plastic energy dissipation.

Shear band thickness depends on essentially two factors: (1) soil particle size, and (2) the boundary conditions for the shear band (that is, does it form entirely within the soil or at an interface between the soil and a structural element). If the interface is rough, the shear band thickness will be of the order of the thickness that forms entirely within soil; however, if the interface is smooth, there is no shear band that forms along the interface: there is only clean sliding of the interface with respect to the soil (Tehrani et al., 2016; Tovar-Valencia et al., 2018). Images of strain localization can be collected through an exposed (transparent) window that allows visualization of soil during loading or, for small specimens, through X-Ray CT (e.g., Desrues et al., 2018). In approximate terms, shear bands in sand are of the order of 5 times the mean particle size for rough interfaces (Tehrani et al., 2016; Tovar-Valencia et al., 2018) to the order of 10 times the mean particle size for shear bands entirely contained in soil (Alshibli & Sture, 1999).

The simplest examples of localization and its impact on the solution of a boundary-value problem can be seen in the context of axially loaded piles, for which localization is known *a priori* to occur along the pile shaft (Han et al., 2017, 2018; Loukidis & Salgado, 2008; Salgado et al., 2017). Figure 7 shows the results of finite element analyses of an axially loaded pile in sand modelled using an advanced constitutive model in terms of the ratio K of the lateral effective stress on the pile shaft to the initial (free-field) vertical effective stress during shearing (Loukidis & Salgado, 2009b).

Knowing K from a battery of finite element analyses, we can compute pile limit unit shaft resistance q_{sL} using:

$$q_{sL} = K\sigma'_v \tan\delta \tag{16}$$

where δ = friction angle of the pile-soil interface. It is seen in the figure that the shaft resistance calculated for a pile depends on the width of the finite elements used immediately next to the pile. As the finite element simulation progresses, shear strain localizes next to the pile in that “column” of elements. Consequently, the shear stress along the pile shaft at any given level of pile settlement depends on the response of that band of soil and how it responds to shearing. Pre-knowledge of what the shear band thickness is in a soil allows the correct calculation of the shaft resistance of the pile. The alternative is more difficult: use of a constitutive model and computational method that inherently have the correct length scale so that the correct final shear band pattern and thickness will emerge.

Pile loading is far from the only problem in which shear strain localization is observed. On the contrary, it is pervasive. It appears in slope failures, behind retaining walls, beneath footings and in other applications at loading stages that would correspond to ultimate limit states or even serviceability limit states (Salgado, 2022b). Shear bands also insure that the critical state is often reached in boundary-value problems of interest, because shear strains in them can be quite large even if boundary displacements are not.

4.2 The shortcoming of not considering shear strain localization

Students are often inundated with coverage of “elastic soil” or elasto-plastic soil following the Mohr-Coulomb or Tresca yield criteria. These are often observed in naïve use of commercial finite element software. An interesting illustration of how analyses using either an elastic soil model or an elasto-plastic soil model without realistic representation of shear strength, strain softening and strain localization fall short comes again from foundation engineering.

Traditional models of pile group interaction relied on modeling soil as an elastic material that transferred stresses between piles in a pile group (Poulos, 1968; Randolph & Wroth, 1979). This work was groundbreaking in highlighting for the first time the interaction between piles in a pile group and the capacity of that group, but led to pile interaction and

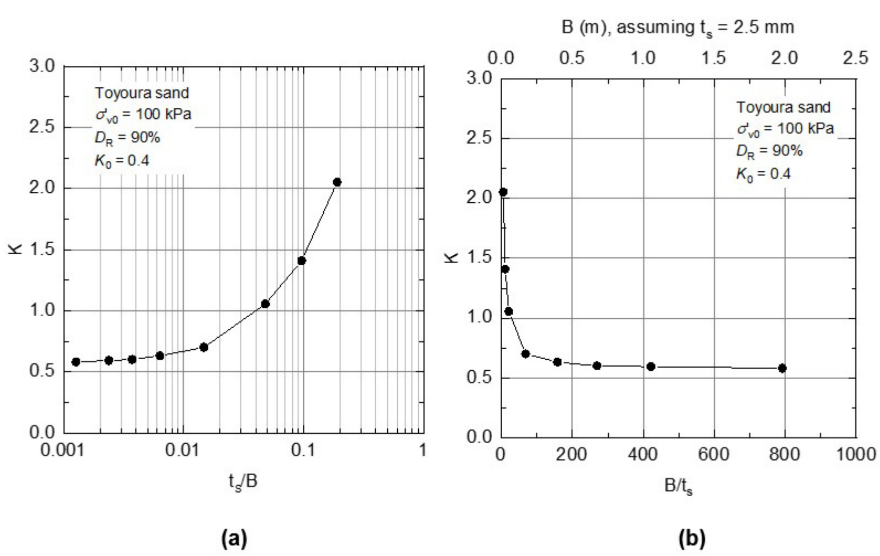

Figure 7. Effect of ratio of shear band thickness t_s to pile diameter B on the ratio K used in the computation of shaft resistance (Salgado et al., 2017): (a) K vs. t_s/B and (b) K vs. B/t_s.

group efficiency coefficients that are unrealistic because the models did not account for strain localization, which significantly reduces interaction between neighboring piles (Han et al., 2019). Figure 8 shows the significant difference in pile interaction within a group and group efficiency resulting from finite element analyses assuming a linear elastic soil, an elasto-plastic soil with a Mohr-Coulomb yield criterion, and a realistic sand model with an appropriately fine finite element mesh. These results show clearly that shear strain localization cannot be ignored if we desire accurate, realistic solutions to geotechnical boundary-value problems.

As a final illustration of the importance of capturing shear strain localization correctly, consider again the bearing capacity problem discussed earlier. Assume that a student or engineer decides to use a modern method of analysis or a commercial computational package to perform calculations for the same problem we discussed earlier. Table 2 shows results for calculations using SNAC (Abbo & Sloan, 2000), OptumG2 (Krabbenhoft et al., 2015) and the material point method (MPM) (Bisht & Salgado, 2018; Woo & Salgado, 2018). The values shown in the table are in reasonable agreement because consistent size for the mesh elements were chosen in these calculations. The SNAC and OptumG2 analyses were done using 15-node triangles with 12-point Gauss quadrature. The MPM analyses were done using Q4 elements with an initial number of material points per element equal to 4 and a B-bar scheme. The MPM analysis with the smallest element size e = 0.025m has approximately the same Gauss point density as the SNAC analysis, and the match between the two is evident. However, use of a coarser mesh, whether in SNAC, OPTUM or MPM would produce higher values of bearing capacity. For example, in the table, MPM with the smallest element size e = 0.1m yields a bearing capacity of 3055 kPa instead of 2241 kPa. This results from the fact that strain localization can only take place to the degree that the mass is discretized. A coarse mesh will lead to thick shear bands and a stiffer response.

4.3 What should be taught instead

Students should be acquainted with realistic stress-strain relationships under various loading paths, both drained and undrained, and should be provided with the opportunity to understand the role density, initial effective stress, dilatancy, and fabric evolution have in shaping these relationships. When exposed to problems in which shear strain localization occurs, and therefore the stress-strain history before localization is determinative of soil response, it is important to explain this and provide students with solutions and design methods based on analyses that do take localization into consideration.

Taking piles again as an example, teaching an analysis that ignores the shear strain localization along the pile shaft will be ineffective in that the value of pile shaft resistance cannot be calculated with any accuracy using such an analysis. Thus, one could teach using directly the results of analysis for piles in sand (e.g., Han et al., 2017; Loukidis & Salgado, 2008) or clay (e.g., Basu et al., 2014;

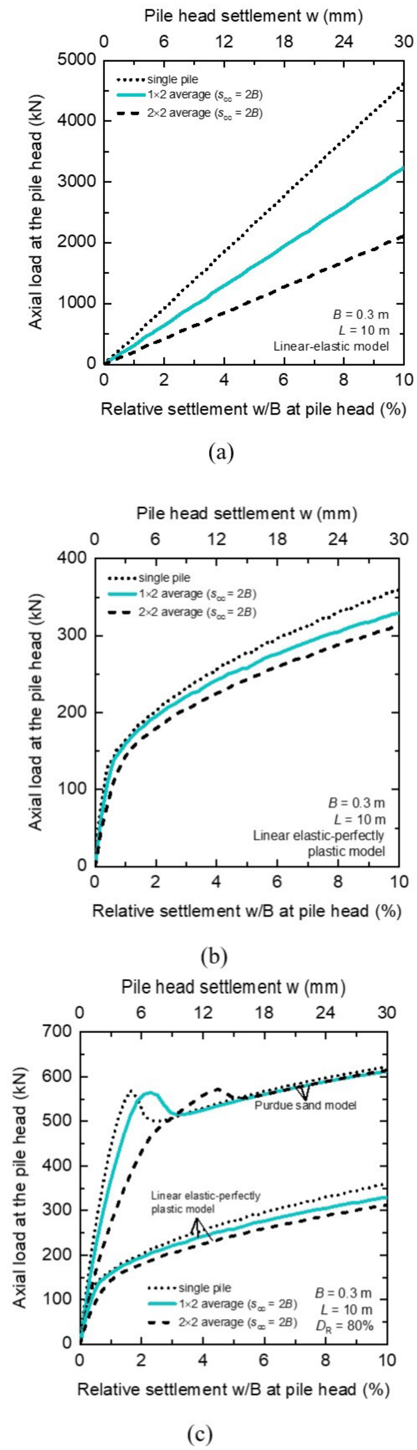

Figure 8. Load-settlement curves obtained from analyses using: (a) a linear-elastic model; (b) a linearly elastic-perfectly plastic model with the Mohr-Coulomb yield criterion; and (c) the Purdue sand model and the linearly elastic-perfectly plastic model with the Mohr-Coulomb yield criterion (Han et al., 2019).

Table 2. Strip footing bearing capacity computed using different numerical schemes and element sizes.

Flow rule	ϕ (°)	ψ (°)	q_{bL} (kN/m^2) (embedment = 0 m, B = 1 m)				
			SNAC	OptumG2	MPM		
					e = 0.1 m	e = 0.05 m	e = 0.025 m
Associated	45	45	2230	2307	3055	2301	2241
Non-associated	45	18	1631	1646	1924	1650	1611

Chakraborty et al., 2013a) that do account for localization and realistic soil response. For undergraduates, the teaching might consist of presenting the equations, explaining why they were formulated with those particular forms, and then having the students apply the equations directly to design problems. At the graduate level, one could go beyond that, and ask the student to read the papers, reproduce results and apply them to more challenging design problems.

As a final illustration of how strain localization can be included in our teaching, we turn again to the pile group example. It is advantageous to introduce students to these problems using the classical papers assuming linear elastic soil (Poulos, 1968; Randolph & Wroth, 1979), which facilitate understanding of the concepts of group pile interaction and group efficiency, but then share with them new results (Han et al., 2019; Salgado et al., 2017) that show that the interaction between the piles is considerably reduced when shear strains localize along the shafts of the piles.

5. Looking for a future: the modeling of soil as a particulate medium, artificial intelligence, and the pressing challenges of a world under stress

We have so far focused on past decisions that influenced the teaching and practice of geotechnical engineering. These decisions came out of research that focused on how to solve the problems found in the practice of geotechnical engineering. The thrust of past efforts has been to develop the science of soil mechanics: how to model soil as a material and how to solve the boundary-value problems of soil mechanics.

In the last 10-15 years, the volume of research in two areas—particulate mechanics and artificial intelligence applications to geotechnical engineering—has increased considerably. In particulate mechanics, soil is not viewed as a solid, but as a collection of particles. The emphasis is on describing particle interactions and letting these interactions, and possibly any mechanical effects on the particles themselves (such as crushing or breakage), determine the behavior of the overall particle assemblage. The main analysis tool used for this is the Discrete Element Method (DEM) (Cundall & Strack, 1979). The enthusiasm with DEM has led to very optimistic statements about its role in the future of geotechnical engineering. We will briefly examine the current viability of DEM as an analysis tool and potential implications of its adoption in both teaching and practice.

DEM is a model of soil and its mechanical response. In this, it does not differ from all the work that has been done in geotechnical engineering to the present and the modeling decisions we discussed in the previous sections. The decision that some researchers sometimes appear to advocate is to abandon solid mechanics as a vehicle to model soils and embrace DEM or, more generally, particulate mechanics for that purpose. We will examine this specific question in our discussion of DEM.

DEM development is a scientific pursuit, with hypotheses made about soil particle interactions, and predictions obtained using these hypotheses coupled with the established laws of mechanics to solve problems. In contrast, pure artificial intelligence is not a model of soil and its mechanical response. It does not explore the connection between variables through the laws of physics. Physical causation is not part of artificial intelligence methods used so far in geotechnical engineering. Instead, artificial intelligence explores correlations. One may attempt to infer causation from correlation, but that is not an immediate AI result. We will explore the implications of this different paradigm for teaching and practicing geotechnical engineering.

Last in this section, we will discuss the coverage in geotechnical engineering courses of how the discipline fits into certain themes related to a planet affected by the consequences of overconsumption: dealing with mine tailings, designing in the context of climate change, and the development of renewable energy infrastructure. This discussion differs from our previous discussions in that we will not address how to model and solve problems, but instead we will discuss the importance of teaching certain applications in geotechnical engineering courses.

5.1 Explicitly accounting for the particulate nature of soil

The Discrete Element Method was proposed roughly 45 years ago, when it was referred to as the "Distinct" Element Method (Cundall & Strack, 1979). Either way, the acronym "DEM" applies. The essence of the method is to model the transmission of forces at the contacts between particles in an assemblage. Loading can be applied in the same way as in any other problem: through activation of gravity and as tractions on the boundaries of the assemblage.

Instead of constitutive relationships relating stress and strain rates at points within a continuum, DEM relates forces and displacements and rotations at particle contacts. Explicit integration of the equation of motion is the norm.

Original DEM models were rather simple, initially cylinders in a two-dimensional assembly (Cundall & Strack, 1979), then gradually evolving to spherical particles and irregular particles made by "gluing" together spherical particles of different sizes (see, e.g., Ferellec & McDowell, 2010), the use of polyhedra of different sizes (Cundall, 1988), the use of "superquadrics" (Williams & Pentland, 1992), and representation of the particles by single elements or meshes of elements (Zhao et al., 2023). Other efforts have included scanning real particle assemblages using X-Ray CT and using the real particle shapes in analysis (see, e.g., Wang et al., 2007). Save for contact points between spheres, contact mechanics has been an ongoing object of research in DEM (Zhao et al., 2023).

One of the main challenges in the application of DEM, certainly in a practical context, has been computing power. The reason this is a challenge is that a large number of particles must be used to simulate even a laboratory soil sample, requiring many calculations of particle interactions at each time step, assuming integration of the equation of motion using an explicit solver. This is compounded if the particles are modeled as complex (or realistic) in shape. Simulating problems at prototype scale with particles of realistic size is generally impractical and likely to remain so for at least some time.

The modeling of real or realistically shaped particles also presents an obstacle of its own: the proper modeling of contact interaction between particles. Whereas contact is simple to model if particles are modeled as spheres, with contact forces with defined point of application and direction, that is far from true if particles have irregular shapes. In the modeling of clay, complications go beyond the modeling of contact forces. In clay, particle interactions involve physical interactions that go beyond just normal stress and shear stress transmission, involving also van der Waals forces, double-layer forces and other long-range forces (Jaradat & Abdelaziz, 2019). Moreover, clay particles can have varied and complex shapes (including plates, membranes, tubes and needles), which are not easily modeled realistically. An additional complication exists when the pore fluid is composed of both liquid and gaseous phases: modeling the interaction of the two phases with particles is not trivial.

Combining the challenges in modeling interparticle forces and particle shapes with the large number of particles in even a small soil volume of clay, which is orders of magnitude greater than for an identical volume of sand, leads to a very challenging computation task if realistic answers are sought. Scaling DEM analysis up to prototype scale appears impractical. Whereas research is in progress to overcome these limitations, it is difficult to see DEM overtaking solid mechanics-based methods—like FEM or MPM (See, e.g., Salgado & Bisht, 2021)—in predictive ability or efficiency in the solution of full-blown boundary-value problems involving clay for some time to come.

So how does particulate soil modeling enter a geotechnical engineering curriculum? Certainly, interest in it is broad, extending beyond just soil mechanics (O'Sullivan, 2011; Zhao et al., 2023), and research in the topic is active, so its teaching in graduate courses on soil mechanics is fully justified. However, given the challenges that exist to its application in practice for likely many years to come, extensive teaching of DEM at the undergraduate level in replacement of continuum mechanics would not be justified. This means that, as a practical matter, geotechnical practice will not likely rely on DEM to any significant extent for some time. The challenges to DEM applications in practice stem from both infrastructure requirements (computer power requirements) and modeling challenges (especially challenging for clays).

To the extent that DEM is taught, emphasis should be placed on the mechanics involved, computational schemes, particle representation, and particle contact/interaction modeling.

5.2 Artificial intelligence and machine learning

There are multiple definitions, not necessarily contradictory, of artificial intelligence. The emphasis of some definitions is on the ability of an AI system to act or think "rationally." Other definitions tend to focus on acting or thinking "like a human" (Kok et al., 2009). In order to do either—think or act like a human (not necessarily rationally) or think or act rationally (not necessarily as a human)—a system would need to have a number of capabilities, starting with the five basic senses: vision, hearing, touch, smell and taste. We immediately see that image recognition and processing, sound and language processing, and sensors that can measure the values of mechanical, physical and chemical variables are all needed, depending on the application. Then the system must be able to process this information, reason using it, and act or communicate the product of that reasoning. To do all this, a variety of technologies are required (see Figure 9).

It is important to understand that, despite the excitement with it in 2023, AI is not omni-capable. In deciding what to deliver in an education setting, it is important to analyze what AI can and cannot do. It would be highly valuable to educate engineers for tasks that AI cannot do, because that means that they cannot be replaced by AI. But it is also valuable to teach them to be users of AI, and that requires understanding the flipside of the issue: what can AI do?

Geotechnical engineers perform a range of tasks. With slight simplification, these tasks include:

- interact with a client to understand a problem;
- read drawings;
- design site investigation or monitoring plans;
- perform site investigation or install instrumentation;
- interpret test or measurement results;

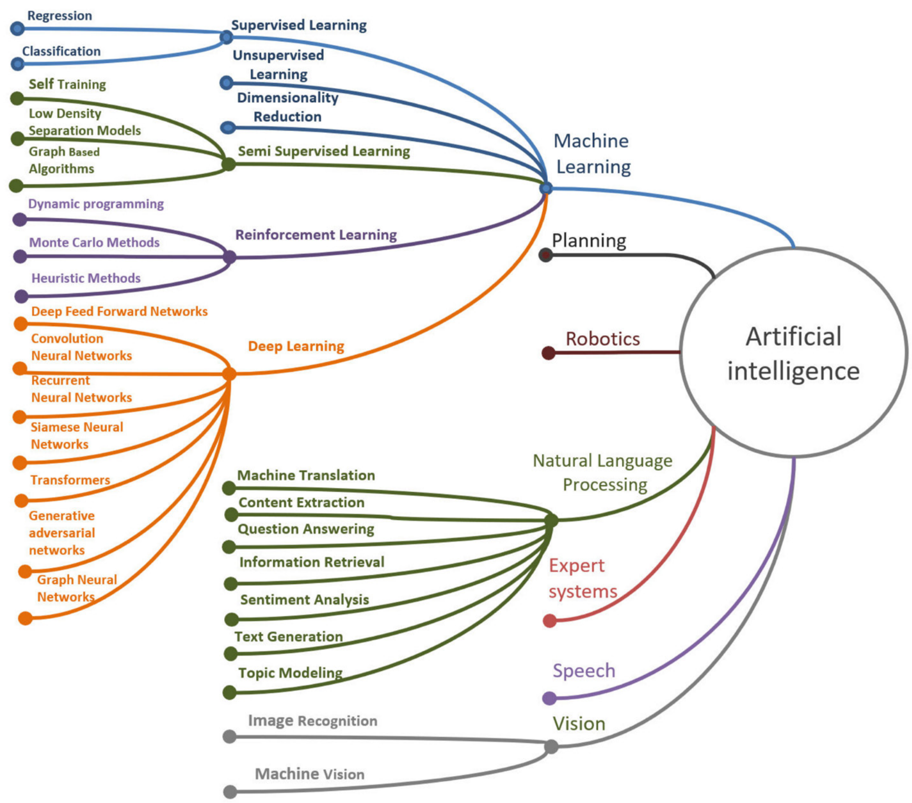

Figure 9. Artificial Intelligence and its various enabling technologies (Mukhamediev et al., 2022).

- perform calculations;
- perform design;
- reduce the design to plans and drawings and a report;
- explain the basis for a design to a client
- interact with other parties, including structural engineers.

A system that can generate new text, images, or sounds when prompted is sometimes called generative AI (Baidoo-Anu & Owusu Ansah, 2023). Such systems will have a significant impact on education and on professional practice, but there are pitfalls. Shoemaker et al. (2023) asked ChatGPT, a generative AI system in which interest spiked in 2023, to solve questions from a professional licensure exam and to perform typical tasks that a geotechnical engineer involved in design activities would perform. It was not clear how the questions were selected, except that none involved figures or charts; ChatGPT answered 67% of the answers correctly. The performance of the design tasks also contained errors. Nonetheless, it is clear from this simple exercise that AI can, even if imperfectly, do certain tasks. As technology improves, it is likely to make fewer errors. This shows that AI will have a role to play in future geotechnical engineering practice.

It is clear that the tasks in the list provided earlier are not uniformly well suited to be done by AI, and that much work remains if we were to rely exclusively on AI to do the work of a geotechnical engineer. Nearly complete replacement of a person would require most of the technologies shown in Figure 9. Image recognition, for example, can be used to calculate deformations or displacements or read and interpret design drawings. Sensors of many types can be used in instrumentation that can be directly connected to an AI-based data acquisition system. Interpretation of various types of information and performance of design tasks would rely on the branch of AI called "Machine Learning."

Machine learning ("ML") is, as the term makes explicit, "learning." We learn to do certain things, like walking, by trial and error. When we learn how to walk, outcomes may range from moving forward or back, fast or slowly, or falling in various ways. We gradually correlate our gait, that is, how we move our feet—such as how high to raise them to overcome an obstacle or slight unevenness of the ground—when attempting to go from one location to another to these outcomes, and learn how to walk naturally and safely. ML is also essentially the finding of correlations between input and output (or outcomes), both described using variables.

The learning that geotechnical engineers do, particularly in a research setting, has an element that is not inherently part of AI: the development of understanding of causal relationships. Science, as discussed earlier, has, as its focus, the establishment of predictive models. These models not only allow us to make successful predictions, but also to understand why the predictions are successful. For example, if a body is deformable, we expect that application of loads to it will lead to deflections and deformation (a hypothesis). We can continuously refine the hypothesis, and perform experiments to confirm the hypothesis, which then becomes a model. For example, we can see that, depending on certain properties of the body, the displacements and deformation that result from applying a given loading to it will be different. We can then establish exactly what these properties are, and then use mathematics to frame this knowledge. Causation is an inherent process of reasoning involved in the scientific method. We know that stress changes cause strain changes. They are not merely correlated if the deformation resulted from increasing loading: we then know that the reason for the observed deformation is the stress change. Temperature changes also cause deformation, because materials tend to change volume upon temperature changes; if this is constrained, stress results instead. Again, in this way of thinking, temperature is not only correlated, but causally linked to deformation. Further inquiry would lead us to understand, at a microscopic level, why that happens. The scientific process forces us to think in this manner: it invites us to understand the reasons for observed outcomes. In the physical sciences, causation is associated with physical processes that research helps us understand. The ML paradigm is different. No hypothesis is made. Observations are fed to the ML system, and it would learn how variables correlate, but the search for causation is neither required nor inherent to it. As often repeated, correlation is not causation (Wright, 1921). This is not to say that causation may not be inferred once correlation is established; it may, but not without more.

ML is not new. The spike in interest in ML in the 2020s can be attributed to the much greater availability of data that has been collected on everything, including people, as the Internet, the World Wide Web, and cell phones have become ever more pervasive. This availability of data has enabled AI to be an effective tool in connection with many areas of human activity. Naturally, the greater storage and computational infrastructure capabilities have also been enabling factors. A question about the application of ML in geotechnical engineering that must be asked is whether there is enough data for ML predictive ability to match that of methods developed based on the rigor of the scientific method. Given how costly data generation is in geotechnical engineering, the answer will frequently be no. But even if there is enough data, challenges remain regarding curating the data and “cleaning” it for training purposes, given that the data would likely be noisy and possibly contaminated by extraneous factors. It is obvious that the availability of large volumes of data on the World Wide Web has not been sufficient to avoid certain AI pitfalls, like “hallucinations” (Ji et al., 2023). Hallucinations are misperception by the AI system, such as generating (often eloquent or convincing) writing that is untrue, or misidentifying objects. The fact that AI does not get the answers in a licensure exam 100% correctly, as discussed earlier, illustrates that it “guesses,” much as a student might do if he did not know the answer to a question. It will try. It will provide an answer to a prompt, but that answer may be completely wrong.

One possible approach to overcome the limited availability of data in geotechnical engineering would be to use the results of simulations performed using continuum mechanics-based or other numerical methods to train the ML engine, much as has been done before to develop relatively simple regressions for design applications. Another possibility is to develop physics-aware deep learning algorithms, fully integrating scientific knowledge (such as the knowledge that a process is driven by a specific differential equation) into the deep learning scheme. Efforts to do this have recently started, but apparently not yet in geotechnical engineering.

Artificial intelligence is a tool with which geotechnical engineers should be familiar, both as users and developers. I expect that writing effective AI prompts (Kumar, 2024) will become an attractive skill, much as search keyword selection has become. Customizing engines for geotechnical engineering applications will also be important. Teaching the essence of AI and ML to geotechnical engineering students, even at the undergraduate level, is desirable. But stressing the limitations of AI and ML, and what must be used instead, is just as important. The most important such limitation is one intricately connected to education and learning: the fact that physical process understanding—the answers to “why” and “how” questions in particular—does not easily follow from AI. In contrast, understanding causal relationships is an integral part of the application of the scientific method to the solutions of engineering problems. In a discipline like geotechnical engineering, the limited volume of data will likely require strategies that bring information learned from the physics of the problem into the ML analysis. But, in some problems in which AI could be most useful, the science itself may yet be entangled, with variables required for the description of the problem not all identified, let alone relationships between them.

5.3 Other timely topics

There are certain themes that are related to geotechnical practice aimed at addressing some of the pressing challenges that our planet currently faces. The talk of voyages to Mars notwithstanding, the Earth remains, and likely will continue to be, the only inhabitable environment that is available to humans. But our planet faces major challenges: 8 billion people consuming limited resources at a high rate and a resulting pollution so significant that it is changing the planet’s climate. Geotechnical engineering is intricately connected to these challenges. I will point to three illustrations of this that deserve to appear in the teaching curriculum: (1) the role of geotechnical engineering in mining and the disposal of mine tailings, (2) the design and installation of offshore foundations and infrastructure, and (3) the mitigation of certain consequences of climate change. There is active research, therefore active decision making, happening in connection with these themes.

Geotechnical engineering plays a major role in mining. Mining activities predominantly involve excavating and drilling into rock and soil. The one aspect of mining that presents the most challenge is however the disposal of mine tailings. This is usually done using tailings dams, and the failure rates of these structures are so high that, in many jurisdictions, the law holds defendants strict liable for damages when there is a dam failure (Salgado, 2022a). Such high failure rates suggest that the profession must approach the design and construction of these structures differently or, alternatively, look for other approaches to disposal of the tailings. Teaching about this topic is important because it raises awareness of the connection between mining and the increasing demand for resources; it also highlights an area of geotechnical engineering that would benefit from performance at a higher level of care.

Offshore geotechnical engineering was originally mostly related to oil and gas exploration and production but has now become an essential component of the envisioned transition from fossil fuels to renewable or clean energy. The same challenges remain. Designing in shallow, intermediate, and deep waters requires different strategies. Whereas the monopile has been the most often used solution for shallow to intermediate-depth waters (see, e.g., Doherty & Gavin, 2012; Hu et al., 2022), future developments will rely on heavier turbines, often installed in deeper waters (Doherty et al., 2011). This development will likely rely on versions of strategies traditionally used for oil production platforms (see Randolph & Gourvenec, 2017), such as floating wind turbines anchored to the seafloor using piles, suction caissons or plate anchors. All viable design strategies for the foundations or anchors of wind turbines should be covered in geotechnical engineering curricula.

As a last example of a timely topic, climate change has led to extremes in temperature and precipitation. In some areas, wildfires have grown in number, size, and intensity. One of the many consequences of wildfires is to change the state of superficial soil (Costa et al., 2023), with important implications for stability of slopes or structures built on it. In others, intense precipitation has led to flooding that has not been seen in many decades and creates risks of coastal erosion, river margin erosion, slope and levee failures, and other similar problems. Additionally, permafrost in some regions of the world is now melting (see, e.g., Jardine, 2020), undermining design strategies relying on what had been viewed as a perennially frozen material that had always been effectively used in these areas.

Introducing these topics to undergraduate classes helps them see the relevance and timeliness of geotechnical engineering, and providing coverage of the same topics in greater depth in graduate courses helps prepare the workforce that will be required in a changing environment.

6. Conclusions

The pioneers of soil mechanics faced some difficult choices. Faced with difficult challenges and limited knowledge, they made some decisions on how to model soil and analyze the boundary-value problems of soil mechanics that have had a significant impact on how the discipline and its teaching evolved.

The three choices that were made that are highlighted in the paper are the use of Terzaghi's "ideal sand" and "ideal clay" models, the use of an associated flow rule with these models, and the neglect of shear strain localization in the solution of boundary-value problems. These choices led to some confusion regarding how soil responds to load, left engineers at a loss as to how to estimate shear strength parameters, and produced solutions to core problems in soil mechanics—such as the bearing capacity problem, the axial loading of a pile or the response of pile groups—that are not as accurate as desirable.

The discipline has overcome these initial modeling choices, and there are now better models and better theories for modeling both soil—the material—and the various engineering problems of interest. These better approaches should be included in textbooks and shared with the community. With the right way of presenting these newer theories, it is possible to teach them to undergraduate, as well as graduate students.

It is interesting to speculate about how decisions that are being made now—particularly regarding how much effort to invest in Artificial Intelligence and Discrete Element Method research—will have on the practice of geotechnical engineering. It seems that AI will have a definite role to play, but it has important limitations that may have to be addressed by making AI engines think not only "like a human," but like a "human with a science background" and one without pathologies or ethical challenges to avoid so-called "hallucinations" and answers that are imperfect "guesses." Research on the Discrete Element Method has led to impressive results, enabling the modeling of soil in accordance with its nature (that of a collection of interacting particles), but limitations remain to its widespread use in practice. These include computational cost and the modeling of clay particles and their interaction.

Finally, overconsumption on our planet has led to many undesirable consequences. Mine tailings, the mining industry version of industrial waste, has been a source of life loss, monetary damages, and environmental damage through frequent tailings dam failures. Climate change has led to a number of challenges—such as changes to the soil caused by wildfires, the melting of permafrost, the susceptibility of structures to floods—that fall clearly within the geotechnical field of knowledge. And the need to transition to clean energy has led to increasing investments in offshore wind energy development, in which offshore geotechnical engineering plays a key role. These are all topics that deserve priority coverage in geotechnical engineering courses.

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Declaration of interest

I have no conflicts of interest or financial interest to report.

Data availability

All data produced or examined in the course of the current study are included in this article.

List of symbols and abbreviations

c	Cohesive intercept in the Mohr Coulomb yield criterion
p'	Mean effective stress
q	Mises stress
q_{bL}	Limit unit shaft resistance
q_0	Overburden stress
q_{sL}	Limit unit bearing capacity
t_s	Shear band thickness
AI	Artificial Intelligence
A_ψ	Parameter in dilatancy correlation
B	Foundation width
DEM	Discrete Element Method
D_p	Plastic dissipation rate
F	Yield function
FEM	Finite Element Method
G	Plastic potential function
I_D	Relative density as a number
I_R	Dilatancy index
$J(\phi,\psi)$	Parameter in bearing capacity equation for soil following non-associated flow rule
K	Coefficient of lateral earth pressure
N_q, N_γ	Bearing capacity factors
ML	Machine Learning
MPM	Material Point Method
Q	Parameter in dilatancy correlation
R	Parameter in dilatancy correlation
SNAC	Finite element analysis software
X-Ray CT	X-Ray Computed Tomography
ε	Strain
ε_a	Axial strain
$\dot{\varepsilon}_{ij}^{\,p}$	Plastic strain rate tensor
$\dot{\varepsilon}_v$	Volumetric strain rate
ϕ	Friction angle
γ	Unit weight
$\dot{\gamma}_{\max}$	Maximum shear strain rate
$\dot{\lambda}$	Plastic multiplier
σ	Stress
σ_1	Major principal stress
σ_3	Minor principal stress
$\sigma_{a,max}$	Peak axial stress
σ_{ij}	Stress tensor
ψ	Dilatancy angle

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