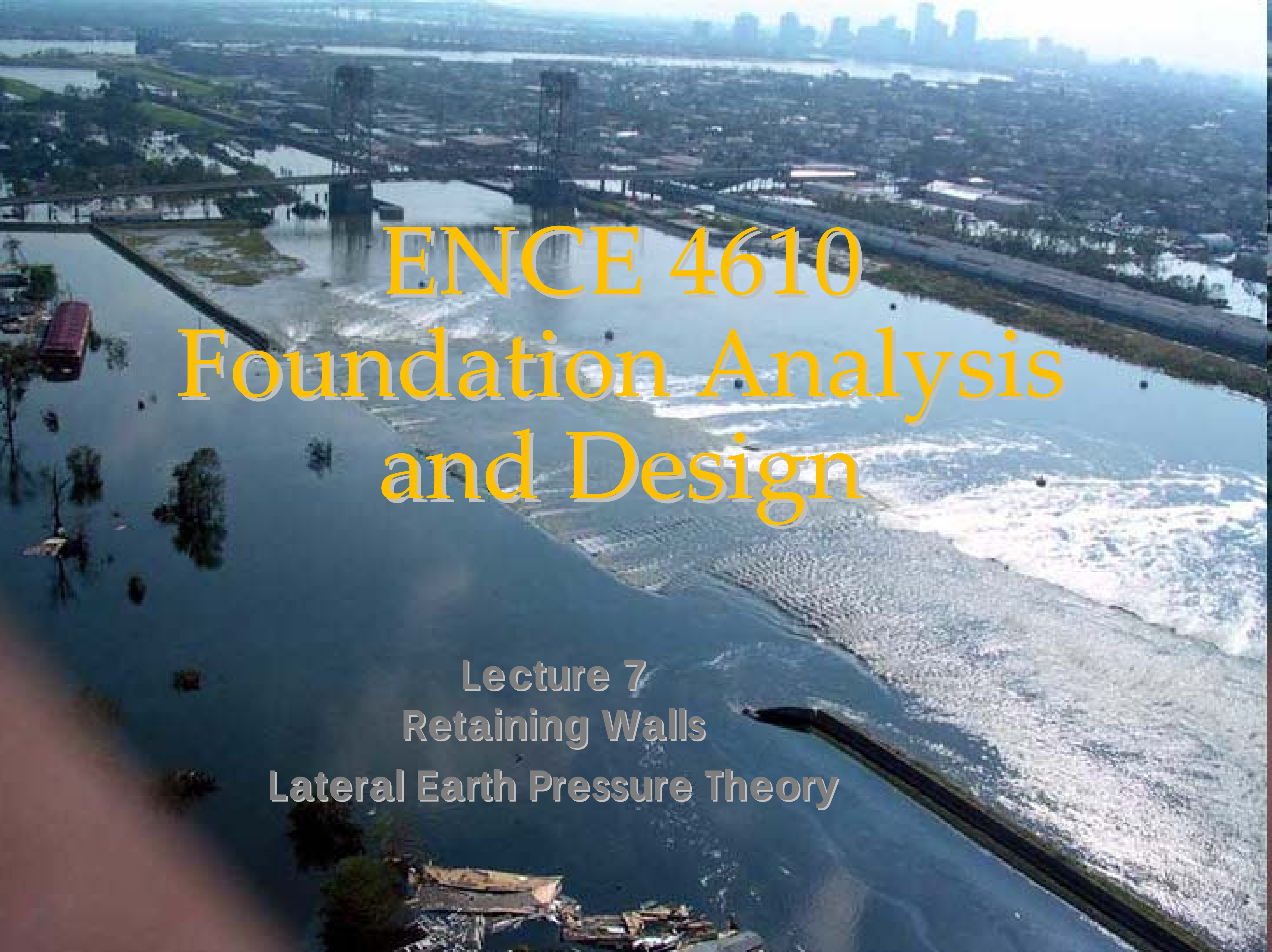


ENCE 4610 Foundation Analysis and Design

Lecture 7
Retaining Walls
Lateral Earth Pressure Theory

Retaining Walls

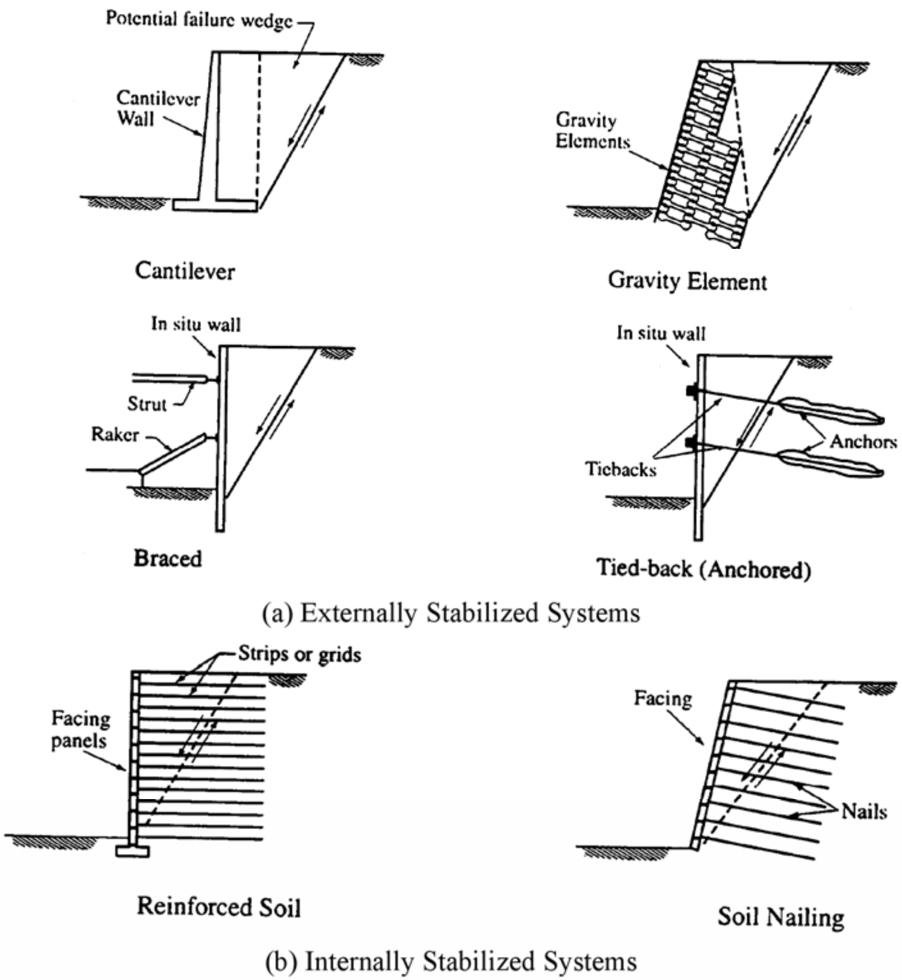


Figure 10-2. Variety of retaining walls (after O'Rourke and Jones, 1990)

- Necessary in situations where gradual transitions either take up too much space or are impractical for other reasons
- Retaining walls are analysed for both resistance to overturning and structural integrity
- Two categories of retaining walls
  - Externally Stabilized
  - Internally Stabilized

Types of Retaining Walls

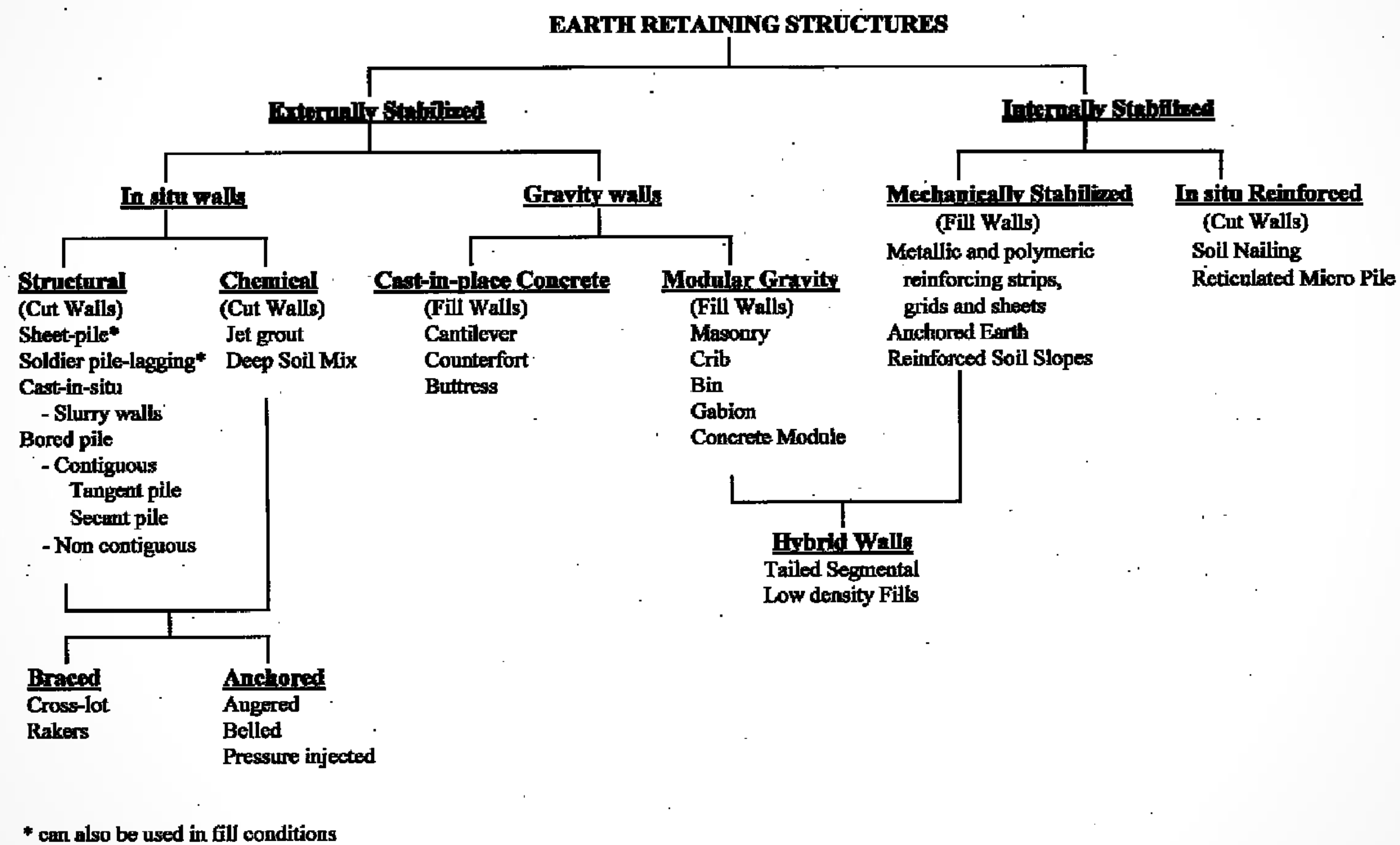


Figure 10-3. Classification of earth retaining systems (after O'Rourke and Jones, 1990).

Outline of Retaining Wall Materials

- Review of Lateral Earth Pressures
- Concrete Gravity Retaining Walls
- MSE (Mechanically Stabilised Earth) Walls
- Sheet Pile Walls
  - Cantilever Walls
  - Anchored Walls
- Braced Cuts

Reference Materials

NAVFAC DM 7.02
Chapter 3

Verruijt Chapters 32-38

FHWA NHI-06-089
Chapter 10

Lateral Earth Pressure Topics

- States of Lateral Earth Pressure
  - At-Rest (Neutral) Earth Pressure
  - Active Earth Pressure
  - Passive Earth Pressure
  - It's possible to have in-between states, but we usually don't discuss these in elementary retaining wall design
- Theories of Lateral Earth Pressure
  - Jaky (At-Rest Pressures)
  - Planar Failure Surface
    - Rankine (Active, Passive)
    - Coulomb (Active, Passive)
  - Non-Planar Failure Surface
    - Log-Spiral Theory (Active, Passive)

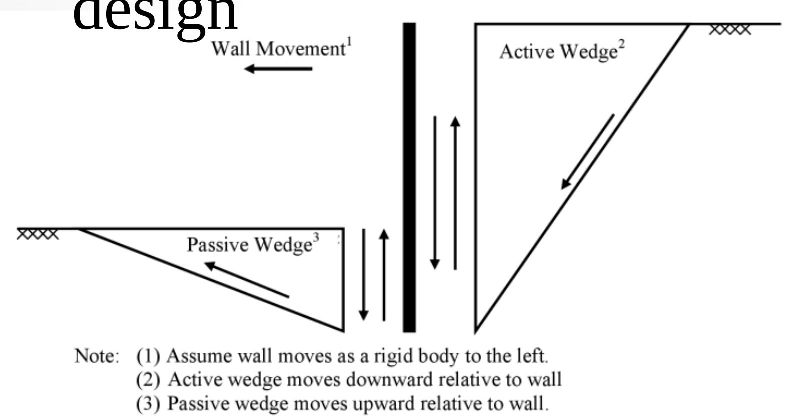


Figure 10-7. Wall friction on soil wedges (after Padfield and Mair, 1984)

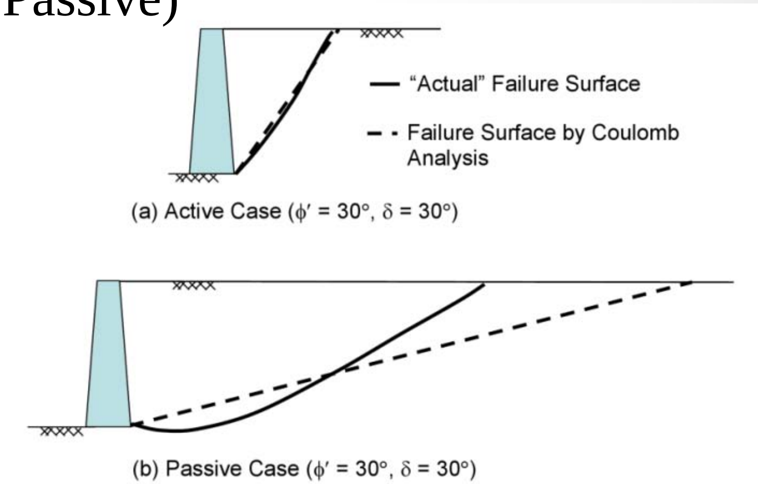


Figure 10-8. Comparison of plane and log-spiral failure surfaces (a) Active case and (b) Passive case (after Sokolovski, 1954) – Note: Depiction of gravity wall is for illustration purpose only.

Lateral Earth Pressure Coefficient

32.1 Coefficient of lateral earth pressure

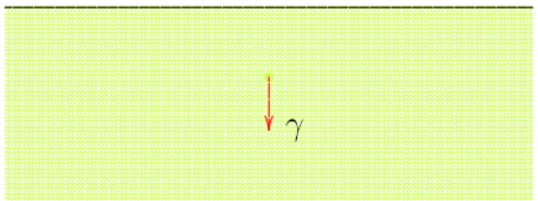


Figure 32.1: Half space.

As stated before, see Chapter 5, even in the simplest case of a semi-infinite soil body, without surface loading, see Figure 32.1, it is impossible to determine all stresses caused by the weight of the soil. It seems reasonable to assume that in a homogeneous soil body with a horizontal top surface the shear stresses σ_{zx}, σ_{zy} and σ_{xy} are zero, and it also seems natural to assume that the vertical normal stress σ_{zz} increases linearly with depth, $\sigma_{zz} = \gamma z$. These assumptions ensure that the condition of vertical equilibrium is satisfied. The horizontal stresses σ_{xx} and σ_{yy}, however, can not be determined unequivocally from the equilibrium conditions.

The stress state described by equations (32.7) – (32.11) can be made somewhat more practical by writing $f(z) = K\sigma_{zz}$, where K is an unknown coefficient, that may depend upon the vertical coordinate z. The horizontal stresses then are

$$\sigma_{xx} = \sigma_{yy} = K\sigma_{zz} = K\gamma z, \tag{32.12}$$

where K is the *coefficient of lateral earth pressure*. It gives the ratio of the lateral normal (effective) stress to the vertical (effective) stress. Theoretically speaking, the problem has not be cleared, because the value of K is still unknown, but it seems to make sense to assume that the horizontal stresses will also increase with depth, if the vertical stresses do so. Thus, it can be assumed that the coefficient K will not vary too much, at least compared to the original function $f(z)$.

A possible approach to the behavior of soils is to consider it as an elastic material. In such a material the stresses and strains satisfy Hooke's law. In a situation in which there can be no lateral deformation, the stresses must satisfy the condition

$$\varepsilon_{xx} = -\frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})] = 0,$$

$$\varepsilon_{yy} = -\frac{1}{E}[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})] = 0,$$

if the z-direction is vertical. In a medium of large horizontal extent it can be expected that $\sigma_{xx} = \sigma_{yy}$. Then

$$\varepsilon_{xx} = \varepsilon_{yy} = 0 : \quad \sigma_{xx} = \sigma_{yy} = \frac{\nu}{1-\nu}\sigma_{zz}, \tag{32.18}$$

or

$$K = \frac{\nu}{1-\nu}. \tag{32.19}$$

If Poisson's ratio varies between 0 and 0.5, the value of K varies from 0 to 1.

At-Rest (Neutral) Earth Pressures

33.4 Neutral earth pressure

It has been found that the possible states of stress in a soil may vary between fairly wide limits, especially if the friction angle is large. For a normal sand, with $\phi = 30^\circ$, the smallest value of the horizontal stress is $\frac{1}{3}$ of the vertical stress (which usually is known from the surcharge and the weight of the overlying soil), and the largest value is 3 times the vertical stress. In case of a rigid retaining wall, the lateral stress against the wall is unknown, at least from a strictly scientific viewpoint. In reality there may be some additional information that may help to determine the probable range of the horizontal stress. If the horizontal displacements of the wall are practically zero, the ratio of the horizontal stress to the vertical stress is denoted as the *neutral earth pressure coefficient* K_0. In an elastic material this value would be $K_0 = \nu/(1-\nu)$, but this is not a very good estimate, as soil is not an elastic material, and the history of the development of stresses in the soil may be more important than the condition of zero lateral deformation. Nevertheless, in many cases it is unlikely that the coefficient K_0 would be larger than 1, as this would require some form of motion of the wall towards the soil mass. Also, the active state of stress (say $\frac{1}{3}$) may also be unlikely, if the wall is rather stiff and strong. All this suggests that the neutral stress coefficient may perhaps vary in the range from 0.5 to 1.0. For soft clay the value may be close to 1, and for sands values of about 0.6 or 0.7 have been found to give reasonable results. Lacking any better information the value may be estimated by the formula proposed in 1938 by the German engineer J. Jaky,

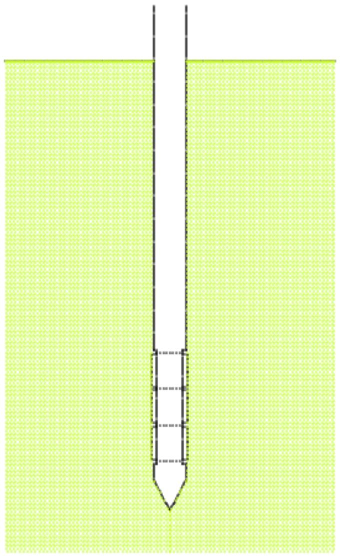


Figure 33.8: CAMKO-meter.

$$K_0 = 1 - \sin\phi, \tag{33.16}$$

but there is no real basis for this formula, except that it gives values between 0 and 1, with the value being close to 1 if the friction angle ϕ is very small (as it is for soft clays).

Active Earth Pressures

33.2 Active earth pressure

It can be expected that the smallest value of the horizontal stress occurs in the case of a retaining wall that is moving away from the soil, see Figure 33.3. The Mohr circle for that case is also shown in the figure. The pole for the vectors normal to the planes is the rightmost point of the circle. This means that the critical shear stress acts on planes making an angle $\frac{1}{4}\pi + \frac{1}{2}\phi$ with the horizontal direction, that is an angle of $\frac{1}{4}\pi - \frac{1}{2}\phi$ with the vertical direction. These planes have been indicated in the left half of Figure 33.3. It is sometimes imagined that the soil indeed slides along these planes in case of failure.

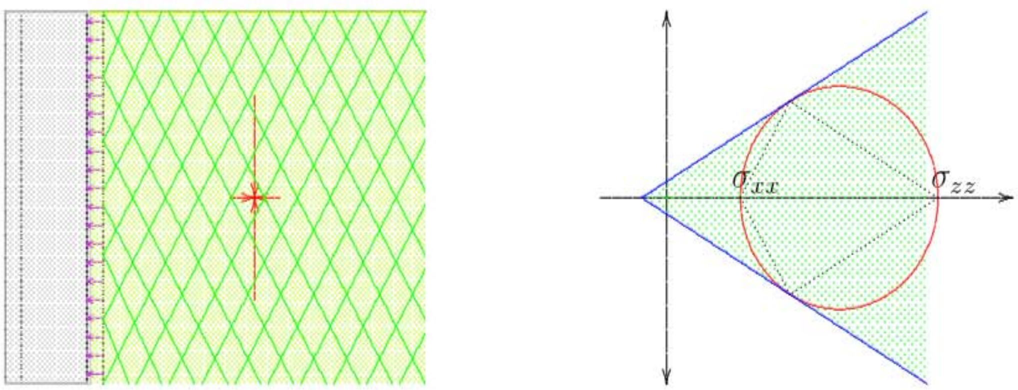


Figure 33.3: Active earth pressure.

The vertical stresses along the wall are

$$\sigma_{zz} = \gamma z, \tag{33.9}$$

in which γ is the volumetric weight of the soil, and z is the depth below soil surface. The horizontal stresses now are, with (33.3),

$$\sigma_{xx} = K_a \gamma z - 2c\sqrt{K_a}. \tag{33.10}$$

The total horizontal force on a wall of height h is obtained by integration from $z = 0$ to $z = h$. This gives

$$Q = \tfrac{1}{2} K_a \gamma h^2 - 2ch\sqrt{K_a}. \tag{33.11}$$

Passive Earth Pressures

33.3 Passive earth pressure

The case of passive earth pressure, in which the horizontal soil stress has its maximum value, can be considered to correspond to a smooth vertical wall that is being pushed in horizontal direction into the soil, see Figure 33.6. Again the Mohr circle has been shown in the figure as well, with the pole in this case being located in the leftmost point of the circle. The critical shear stress $\tau = \tau_f = c + \sigma \tan\phi$ occurs on planes making an angle $\frac{1}{4}\pi - \frac{1}{2}\phi$ with the horizontal direction. These planes have also been indicated in the left half of the figure. In this case the potential sliding planes are shallower than 45°.

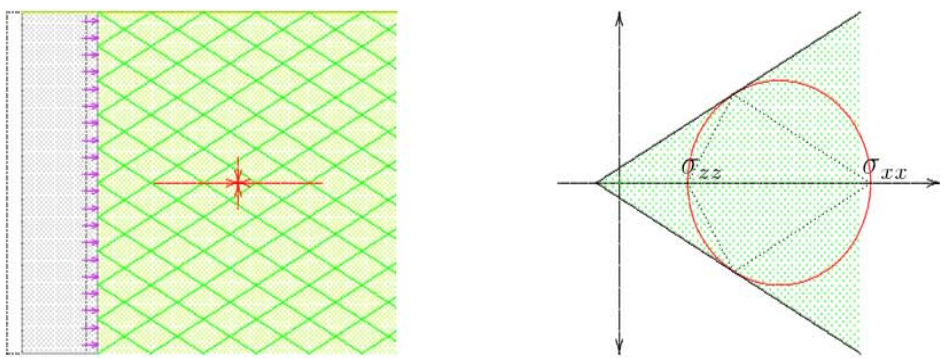


Figure 33.6: Passive earth pressure.

In this case the horizontal stresses are

$$\sigma_{xx} = K_p \gamma z + 2c\sqrt{K_p}. \tag{33.14}$$

The total horizontal force on a wall of height h is obtained by integration of the horizontal stresses from $z = 0$ to $z = h$. This gives

$$Q = \tfrac{1}{2} K_p \gamma h^2 + 2ch\sqrt{K_p}. \tag{33.15}$$

In the passive case the cohesion c appears to lead to a constant factor in the expression for the horizontal stresses. There is no reason for cracks to appear, as there are no tensile stresses in this case.

The two extreme states of stress considered here are often denoted as the *Rankine states*, after the Scottish

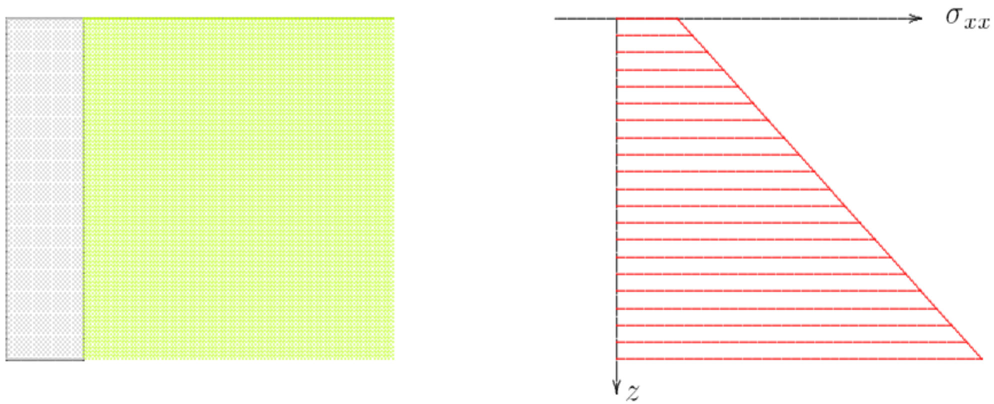


Figure 33.7: Horizontal stresses in case of passive earth pressure.

Relationship of the Different Stress States

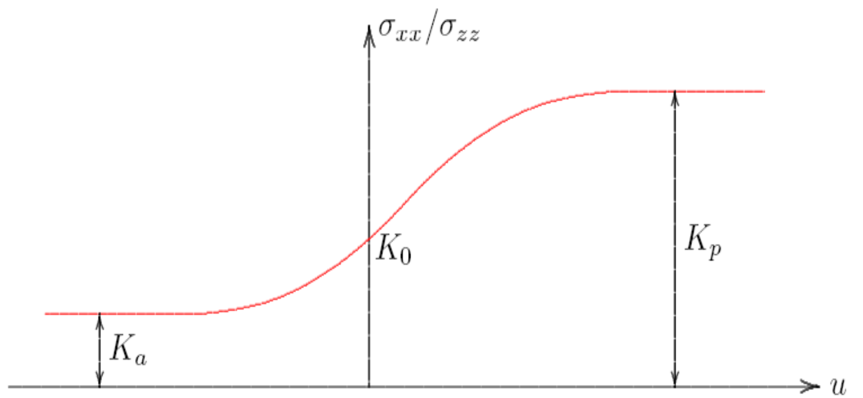
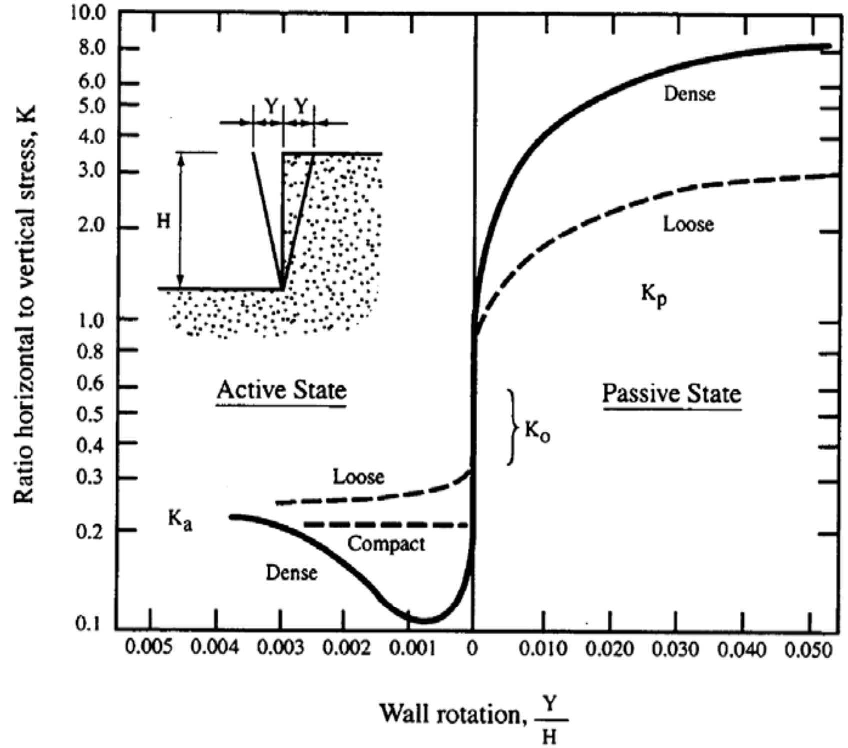


Figure 33.9: Horizontal stress as a function of the displacement.



Magnitude of Wall Rotation to Reach Failure

Soil type and condition	Rotation, Y/H	
	Active	Passive
Dense cohesionless	0.001	0.02
Loose cohesionless	0.004	0.06
Stiff cohesive	0.010	0.02
Soft cohesive	0.020	0.04

Figure 10-4. Effect of wall movement on wall pressures (after Canadian Geotechnical Society, 1992).

Rankine Earth Pressure

- Assumes Mohr-Coulomb failure conditions for the soil
- Does not include effects of wall-soil friction
- Can be extended to include sloping backfill (see Coulomb Theory)
- Also includes provision for cohesive soils (see chart at right)

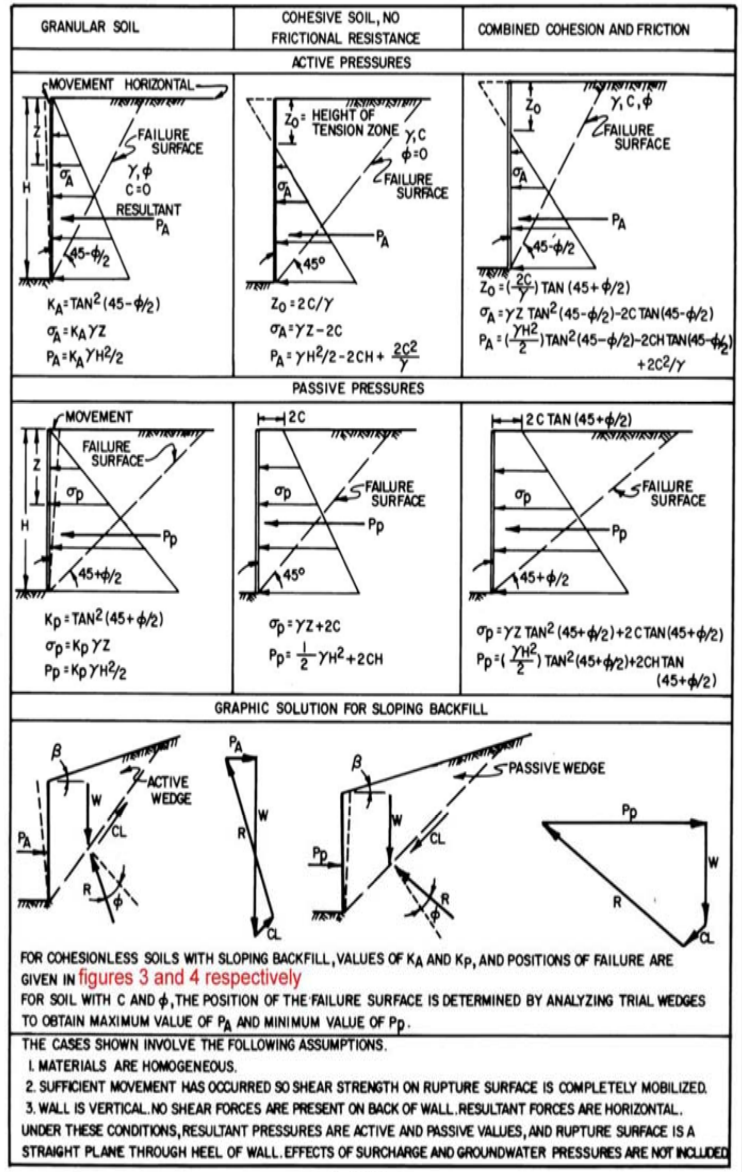
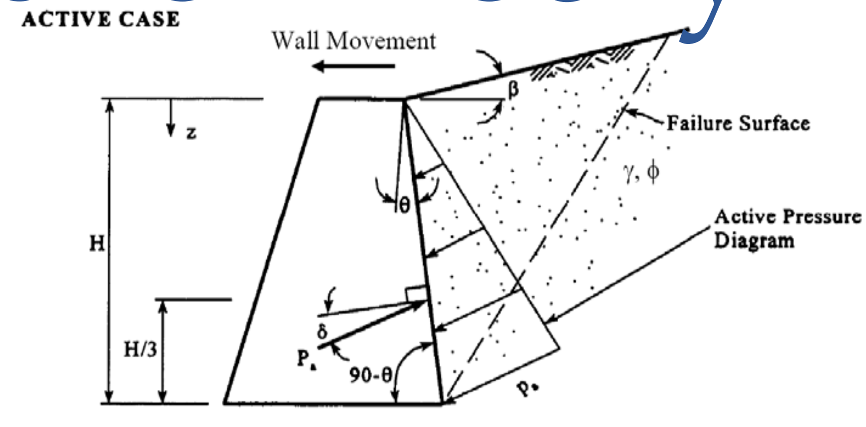


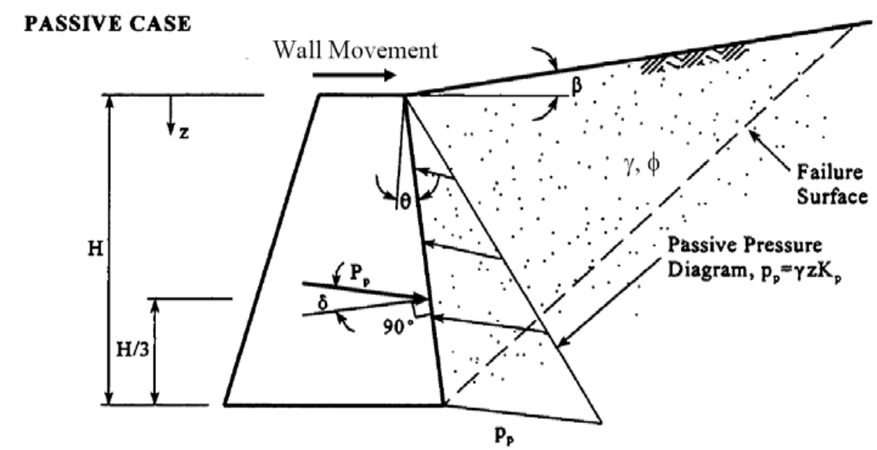
FIGURE 2
Computation of Simple Active and Passive Pressures

Coulomb Failure Theory

- Includes effects of wall-soil friction through use of failure wedge theory
- Can also be used with sloping backfill walls
- Is very UNCONSERVATIVE for passive pressures with high values of wall friction
- Formulas to the right assume that δ is positive for both active and passive pressures



$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta\cos(\theta+\delta)\left[1 + \sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\beta)}{\cos(\theta+\delta)\cos(\theta-\beta)}}\right]^2}$$



$$K_p = \frac{\cos^2(\theta + \phi)}{\cos^2\theta\cos(\theta-\delta)\left[1 - \sqrt{\frac{\sin(\phi+\delta)\sin(\phi+\beta)}{\cos(\theta-\delta)\cos(\theta-\beta)}}\right]^2}$$

Figure 10-5. Coulomb coefficients K_a and K_p for sloping wall with wall friction and sloping cohesionless backfill (after NAVFAC, 1986b).

Coulomb Earth Pressure Theory

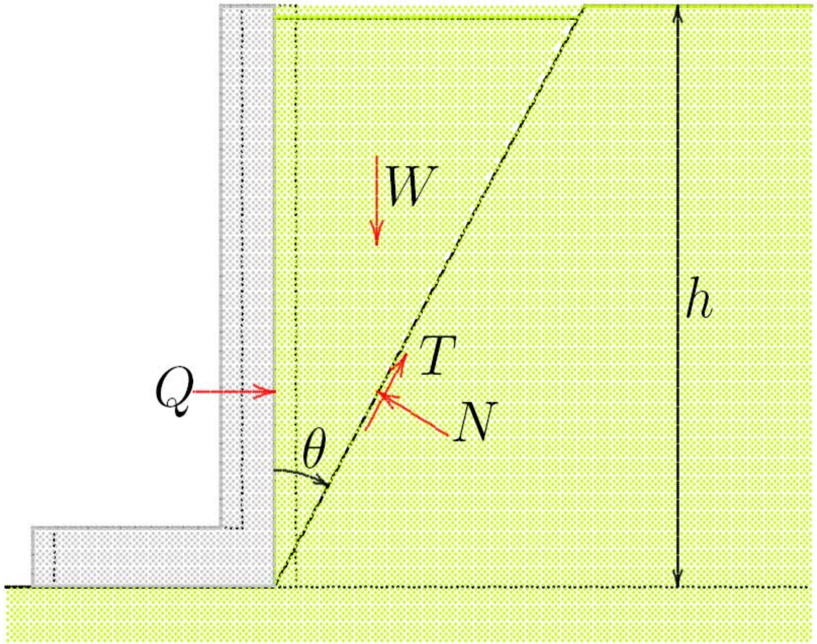


Figure 34.1: Active earth pressure.

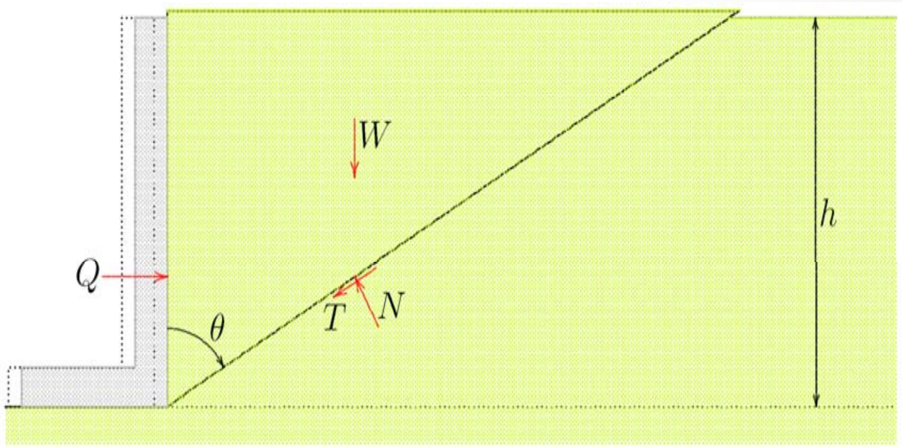
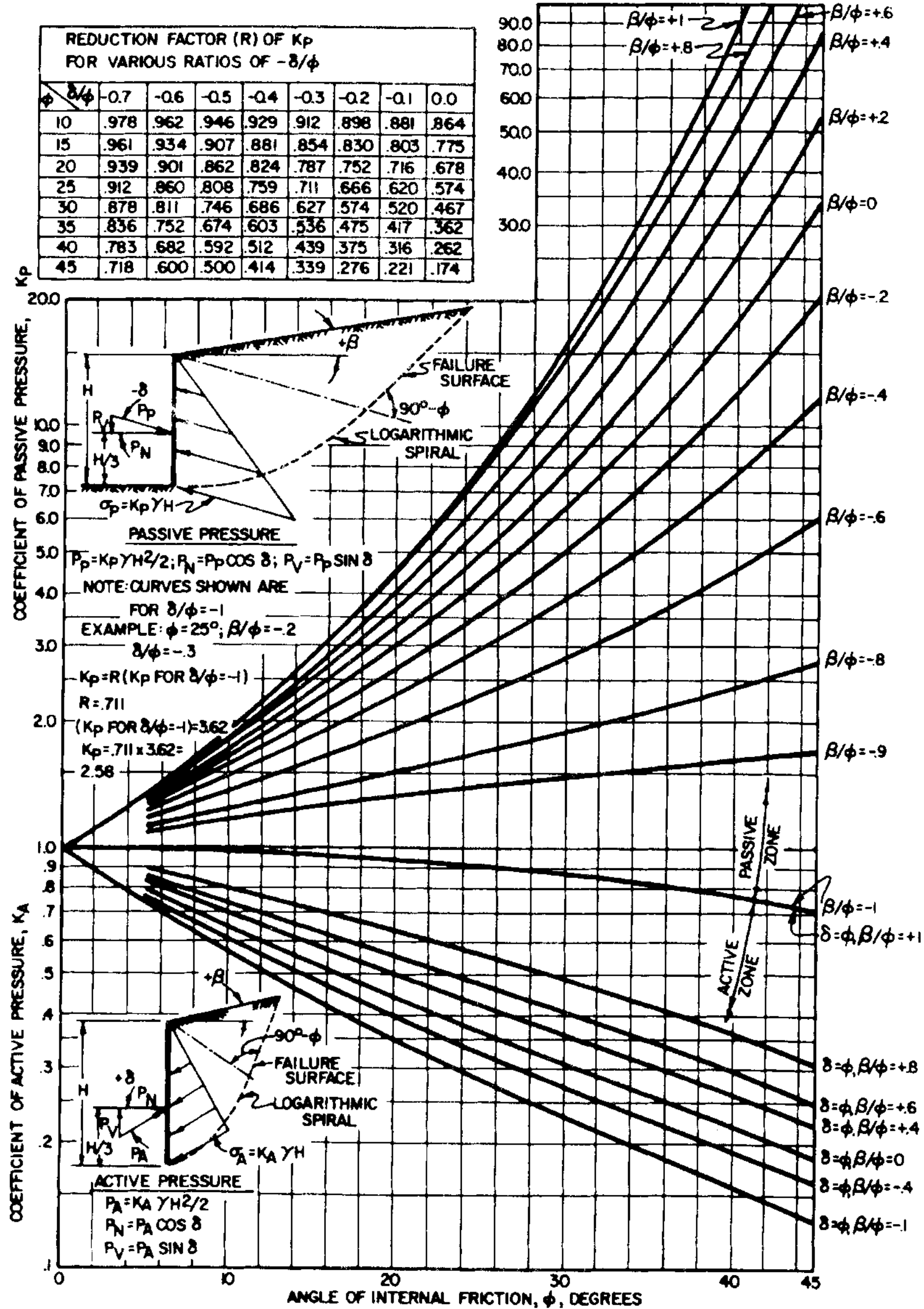


Figure 34.2: Passive earth pressure.

Log-Spiral Model



REDUCTION FACTOR (R) OF K_P FOR VARIOUS RATIOS OF $-\delta/\phi$

ϕ \ δ/ϕ	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

- Assumes log spiral failure surface behind wall
- Requires use of suitable chart for K_A and K_P
- Requires different chart for vertical wall and horizontal backfill (vertical wall shown at left)
- With AASHTO specifications, only log-spiral charts for K_P are necessary

REDUCTION FACTOR (R) OF K_p FOR VARIOUS RATIOS OF $-\delta/\phi$								
ϕ \ δ/ϕ	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
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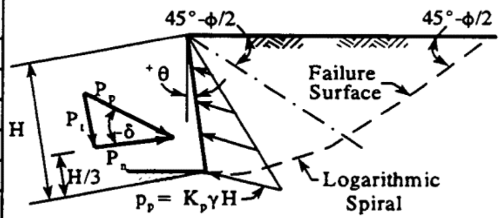
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

Note: Curves shown are for $\delta/\phi = -1$



$P_p = \frac{K_p \gamma H^2}{2}$

$P_n = P_p \cos\delta$

$P_t = P_p \sin\delta$

Example: $\phi = 30°$; $\theta = -10$; $\delta/\phi = -0.6$

$K_p = R(K_p \text{ for } \delta/\phi = -1) = (0.811)(8.2) = 6.65$

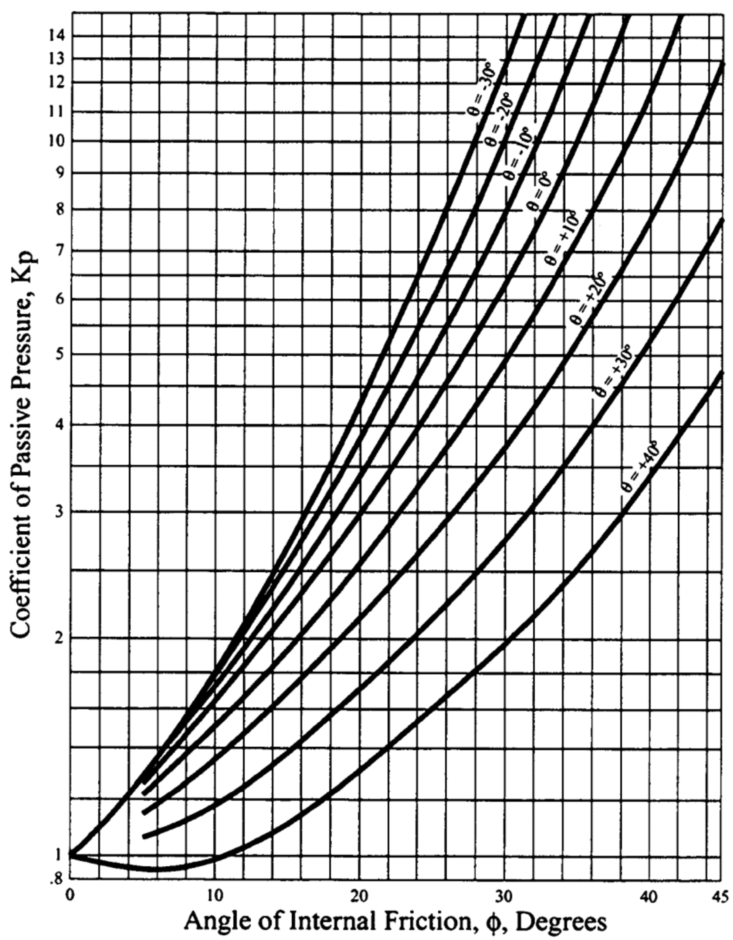
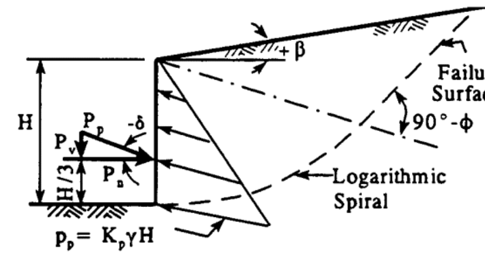


Figure 10-9. Passive coefficients for sloping wall with wall friction and horizontal backfill (Caquot and Kerisel, 1948; NAVFAC, 1986b).

REDUCTION FACTOR (R) OF K_p FOR VARIOUS RATIOS OF $-\delta/\phi$								
ϕ \ δ/ϕ	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

Note: Curves shown are for $\delta/\phi = -1$



$P_p = K_p \gamma H^2/2$; $P_n = P_p \cos\delta$; $P_v = P_p \sin\delta$

Example: $\phi = 25°$; $\beta/\phi = -0.2$; $\delta/\phi = -0.3$

$K_p = R(K_p \text{ for } \delta/\phi = -1) = (0.711)(3.62) = 2.58$

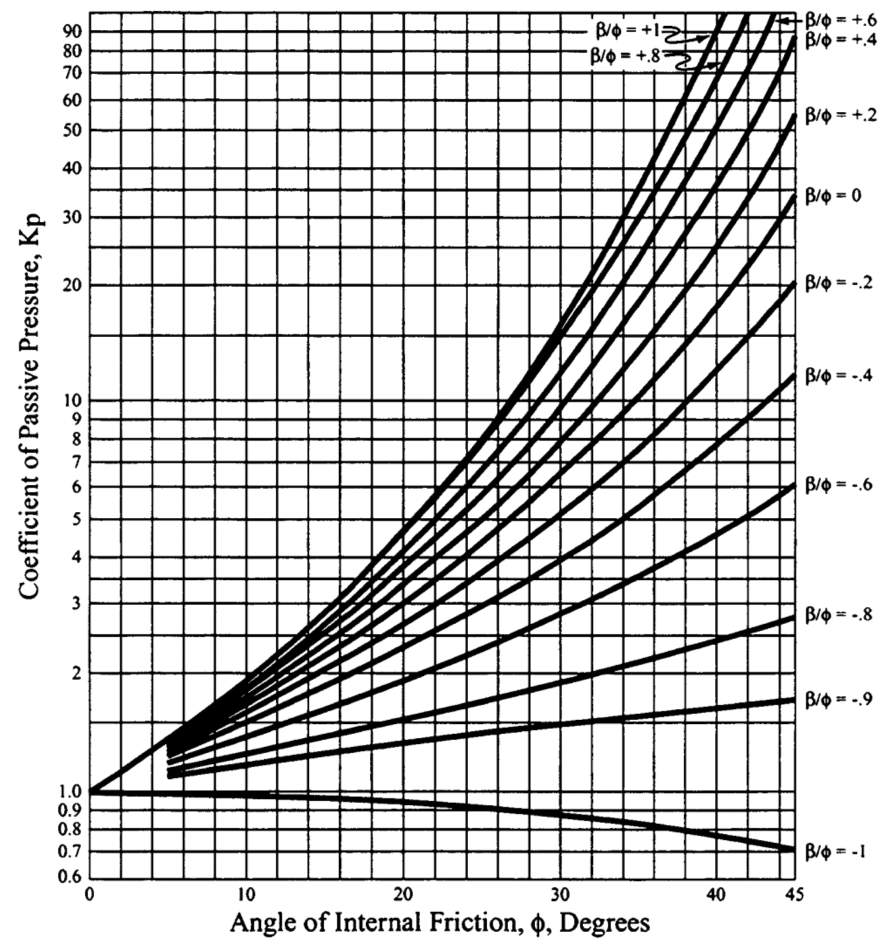


Figure 10-10: Passive coefficients for vertical wall with wall friction and sloping backfill (Caquot and Kerisel, 1948; NAVFAC, 1986b).

Typical Values of Wall Friction

Maximum wall friction suggested for design:

Active: $\delta = 2\varphi'/3$

Passive: $\delta = \varphi'/2$

Also: $\delta = \tan^{-1}(\sin(\varphi))$

Table 10-1

Wall friction and adhesion for dissimilar materials (after NAVFAC, 1986b)

Interface Materials	Friction Factor, tan δ	Friction angle, δ degrees
Mass concrete on the following foundation materials:		
Clean sound rock	0.70	35
Clean gravel, gravel sand mixtures, coarse sand	0.55 to 0.60	29 to 31
Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel	0.45 to 0.55	24 to 29
Clean fine sand, silty or clayey fine to medium sand	0.35 to 0.45	19 to 24
Fine sandy silt, nonplastic silt	0.30 to 0.35	17 to 19
Very stiff and hard residual or preconsolidated clay	0.40 to 0.50	22 to 26
Medium stiff and stiff clay and silty clay (Masonry on foundation materials has same friction factor)	0.30 to 0.35	17 to 19
Steel sheet piles against the following soils:		
Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls	0.40	22
Clean sand, silty sand-gravel mixtures, single size hard rock fill	0.30	17
Silty sand, gravel or sand mixed with silt or clay	0.25	14
Fine sandy silt, nonplastic silt	0.20	11
Formed concrete or concrete sheet piling against the following soils:		
Clean gravel, gravel-sand mixture, well-graded rock fill with spalls	0.40 to 0.50	22 to 26
Clean sand, silty sand-gravel mixture, single size hard rock fill	0.30 to 0.40	17 to 22
Silty sand, gravel or sand mixed with silt or clay	0.30	17
Fine sandy silt, nonplastic silt	0.25	14
Various structural materials:		
Masonry on masonry, igneous and metamorphic rocks:		
Dressed soft rock on dressed soft rock	0.70	35
Dressed hard rock on dressed soft rock	0.65	33
Dressed hard rock on dressed hard rock	0.55	29
Masonry on wood (cross grain)	0.50	26
Steel on steel at sheet pile interlocks	0.30	17

Interface Materials (Cohesion)	Adhesion c_a (kPa)
Very soft cohesive soil (0 - 12 kPa)	0 - 12
Soft cohesive soil (12 - 24 kPa)	12 - 24
Medium stiff cohesive soil (24 - 48 kPa)	24 - 36
Stiff cohesive soil (48 - 96 kPa)	36 - 45
Very stiff cohesive soil (96 - 192 kPa)	45 - 62

Walls with Cohesive Backfill

- Retaining walls should generally have cohesionless backfill, but in some cases cohesive backfill is unavoidable
  - Cohesive soils present the following weaknesses as backfill:
    - Poor drainage
    - Creep
    - Expansiveness
- Most lateral earth pressure theory was first developed for purely cohesionless soils (c = 0) and has been extended to cohesive soils afterward

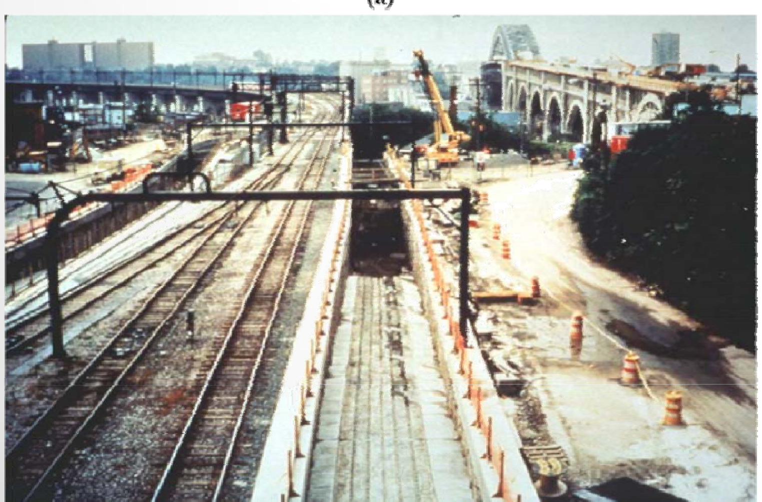
Table 10-3

Suggested gradation for backfill for cantilever semi-gravity and gravity retaining walls

Sieve Size	Percent Passing
3 in. (76.2 mm)	100
No. 4 (4.75 mm)	35 – 100
No. 30 (0.6 mm)	20 – 100
No. 200 (0.075 mm)	0 – 15

Effects of Surface Loading

(a)



(b)

Figure 10-13: (a) Retaining wall with uniform surcharge load and (b) Retaining wall with line loads (railway tracks) and point loads (catenary support structure).

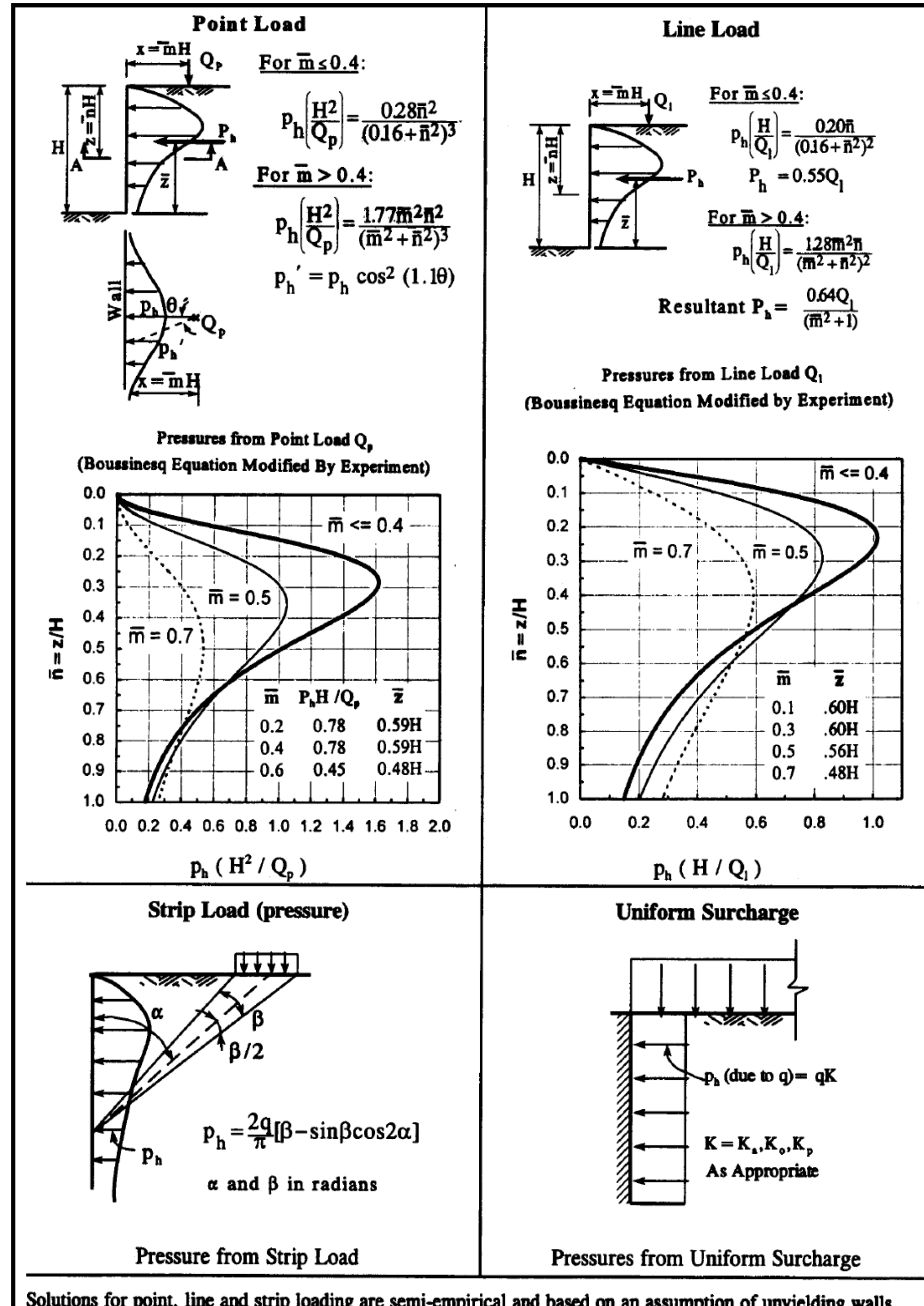
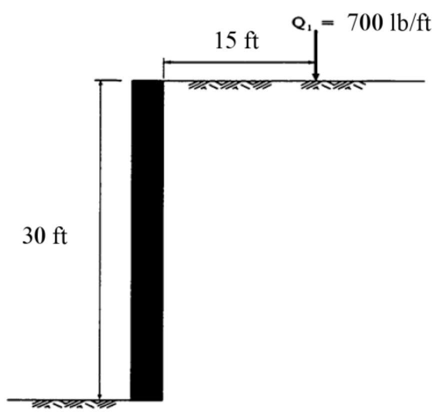


Figure 10-14. Lateral pressure due to surcharge loadings (after USS Steel, 1975)

Example of Surcharge Loading

Example 10-2: Construct the lateral pressure diagram due to a line load of 700 lb/ft located 15 ft behind the top of a 30 ft high unyielding wall shown below.



Geometry of the Example Problem 10-2

Solution:

The procedure to calculate the lateral pressures due to a line load is given in Figure 10-14. From this figure the lateral pressure can be found as follows:

$$\bar{m} = \frac{15\,\text{ft}}{30\,\text{ft}} = 0.5 > 0.4$$

For $\bar{m} > 0.4$, the lateral pressure is given by:

$$P_h = 1.28\left(\frac{Q_l}{H}\right)\left[\frac{\bar{m}^2\bar{n}}{\left(\bar{m}^2 + \bar{n}^2\right)^2}\right]$$

For $\bar{m} = 0.5$, Q_l=700 lb/ft and H = 30 ft, the lateral pressure is given by:

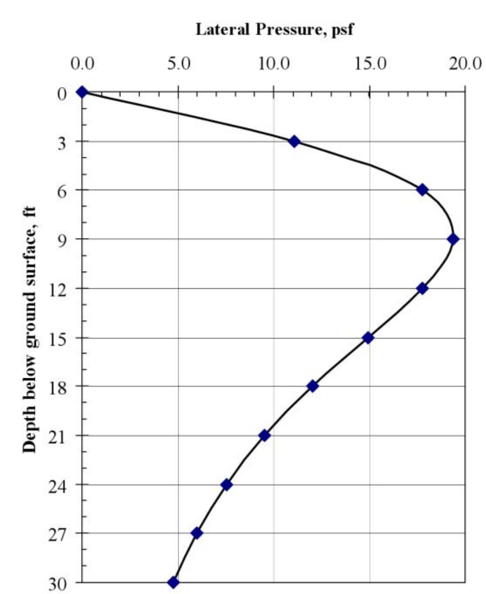
$$P_h = 1.28\left(\frac{700\,\text{lb/ft}}{30\,\text{ft}}\right)\left[\frac{0.5^2\bar{n}}{\left(0.5^2 + \bar{n}^2\right)^2}\right] \rightarrow P_h = 29.9\left[\frac{0.25\bar{n}}{\left(0.25 + \bar{n}^2\right)^2}\right]$$

Lateral pressures computed at various depths by using the above formula and the chart for line loads in Figure 10-14 are tabulated below.

Computation of Lateral Earth Pressures Due To Line Load

$\bar{n} = z/H$	Depth below top of wall (ft)	P_h (psf)
0	0	0.00
0.1	3	11.0
0.2	6	17.8
0.3	9	19.4
0.4	12	17.8
0.5	15	14.9
0.6	18	12.0
0.7	21	9.5
0.8	24	7.5
0.9	27	6.0
1.0	30	4.8

The information in the table is used to construct the curve of depth vs. lateral pressure shown below.



Groundwater Effects

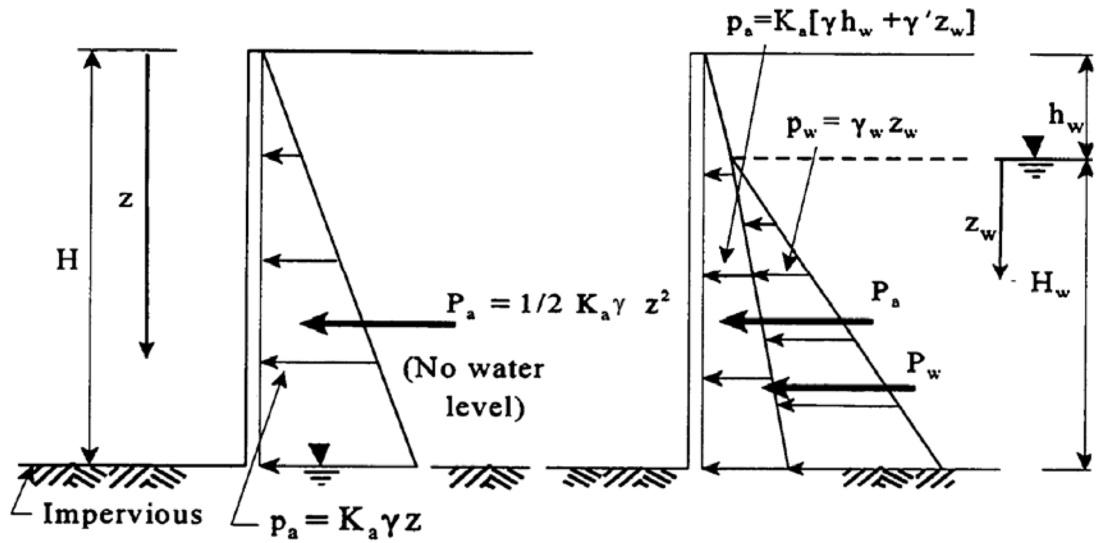
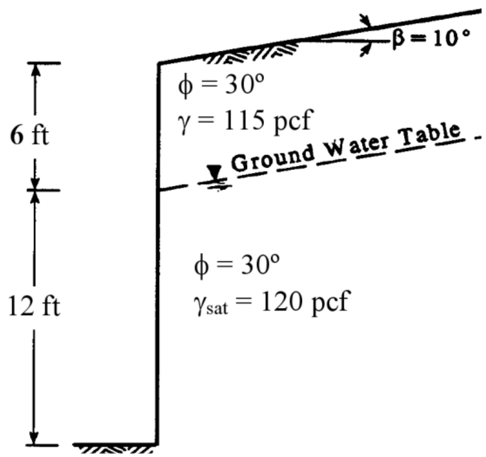


Figure 10-12. Computation of lateral pressures for static groundwater case.

- Steps to properly compute horizontal stresses including groundwater effects:
  - Compute total vertical stress
  - Compute effective vertical stress by removing groundwater effect through submerged unit weight; plot on P_o diagram
  - Compute effective horizontal stress by multiplying effective vertical stress by K
  - Compute total horizontal stress by directly adding effect of groundwater unit weight to effective horizontal stress

Groundwater Example

Example 10-1: For the wall configuration shown below, construct the lateral pressure diagram. Assume the face of the wall to be smooth ($\delta = 0$, $c_w = 0$).



Solution:

Use the Coulomb method (Figure 10-5) for $\phi = 30^\circ$, $\beta = 10^\circ$, $\theta = 0$, and $\delta = 0$:

$$K_a = 0.374$$

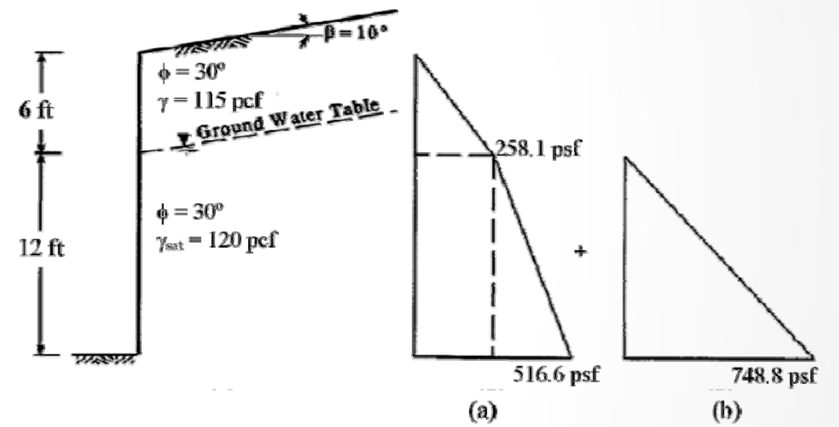
The pressures at various depths can then be calculated as shown in a tabular format as follows. Based on the values in the table, the lateral pressure diagrams due to earth and water can be constructed as shown below. The total lateral pressure diagram is the sum of the two lateral pressure diagrams shown in the figure accompanying this example.

Effective Lateral Earth Pressures, p'_a

z, ft	p_o, psf		$p_a = K_a\, p_o$, psf	
0	0			0
6	(115 pcf) (6 ft)	= 690.0 psf	0.374(690.0 psf) =	258.1 psf
18	690 psf +(120 pcf-62.4 pcf)(12 ft)	= 1381.2 psf	0.374(1381.2 psf) =	516.6 psf

Hydrostatic Pressure, $u = K_w\, u_w$

z, ft	z_w, ft	$u_w = z_w\, \gamma_w$, psf	Lateral water pressures, u psf	
0	0	0	=	0
6	0	0	=	0
18	12	12 ft (62.4 pcf) = 748.8 psf	1.0(748.8 psf) =	748.8 psf



(a) Lateral effective earth pressure diagram and (b) Lateral water pressure diagram.

Questions