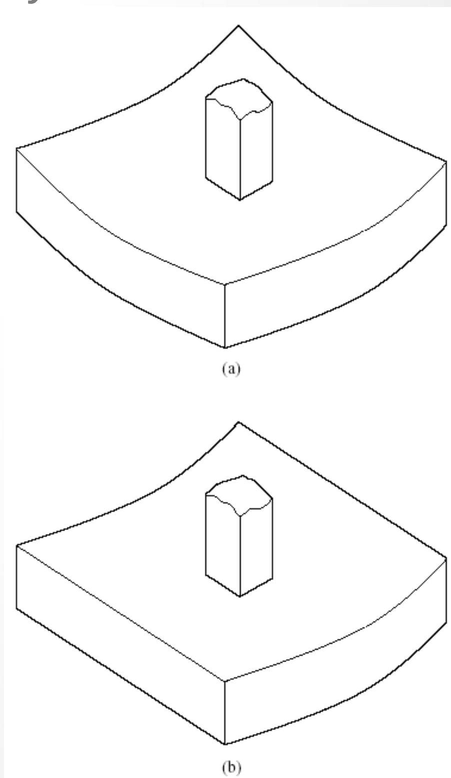
ENCE 4610
Foundation Analysis and Design

Lecture 23
Spread Footings: Structural Design, Flexural Design

Example of Flexure Design (Step 7)

- Design for Flexure
  - Use only the reinforcing steel for flexure considerations
  - Determine the total steel area
  - Check reinforcement development length
  - Determine the number and size of reinforcing bars necessary
- Given
  - Foundation design determined by two-way shear design example, T = 30"
- Find
  - Required steel area for flexure
  - Number and size of rebars used

- One vs. Two Way Bending
  - Footings in reality bend in two perpendicular directions
  - Footings are designed as if they bend in only one direction



Justification of One-Way Slab Assumption, Steel Area (Step 7)

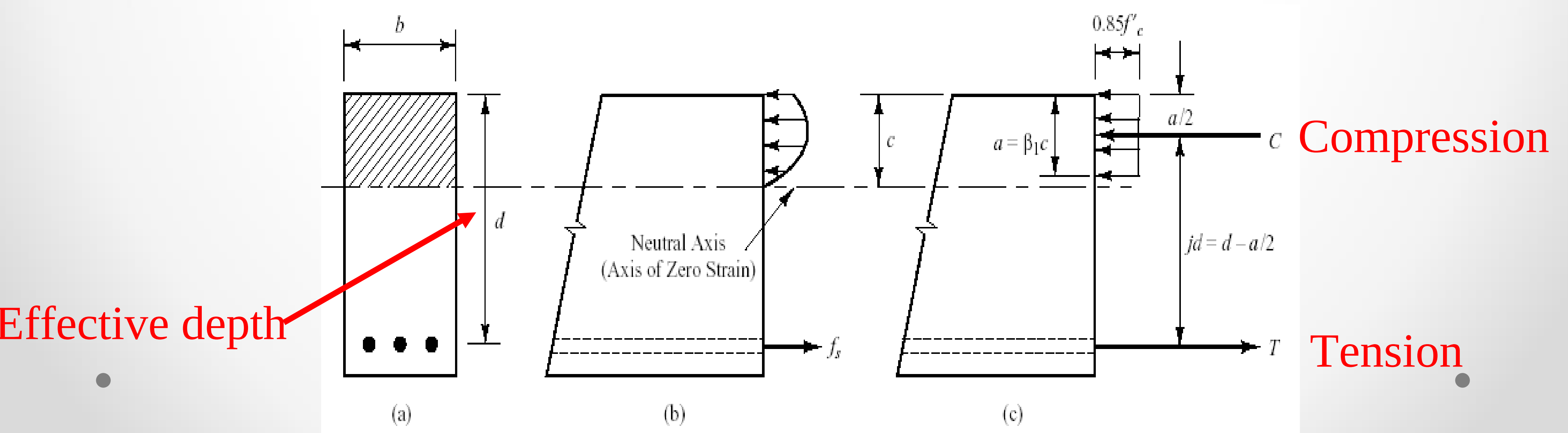
- Steel Area
  - Usual procedure is to prepare a moment diagram and select an appropriate amount of steel for each portion of the member
  - For spread footings, we can simplify this by identifying a *critical section for bending* and use the moment determined there to design the steel for the entire footing
  - Location of critical section for bending depends upon the type of column being used

- Justification of One-Way Slab Assumption
  - The full-scale load tests on which this analysis method is based were interpreted this way
  - Foundations should be designed more conservatively than the superstructure
  - Flexural stresses are low, so amount of steel required is nominal and often governed by ρ_{min}
  - Additional construction cost due to this simplified approach is minimal
  - Once footing is analysed one way, we place the same steel area in the perpendicular direction

Basic Considerations for Flexural Loads (Step 7)

- Factored moment on the critical surface M_{uc} determines the necessary dimensions of the member and the necessary size and location of the reinforcing bars
- This can be a complex process; however, geotechnical considerations tend to simplify the design process, as it dictates some of the options
- Amount of the steel required flexure depends upon the effective depth d

- Reinforcing Steel
  - Concrete's weakness in tension; thus, reinforcing steel must be added when tension is anticipated, which is virtually guaranteed with flexural loading
  - Reinforcing steel in foundation almost inevitably involves use of reinforcing bars (rebar); welded wire fabric, needles, etc., are not generally used
    - Since flexural stresses are usually small, Grade 40 (Metric Grade 300, f_y = 40 ksi or 300 MPa) steel is usually adequate, although unavailable for bars larger than #6, in which case Grade 60 (Metric Grade 420, f_y = 60 ksi or 420 MPa) steel may have to be used



Moment at Critical Bending Section (Step 7)

- Factored bending moment

Assumes P_u acts through centroid of footing

$$M_{uc} = \frac{P_u l^2}{2B} + \frac{2 M_u l}{B}$$

Based on soil bearing pressure with assumed eccentricity of B/3

- M_{uc} = factored moment at critical section for bending
- P_u = factored compressive load from column
- M_u = factored moment load from column
- l = cantilever distance
- B = footing width

Design Cantilever Distances (Step 7)

- Notes
  - Treat timber columns in same way as concrete columns
  - If column has circular, octagonal or other similar shape, use a square with equivalent cross-sectional area
  - If column has a circular or regular polygon cross-sectional area, base the analysis on an equivalent square

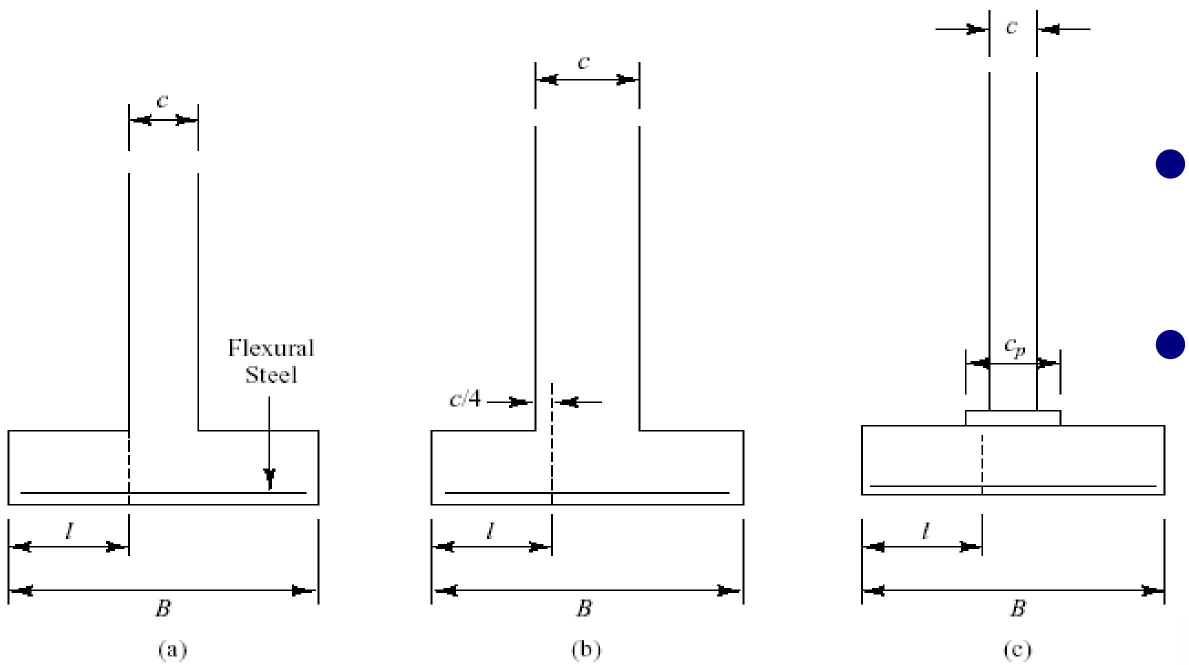


Figure 9.11 Location of critical section for bending: (a) with a concrete column; (b) with a masonry column; and (c) with a steel column.

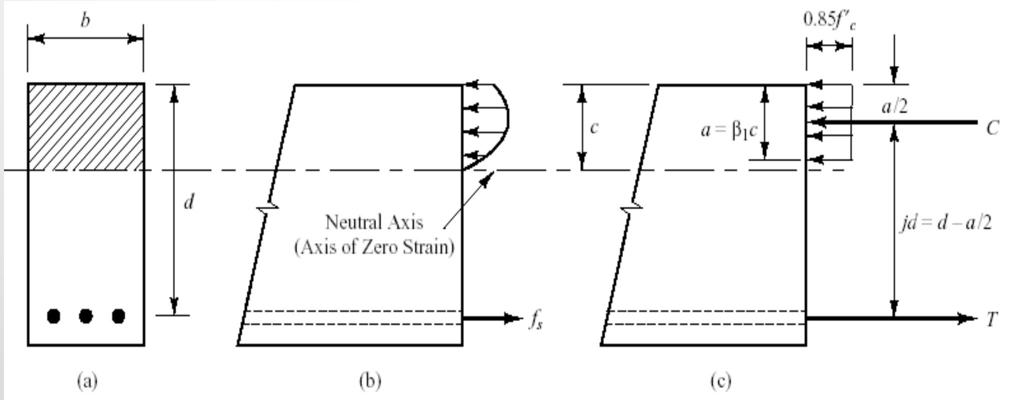
- Location of Critical Section for Bending
  - Concrete Columns
    - l = (B-c)/2
  - Masonry Columns
    - l = (B – c/2)/2
  - Steel Columns
    - l = (2B – (c + c_p))/4
  - Variables
    - B = footing width
    - c = column width
    - c_p = base plate width

$$l = \frac{B - c}{2} = \frac{126 - 21}{2} = 52.5 \text{ inches}$$

Determine the Total Steel Area (Step 7)

- Setting $M_{uc} = \varphi M_n$ (where M_{uc} = factored moment at the section being analysed), A_s can be solved to

$$A_s = \left(\frac{f'_c b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353 M_{uc}}{\varphi f'_c b}}\right)$$



- Nominal moment capacity of a flexural member made of reinforced concrete with f'_c < 30 MPa (4 ksi)

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$a = \frac{\rho d f_y}{0.85 f'_c}$$

$$\rho = \frac{A_s}{bd}$$

Equation for Steel Area (Step 7)

$$A_s = \left(\frac{f'_c b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353 M_{uc}}{\varphi f'_c b}}\right)$$

- Variables for equation:
  - A_s = cross-sectional area of reinforcing steel (sq. in., m^2)
  - f'_c = 28-day compressive strength of concrete (psi, MPa)
  - f_y = yield strength of reinforcing steel (psi, MPa)
  - ρ = steel ratio
  - b = width of flexural member (in, m)
  - d = effective depth (in., m)
  - φ = 0.9 for flexure in reinforced concrete
  - M_{uc} = factored moment at the section being analysed (in-lbs, MJ)

Steel Cross-Sectional Area, Spacing and Development Length (Step 7)

- Spacing of Rebar
  - Selection of reinforcing bar size and spacing must satisfy the following minimum and maximum spacing requirements
    - Clear space between bars must be at least equal to d_b, 25 mm (1"), or 4/3 times the nominal aggregate size
    - Centre-to-centre spacing of the reinforcement must not exceed 3T or 500 mm (18"), whichever is less
- Development Length
  - Development length l_d is the length rebars must extend through the concrete in order to develop proper anchorage
  - Assumptions for calculations of minimum development length
    - Clear spacing between the bars is at least $2d_b$
    - Concrete cover is at least d_b
- Minimum Steel Requirements (ACI 10.5.4 and 7.12.2)
  - $\rho = A_s/A_g$
    - A_s = Cross-sectional area of the steel
    - $A_g = Bd$ = Gross Cross-sectional area
  - For Grade 40 (Metric Grade 300) Steel: $\rho \geq 0.002$
  - For Grade 60 (Metric Grade 420) Steel: $\rho \geq 0.0018$
  - If $\rho < \rho_{min}$, use ρ_{min}. ρ is rarely larger than 0.004
- Carry the flexural steel to a point 70 mm (3") from the edge of the footing
- Maximum Steel Requirements (ACI 10.3) never govern the design of spread footings, but may be of concern in combined footings and mats

Find Required Steel Area (Step 7)

- Determine factored bending moment at critical section

$$M_{uc} = \frac{P_u l^2}{2B} + \frac{2M_u l}{B}$$

$$M_{uc} = \frac{991{,}000 \times 52.5^2}{2 \times 126} + 0$$

$$M_{uc} = 9{,}197{,}000 \text{ in - lbs}$$

- Determine steel area based on factored bending moment at critical section

$$A_s = \left(\frac{f'_c b}{1.176 f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353 M_{uc}}{\varphi f'_c b}}\right)$$

$$A_s = \frac{4000 \times 126}{1.176 \times 60{,}000}\left(24 - 24^2\sqrt{\frac{2.353 \times 9{,}197{,}000}{0.9 \times 4000 \times 126}}\right)$$

$$A_s = 7.25 \; in^2$$

Check for Minimum Steel Area and Determine Number and Size of Bars (Step 7)

- Check computed steel area against minimum steel requirements
  - $\rho = A_s/A_g = 7.25/(24 \times 126) = 0.0024$
  - For Grade 60 (Metric Grade 420) Steel: $\rho \geq 0.0018$
  - Since 0.0024 > 0.0018, OK

- Determine Size and Number of Rebars
  - Use #8 (1") bars: Area of each bar = 0.79 sq. in.
  - Number of bars = 8.94/0.79 = 9.17 so use 10 bars
  - Space between bars = 126/10 = 12.6"
  - Minimum spacing = 18" or 3T = (3)(27) = 81" so OK either way

Imperial Bar Size	"Soft" Metric Size	Weight (lb/m)	Weight (kg/m)	Nominal Diameter (in)	Nominal Diameter (mm)	Nominal Area (in^2)	Nominal Area (mm^2)
#3	#10	0.376	0.561	0.375	3	.11	71
#4	#13	0.668	0.996	0.500	4	.20	129
#5	#16	1.043	1.556	0.625	5	.31	200
#6	#19	1.502	2.240	0.750	6	.44	284
#7	#22	2.044	3.049	0.875	7	.60	387
#8	#25	2.670	3.982	1.000	8	.79	509
#9	#29	3.400	5.071	1.128	9	1.00	645
#10	#32	4.303	6.418	1.270	10	1.27	819
#11	#36	5.313	7.924	1.410	11	1.56	1006
#14	#43	7.650	11.41	1.693	14	2.25	1452
#18	#57	13.60	20.284	2.257	18	4.00	2581

Computation of Development Length (Step 7)

- The development length is measured from the critical section for bending to the end of the bars (usually 70 mm (3") from the end of the footing, even if loads don't require it)
- Supplied development length

$$(I_d)_{supplied} = l - 70\ mm\ (3\text{in})$$

  - $(I_d)_{supplied}$ = supplied development length
  - l = cantilever distance
  - This length must be greater than the required development length. If not, best solution is to use smaller rebars with shorter development lengths

Development Length (Step 7)

$$\frac{I_d}{d_b} = \frac{9}{10}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \text{ (SI Units)}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{10\, sn} = 0 \text{ (for spread footings)}$$

$$\frac{I_d}{d_b} = \frac{9}{10}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda d_b}{c}$$

$$\frac{I_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \text{ (US Units)}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500\, sn} = 0 \text{ (for spread footings)}$$

$$\frac{I_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda d_b}{c}$$

Development Length (Step 7)

- Variables for development length variables
  - α = reinforcement location factor
    - α = 1.3 for horizontal reinforcement with more than 300 mm (12") of fresh concrete below the bar
    - α = 1.0 for all other cases
  - β = coating factor
    - β = 1.5 for epoxy coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$
    - β = 1.2 for other epoxy coated bars or wires
    - β = 1.0 for uncoated bars or wires

- Variables for development length variables
  - l_d = minimum required development length (in., mm)
  - d_b = nominal rebar diameter (in, mm)
  - f_y = yield strength of reinforcing steel (psi, MPa)
  - f_{yt} = yield strength of transverse reinforcing steel (psi, MPa)
  - f'_c = 28-day compressive strenght of concrete (psi, MPa)

Development Length (Step 7)

- Variables for development length variables
  - A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and which crosses the potential place of splitting through the reinforcement being development (in^2, mm^2) – may conservatively be taken to be zero
  - s = maximum centre-to-centre spacing of transverse reinforcement within l_d (in, mm)
- The term $(c + K_{tr})/d_b < 2.5$
- Product $\alpha\ \beta < 1.7$
- Development length l_d > 300 mm (12")

- Variables for development length variables
  - γ = reinforcement factor
    - γ = 0.8 for #6 (metric #19) and smaller bars
    - γ = 1.0 for #7 (metric #22) and larger bars
  - λ = lightweight concrete factor = 1.0 for normal concrete (lightweight concrete is not used in foundations)
  - c = spacing or cover dimension (in, mm) = the smaller of the distance from the centre of the bar to the nearest concrete surface or one-half the centre-to-centre spacing of the bars

Development Length (Step 7)

- Check development length

$$\frac{l_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}$$

$$\frac{l_d}{d_b} = \frac{3}{40}\frac{60000}{\sqrt{4000}}\frac{1\times 1\times 1\times 1}{2.5}$$

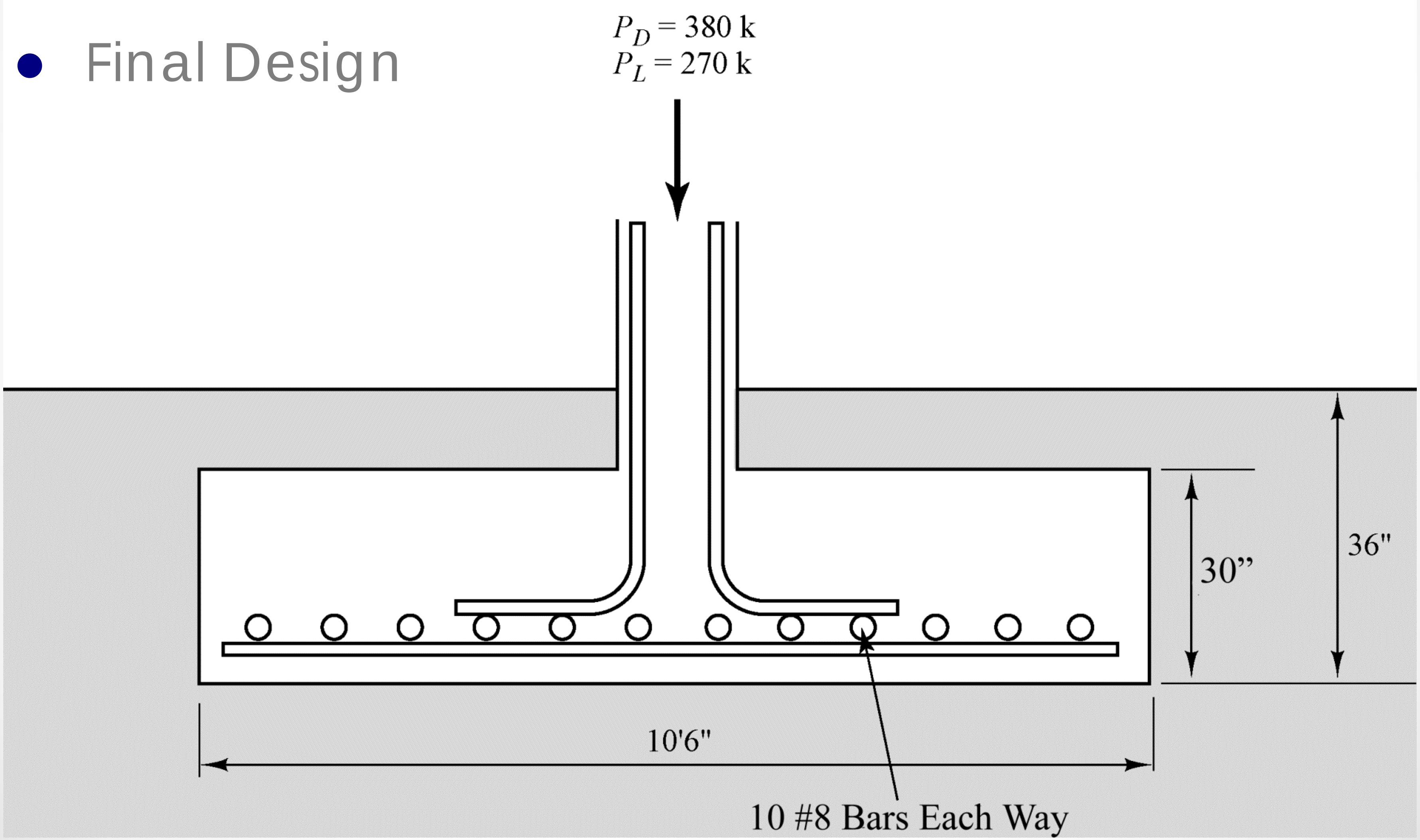
$$\frac{l_d}{d_b} = 28$$

- For 1" bars, l_d = 28" Since $(l_d)_{applied}$ = 49.5, development length is OK

- Check development length
  - $(l_d)_{supplied}$ = l – 3 = 52.5 – 3 = 49.5"
  - Check required inequalities
    - The term $(c + K_{tr})/d_b < 2.5$: (3.5 + 0)/1 = 3.5 > 2.5
    - Product $\alpha\beta < 1.7$: (1)(1) = 1 < 1.7 (using 3" cover and uncoated bars)
    - Development length l_d > 300 mm (12"): 49.5 > 12, so development length meets this criterion

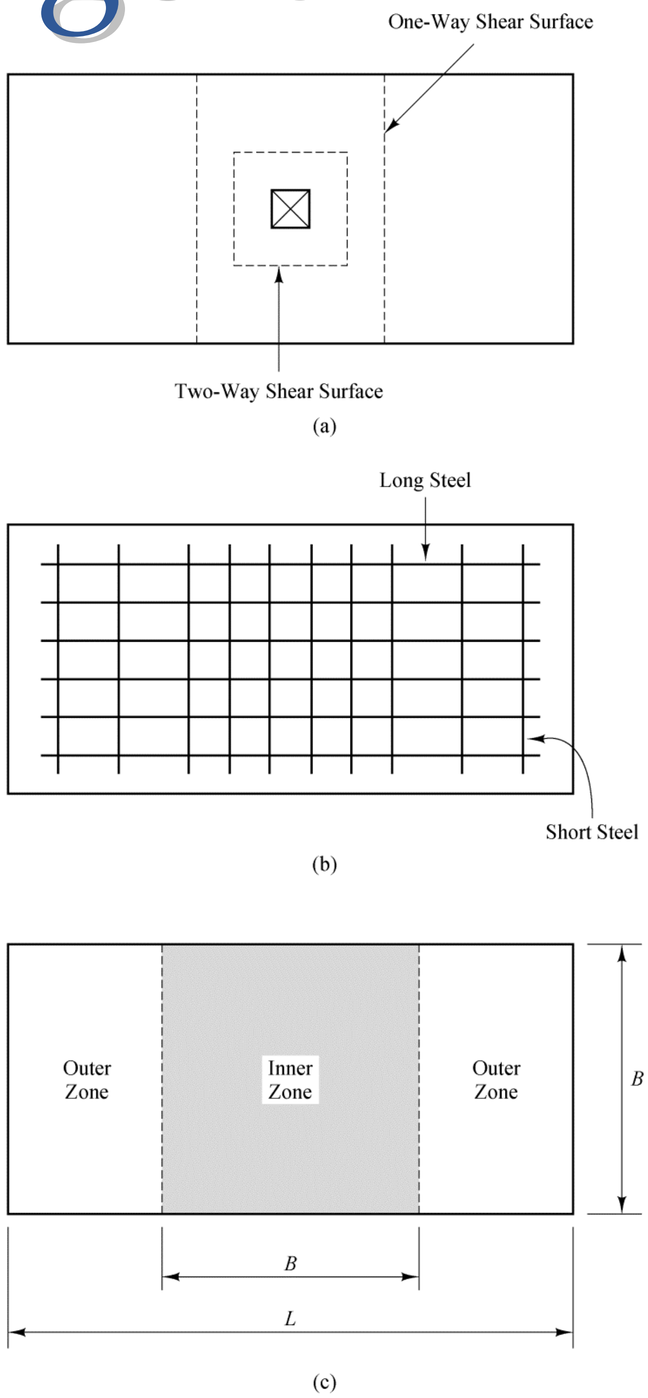
Example of Flexure Design

- Final Design



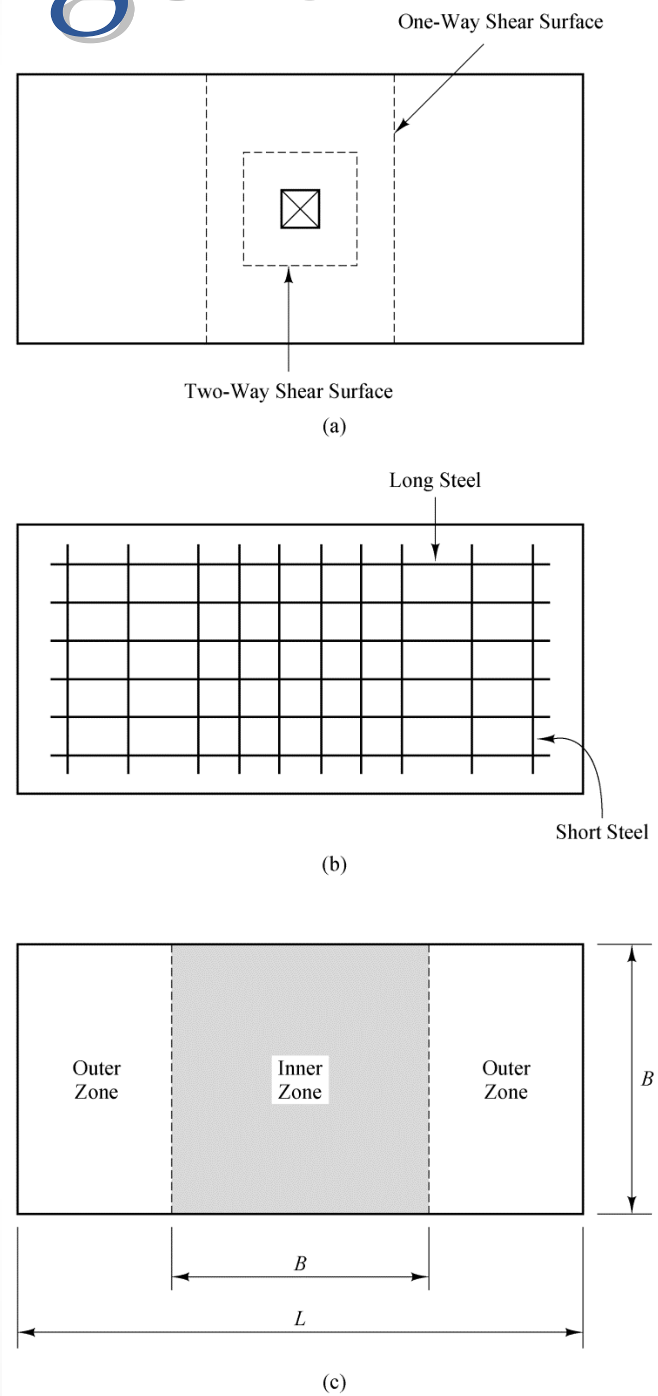
Design of Rectangular Footings

- Check for one and two way shear using the critical shear surfaces shown at right top and the formulae given in the previous slide presentation
- Design the long steel using the following values for l:
  - Concrete Columns
    - l = (L - c)/2
  - Masonry Columns
    - l = (L – c/2)/2
  - Steel Columns
    - l = (2L – (c + c_p))/4
  - Also use the following equation for M_{uc}:
- $$M_{uc} = \frac{P_u l^2}{2L} + \frac{2M_u l}{L}$$

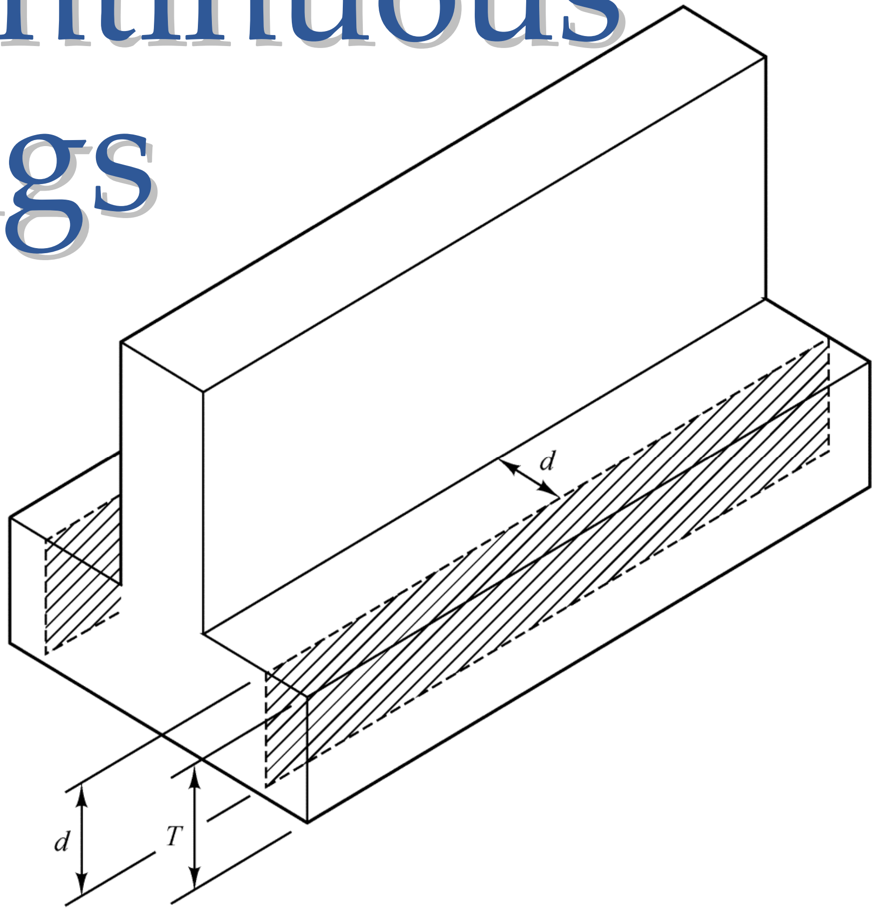


Design of Rectangular Footings

- Design the short steel using the same equations for square foundations, and the following equation for steel area:
  - $A_s = \rho L d$
- Since the flexural stresses tend to be concentrated in the center of the foundation, place more of the short steel in the inner zone. The portion of the total short steel area to be place in the inner zone is given as 2/(L/B+1)
- Distribute the rest of the steel evenly between the two outer zones.
-



Design of Continuous Footings



- Design for Shear
  - With continuous footings, we only have one-way shear
  - Use the shear formulae given in the previous slide presentation and solve for d, which will give you the minimum thickness T
- Design for Flexure
  - Longitudinal steel can be configured by a cross-sectional area of $.0018A_g < As < .0020A_g$ with at least two (2) #4 bars (#13 metric bars)
  - Lateral steel is unnecessary for "narrow" continuous footings but can be important for wider foundations
  - Lateral steel design for continuous footings is basically the same as for "finite" footings except that the force/moment per unit length is substituted for the force/moment

$$M_{uc}/b = \frac{P_u/b \; l^2}{2B} + \frac{2\, M_u/b \; l}{B}$$

$$A_s = \left(\frac{f'_c\, b}{1.176\, f_y}\right)\left(d - \sqrt{d^2 - \frac{2.353\, M_{uc}/b}{\varphi f'_c}}\right)$$

Structural Design of Footings (pp. 144-145)

REINFORCED CONCRETE DESIGN ACI 318-05

US Customary units

Definitions

- a = depth of equivalent rectangular stress block, in.
- A_g = gross area of column, in^2
- A_s = area of tension reinforcement, in^2
- A_s' = area of compression reinforcement, in^2
- A_{st} = total area of longitudinal reinforcement, in^2
- A_v = area of shear reinforcement within a distance s, in.
- b = width of compression face of member, in.
- b_e = effective compression flange width, in.
- b_w = web width, in.
- β_1 = ratio of depth of rectangular stress block, a, to depth to neutral axis, c

$$= 0.85 \geq 0.85 - 0.05\left(\frac{f'_c - 4{,}000}{1{,}000}\right) \geq 0.65$$

- c = distance from extreme compression fiber to neutral axis, in.
- d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in.
- d_t = distance from extreme compression fiber to extreme tension steel, in.
- E_c = modulus of elasticity = $33w_c^{1.5}\sqrt{f'_c}$, psi
- ε_t = net tensile strain in extreme tension steel at nominal strength
- f'_c = compressive strength of concrete, psi
- f_y = yield strength of steel reinforcement, psi
- h_f = T-beam flange thickness, in.
- M_c = factored column moment, including slenderness effect, in.-lb
- M_n = nominal moment strength at section, in.-lb
- ϕM_n = design moment strength at section, in.-lb
- M_u = factored moment at section, in.-lb
- P_n = nominal axial load strength at given eccentricity, lb
- ϕP_n = design axial load strength at given eccentricity, lb
- P_u = factored axial force at section, lb
- ρ_g = ratio of total reinforcement area to cross-sectional area of column = A_{st}/A_g
- s = spacing of shear ties measured along longitudinal axis of member, in.
- V_c = nominal shear strength provided by concrete, lb
- V_n = nominal shear strength at section, lb
- ϕV_n = design shear strength at section, lb
- V_s = nominal shear strength provided by reinforcement, lb
- V_u = factored shear force at section, lb

ASTM STANDARD REINFORCING BARS

BAR SIZE	DIAMETER, IN.	AREA, IN^2	WEIGHT, LB/FT
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.60

LOAD FACTORS FOR REQUIRED STRENGTH

$U = 1.4\,D$

$U = 1.2\,D + 1.6\,L$

SELECTED ACI MOMENT COEFFICIENTS

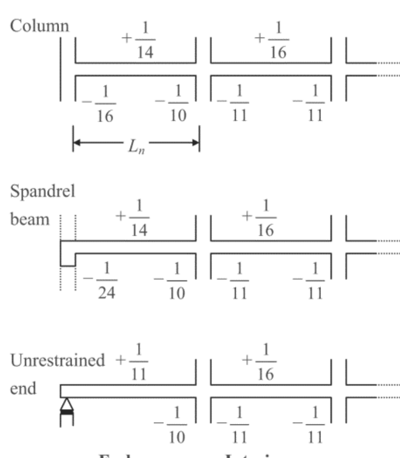
Approximate moments in continuous beams of three or more spans, provided:

1. Span lengths approximately equal (length of longer adjacent span within 20% of shorter)
2. Uniformly distributed load
3. Live load not more than three times dead load

M_u = coefficient * w_u * L_n^2

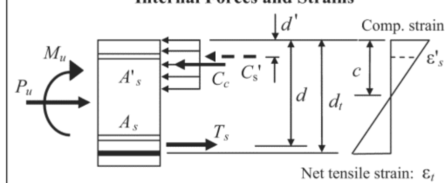
w_u = factored load per unit beam length

L_n = clear span for positive moment; average adjacent clear spans for negative moment

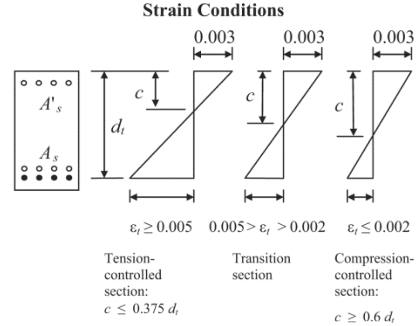


UNIFIED DESIGN PROVISIONS

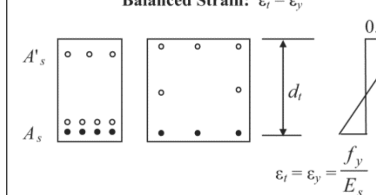
Internal Forces and Strains



Strain Conditions



Balanced Strain: $\varepsilon_t = \varepsilon_y$



RESISTANCE FACTORS, ϕ

Tension-controlled sections ($\varepsilon_t \geq 0.005$):	$\phi = 0.9$
Compression-controlled sections ($\varepsilon_t \leq 0.002$):	
Members with spiral reinforcement	$\phi = 0.70$
Members with tied reinforcement	$\phi = 0.65$
Transition sections ($0.002 < \varepsilon_t < 0.005$):	
Members with spiral reinforcement	$\phi = 0.57 + 67\varepsilon_t$
Members with tied reinforcement	$\phi = 0.48 + 83\varepsilon_t$
Shear and torsion	$\phi = 0.75$
Bearing on concrete	$\phi = 0.65$

BEAMS – FLEXURE: $\phi M_n \geq M_u$

For all beams

Net tensile strain: $a = \beta_1 c$

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} = \frac{0.003(\beta_1 d_t - a)}{a}$$

Design moment strength: ϕM_n

where: $\phi = 0.9$ [$\varepsilon_t \geq 0.005$]

$\phi = 0.48 + 83\varepsilon_t$ [$0.004 \leq \varepsilon_t < 0.005$]

Reinforcement limits:

$A_{S,\,max}$ $\varepsilon_t = 0.004$ @ M_n

$$A_{S,\,min} = \text{larger}\left\{\frac{3\sqrt{f'_c}\,b_w d}{f_y} \text{ or } \frac{200 b_w d}{f_y}\right.$$

$A_{s,min}$ limits need not be applied if A_s (provided) $\geq 1.33\,A_s$ (required)

Singly-reinforced beams

$$A_{s,max} = \frac{0.85 f_c' \beta_1 b}{f_y}\left(\frac{3d_t}{7}\right)$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_n = 0.85 f_c'\, a\, b\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

Doubly-reinforced beams

Compression steel yields if:

$$A_s - A_s' \geq \frac{0.85\beta_1 f_c' d' b}{f_y}\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

If compression steel yields:

$$A_{s,max} = \frac{0.85 f_c' \beta_1 b}{f_y}\left(\frac{3 d_t}{7}\right) + A_s'$$

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b}$$

$$M_n = f_y\left[(A_s - A_s')\left(d - \frac{a}{2}\right) + A_s'(d - d')\right]$$

If compression steel does not yield (two steps):

1. Solve for c:

$$c^2 + \left(\frac{(87{,}000 - 0.85 f_c') A_s' - A_s f_y}{0.85 f_c' \beta_1 b}\right) c - \frac{87{,}000\, A_s' d'}{0.85 f_c' \beta_1 b} = 0$$

Structural Design of Footings (p. 146)

BEAMS – FLEXURE: $\phi M_n \geq M_u$ (CONTINUED)

Doubly-reinforced beams (continued)

Compression steel does not yield (continued)

2. Compute M_n:

$$M_n = 0.85\, b\, c\, \beta_1 f_c' \left(d - \frac{\beta_1 c}{2}\right) + A_s' \left(\frac{c - d'}{c}\right)(d - d')\ 87{,}000$$

T-beams – tension reinforcement in stem

Effective flange width:

$$b_e \underset{\text{smallest}}{=} \begin{cases} 1/4 \bullet \text{span length} \\ b_w + 16 \bullet h_f \\ \text{beam centerline spacing} \end{cases}$$

Design moment strength:

$$a = \frac{A_s f_y}{0.85 f_c' b_e}$$

If $a \leq h_f$:

$$A_{s,max} = \frac{0.85 f_c' \beta_1 b_e}{f_y}\left(\frac{3 d_t}{7}\right)$$

$$M_n = 0.85 f_c' a\, b_e \left(d - \frac{a}{2}\right)$$

If $a > h_f$:

$$A_{s,max} = \frac{0.85 f_c' \beta_1 b_w}{f_y}\left(\frac{3 d_t}{7}\right) + \frac{0.85 f_c' (b_e - b_w) h_f}{f_y}$$

Redefine a: $$a = \frac{A_s f_y}{0.85 f_c' b_w} - \frac{h_f (b_e - b_w)}{b_w}$$

$$M_n = 0.85 f_c' \left[h_f (b_e - b_w)\left(d - \frac{h_f}{2}\right) + a\, b_w \left(d - \frac{a}{2}\right)\right]$$

BEAMS – SHEAR: $\phi V_n \geq V_u$

Beam width used in shear equations:

$$b_w = \begin{cases} b \text{ (rectangular beams)} \\ b_w \text{ (T-beams)} \end{cases}$$

Nominal shear strength:

$$V_n = V_c + V_s$$

$$V_c = 2\, b_w\, d \sqrt{f_c'}$$

$$V_s = \frac{A_v f_y d}{s} \quad \left(\text{may not exceed } 8\, b_w\, d \sqrt{f_c'}\right)$$

Required and maximum-permitted stirrup spacing, s

$V_u \leq \frac{\phi V_c}{2}$: No stirrups required

$V_u > \frac{\phi V_c}{2}$: Use the following table (A_v given):

	$\frac{\phi V_c}{2} < V_u \leq \phi V_c$	$V_u > \phi V_c$
Required spacing	Smaller of: $s = \frac{A_v f_y}{50 b_w}$ $s = \frac{A_v f_y}{0.75 b_w \sqrt{f_c'}}$	$V_s = \frac{V_u}{\phi} - V_c$ $s = \frac{A_v f_y d}{V_s}$
Maximum permitted spacing	Smaller of: $s = \frac{d}{2}$ OR $s = 24"$	$V_s \leq 4\, b_w\, d \sqrt{f_c'}$ Smaller of: $s = \frac{d}{2}$ OR $s = 24"$ $V_s > 4\, b_w\, d \sqrt{f_c'}$ Smaller of: $s = \frac{d}{4}$ $s = 12"$

Questions?