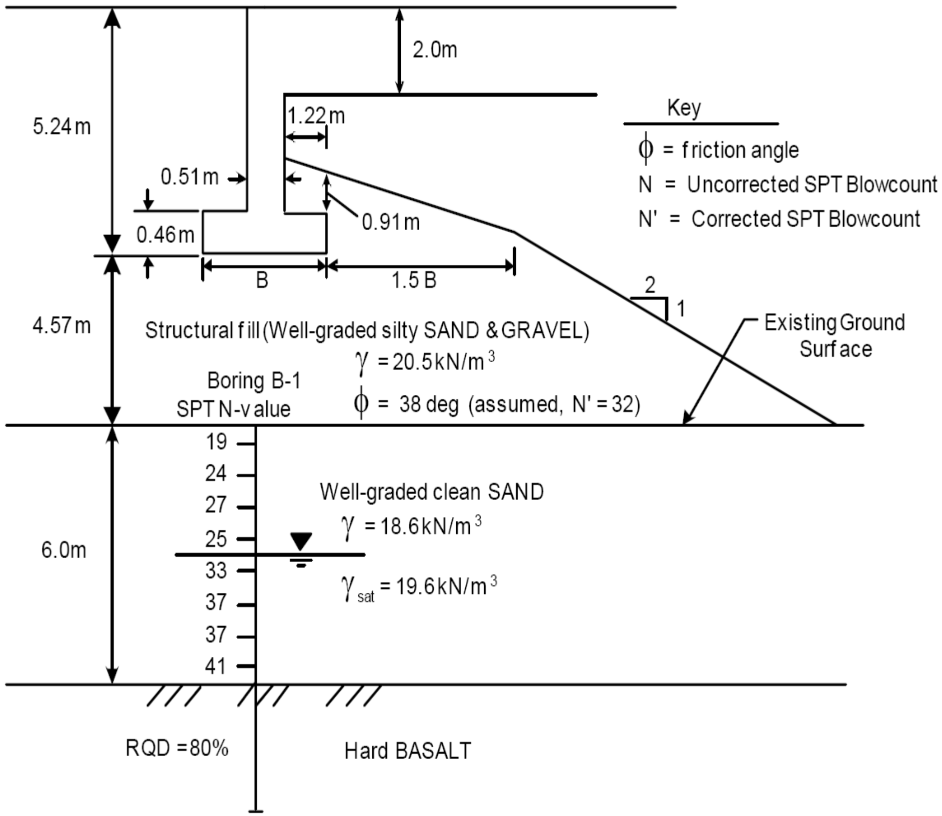
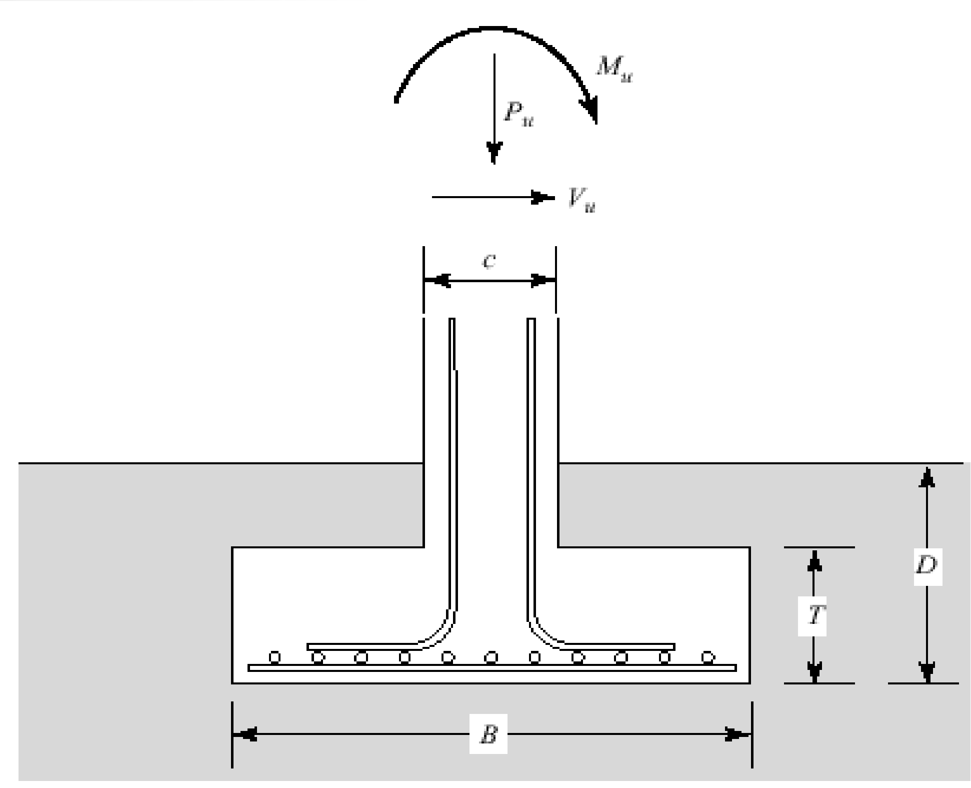


ENCE 4610
Foundation Analysis and Design

Lecture 22
Spread Footings
Structural Design

Structural Design of Shallow Footings

- Structural Design Loads



- Once geotechnical suitability (stability, bearing capacity, settlement, etc.) is established, the designer must turn to the structural capabilities of the foundation

History and Rationale of Design Methods

- Background
  - First studies on foundation structural failures were done by Talbot at the University if Illinois in 1913
  - Advances in the next 50 years include F.E. Richart's tests at the University of Michigan
  - Results were synthesised into the methodology used today by a committee sponsored by the ACI and ASCE. This was published in 1962.
  - Although work has been done since, changes in the methodology are not imminent

- Rationale
  - Spread footings are inexpensive; conservative design is also
  - Additional weight does not bear on any other member
  - Construction tolerances are so wide that precision in design would probably be lost
  - Structural methods are more precise than the geotechnical methods
  - Performance has been good
  - Additional weight provides uplift load resistance

Outline of Structural Design of Shallow Footings

1. Determine Net Unfactored (ASD) Bearing Capacity of Foundation, and its Distribution Across Foundation
   - Insure that resultant force is within the kern; do not proceed if it is not
2. Determine minimum area and dimensions for geotechnical capacity
3. Determine factored (LRFD) axial loads and moments on foundation
   - For structural analysis, it's best to compute the force at the centroid and then have an applied moment
4. Establish minimum cover, standard dimension requirements and material specifications

Outline of Structural Design of Shallow Footings

5. Design for One-Way (Beam) Shear
   - Neglect any effect of reinforcing steel
6. Design for Two-Way (Column Punching) Shear
   - Neglect any effect of reinforcing steel
   - Unnecessary when the foundation can only be loaded one way, i.e., strip foundations
7. Design for Flexure
   - Use only the reinforcing steel for flexure considerations
   - Determine the total steel area
   - Check reinforcement development length
   - Determine the number and size of reinforcing bars necessary

Shallow Foundation Structural Design Example

- Given
  - 21" square reinforced concrete column
    - Vertical dead load of 380 kips
    - Vertical live load of 270 kips
  - Supported on square spread footing
    - Soil allowable bearing pressure q_a = 6500 psf
    - Groundwater table well below the bottom of the foundation
    - f'_c = 4000 psi
  - Find
    - Required width B, thickness T, and effective depth d of the square foundation

Shallow Foundation Structural Design Example
Step 1: Design foundation to carry geotechnical load
Step 2: Determine Minimum Dimensions for Geotechnical Capacity

- Design foundation to carry geotechnical load
  - Use ASD or LRFD methods for geotechnical analysis
  - In this case, we will simply add the two loads to produce an ASD load for geotechnical purposes
  - $P = P_D + P_L$ =380 kips + 270 kips = 650 kips
  - Since load is concentric, it's within the kern (always check to make sure whether this is so or not)

- Determine minimum depth for the foundation
  - Use table below as a guide
  - Design depth for 650 kip foundation = 36"

Load P, kips	Minimum D, in.		Load P, kN	Minimum D, mm
0-65	12		0-300	300
65-140	18		300-500	400
140-260	24		500-800	500
260-420	30		800-1100	600
420-650	36		1100-1500	700
			1500-2000	800
			2000-2700	900
			2700-3500	1000

Shallow Foundation Structural Design Example
Step 2: Determine Minimum Dimensions for Geotechnical Capacity

- Knowing the depth of the foundation, determine the pressure due to the weight of the foundation
  - $p_f = \gamma D$ = (150 pcf)(3') = 750 psf
- Subtract this value for the net foundation pressure
  - $p_{net} = q_a - p_f$ = 6500 – 750 = 5750 psf
- Determine the minimum foundation area:
  - $A_{min} = P/p_{net}$ = (650 x 1000)/5750 = 113.04 ft^2
- Determine dimensions of foundations
  - In general, $BL = A_{min}$
  - For square foundations, $A_{min} = B^2$, or $B = (A_{min})^{1/2}$
  - $B = (113.04)^{1/2}$ = 10.63' → 10' 9"

Shallow Foundation Structural Design Example
Step 3: Determine factored (LRFD) axial loads and moments on foundation

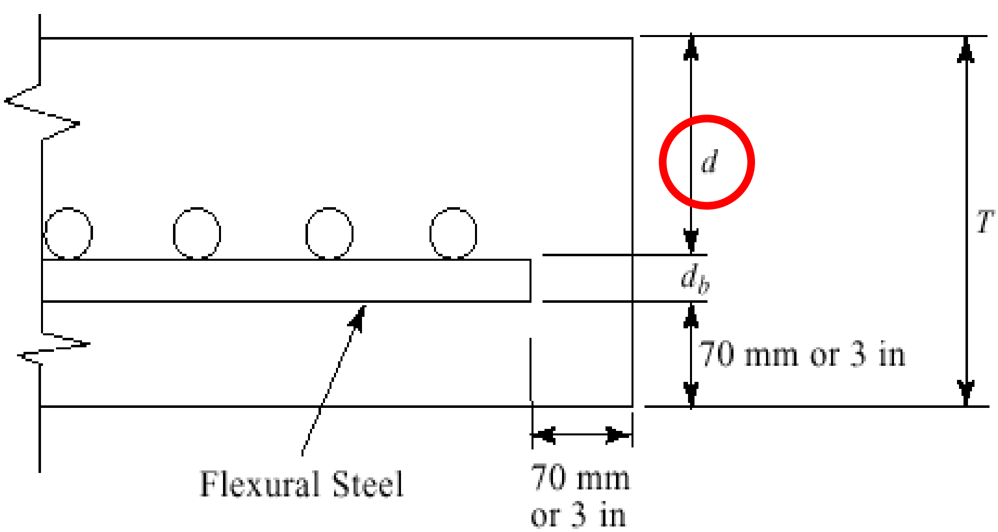
- Determine factored load for LRFD computations
  - Use ACI Section 9.2.1 load factors
  - For all problems in this course, we will only consider dead and live loads on the foundations
  - Thus we only consider one of two cases:
    - U = 1.4 D
    - U = 1.2 D + 1.6 L
      - U = Factored load on foundation
      - D = dead load on foundation
      - L = live load on foundation
- P_u = (1.2)(380) + (1.6)(270) = 881 kips = 881,000 lbs
- It is not necessary to include the foundation weight for structural design purposes
- In this case, there are no moments, only concentric axial forces
- Determine concrete and steel rebar properties

Shallow Foundation Structural Design Example
Step 4: Establish minimum cover, standard dimension requirements and material specifications

- Structural members frequently use concrete that has 20 MPa < f'_c < 35 MPa and in some cases up to 70 MPa
  - The stronger concrete enables weight reduction in the structure
- For foundations, since the geotechnical design usually drives the basic dimensions, and the strength of the soil is limited, higher strength concrete is usually not appropriate
  - Shallow foundations are frequently used to « ballast » structures against uplift loads, so weight can be useful
- Spread footings are usually designed so that 15 MPa < f'_c < 20 MPa (2 ksi < f'_c < 3 ksi).
- In cases where the footing carries loads greater than 2 MN (500 kips), an f'_c = 35 MPa (5 ksi) might be justified
- Reinforcing Bars
  - Since flexural stresses are usually small, Grade 40 (Metric Grade 300) steel is usually adequate, although unavailable for bars larger than #6, in which case Grade 60 (Metric Grade 420) steel may have to be used
- For this example, concrete and steel rebar properties are as follows:
  - f'_c = 4000 psi = 4 ksi
  - f_y = 60,000 psi = 60 ksi

Shallow Foundation Structural Design Example
Step 4: Establish minimum cover, standard dimension requirements and material specifications

- Leveling Slab
  - Sometimes contractors place a thin layer of lean concrete (mud slab or leveling slab) to provide a smooth working surface for the foundation
- Design Depth d
  - For design purposes, we ignore the concrete under the reinforcing bars. Only the concrete depth between the top of the footing and the rebars is considered (effective depth, d)
- Minimum Cover Requirements
  - ACI code specifies that at least 70 mm (3 inches) of concrete cover must be included from ground contact
  - This takes into consideration irregularities in the excavation and corrosion factors
  - In some cases (such as loose sands or soft clays where level excavation is hard to maintain,) more cover may be appropriate

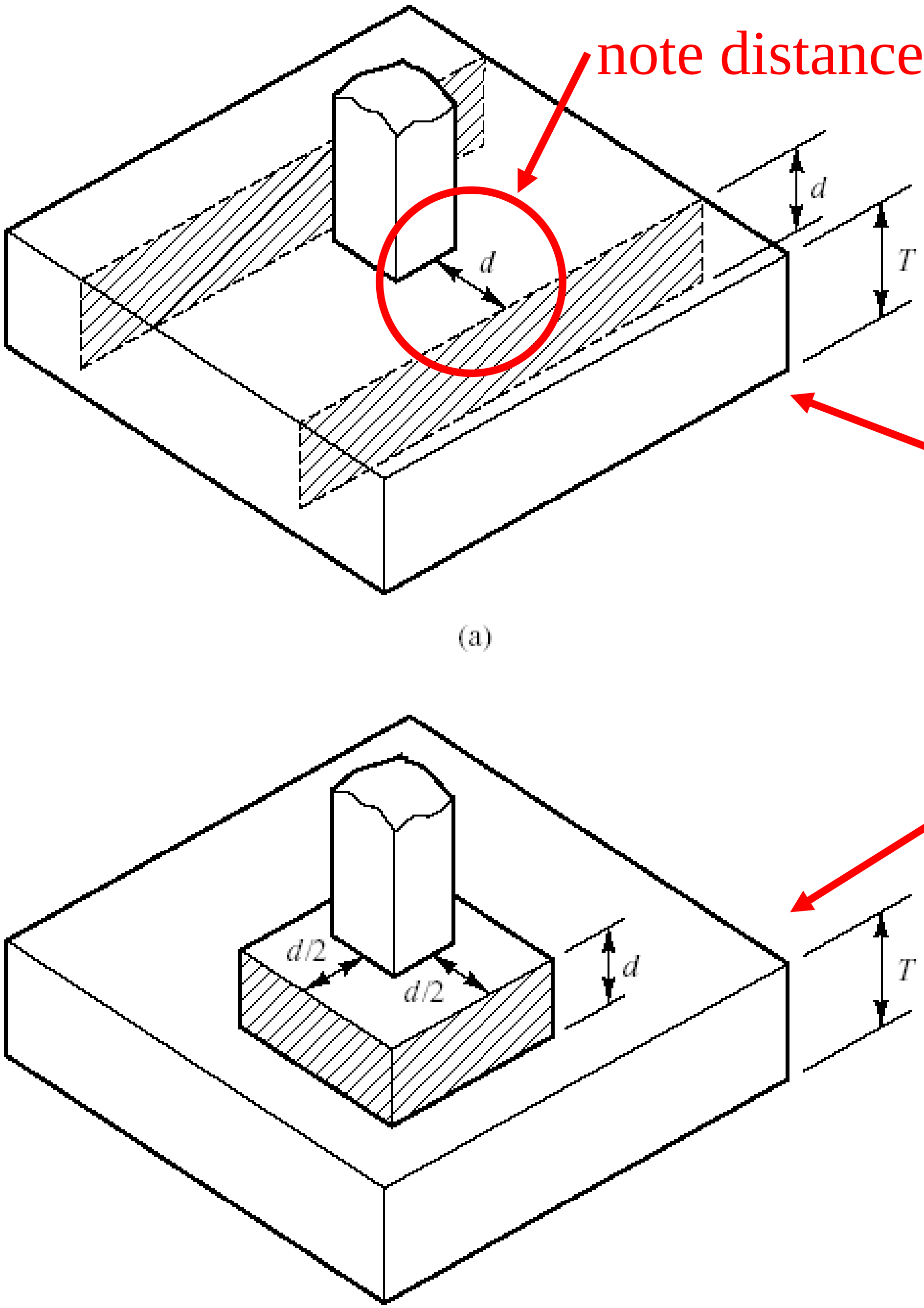


$$d = T - 3in. - d_b$$

$$d = T - 70\ mm - d_b$$

Shallow Foundation Structural Design Example
Step 4: Establish minimum cover, standard dimension requirements and material specifications

- Assumptions for this problem:
  - Cover = 3”
  - Assume a value of of d_b = 1“ for shear analysis
  - This may be changed in flexural analysis.
  - There is no need to go back and redo the shear analysis for larger d_b
  - d = T - 4”
- High precision in specifying the depth of excavation is unnecessary because of the capabilities of excavators
- Standard thicknesses T:
  - English Units: Multiples of 3": 12", 15", 18"...
  - SI Units: Multiples of 100 mm :300 mm, 400 mm, 500 mm...



(b)

One vs. Two Way Shear (Steps 5 & 6)

- ACI defines two modes for shear failure in square footings
  - One-way shear (beam shear or wide-beam shear)
  - Two-way shear (diagonal tension shear, or punching shear of column through foundation)

Design Conditions for Shear (Steps 5 & 6)

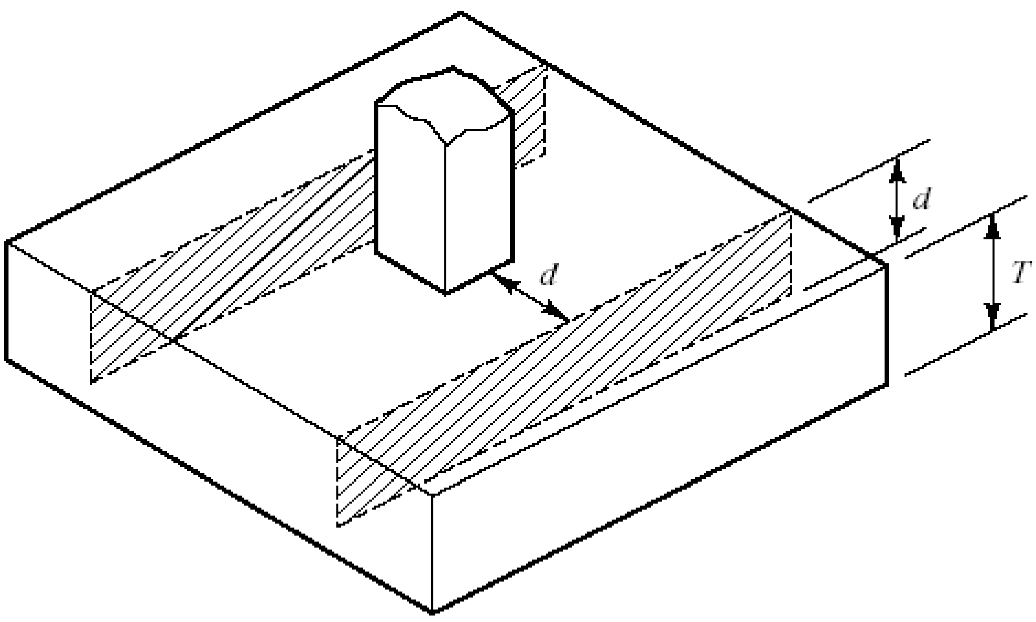
- Footing design is satisfactory for shear when

$$V_{uc} \leq \varphi V_{nc}$$

  - V_{uc} = factored shear force on critical surface
  - φ = resistance factor for shear = 0.75
  - V_{nc} = nominal shear capacity on the critical surface

$$V_{nc} = V_c + V_s$$

  - V_c = nominal shear load capacity of concrete
  - V_s = nominal shear load capacity of steel (neglected)



Shallow Foundation Structural Design Example

Step 5: Design for One-Way (Beam Action) Shear

- Assumptions
  - The applied normal, moment and shear loads must be multiplied by (B – c – 2d)/B before applying them to the critical vertical planes. This factor is the ratio of the footing base area outside the critical planes to the total area, and thus reflects the percentage of the applied loads that must be transmitted through the critical vertical planes
  - The maximum shear stress on the critical vertical surfaces is the vector sum of those due to the applied normal, moment and shear loads
  - Shear stress caused by the applied shear load is uniformly distributed across the planes

- Assumptions
  - Shear stress caused by the applied vertical load P_u is uniformly distributed across the two vertical planes as shown in the previous slide
  - Shear stress on the vertical planes caused by the applied moment load M_u is expressed by the flexure formula $\tau = Mc/I$, and thus is the greatest in the left and right edges of these planes
  - The factored shear stress on the critical vertical surfaces is the greatest shear stress multiplied by the area of the shear surfaces. This may be greater than the integral of the shear stress across the shear surfaces, but is useful because it produces a design that keeps the maximum shear stress within acceptable limits.

Shallow Foundation Structural Design Example
Step 5: Design for One-Way (Beam Action) Shear

- Factored shear force on the critical vertical surfaces

$$V_{uc} = \frac{A_{out}}{A_{tot}} P_u$$

  - V_{uc} = shear force on critical shear surfaces (lb,N)
  - A_{out} = outer area of foundation (ft^2, m^2)
  - A_{tot} = total area of foundation (ft^2, m^2)
  - P_u = applied normal load
  - M_u = applied moment load
  - V_u = applied shear load

- Nominal one-way shear load capacity on the critical section

$$V_{nc} = V_c = 144\, A_\tau \sqrt{f'_c} \text{ (U.S. Units)}$$

$$V_{nc} = V_c = \frac{1\times 10^6}{12} A_\tau \sqrt{f'_c} \text{ (S.I. Units)}$$

  - V_{nc} = nominal one-way shear capacity on the critical section (lb, N)
  - V_c = nominal one-way shear capacity of concrete (lb, N)
  - A_τ = shear area of concrete (see chart at end of presentation) (ft^2, m^2)
  - f'_c = 28-day compressive strength of concrete (psi, MPa)

Shallow Foundation Structural Design Example Step 5: Design for One-Way (Beam Action) Shear

- Substituting into equations for shear,

$$A_{tot} = B^2 = (10.5)^2 = 110.25\,\text{ft}^2$$

$$A_{out} = B(B-(c+2d)) = 10.5(10.5-(1.75+2d))\,\text{ft}^2$$

$$V_{uc} = 881{,}000\frac{10.5(10.5-(1.75+2d))}{110.25}$$

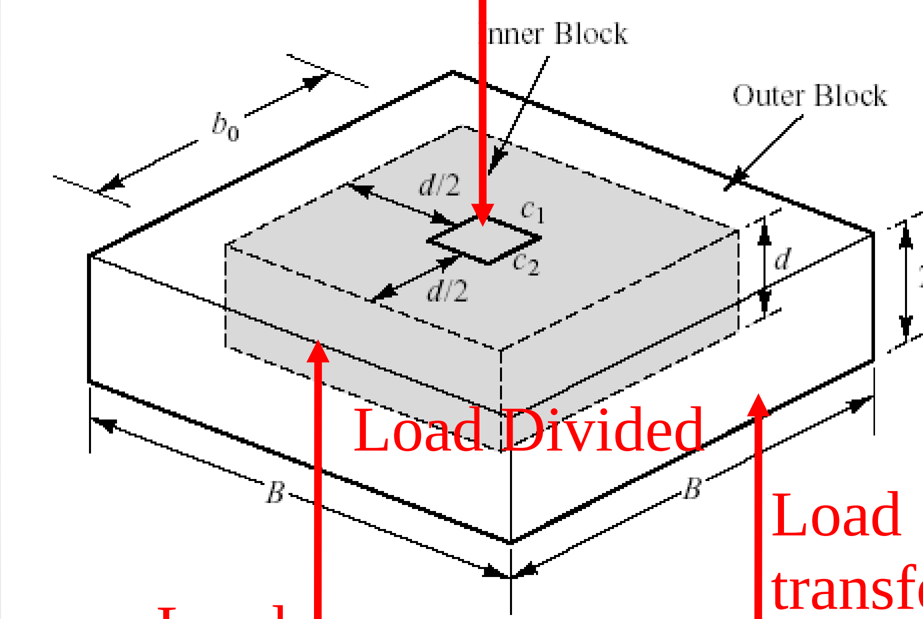
$$V_{nc} = (0.75)(144)(10.5)d\sqrt{4000}$$

$$881{,}000\frac{10.5(10.5-(1.75+2d))}{110.25} = (0.75)(144)(10.5)d\sqrt{4000}$$

  - (this formulation only good for US units, lbs and ft, using f'_c in psi)
- Solving for, d = 1.608' = 19.3" → 21"
  - d can be solved either by trial and error or Goal Seek, or explicitly solving for d (equation is linear)

Two-Way Shear (Step 6)

Factored Normal Load P



Load Divided

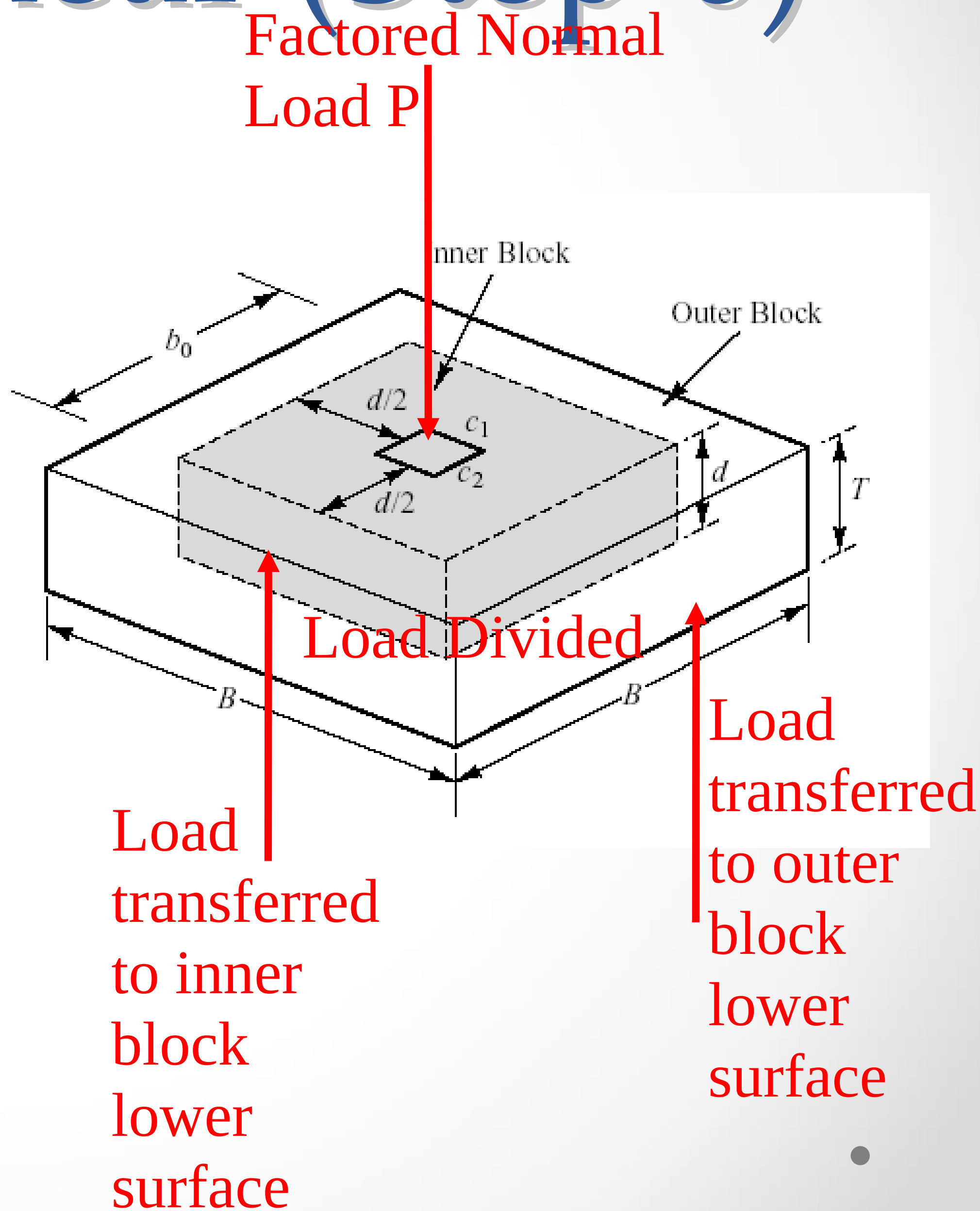
Load transferred to inner block lower surface

Load transferred to outer block lower surface -- only this load produces shear on critical shear surface

- Two-way shear can be caused by the column load P, moment M and horizontal shear V on the foundation
- To analyse the shear force, we divide the footing into two blocks

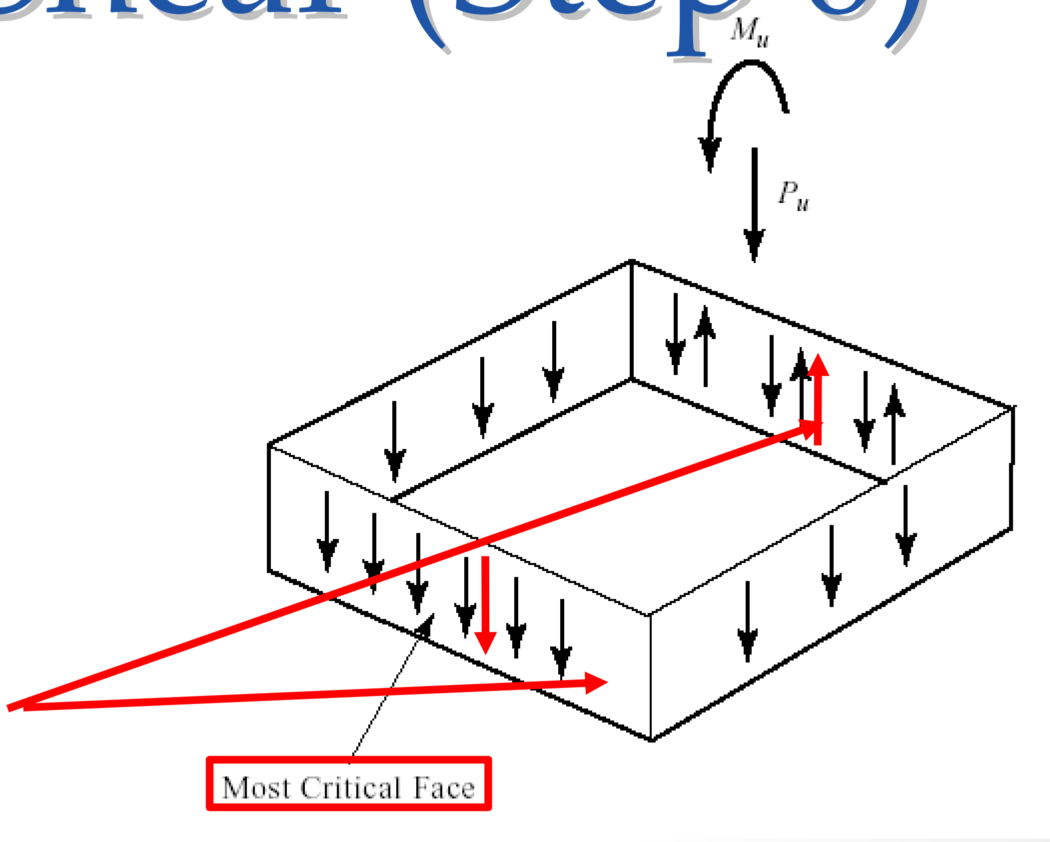
Two-Way Shear (Step 6)

- Foundation Base Areas for Calculation
  - Total area of footing $A_{tot} = BL$
  - Area inside of shear surface $A_{in} = (c+d)^2$ (always square)
  - Area outside of shear surface $A_{out} = A_{tot} - A_{in}$
- Shear Force for Two-Way Shear, Concentric Load Only
  - $V_{uc} = P_u (A_{out}/A_{tot}) = P_u (1-A_{in}/A_{tot})$
  - This is the total shear; for cases when we consider the shear on one face only, this value is divided by four



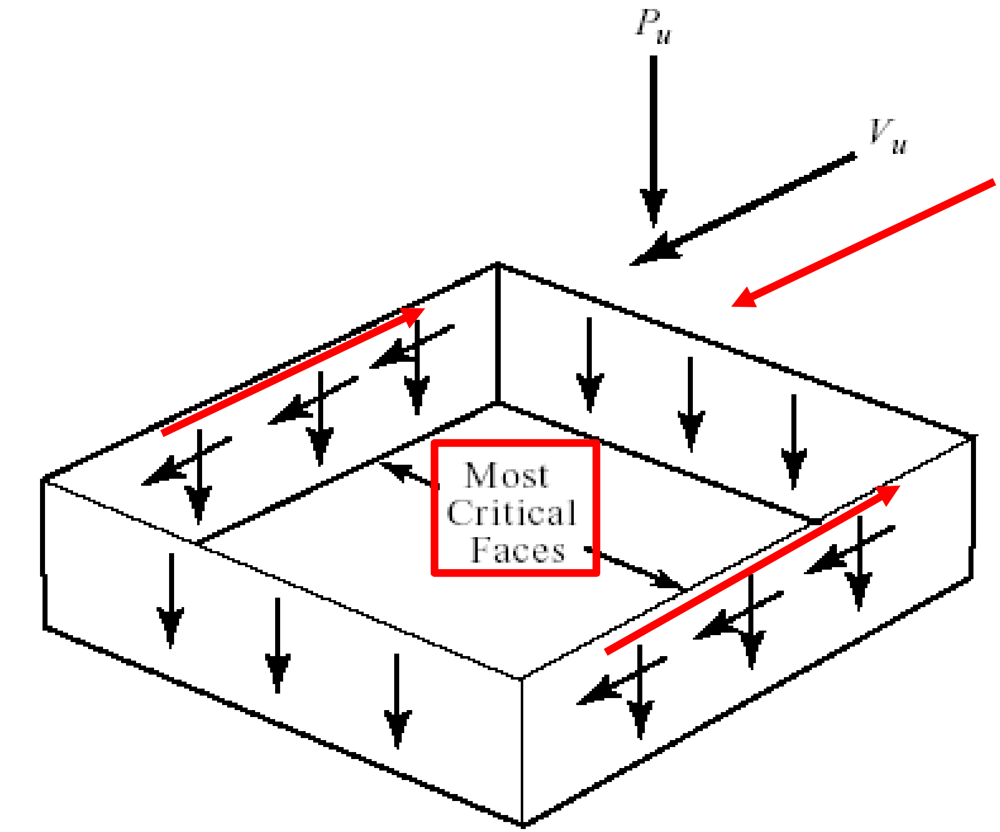
Two-Way Shear (Step 6)

- The percentage of P_u that produces shear along the critical surfaces is the ratio of the base area of the outer block to the total base area
- An applied moment load M_u will produce an additional shear force on the two opposing faces of the inner block



$$V_{uc} = 4\left(\frac{P_u}{4} + \frac{M_u}{c+d}\right)\frac{A_{out}}{A_{tot}}$$

Two-Way Shear (Step 6)



- In the presence of an applied shear load V_u, if it acts in the same direction as the moment load, it produces a shear load on the other two faces
- Shear force on each critical face with an applied shear load

$$V_{uc} = 4\frac{A_{out}}{A_{tot}}\sqrt{\left(\frac{P_u}{4}\right)^2 + \left(\frac{V_u}{2}\right)^2}$$

- Variables for this and the previous equation
  - V_{ac} = factored shear force on the most critical face
  - P_u = applied normal load
  - M_u = applied moment load
  - V_u = applied shear load
  - c = column width or diameter (for concrete columns) or base plate width (for steel columns)
  - d = effective depth
  - B = footing width

Two-Way Shear (Step 6)

- Variables
  - V_{nc} = nominal two-way shear capacity on the critical section (lb, N)
  - V_c = nominal two-way shear capacity of concrete (lb, N)
  - b_o = length of critical shear surface = length of one face of inner block (in., mm)
  - c = column width (in., mm)
  - d = effective depth (in., mm)
  - f'_c = 28-day compressive strength of concrete (psi, MPa)
  - d_r = diameter of rebar, in. or mm

- Nominal two-way shear capacity for square footings supporting square or circular columns located in the interior (not the edge or corner)

$$V_{nc} = V_c = 4 b_o d \sqrt{f'_c} \; (English)$$

$$V_{nc} = V_c \frac{1}{3} b_o d \sqrt{f'_c} \; (SI)$$

$$b_o = c + d$$

$$d = T - d_r - 3$$

$$b_o = c + T - d_r - 3$$

Solution of Shear in Square Footings (Step 6)

- Both V_{uc} and V_{nc} depend upon the effective depth d which are determined by the equations given for two-way shear
- The effective depth is related
- There is no direct solution for d from these equations
- It is necessary to use an iterative solution to determine the value of d
- The assumed value of d_b = 1" (assumed diameter of rebar) may be changed in flexural analysis. There is no need to go back and redo the shear analysis for larger d_b

Two-Way Shear Design Example (Step 6)

- Equate V_{uc} and V_{nc} using LRFD equation to solve for minimum T
- Other equations for substitution
- Equality after substituting (for square foundation)

$$P_u \frac{A_{out}}{A_{tot}} = \phi 4 b_o d \sqrt{f_c'}$$

$$P_u \frac{B^2 - \left(c + T - d_r - 3\right)^2}{B^2} = 4\phi\left(c + T - d_r - 3\right)\left(T - d_r - 3\right)\sqrt{f_c'}$$

Two-Way Shear Design Example (Step 6)

- Substitute known quantities (note use of pounds and inches consistently!)
  - P_u = 881,000 lb.
  - d_r = 1”
  - M = 0
  - B = 126”
  - f'_c = 4000 psi
  - c = 21”
  - ϕ = 0.75
- Resulting equality after substitution
  - $220250 - (110125/8712)(17+T)^2 = 3(17+T)(T-4)(4000)^{1/2}$
- Positive solution for T = 27.271" use 30"
  - d = 23.271" or use 24“
  - Larger than one-way shear
- Check computed foundation thickness against assumed depth of foundation
  - Depth of foundation = 3' = 36" > 30" thickness so OK
  - Unless conditions dictate otherwise, depth of foundation could be decreased
  - If minimum thickness is greater than assumed depth, we need to increase assumed depth to properly account for foundation weight

Shape	Type of Shear	Total Area A_{tot}	Inner Area A_{in}	Outer Area A_{out}	Total Shear Area A_{τ}
Square	One-Way	B^2	$B(c+2d)$	$B^2 - B(c+2d)$	$2Bd$
	Two-Way	B^2	$(c+d)^2$	$B^2 - (c+d)^2$	$4d(c+d)$
Rectangle	One-Way	BL	$B(c+2d)$	$BL - B(c+2d)$	$2Bd$
	Two-Way	BL	$(c+d)^2$	$BL - (c+d)^2$	$4d(c+d)$
Continuous	One-Way	B	$(c+2d)$	$B - (c+2d)$	$2d$

Variables:

- B = basic foundation dimension, ft or m
- L = length of foundation, ft or m
- c = basic dimension of column, ft or m[1]
- d = distance from top of foundation slab to top of rebar, ft or m
- A_{in}, A_{out}, A_{tot}, A_{τ}=Inside, Outside, Total and Shear Area, ft^2 or m^2
- f_c' = 28-day strength of concrete, psi or MPa
- P_u = factored axial compressive load, lbs or N (lbs/ft or N/m for continous footings)
- ϕ = resistance factor (usually 0.75)

Formula for driving shear force:[2]

$$V_{uc} = P_u \frac{A_{out}}{A_{tot}}$$

Formula for resisting shear force:

$$V_{nc} = 144 A_{\tau} \sqrt{f_c'}\,(U.S.\,units)$$

$$V_{nc} = \frac{1 \times 10^6}{12} A_{\tau} \sqrt{f_c'}\,(SI\,units)$$

LRFD Formula to compare the two:

$$V_{uc} \leq \phi V_{nc}$$

[1] Formulae assume square column.
[2] Assume concentric axial load only. V_{uc} and P_u in units of force (finite foundations) or force per unit length (continous foundations.)

Questions