ENCE 4610
Foundation Analysis and Design

Lecture 2
Shallow Foundations:
Overview
Basic Bearing Capacity Equations

Types of Shallow Foundations

- Shallow foundations are usually placed within a depth D beneath the ground surface less than the minimum width B of the foundation
- Shallow foundations consist of:
  - Spread and continuous footings
  - Square, Rectangular or Circular Footings
  - Continuous footings
  - Ring Foundations
  - Strap Footings
- Wall footings
- Mats or Rafts

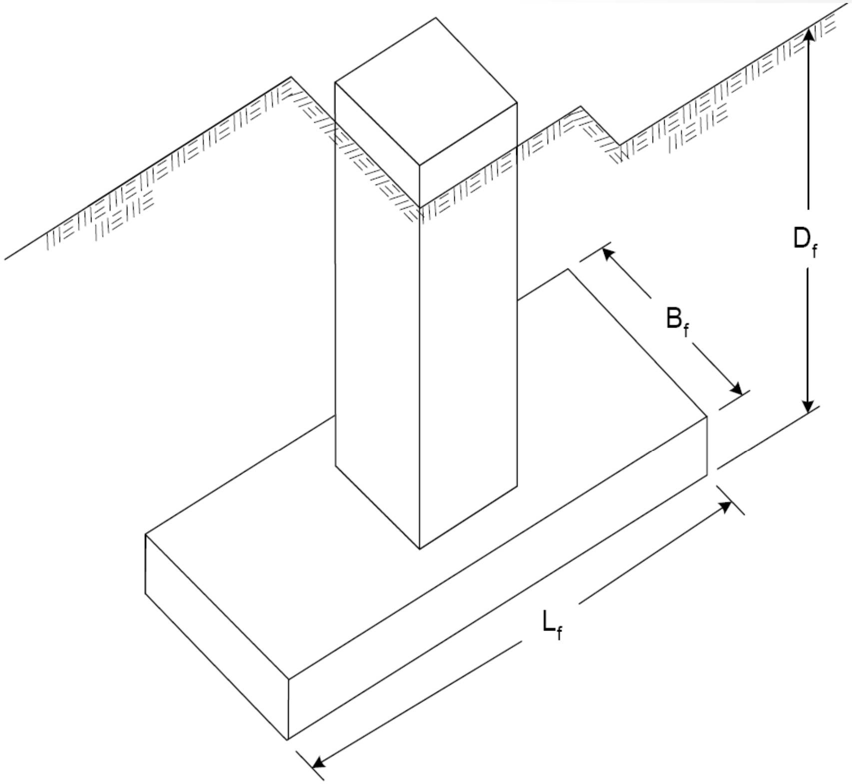

Figure 8-1. Geometry of a typical shallow foundation (FHWA, 2002c, AASHTO 2002).

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Footings

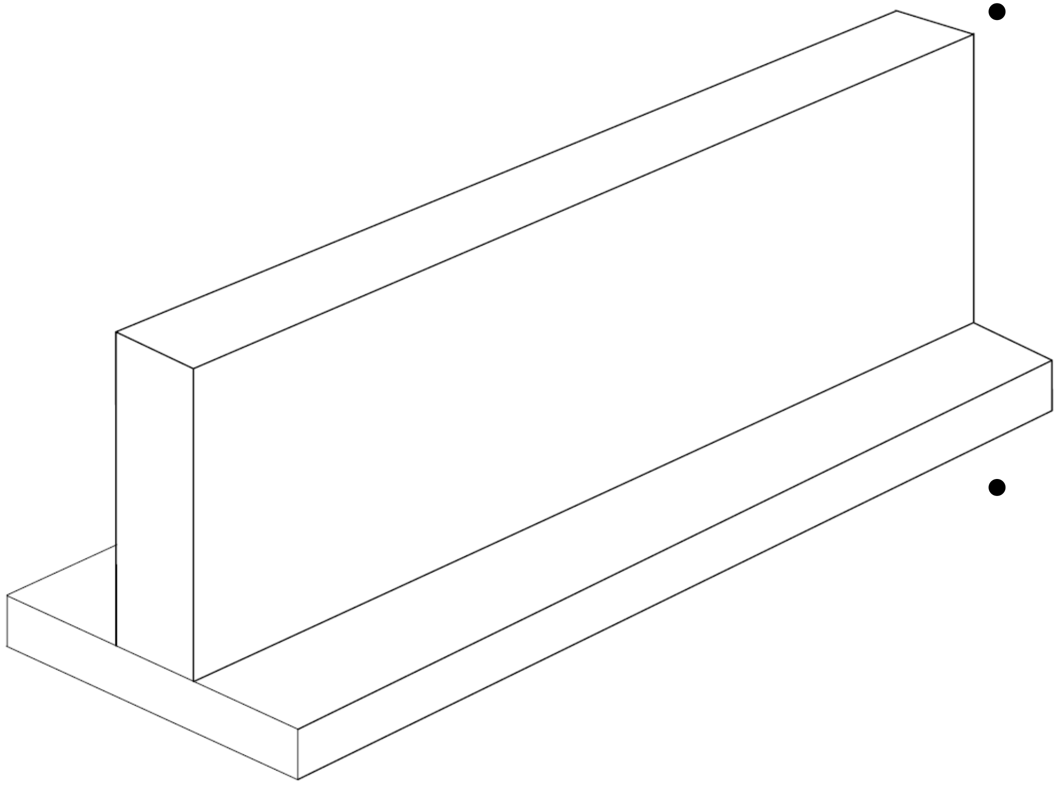
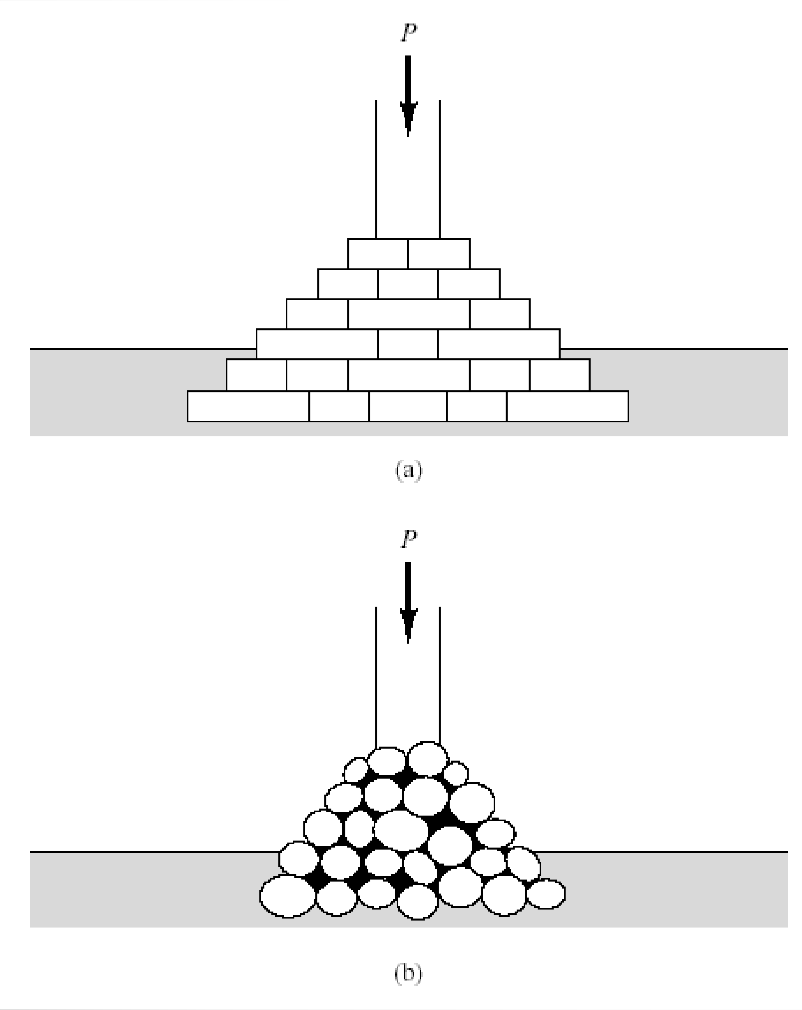


Figure 8-3. Continuous strip footing (FHWA, 2002c).

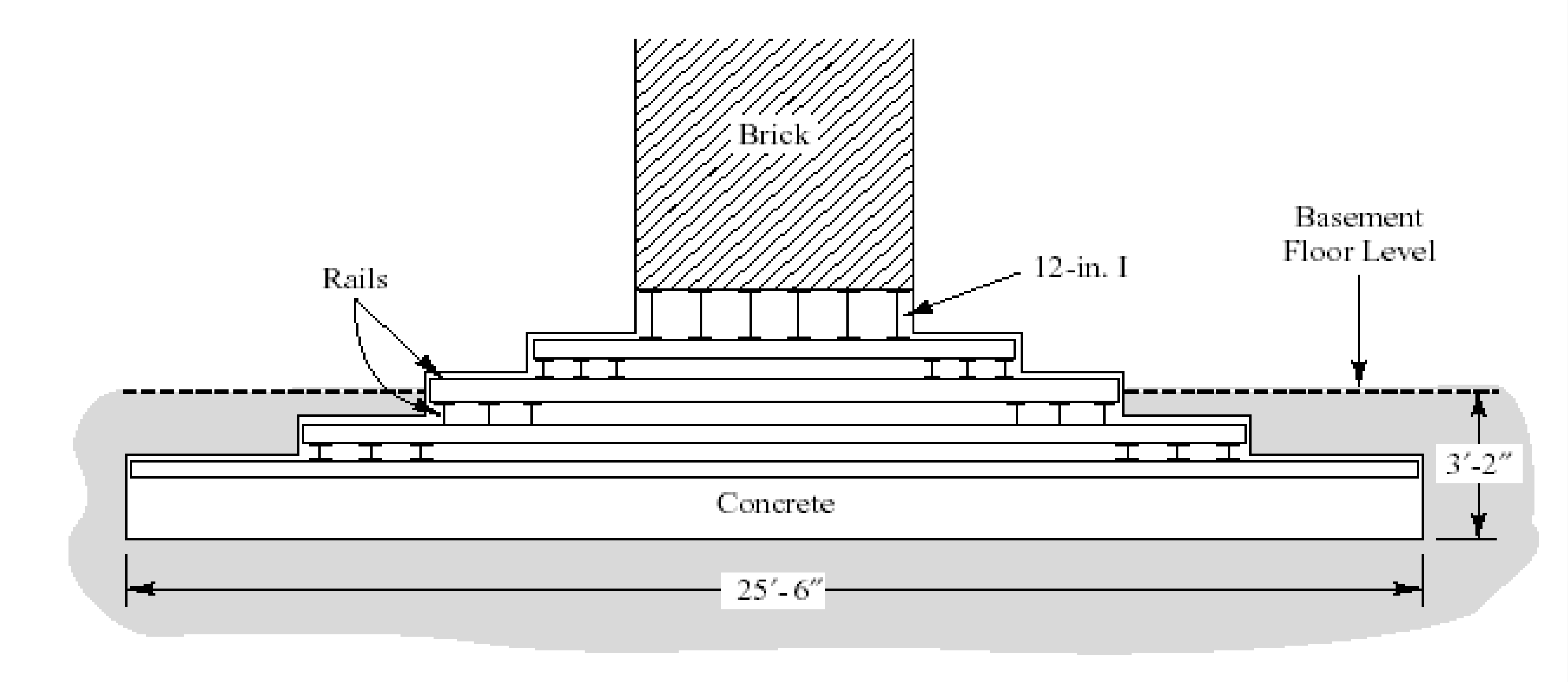
- A finite spread footing is a shallow foundation that transmits loads and has an aspect ratio of $1 \leq {}^{L}/_{B} \leq 10$
- A continuous spread footing is an “infinite” footing where ${}^{L}/_{B} > 10$ and the effects of L are ignored

Dimension and Rubble Stone Footings



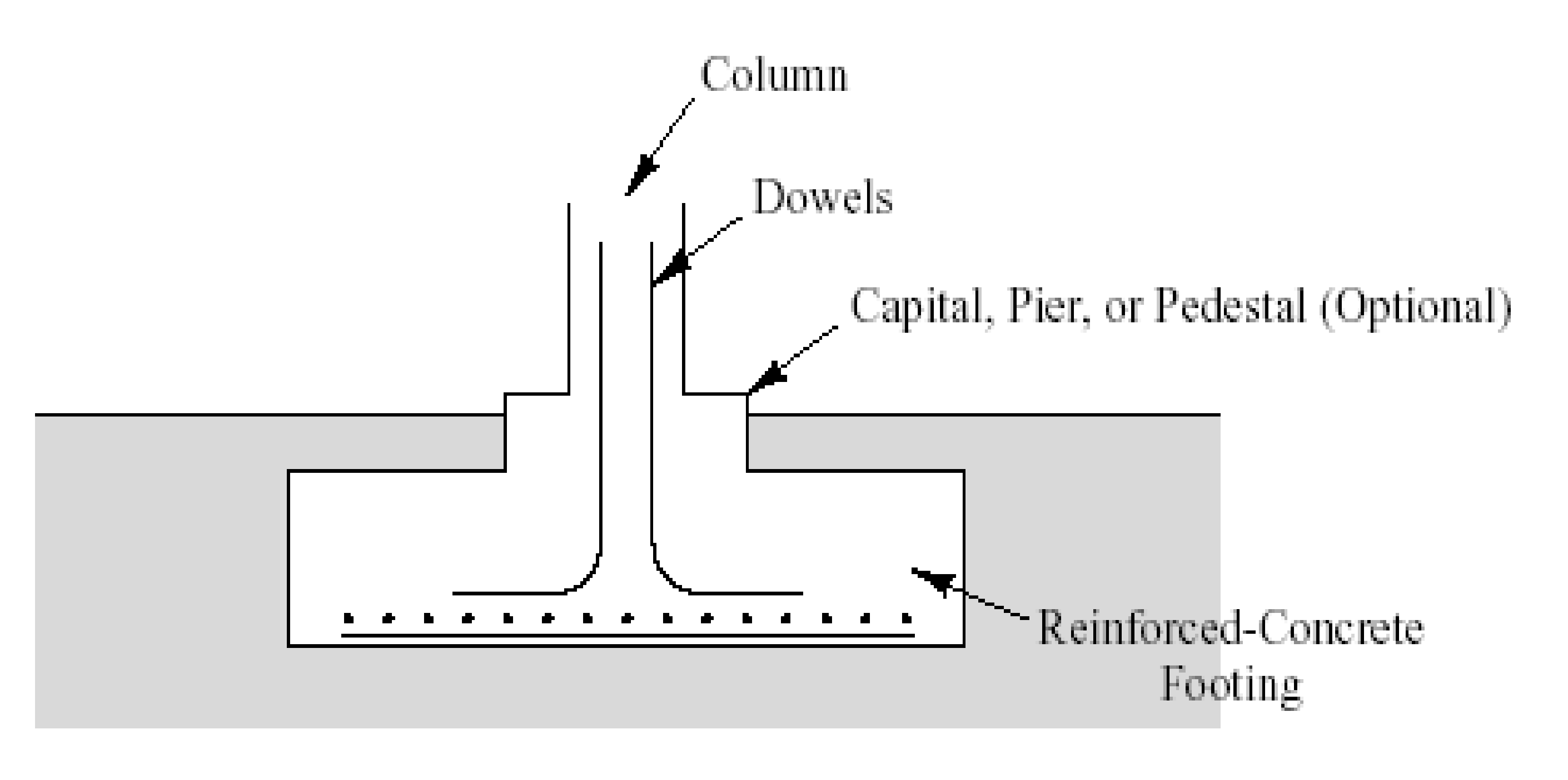
- Before 1800, most all footings were unreinforced masonry, as shown
  - Dimension stone footings
  - Rubble stone footings
- Satisfactory for lighter structures, they were too heavy for the larger structures of the 19th century

Steel Grillage Footings



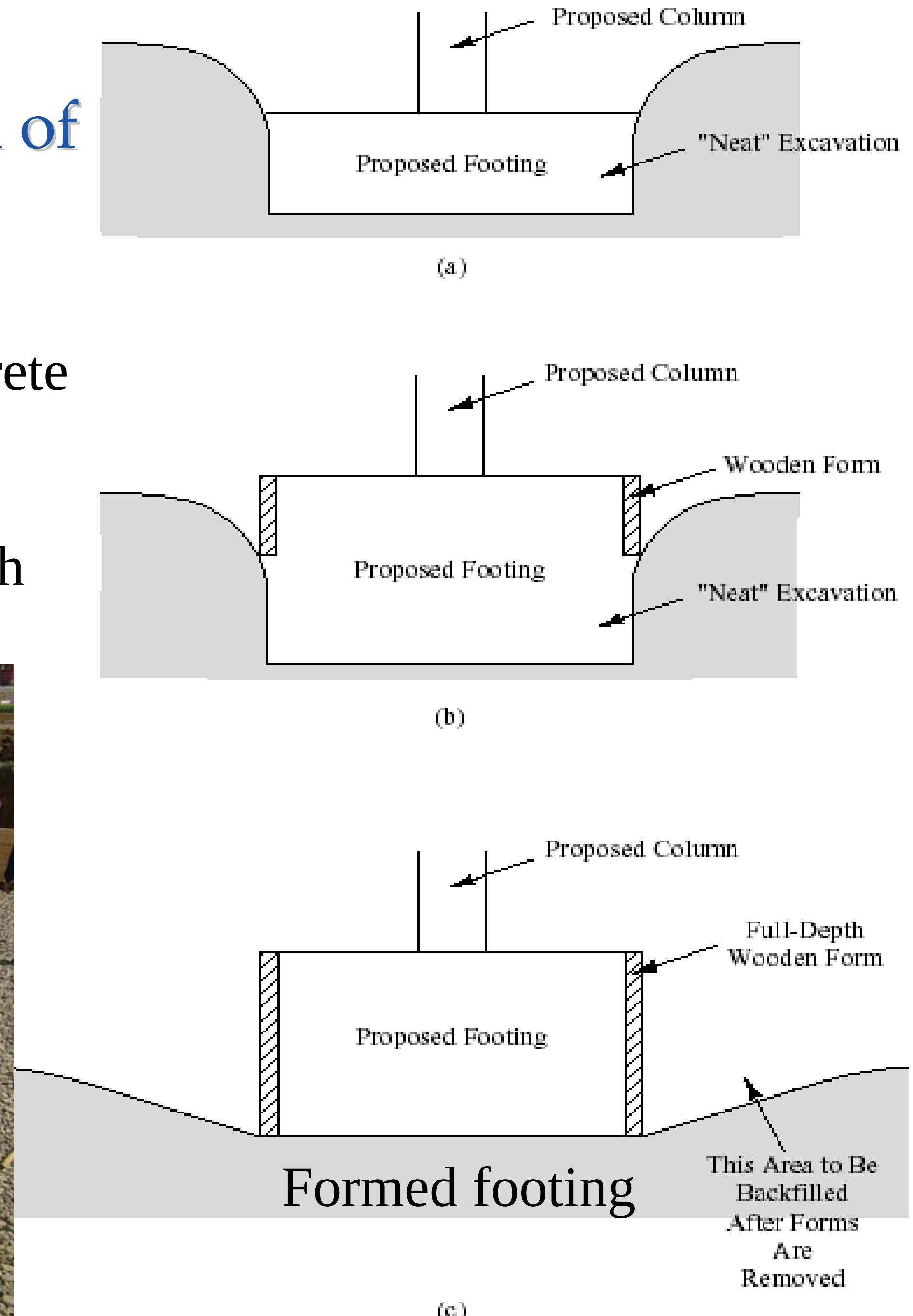
Used first with the Montauk Block Building in Chicago (1882). First foundation type specifically designed for flexure.

Typical Concrete Footing



Methods of Construction of Concrete Footings

Once form is made, before concrete is poured either anchor bolts or dowels are placed to enable connection of the foundation with the building.



Formed footing

Mat Foundations

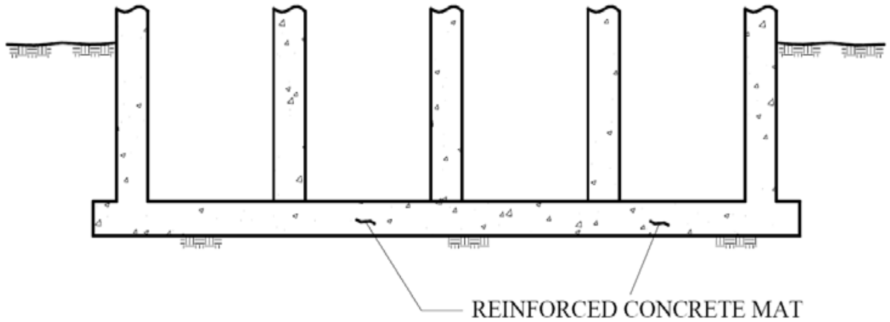
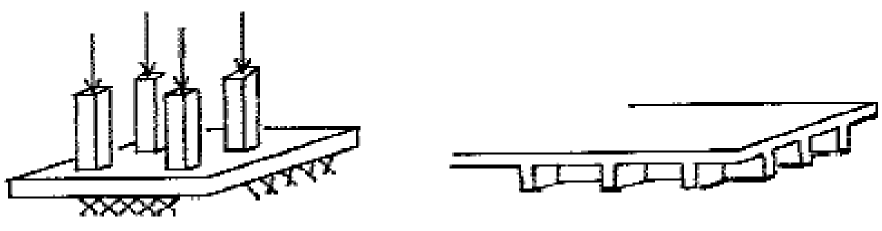


Figure 8-9. Typical mat foundation (FHWA, 2002c).



- A mat is continuous in two directions capable of supporting multiple columns, wall or floor loads. It has dimensions from 20 to 80 ft or more for houses and hundreds of feet for large structures such as multi-story hospitals and some warehouses
- Ribbed mats, consisting of stiffening beams placed below a flat slab are useful in unstable soils such as expansive, collapsible or soft materials where differential movements can be significant (exceeding 0.5 inch).

Conditions for Mat Foundations

- Structural loads require large area to spread the load
- Soil is erratic and prone to differential settlements
- Structural loads are erratic
- Unevenly distributed lateral loads
- Uplift loads are larger than spread footings can accommodate; weight of the mat is a factor here
- Mat foundations are easier to waterproof

Example: Chase Tower, Houston, TX

Mat foundation is 3 metres thick and bottomed at 19.2 m below street level

Distribution of Bearing Pressure

- Distribution of bearing pressure depends on
  - Eccentricity, if any, of applied load
  - Magnitude of the applied moment, if any
  - Structural rigidity of the foundation
  - Stress-strain properties of the soil
  - Roughness of the bottom of the foundation
- Spread footings are nearly rigid; effects of foundation/soil flexibility usually ignored
- Mat foundations are more flexible; flexibility an important factor

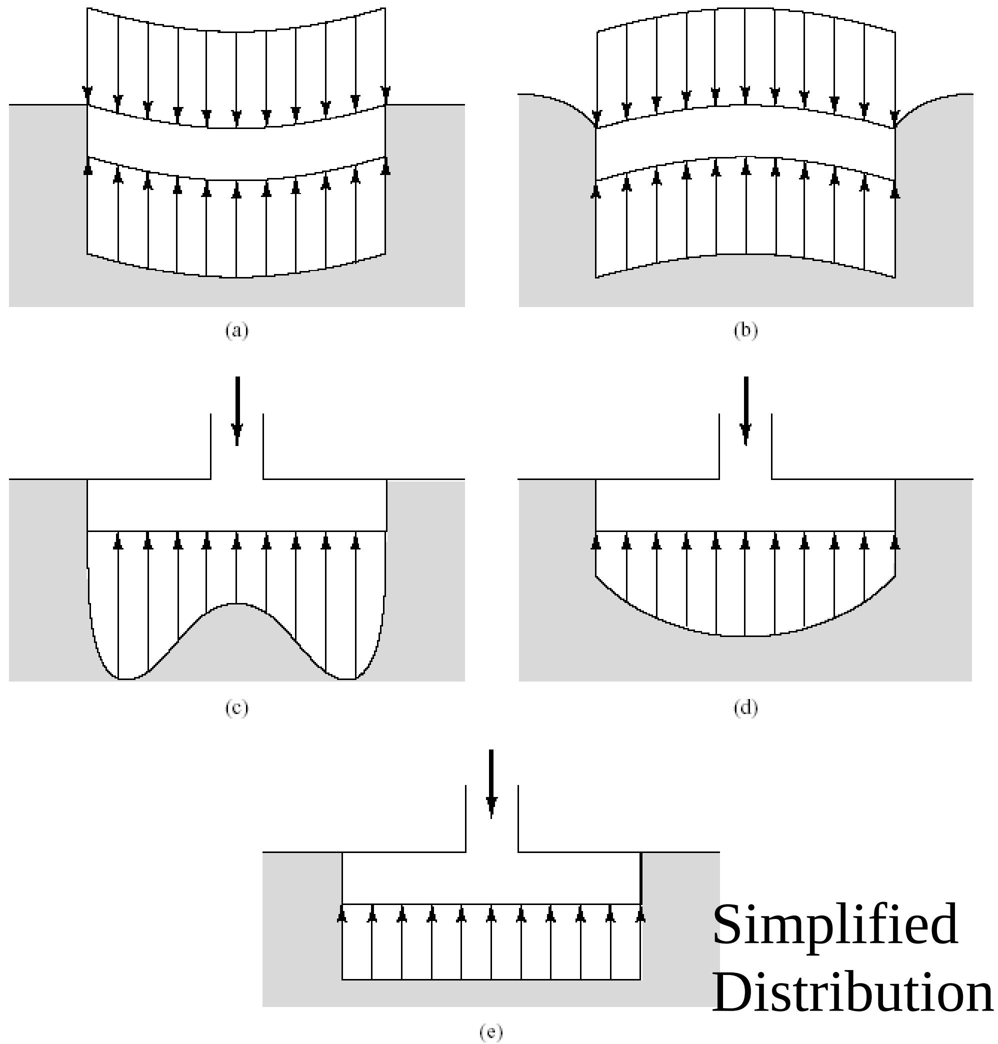
Bearing Pressure Distribution
Concentric Loads

Flexible foundation on clay

Flexible Foundation on Sand

Rigid foundation on clay

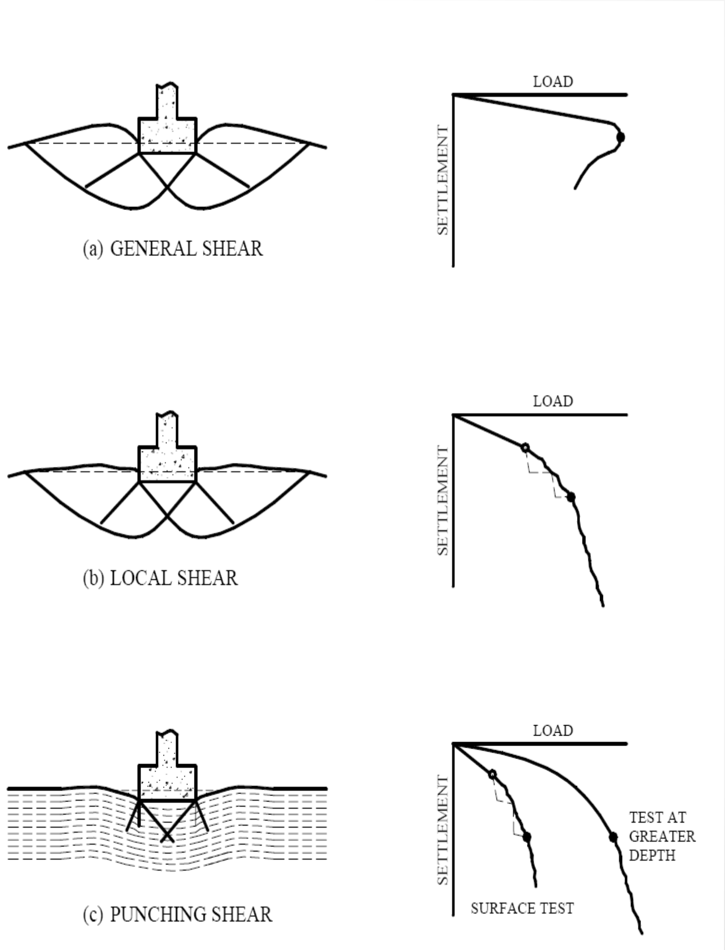
Rigid Foundation on Sand



Simplified Distribution

Types of Bearing Capacity

- Traditionally, bearing capacity has been classified as follows:
  - General Shear
  - Local Shear
  - Punching Shear
  - Which one takes place depends upon consistency or density of soil, which decreases from general to local to punching
- For our purposes, we will only consider general shear; other modes are better predicted using settlement analysis

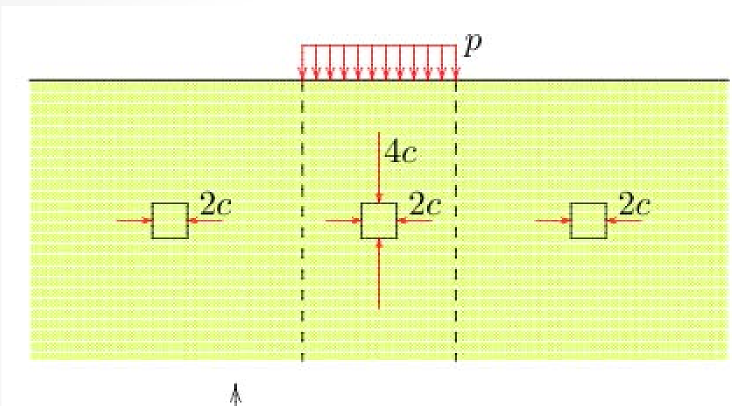


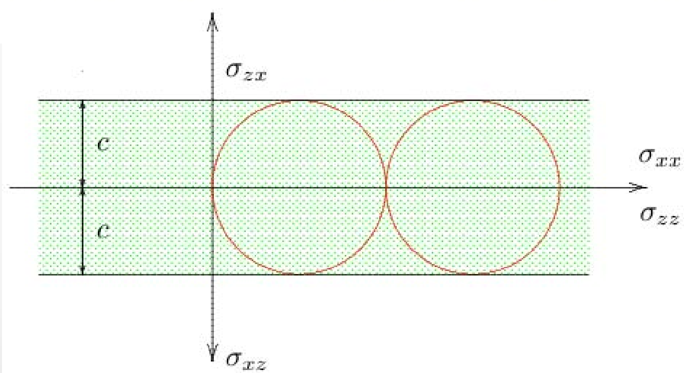
Plasticity: Lower and Upper Bound Solutions

- Review of Concept
  - Lower Bound: The true failure load is larger than the load corresponding to an equilibrium system. The system has failed in at least one place.
  - Upper Bound: The true failure load is smaller than the load corresponding to a mechanism, if that load is determined using the virtual work principle. The system has failed "in general."
- The idea is that the true solution is somewhere between the two
- We saw this when we went through unsupported cuts in purely cohesive soils
- In principle, only applicable to purely cohesive soils without friction, due to volume expansion considerations
- We will begin by considering strip (infinite or continuous) foundations only) in cohesive soils

Lower Bound Solution

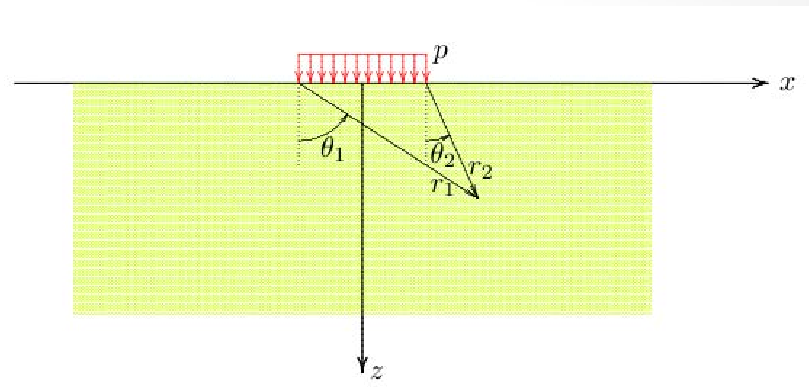
- By direct application and Mohr's Circle





$p_c \geq 4c.$

- By theory of elasticity



$$\tau = \frac{p}{\pi} \mid \sin(\theta_1 - \theta_2) \mid$$

The maximum value of $\mid \sin(\theta_1 - \theta_2) \mid$ is 1

$$p = \pi c.$$

- The more realistic lower bound

Upper Bound Limit Equilibrium Method

(Circular Failure Surface, Cohesive Soil)

$$\sum M_p = q_{ult} Bb \times \frac{B}{2} - \pi c Bb \times B - \sigma_0 Bb \times \frac{B}{2} = 0$$

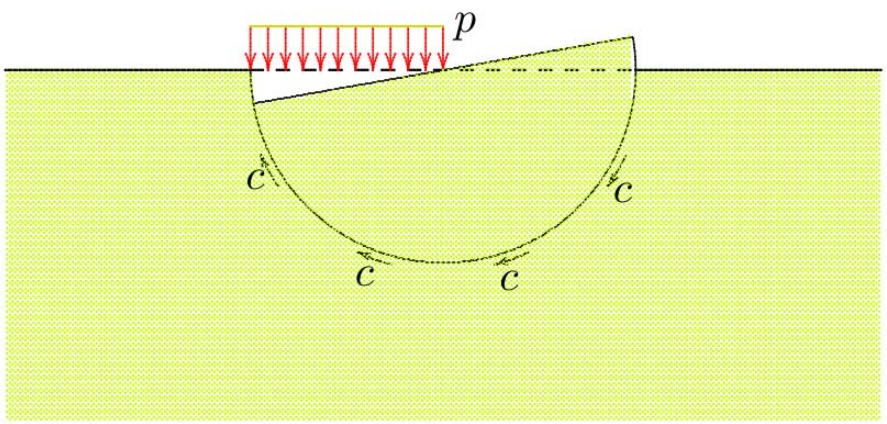


Figure 40.5: Mechanism 1.

$$q_{ult} = 2\pi c + \sigma_0$$

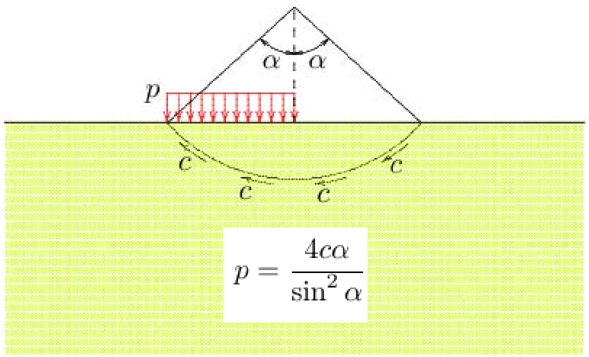
$$N_c = 2\pi \approx 6.28$$

$$q_{ult} = N_c c + \sigma_0$$

Assume: No soil strength due to internal friction (cohesive soil,) shear strength above foundation base neglected

We add the effect of the weight of the soil (effective stress) acting on the top of the right side of the circle against rotation.

More Realistic Upper Bound Case



The smallest value is obtained for $\alpha = 1.165562$, or $\alpha = 66.78°$. The center of the circle then is located at a height $0.429a$. The corresponding value of p is $5.52c$. This is an upper bound, hence

$$p_c \leq 5.52c. \tag{40.9}$$

So the solution is bounded by

$$3.14c \leq p \leq 5.52c$$

Surface (Upper Bound Condition)

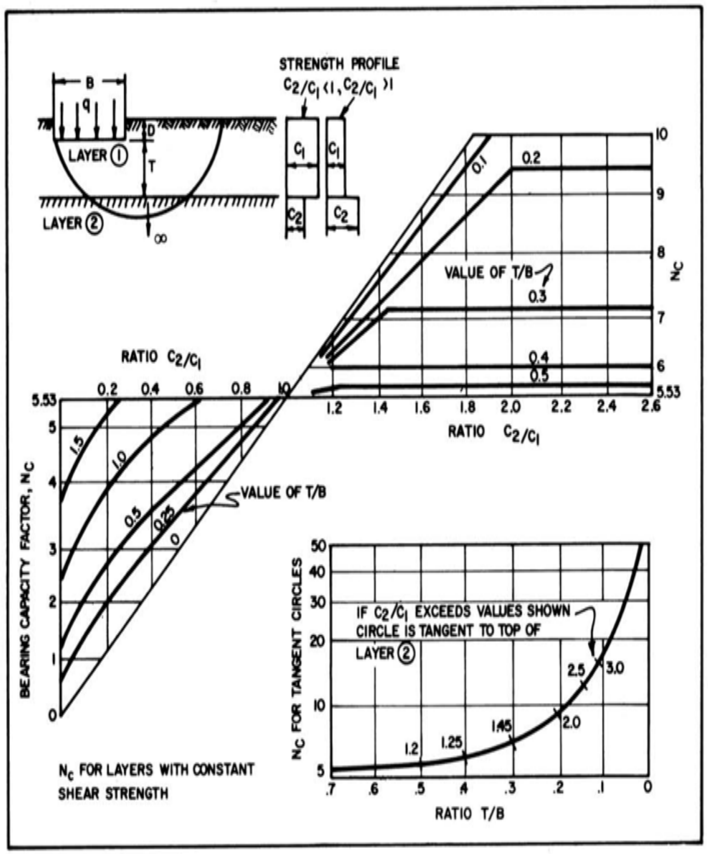


FIGURE 5
Ultimate Bearing Capacity of Two Layer Cohesive Soil (Ø=0)

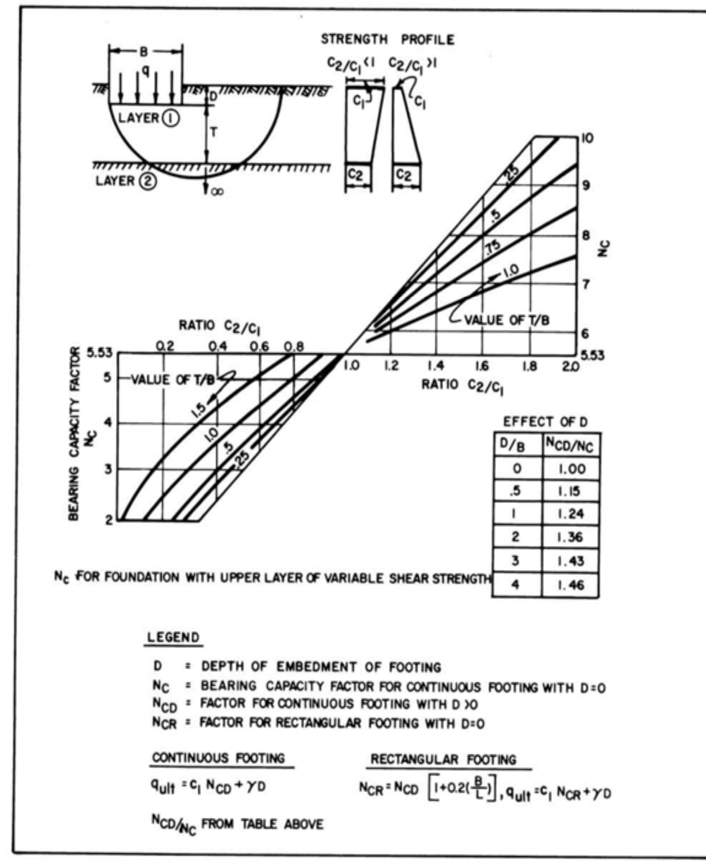


FIGURE 5 (continued)
Ultimate Bearing Capacity of Two Layer Cohesive Soil (Ø=0)

Example Using Circular Failure Surface

- Circular failure surface is especially useful when considering soft clays
- Derived from upper bound limit equilibrium solution
- Adapted in NAVFAC DM 7.02 to two-layer soils
- Correction for rectangular (and square footing) shown in charts

- Example Problem
  - Continuous Shallow Foundation
    - $B = 2$ m
    - $D = 0.5$ m
  - Soil Profile
    - Layer 1: $c = 50$ kPa, depth = 1.5 m, $\gamma = 18$ kN/m^3
    - Layer 2: $c = 110$ kPa
  - Find: q_{ult} for foundation

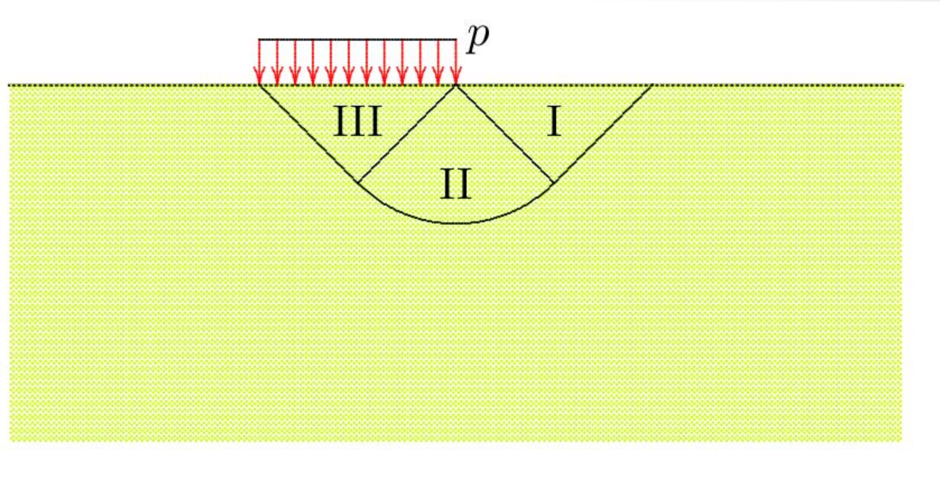
Example of Circular Failure Surface

- Solution:
  - Since centre of failure circle is assumed to be at the surface, the bottom of the circle is 0.5 m below the layer boundary, thus the “depth” of Layer 2 = 0.5 m
  - T = Depth of Layer 1 – D = 1.5 – 0.5 = 1 m
  - T/B = ½ = 0.5
  - C_2/C_1 = 110/50 = 2.2
  - D/B = 0.5/2 = 0.25
- N_c = 5.7 (from chart)
  - However, the formula in the chart calls for using N_{CD}, which is modified for the depth of the foundation
  - $N_{CD}/N_c = 1.08$ (using interpolation)
  - $N_c = (5.7)(1.08) = 6.16$
- Compute overburden pressure at base of foundation
  - $\sigma_0 = \gamma D = (18)(0.5) = 9$ kPa (assumes no water table involved)
- Compute ultimate bearing capacity
  - $q = N_{CD}c + \sigma_0$
  - q = (6.16)(50) + 9 = 316.8 kPa
  - Q = (316.8)(2) = 633.6 kN/m

Development of Prandtl Bearing Capacity Theory

- Application of limit equilibrium methods first done by Prandtl on the punching of thick masses of metal (materials with no internal frictional effects)
- Prandtl's methods first adapted by Terzaghi to bearing capacity failure of shallow foundations (specifically, he added the effects of frictional materials)
- Vesić and others (Meyerhof, Brinch Hansen, etc.) improved on Terzaghi's original theory and added other factors for a more complete analysis

- Note the three zones, the foundation fails along the lower boundary of these zones



Assumptions for Bearing Capacity Methods

- Foundation-Soil Interface Assumptions
  - Foundation is very rigid relative to the soil
  - No sliding occurs between foundation and soil (rough foundation)
- Loading Assumptions
  - Applied load is compressive and applied vertically to the centroid of the foundation*
  - No applied moments present
- Geometric assumption
  - Depth of foundation is less than or equal to its width
- Geotechnical Assumptions
  - Soil beneath foundation is homogeneous semi-infinite mass*
  - Mohr-Coulomb model for soil
  - General shear failure mode is the governing mode
  - No soil consolidation occurs
  - Soil above bottom of foundation has no shear strength; is only a surcharge load against the overturning load*

* We will discuss “workarounds” to these assumptions

Loads and Failure Zones on Strip Foundations

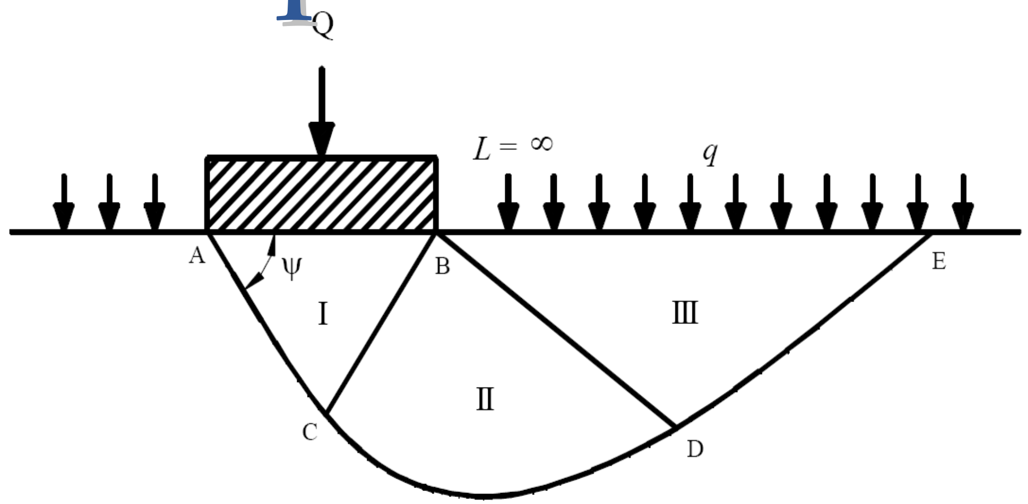
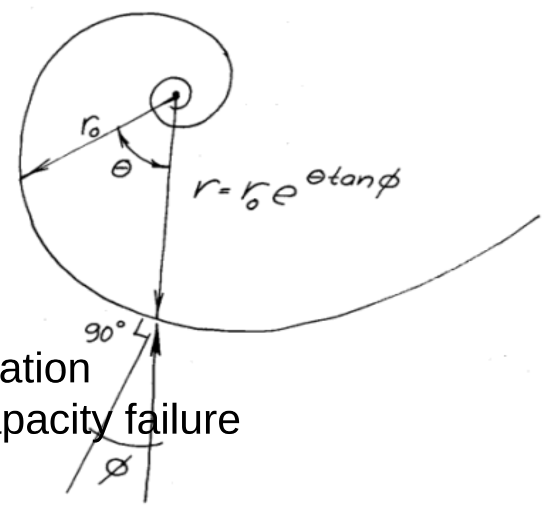


Figure 8-13. Boundaries of zone of plastic equilibrium after failure of soil beneath continuous footing (FHWA, 2002c).



- Q = load/unit length on foundation
- q = load/unit area due to effective stress at base of foundation
- I, II, III = regions of failure in Prantdl theory for bearing capacity failure

Basic Equation of Bearing Capacity

8.4.2 Bearing Capacity Equation Formulation

In essence, the bearing capacity failure mechanism is similar to the embankment slope failure mechanism discussed in Chapter 6. In the case of footings, the ultimate bearing capacity is equivalent to the stress applied to the soil by the footing that causes shear failure to occur in the soil below the footing base. For a concentrically loaded rigid strip footing with a rough base on a level homogeneous foundation material without the presence of water, the gross ultimate bearing capacity, q_{ult}, is expressed as follows (after Terzaghi, 1943):

$$q_{ult} = \underbrace{c(N_c)}_{\text{“Cohesion” term}} + \underbrace{q(N_q)}_{\text{“Surcharge” term}} + \underbrace{0.5(\gamma)(B_f)(N_\gamma)}_{\text{Foundation soil “Weight” term}} \qquad 8\text{-}1$$

where: c = cohesion of the soil (ksf) (kPa)

q = total surcharge at the base of the footing = $q_{appl} + \gamma_a D_f$ (ksf) (kPa)

q_{appl} = applied surcharge (ksf)(kPa)

γ_a = unit weight of the overburden material above the base of the footing causing the surcharge pressure (kcf) (kN/m^3)

D_f = depth of embedment (ft) (m) (Figure 8-1)

γ = unit weight of the soil under the footing (kcf) (kN/m^3)

B_f = footing width, i.e., least lateral dimension of the footing (ft) (m) (Figure 8-1)

N_q = bearing capacity factor for the “surcharge” term (dimensionless)

$$= e^{\pi \tan\phi} \tan^2(45° + \frac{\phi}{2}) \qquad 8\text{-}2$$

N_c = bearing capacity factor for the “cohesion” term (dimensionless)

$$= (N_q - 1)\cot\phi \qquad \text{for } \phi > 0° \qquad 8\text{-}3$$

$$= 2 + \pi = 5.14 \qquad \text{for } \phi = 0° \qquad 8\text{-}4$$

N_γ = bearing capacity factor for the “weight” term (dimensionless)

$$= 2(N_q + 1)\tan(\phi) \qquad 8\text{-}5$$

Basic Equation of Bearing Capacity

- Basic bearing capacity equation:

$$p = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma$$

- p = unit pressure on foundation
- q = effective stress at base of foundations
- γ = (average) soil unit weight under foundation
- B = basic width of foundations
- Strictly speaking, applicable to continuous foundations only

- Bearing Capacity Factors
- Several variations on the basic theory exist
- Values of N_c, N_q mostly the same:

$$N_q = \frac{1+\sin\phi}{1-\sin\phi}\exp(\pi\tan\phi),$$

$$N_c = (N_q - 1)\cot\phi.$$

Basic Equation of Bearing Capacity

- Values of N_c, N_q mostly the same. Values of N_γ depend upon theory
- Brinch-Hansen:

$$N_\gamma = 2(N_q - 1)\tan\phi.$$

- CFEM:

$$N_\gamma = 1.5(N_q - 1)(\tan\phi')$$

- Vesić:

$$N_\gamma = 2(N_q + 1)(\tan\phi')$$

- DIN

$$N_\gamma = 2(N_q - 1)\ (\tan\phi')$$

Vesić-AASHTO Factors:

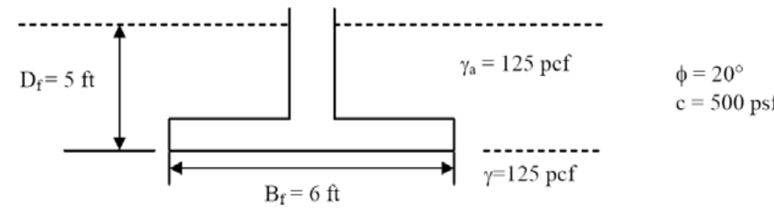
Table 8-2

Bearing Capacity Factors (AASHTO, 2004 with 2006 Interims)

ϕ	N_c	N_q	N_γ	ϕ	N_c	N_q	N_γ
0	5.14	1.0	0.0	23	18.1	8.7	8.2
1	5.4	1.1	0.1	24	19.3	9.6	9.4
2	5.6	1.2	0.2	25	20.7	10.7	10.9
3	5.9	1.3	0.2	26	22.3	11.9	12.5
4	6.2	1.4	0.3	27	23.9	13.2	14.5
5	6.5	1.6	0.5	28	25.8	14.7	16.7
6	6.8	1.7	0.6	29	27.9	16.4	19.3
7	7.2	1.9	0.7	30	30.1	18.4	22.4
8	7.5	2.1	0.9	31	32.7	20.6	26.0
9	7.9	2.3	1.0	32	35.5	23.2	30.2
10	8.4	2.5	1.2	33	38.6	26.1	35.2
11	8.8	2.7	1.4	34	42.2	29.4	41.1
12	9.3	3.0	1.7	35	46.1	33.3	48.0
13	9.8	3.3	2.0	36	50.6	37.8	56.3
14	10.4	3.6	2.3	37	55.6	42.9	66.2
15	11.0	3.9	2.7	38	61.4	48.9	78.0
16	11.6	4.3	3.1	39	67.9	56.0	92.3
17	12.3	4.8	3.5	40	75.3	64.2	109.4
18	13.1	5.3	4.1	41	83.9	73.9	130.2
19	13.9	5.8	4.7	42	93.7	85.4	155.6
20	14.8	6.4	5.4	43	105.1	99.0	186.5
21	15.8	7.1	6.2	44	118.4	115.3	224.6
22	16.9	7.8	7.1	45	133.9	134.9	271.8

Bearing Capacity Example

Example 8-1: Determine the ultimate bearing capacity for a rigid strip footing with a rough base having the dimensions shown in the sketch below. Assume that the footing is concentrically loaded and that the total unit weight below the base of the footing is equal to the total unit weight above the base of the footing, i.e., in terms of the symbols used previously, $\gamma = \gamma_a$. First assume that the ground water table is well below the base of the footing and therefore it has no effect on the bearing capacity. Then, assume that the groundwater table is at the base of the footing and recompute the ultimate bearing capacity.



Solution:

Assume a general shear condition and enter Table 8-2 for ϕ= 20° and read the bearing capacity factors as follows:

$N_c = 14.8$, $N_q = 6.4$, $N_\gamma = 5.4$. These values can also be read from Figure 8-15.

$$q_{ult} = c(N_c) + \gamma_a(D_f)(N_q) + 0.5(\gamma)(B_f)(N_\gamma)$$

$$\begin{aligned} q_{ult} &= (500 \text{ psf})(14.8) + (125 \text{ pcf})(5 \text{ ft})(6.4) + 0.5(125 \text{ pcf})(6 \text{ ft})(5.4) \\ &= 7{,}400 \text{ psf} + 4{,}000 \text{ psf} + 2{,}025 \text{ psf} \\ q_{ult} &= 13{,}425 \text{ psf} \end{aligned}$$

Effect of water: If the ground water table is at the base of the footing, i.e., a depth of 5 ft from the ground surface, then effective unit weight should be used in the "weight" term as follows:

$$\begin{aligned} q_{ult} &= (500 \text{ psf})(14.8) + (125 \text{ pcf})(5 \text{ ft})(6.4) + 0.5(125 \text{ pcf} - 62.4 \text{ pcf})(6 \text{ ft})(5.4) \\ &= 7{,}400 \text{ psf} + 4{,}000 \text{ psf} + 1{,}014 \text{ psf} \\ q_{ult} &= 12{,}414 \text{ psf} \end{aligned}$$

Sections 8.4.2.1 and 8.4.3.2 further discuss the effect of water on ultimate bearing capacity.

Questions?