U.S. Department
of Transportation

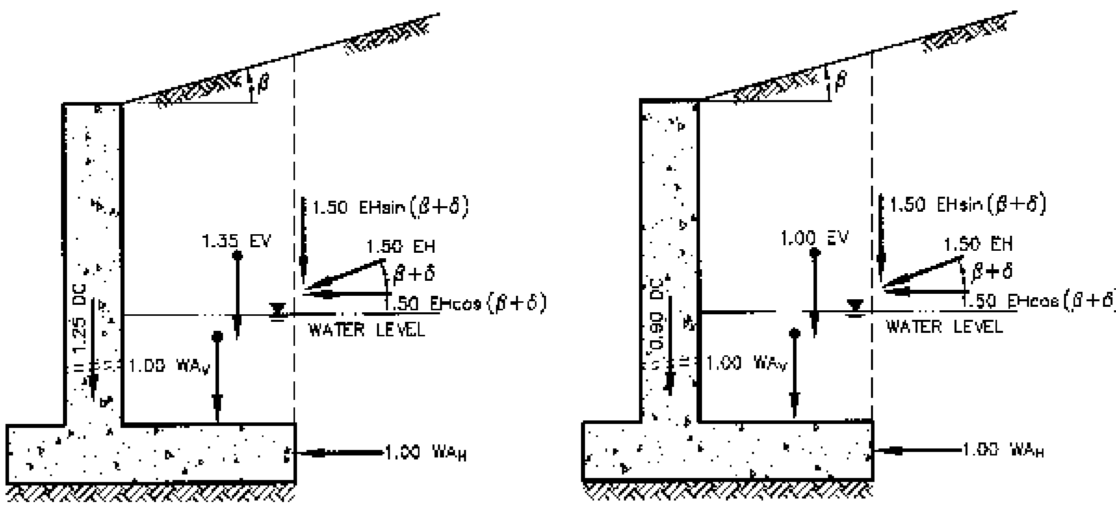
**Federal Highway
Administration**

Publication No. FHWA HI-98-032
May 2001

NHI Course No. 132068

Load and Resistance Factor Design (LRFD) for Highway Bridge Substructures

Reference Manual and Participant Workbook



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PREFACE

For many years, engineers have designed foundations, walls and culverts for highway and other applications using allowable stress design (ASD) methods. In ASD, all uncertainty in loads and material resistance is combined in a factor of safety or allowable stress. For most highway design, the American Association of State Highway and Transportation Officials (AASHTO) *Standard Specifications for Highway Bridges* represents the primary source document for ASD of substructures. In 1994, AASHTO approved Load and Resistance Factor Design (LRFD) in the *LRFD Highway Bridge Design Specifications*. In LRFD, the uncertainty in load is represented by a load factor and the uncertainty in material resistance is represented by a resistance factor. Due to the fundamental differences between the substructure design process by ASD and LRFD, this course has been developed to present the fundamentals of LRFD for the geotechnical design of highway bridge substructures. Because this document was prepared for engineers and others who are already familiar with the design of substructures using ASD, it is intended for use in conjunction with other documents describing standard geotechnical design procedures.

The objectives of this reference manual are to provide the basis for an understanding of the:

- Differences between ASD and LRFD for substructure design
- Benefits of LRFD for substructure design
- Importance of site characterization and selection of geotechnical design parameters
- Process for design of substructure elements by LRFD using the AASHTO LRFD Specifications as a guide
- Process for selection and application of load factors and load combinations
- Methods available for calibration of resistance factors
- Basis for calibration of the AASHTO LRFD resistance factors for substructure design
- Procedures available for modifying or developing resistance factors to achieve designs comparable to ASD

This Reference Manual is intended principally to serve as a supplement to the Participant's Workbook for the FHWA's National Highway Institute Course on Load and Resistance Factor Design for Highway Bridge Substructures. The Reference Manual can also serve as a primary reference for engineers in modifying or developing resistance factors to achieve designs comparable to ASD and for designing foundations, earth retaining structures and culverts through the numerous design examples included throughout the design chapters in the manual.

The authors acknowledge the efforts of the project technical managers, Mr. Jerry DiMaggio and Mr. Richard Cheney, FHWA Senior Geotechnical Engineers who provided constructive guidance and input for this document, and Mr. Chien-Tan Chang, FHWA's Contracting Officer's Technical Representative who provided administrative oversight and guidance.

The authors also wish to acknowledge contributions of the following members of the Technical Working Group listed in alphabetical order:

- Chris Benda - Vermont Department of Transportation
- James Cuthbertson - Washington State Department of Transportation
- Richard Engle - Ohio Department of Transportation
- Ashton Lawler - Virginia Department of Transportation
- James Lyons - Federal Highway Administration
- Paul Passe - Florida Department of Transportation
- Tom Saad - Federal Highway Administration
- James Schluter - CONTECH Construction Products

Lastly, the authors wish to thank the following employees from D'Appolonia for their vital contributions and significant effort in preparing this document: Ms. Tara Heyer, Mr. Robert Whitaker, Mr. David Mudry, Mr. Christopher Lewis, Mr. James Wang and Dr. Yasser A. Hegazy.

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LIST OF SYMBOLS

A = Acceleration coefficient for earthquake load evaluation (dim)
A = Cross-sectional area of soil reinforcement less any sacrificial thickness (m^2)
A = Cross-sectional area of structural element (m^2)
A = Cross sectional area of pipe wall (m^2)(15.2.1)
A = Wall area per unit length (m^2/m)(15.3.2)
a = Net area ratio for cone penetrometer (dim)
A' = Effective footing area (m^2)
a_B = Barge bow damage length (m)
A_b = Surface area of transverse reinforcement in bearing, less sacrificial thickness of cross bars (m^2)
A_D = Distributed area at depth D_E (m^2)
AF = Annual frequency of collapse of a bridge component (dim)
A_g = Gross area of concrete section (m^2)
A_p = Area of pile or shaft tip (m^2)
A_s = Net cross-sectional area reduced for corrosion (m^2)(14.2.2)
A_s = Total top and bottom surface area of reinforcement along, $L_{eff''}$ less any sacrificial thickness (m^2)
A_{st} = Area of reinforcing steel (m^2)
AL = Group axle load (kN)
A_s = Area of pile or shaft side surfaces (m^2)
B = Width of footing (m)
B = Base width (m)
B = Effective backslope (deg)
B = 1 - 0.33 h_w/h (dim)
B' = Effective width of footing (m)
B_{BP} = Bearing pad width (m)
B_c = Outside diameter or width of pipe (m)
B'_c = Out-to-out vertical rise of pipe (m)
B_{FE}, B_{FLL} = Bedding factor (dim)
BR = Vehicular braking force (kN)
c = Cohesion of soil (kPa)
C_a = Coefficient for effect of pier width to ice thickness ratio where flow fails by crushing (dim)
C_C = Compression Index (dim)
C_{cr} = Recompression Index (dim)
CE = Vehicular centrifugal force (kN)
C_H = Hydrodynamic mass coefficient (dim)
C_o = Unconfined compressive strength of rock core (kPa)
COV = Coefficient of variation (dim)
$COV_{INHERENT}$ = Coefficient of variation of inherent soil variability (dim)
$COV_{MEASUREMENT}$ = Coefficient of variation of property measurement (dim)
COV_{MODEL} = Coefficient of variation of predictive model (dim)
COV_Q = Coefficient of variation of load (dim)
COV_{QD} = Coefficient of variation of dead load (dim)

LIST OF SYMBOLS
(continued)

COV_{QL}	=	Coefficient of variation of live load (dim)
COV_R	=	Coefficient of variation of resistance (dim)
COV_{RA}	=	Coefficient of variation for soil internal friction angle using Nordlund Method (dim)
COV_{RN}	=	Coefficient of variation for pile capacity using Nordlund Method (dim)
COV_λ	=	Coefficient of variation of overall bias (dim)
$COV_{\lambda i}$	=	Coefficient of variation of a bias factor (dim)
C_n	=	Horizontal inclination of pier nose to a vertical line (dim)
CR	=	Force effects due to creep (kN)
CRF	=	Creep reduction factor from creep tests or Table 14-5 (dim)
C_{sm}	=	Elastic seismic response coefficient for m^{th} mode of vibration (dim)
CT	=	Vehicular collision force (kN)
C_{ur}	=	Recompression Index (dim)
CV	=	Vessel collision force (kN)
C_v	=	Coefficient of consolidation (m^2/sec)
D	=	Pile, shaft or culvert diameter (m)
D	=	Diameter of vane (mm)
D	=	D-load (three-edge bearing resistance of reinforced concrete pipe, corresponding to an experimentally observed 0.25 mm width crack or to ultimate load capacity of culvert (kN/m)
DC	=	Dead load of structural components and non-structural attachments (kN)
DD	=	Downdrag load (kN)
D_E	=	Depth below road surface (m)
DLP	=	Distributed lane pressure (kPa)
DTP	=	Distributed truck/tandem pressure (kPa)
D_r	=	Relative density (percent)
DW	=	Dead load of wearing surfaces and utilities (kN)
DWT	=	Dead weight tonnage of vessel (metric ton)
D_{10}	=	Particle diameter for which material is 10% finer by weight (mm)
D_{50}	=	Mean particle diameter (mm)
E	=	Young's modulus (kPa)
e	=	Eccentricity (m)
e_B, e_L	=	Eccentricity of load resultant with respect to centroid of footing (m)
E_c	=	Thickness of metal reinforcement at end of service life (mm)
ecc	=	Reduction factor for accidental load eccentricity (dim)
E_D	=	Dilatometer modulus averaged over a depth interval of 5m (kPa)
EH	=	Horizontal earth pressure load (kN)
E_m	=	Modulus of elasticity of culvert material (kPa)
e_{max}	=	Maximum allowed eccentricity (m)
E_n	=	Nominal thickness of steel reinforcement at construction (mm)
E_{PMT}	=	Pressuremeter modulus (kPa)
ES	=	Earth surcharge load (kN)
E_s	=	Sacrificial metal thickness expected to be lost by uniform corrosion during service life of structure (mm)
EQ	=	Earthquake load (kN)
E_u	=	Undrained modulus in cohesive soils (kPa)

LIST OF SYMBOLS
(continued)

EV	=	Vertical pressure from dead load of earth fill (kN)
F	=	Horizontal force caused by ice flows (kN)
F_b	=	Horizontal force caused by ice flows which fail by bending (kN)
FC	=	Construction damage factor for geosynthetic reinforcement ranging from 1.1 to 3.0, and taken as 3.0 without tests (dim)
F_c	=	Horizontal force caused by ice flows which fail by crushing (kN)
f'_c	=	Concrete compressive strength (kPa)
f_{cr}	=	Buckling strength of culvert wall (kPa)
FD	=	Durability factor for geosynthetic reinforcement ranging from 1.1 to 2.0, and taken as 2.0 without tests (dim)
f_d	=	Coefficient of resistance to direct sliding of reinforcement (dim)
F_e	=	Soil-structure interaction factor (dim) (4.5)
FF	=	Flexibility Factor (m/kN)
FF_{max}	=	Suggested Maximum Permissible Flexibility Factor (m/kN)
f_L	=	Material limit stress (kPa)
f_{pe}	=	Concrete compressive stress due to prestressing (kPa)
FR	=	Frictional force effects due to sliding between structure members (kN)
FS	=	Factor of safety (dim)
f_s	=	Cone penetrometer side resistance (kPa)
FS_{ASD}	=	ASD Factor of Safety (dim)
FS_{LRFD}	=	LRFD equivalent factor of safety (dim)
f_u	=	Specified minimum tensile strength (kPa)
$f_x(x)$	=	Probability density function
F_y	=	Minimum yield stress (kPa)
f_y	=	Yield strength of metal (kPa)
f*	=	Apparent coefficient of friction at each reinforcement level (dim)
g	=	Gravitational acceleration ($9.8/m/s^2$)
func {overline g}	=	Mean value of g(R,Q)
g(R,Q)	=	Combined probability density function
H	=	Design height of wall and height of backfill above culvert crown (m) (4.5)
h	=	Nominal height of horizontal earth pressure diagram (m)
h	=	Height of ground surface above culvert (m)
h	=	Depth of reinforced soil (m)
HDF	=	Reduction factor for hidden defects due to damage during pile driving (dim)
h_{eq}	=	Equivalent height of soil for design live load (m)
h_i	=	Height of reinforced zone contributing horizontal load to reinforcement at level i (m)
H_{TOT}	=	Total horizontal force per unit length (kN/m)
h_w	=	Height of water surface above culvert (m)
I	=	Slope of backfill behind wall (deg)
I	=	Impact Factor (dim)
I	=	Moment of inertia of culvert wall (m^4/m)
IC	=	Ice load (kN)
ID	=	Inside diameter (m) (4.5)
IM	=	Dynamic Load Allowance (dim)
I_S	=	Point load strength (kPa)

LIST OF SYMBOLS
(continued)

k	=	Horizontal earth pressure coefficient (dim)
k	=	Coefficient of permeability (m/s)
k	=	Soil correlation factor (dim)
k_a	=	Active earth pressure coefficient (dim)
K_{ae}	=	Seismic active earth pressure coefficient (dim)
KE	=	Vessel collision energy (joule)
k_h	=	Lateral earth pressure coefficient (dim)
k_h	=	Horizontal acceleration coefficient (dim) (4.2.1.3)
k_o	=	At-rest earth pressure coefficient (dim)
K_{pe}	=	Seismic passive earth pressure coefficient (dim)
k_s	=	Coefficient of earth pressure due to surcharge (dim)
k_v	=	Vertical acceleration coefficient (dim)
L	=	Footing length, anchor bond length and span length of culvert (m)
L	=	Horizontal spacing between vertical wall elements (m)(13.3.6.1)
L	=	Soil reinforcement length (m)
l	=	Length of tire contact in direction of travel (m)
$l = L_e$	=	Length of geogrid beyond failure plane (m)
L'	=	Effective footing length (m)
ℓ	=	Incremental length of sliding surface in slope stability analysis (m)
L_D	=	Distributed length of wheel load (m)
$L_{eff} = L_e$	=	Effective soil reinforcement length (m)
LF	=	Load factor (dim)
LI	=	Liquidity Index (dim)
LL	=	Vehicular live load (kN)
LS	=	Live load surcharge (kN)
M	=	Vessel displacement tonnage (metric ton)
M	=	Constrained modulus (kPa)
m	=	Multiple presence factor (dim)
M_{dt}	=	Tangent constrained drained modulus (kPa)
M_{max}	=	Factored maximum flexural moment in facing or vertical wall element (kN-m/m)
M_s	=	Confined modulus of soil (kPa)
N	=	Number of data values (dim)
N	=	SPT blow count (blows per 30 cm)
N	=	Normal force (kN)
n	=	Sample size
n	=	Number of transverse bearing members behind failure plane (dim)
N_k	=	Cone bearing factor (dim)
N_p	=	Passive resistance factor based on site-specific pullout tests, or as defined by Figure 14-2 (dim)
OCR	=	Overconsolidation ratio (dim)
P	=	Wheel load (kN)
p	=	Effective ice crushing strength (kPa) (4.2.2.12)
p	=	Live load intensity (kPa)
p	=	Average factored lateral pressure acting on vertical wall element or facing (kPa)
PA	=	Probability of vessel aberrancy (dim)
P_a	=	Lateral earth pressure resultant force per unit wall length (kN/m)

LIST OF SYMBOLS
(continued)

p_a	=	Lateral earth pressure (kPa)
p_a	=	Atmospheric pressure (kPa)
P_{ae}	=	Seismic active earth pressure (kN/m)
P_{all}	=	Allowable axial, tensile or flexural structural capacity (kN)
$P_{all(LRFD)}$	=	LRFD equivalent allowable tensile load (kN)
P_B	=	Base wind pressure (kPa) (4.2.2.8)
P_B	=	Equivalent static vessel impact force (kN)
PC	=	Probability of bridge collapse due to a collision with an aberrant vessel (dim)
P_D	=	Design wind pressure (kPa)
p_f	=	Probability of failure (dim)
P_{fg}	=	Nominal pullout resistance of steel grid reinforcement (kN)
P_{fs}	=	Nominal pullout capacity of ribbed or smooth steel reinforcing strips (kN)
P_H	=	Horizontal load (kN)
P_h	=	Horizontal component of lateral earth pressure per unit length of wall (kN/m)
p_h	=	Horizontal stress (kPa)
PI	=	Plasticity Index (dim)
PL	=	Pedestrian live load (kN)
P_L	=	Factored crown pressure (kPa)
P_N	=	Wind pressure normal to structure component (kPa)
$P_n=P_{ult}$	=	Ultimate (nominal) structural resistance (kN)
P_n	=	Ultimate anchor tensile resistance or GUTS (kN)(13.2.2)
P_r	=	Factored structural resistance of reinforcements, facing and connections (kN)
P_P	=	Wind pressure parallel to structure (kPa)
P_{pe}	=	Seismic passive earth pressure (kN/m)
P_r	=	Factored structural resistance (kN)
p_s	=	Probability of survival (dim)
P_v	=	Vertical component of lateral earth pressure resultant per unit wall length (kN/m)
Q	=	Load or design load (kN)
q	=	Surcharge pressure (kPa)
q	=	Average pressure of an assumed rectangular distribution (kPa)
func {overline Q}	=	Mean load (kN)
func {overline q}	=	Magnitude of uniform distribution of soil pressure (kPa)
Q_a	=	Ultimate unit resistance between grout/soil/rock in anchor bond zone per unit length (kN/m)
Q_{all}	=	Allowable design load (kN)
q_c	=	Cone penetrometer tip resistance (kPa)
Q_D	=	Dead load (kN)
Q_{ep}	=	Nominal passive resistance of foundation material (kN)
Q_i	=	Force effect, stress or stress resultant (kN or kPa)
Q_L	=	Live load (kN)
q_{max}	=	Maximum unit bearing stress (kPa)
q_{min}	=	Minimum unit bearing stress (kPa)
Q_n	=	Nominal sliding resistance of footing (kN)
Q_p	=	Ultimate pile tip resistance (kN)
q_p	=	Unit tip resistance (kPa)
Q_R	=	Factored geotechnical resistance (kN)

LIST OF SYMBOLS
(continued)

q_R	=	Factored unit bearing resistance (kPa)
Q_s	=	Ultimate pile side resistance (kN)
q_s	=	Uniform surcharge applied to upper surface of active earth wedge (kPa)
q_s	=	Vehicular live load surcharge (kPa)
q_s	=	Unit shear resistance (kPa)
q_T	=	Corrected cone tip resistance (kPa)
Q_{ult}	=	Ultimate geotechnical resistance (kN)
q_{ult}	=	Ultimate unit bearing resistance (kPa)
$q_{uniform}$	=	Rectangular distribution of soil pressure (kPa)
Q_τ	=	Nominal shear resistance between footing and foundation material (kN)
R	=	Resistance (kN)
R	=	Radius of curvature of traffic lane (m) (4.2.2.3)
R	=	Earthquake response modification factor (dim) (4.2.2.11)
R	=	ADTT reduction factor (dim) (4.5)
R	=	Schmidt Hammer Rebound (dim)
R	=	Radius of culvert cross section at neutral axis (m)
r	=	Radius of gyration of culvert wall (m/m)
r^2	=	Coefficient of determination (dim)
func {overline R}	=	Mean resistance (kN)
R_m	=	Measured resistance (kN)
R_n	=	Nominal (ultimate) resistance (kN)
R_r	=	Factored resistance (kN or kPa)
S	=	Site coefficient for earthquake load evaluation (dim) (4.2.2.11)
S	=	Diameter or span of culvert (m) (4.5)
S	=	Soil spreading factor (dim) (4.5)
S_c	=	Estimated total consolidation settlement (mm)
S.D.	=	Standard deviation
SE	=	Force effects due to settlement (kN)
SH	=	Force effects due to shrinkage (kN)
S_i	=	Internal diameter of culvert (m)
SS	=	Seam strength (kN/m)
S_u	=	Undrained shear strength (kPa)
T	=	Limit State tensile capacity from creep tests (kN)
T	=	Tangential force (kN)
t	=	Ice thickness (m)
T_L	=	Factored thrust in culvert wall (kN/m)
T_n	=	Ultimate wide width tensile yield strength (kN)
T_o	=	Tensile Strength (kPa)
T_w	=	Serviceability State tensile capacity at which total strain is not expected to exceed 5% based on wide-width tensile test design life of structure (kN)
T_ℓ	=	Highest load level at which log time-creep-strain rate continues to decrease with time within required lifetime without either brittle or ductile failure (kN)
T_5	=	Tension level at 5% strain based on wide width tensile test (kN)
t_i	=	Initial thickness of reinforcing (m)
t_{100}	=	Thickness of reinforcing after 100 years (m)
T_m	=	Period of vibration in m^{th} mode (sec)

LIST OF SYMBOLS
(continued)

TG	=	Force effects due to temperature gradient deformation (kN)
TU	=	Force effects due to uniform temperature deformation (kN)
u_{bt}	=	Pore Pressure measured behind cone tip (kPa)
v	=	Highway design speed (m/s)
V	=	Vessel impact velocity (m/s)
VAF	=	Vertical Arching Factor (dim) (4.5)
V_B	=	Base wind velocity of 160 km/hr
V_{DZ}	=	Design wind velocity at design elevation (km/hr)
V_{est}	=	Most likely value of property
V_{max}	=	Largest conceivable value of property
V_{min}	=	Lowest conceivable value of property
V_P	=	Compression Wave Velocity (m/s)
V_{10}	=	Wind velocity at 10 m above low ground or design water level (km/hr)
V_0	=	Friction velocity (km/hr)
w	=	Width perpendicular to direction of travel of surface tire contact area (m)
w	=	Width of grid reinforcement mat (m)
w_i	=	Initial width of reinforcing (m)
w_L	=	Liquid limit (%)
w_n	=	Natural water content (%)
w_P	=	Plastic limit (%)
w_{100}	=	Width of reinforcing after 100 years (m)
WA	=	Water load and stream pressure (kN)
W_D	=	Width perpendicular to direction of travel for distributed area (m)
W_E	=	Total unfactored earth load per unit length (kN/m)
W_L	=	Total unfactored live load per unit length (kN/m)
W_T	=	Weight of soil slice in slope stability analysis (kN)
W_T	=	$W_E + W_L$ (kN/m)
WL	=	Horizontal wind pressure effects on vehicles (kN)
WS	=	Wind load on structure (kN)
x	=	Height of section for vertical element being considered (m)
func {overline x}	=	Mean value of data
x_i	=	Data set value
X_o	=	Location of resultant from toe of wall (m)
Z	=	Structure height above low ground or design water level (m)
Z	=	Depth below effective top of wall to reinforcement (m)
z	=	Depth from ground surface (m) (4.2.1.3)
Z_o	=	Friction length of upstream fetch (m)
α	=	Inclination of pier nose to vertical (deg)
α	=	Constant (dim)
α	=	Adhesion (dim)
β	=	Reliability index, func {overline g~/ζ_g} (dim)
β	=	Slope of wall backface (deg)
β	=	Coefficient for determining downdrag force by β method (dim)
β	=	Load combination coefficient (dim)
β_T	=	Target reliability index (dim)

LIST OF SYMBOLS
(continued)

γ = Load factor (dim)
γ = Total unit weight (kN/m^3)
func {overline γ} = Average load factor (dim)
γ_D = Load factor applied to dead load (dim)
γ_d = Dry unit weight (kN/m^3)
γ_{EH} = Load factor applied to horizontal earth pressure (dim)
γ_{EV} = Load factor applied to vertical earth pressure (dim)
γ_{LS} = Load factor applied to live surcharge load (dim)
γ_s = Soil density (kN/m^3)
γ_i = Load factor (dim)
γ_L = Load factor applied to live load (dim)
γ_p = Load factor for permanent load (dim)
γ_s = Unit weight of soil (kN/m^3)
γ'_s = Effective unit weight of soil (kN/m^3)
Δ = Lateral movement needed to develop active or passive earth pressure (m)
δ = Angle of friction between soil and wall (deg)
δ_i = Estimated displacement or differential displacement (mm)
δ_{max} = Maximum tolerable settlement (mm)
δ_n = Tolerable or differential displacement or movement (mm)
Δp = Constant horizontal earth pressure due to uniform surcharge (kPa)
Δ_P = Constant horizontal earth pressure due to uniform surcharge (kPa)
Δp_h = Horizontal earth pressure due to surcharge (kPa)
δ_{tol} = Tolerable displacement or differential displacement (mm)
ζ = Standard deviation of a lognormally distributed data set
θ = Angle between wind direction and normal to structure component (deg.) (4.2.2.8)
θ = Skew angle (deg) (4.4)
η = Load modifier to account for effects of ductility, redundancy and operational importance (dim)
η = Efficiency factor for piles (dim)
η_D = Load modifier account for effects of ductility (dim)
η_I = Load modifier account for effects of operational importance (dim)
η_R = Load modifier account for effects of redundancy (dim)
λ = Bias factor (dim)
λ = Combined bias factor (dim)
λ_i = Individual bias factor (dim)
λ_{QD} = Overall bias for dead load (kN)
λ_{QL} = Overall bias for live load (kN)
λ_R = Overall bias for resistance (kN)
λ_{RA} = Bias factor for soil internal friction angle using Nordlund Method (dim)
λ_{RN} = Bias factor for pile capacity using Nordlund Method (dim)
μ = Correction factor (dim)
ν = Poisson's Ratio (dim)
ξ_m = Lognormal mean
ρ = Average elastic settlement (mm)

LIST OF SYMBOLS
(continued)

ρ_{tol} = Tolerable elastic settlement (mm)
σ = Standard deviation of data set
σ_{all} = Allowable stress (kPa)
σ_{all} = Allowable axial, tensile or flexural stress (kPa)
$\sigma_{all(LRFD)}$ = LRFD equivalent allowable anchor tensile stress (kPa)
σ_g = Standard deviation of g(R,Q), combined probability density function
σ_H = Horizontal stress at reinforcing layer 1 = $\gamma_p \sigma_v k$(kPa)
σ_n = Ultimate (yield) strength of steel (kPa)
σ_n = Ultimate tensile capacity (kPa)
σ_p = Preconsolidation stress (kPa)
σ_{ult} = Limit stress in culvert material (kPa)
σ_V = Pressure due to resultant vertical forces at reinforcement level (kPa)
σ'_v = Vertical effective stress (kPa)
σ_{vo} = Total overburden stress (kPa)
σ'_{vo} = Effective vertical overburden stress (kPa)
τ_{av} = Average cyclic stress (kPa)
ϕ = Resistance factor (dim)
ϕ = Internal friction angle (4.2.2.11)
ϕ = Reduction factor to account for manufacturing variability (dim) (9.2.1)
ϕ' = Effective stress friction angle (deg)
ϕ_{ep} = Resistance factor for passive earth pressure component of sliding resistance (dim)
ϕ_f = Internal angle of friction of reinforced soil zone (deg)
ϕ'_f = Effective stress friction angle (deg)
ϕ_m = Modified resistance factor (dim)
ϕ_{qp} = Resistance factor for tip resistance (dim)
ϕ_{qs} = Resistance factor for side resistance (dim)
ϕ_R = Resistance factor for overall stability (dim)
ϕ'_{tc} = Triaxial compression effective stress friction angle (deg)
ϕ_τ = Resistance factor for shear between footing and foundation material (dim)
Ψ = Soil-reinforcement angle of friction (deg)

CHAPTER 1
INTRODUCTION

1.1 Manual Objectives

In the US, the design of foundations, walls and culverts has traditionally been performed using allowable stress design (ASD) in which all uncertainty in loads and material resistance is combined in a factor of safety. For most highway engineering applications, a source document for ASD of substructures has been Division I of the AASHTO "Standard Specifications for Highway Bridges" (1996). In 1989, work began on an entirely new specification in which the uncertainty in load(s) is represented by a load factor(s) which generally has a value greater than one, and the uncertainty in material resistance(s) is represented by a resistance factor(s) which generally has a value less than one. This effort involved a team of about 50 consultants and contractors under the direction of Modjeski and Masters, Inc., and review by a project panel, AASHTO technical subcommittees, state highway departments, transportation authorities, and industry representatives. The AASHTO Load and Resistance Factor Design (LRFD) Specification was approved for use in 1994.

Due to the differences between the substructure design process by ASD and LRFD, FHWA sponsored development of this training course to present the fundamentals of LRFD to bridge design engineers, geotechnical engineers, engineering geologists and others who are responsible for design of bridge substructures using the AASHTO LRFD Specification. This manual focuses on the geotechnical aspects of substructure design.

The objectives of this manual are to provide the basis for an understanding of the:

- Differences between ASD and LRFD for substructure design
- Benefits of LRFD for substructure design
- Importance of site characterization and selection of geotechnical design parameters
- Process for design of substructure elements by LRFD using the AASHTO LRFD Specifications as a guide
- Process for selection and application of load factors and load combinations
- Methods available for calibration of resistance factors
- Basis for calibration of the AASHTO LRFD resistance factors for substructure design
- Procedures available for modifying or developing resistance factors to achieve designs comparable to ASD

Although the design procedures differ between ASD and LRFD, the methods used to estimate

design loads and ultimate resistance remain essentially unchanged. Therefore, verification of the quality of design and construction is equally important for ASD and LRFD.

1.2 Manual Outline

This manual consists of 17 chapters which describe the background, development and use of LRFD for highway bridge substructure design, and which present worked example problems for the majority of substructure types. The manual is intended as a reference document to supplement the Participant's Workbook prepared for a National Highway Institute (NHI) two-day short course on the design of substructures using LRFD. It is hoped that this reference manual and associated short course will facilitate the implementation of LRFD for highway bridge substructures and provide both the basis and impetus for further development and refinement of the AASHTO and state transportation department specifications for design by LRFD.

In addition to this Introduction and a Summary at the end of the manual, the content of the chapters includes the following topics:

- Chapter 2 - Transition to LRFD for Substructure Design
- Chapter 3 - Principles of Limit State Design
- Chapter 4 - Loads
- Chapter 5 - Geotechnical Site Characterization
- Chapter 6 - Geotechnical Design Parameter Selection
- Chapter 7 - Calibration as Part of the Design Process
- Chapter 8 - Spread Footing Design
- Chapter 9 - Driven Pile Design
- Chapter 10 - Drilled Shaft Design
- Chapter 11 - Conventional Retaining Wall and Abutment Design
- Chapter 12 - Prefabricated Modular Wall Design
- Chapter 13 - Anchored Wall Design
- Chapter 14 - Mechanically-Stabilized Earth Wall Design
- Chapter 15 - Flexible Culvert Design
- Chapter 16 - Rigid Culvert Design

Chapters 2, 3 and 4 focus on limit state design principles and development of loads and load combinations needed for substructure design. Chapters 5 and 6 cover aspects of geotechnical site characterization and selection of geotechnical parameters for design. Chapters 5 and 6, which address aspects of material variability, are included in this manual because the development and selection of soil and rock properties are an integral part of both Load and Resistance Factor Design (LRFD) and Allowable Stress Design (ASD) methods for substructure design. Calibration methods used to develop the AASHTO LRFD Specification (1997a) are presented in Chapter 7 as an introduction to the design chapters and to provide a reference to agencies for future additions to the specification. Chapters 8 through 16 cover the design of foundations, retaining walls and culverts by LRFD. In addition, these chapters provide guidance for transforming current ASD methods to LRFD. Chapters 8 through 16 are not intended to provided instruction in applying the LRFD Specification, but instead highlight differences in substructure design between LRFD and ASD. Each design chapter provides a brief discussion and comparison of the general elements of substructure design by ASD and LRFD, and identifies the performance limits important for substructure design (e.g., settlement, bearing capacity, sliding and overturning for spread footing foundations). Application of LRFD principles is illustrated in each chapter by example problems. In addition, several short student exercises are included to test your understanding of the information presented in the manual.

1.3 General References

Copies of the following references are available for use during the class:

AASHTO, 1994, *LRFD Bridge Design Specifications*, American Association for Transportation and Highway Officials, Washington, DC, 1st Ed.

AASHTO, 1997a, *1997 Interims to LRFD Highway Bridge Design Specifications*, SI Units, American Association of State Highway and Transportation Officials, Washington, D.C., First Edition (1997 LRFD Interims)

AASHTO, 1996, *Standard Specifications for Highway Bridges*, American Association of State Highway and Transportation Officials, Washington, D.C., 16th Edition (1996 ASD)

AASHTO, 1997b, *1997 Interims to Standard Specifications for Highway Bridges*, American Association of State Highway and Transportation Officials, Washington, D.C., 16th Edition (1997 ASD Interims)

Barker, R.M., J.M. Duncan, K.B. Rojiani, P.S.K. Ooi, C.K. Tan and S.G. Kim, 1991, *Manuals for the Design of Bridge Foundations*, National Cooperative Highway Research Program Report 343, Transportation Research Board, Washington, D.C., 308p.

Barker, et al. (1991) was published as a result of research conducted in the Department of Civil Engineering at Virginia Polytechnic and State University in Blacksburg, Virginia. That work formed for basis for development of Section 10 - Foundations and Section 11 - Abutments, Piers and Walls found in the AASHTO LRFD Specification.

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CHAPTER 2
TRANSITION TO LRFD FOR SUBSTRUCTURE DESIGN

2.1 Introduction

Highway substructures (i.e., foundations and abutments) have traditionally been designed using Allowable Stress Design (ASD) methods whereas superstructure components have been designed used Load Factor Design (LFD) methods. This application of ASD for substructure design and LFD for superstructure design leads to uncertain and incompatible safety margins in the design of structure components. Solution of this problem requires a coherent method of design for the structure system. As described in subsequent chapters, Load and Resistance Factor Design (LRFD) represents an approach in which applicable failure and serviceability conditions can be evaluated considering the uncertainties associated with loads and material resistances. This chapter:

- Describes briefly the general process of allowable stress design
- Introduces the general process of load and resistance factor design
- Identifies the limit states that must be evaluated for structure design
- Briefly describes procedures used to calibrate LRFD and the AASHTO LRFD Specification (AASHTO, 1997a)

2.2 Allowable Stress Design

Existing practice for the geotechnical design of substructures follows the ASD approach, wherein all uncertainty in the variation of applied loads transferred to the foundation and the ultimate geotechnical capacity of the soil and rock to support the loads are incorporated in a factor of safety, FS. The factor of safety is an empirical, but arbitrary, measure used to reduce the potential for adverse performance (e.g., sliding failure of a footing and bearing failure of driven pile). The general relationship used in applying ASD takes the general form:

$$\frac{R_n}{FS} \geq \sum Q \qquad \text{(Eq. 2-1)}$$

where:

R_n = Nominal (ultimate) resistance
FS = Factor of safety
$\sum Q$ = Summation of force effects

In practice, FS can range from values of about 1.2 to 6 depending on factors such as the type of problem being evaluated, the model used to estimate resistance and the experience of the designer. Graphically, the ASD process can be illustrated as shown in Figure 2-1.

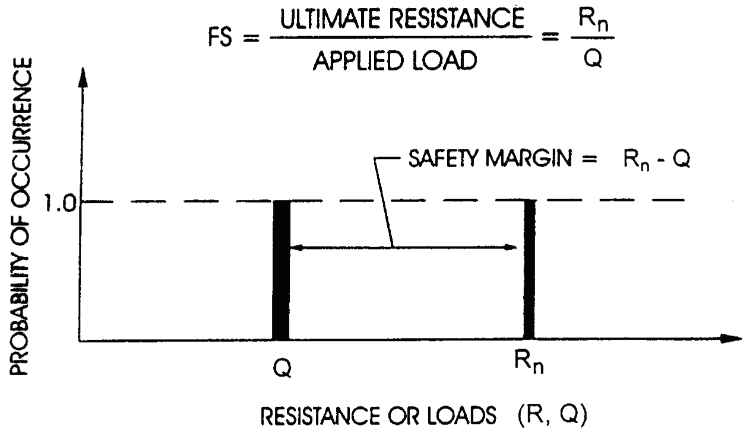

Figure 2-1
Factor of Safety

Figure 2-1 illustrates one of the principal limitations of ASD, wherein the values of Q and R_n are assumed to be unique such that they have a probability of occurrence of unity. In addition, selection of FS is subjective, depends on the design models used and material parameters chosen, and is not inherently related to the probability of component failure. For geotechnical engineering, a rational design approach should consider the uncertainties associated with:

- Variability of engineering properties with aerial and vertical extent, and with time
- Reliability and applicability of property measurements
- Sufficiency and applicability of sampling and testing methods
- Errors in prediction models used
- Errors in measuring material resistance
- Variability in load prediction estimates

Although many or all of these sources of variability are usually considered by the designer using ASD, their consideration is generally qualitative rather than quantitative, leading to a wide range of failure probabilities among designs.

2.3 Load and Resistance Factor Design

Load and resistance factor design represents a more rational approach by which the more significant uncertainties listed above (i.e., load and material resistance) can be incorporated quantitatively into the design process. As used in the AASHTO LRFD Specification (AASHTO, 1997a), the basic LRFD equation or relationship is defined by:

$$\sum \gamma_i Q_i \leq \phi R_n \qquad \text{(Eq. 2-2)}$$

where:

γ_i = Statistically-based load factor generally greater than one
Q_i = Load
R_n = Nominal (ultimate) resistance
ϕ = Statistically-based resistance factor generally less than one

Application of Eq. 2-2 is illustrated in Figure 2-2.

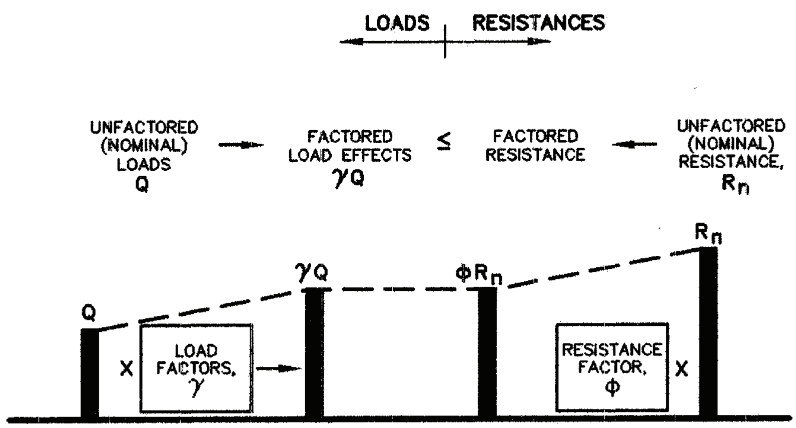

Figure 2-2
LRFD Design Approach

When applying LRFD, the estimated magnitudes of the various types of load effects are multiplied by appropriate load factors to determine the factored load effects, and the estimated nominal (ultimate) resistance is multiplied by a resistance factor prescribed for the model used to estimate material resistance and the field and/or laboratory test methods used to develop the material properties. For simplicity, the nominal and factored loads and the nominal and factored resistances are shown in Figure 2-2 as unique values. Of course, the load and material resistances vary, such as shown in Figure 2-3.

Figure 2-3 shows a possible variation of load and resistance as a function of the frequency or probability of occurrence. If the peak values of load, Q, and resistance, R, are defined by their respective mean values, $\overline{Q}$ and $\overline{R}$, the equivalent ASD factor of safety is the ratio of $\overline{R}$ to $\overline{Q}$ as shown in the figure, and the margin of safety is the difference between $\overline{R}$ and $\overline{Q}$. However, the figure also points out that some potential for failure exists in the area where the distributions of Q and R overlap. Therefore, unless very high factors of safety are used, some probability of failure will always exist.

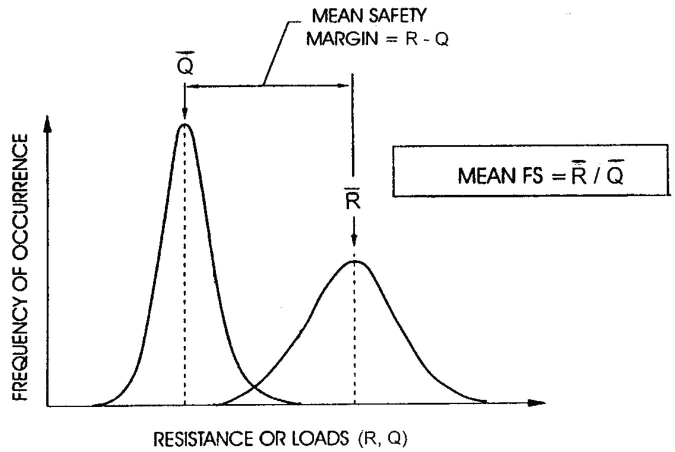

Figure 2-3
Variation of Load and Resistance

Figure 2-3 shows a possible variation of load and resistance as a function of the frequency or probability of occurrence. If the peak values of load, Q, and resistance, R, are defined by their respective mean values, $\overline{Q}$ and $\overline{R}$, the equivalent ASD factor of safety is the ratio of $\overline{R}$ to $\overline{Q}$ as shown in the figure, and the margin of safety is the difference between $\overline{R}$ and $\overline{Q}$. However, the figure also points out that some potential for failure exists in the area where the distributions of Q and R overlap. Therefore, unless very high factors of safety are used, some probability of failure will always exist.

Another factor which must be considered is the distribution of load and resistance. Figure 2-3 represents only one pair of distributions. Figure 2-4 shows that using the ratio of $\overline{R}$ to $\overline{Q}$ to define safety can be misleading. Two pair of load and resistance distributions are presented which have identical values of $\overline{Q}$ and $\overline{R}$. The upper distribution for resistance is relatively small with steep flanks about $\overline{R}$, whereas the lower distribution for resistance is broad with flat flanks, also about $\overline{R}$. As a result, the area of overlap between the upper distributions of R and Q is small, representative of a small probability of failure. Conversely, the area of overlap between the lower distributions is large, representative of a greater probability of failure.

For substructure design, the majority of loads which must be supported are prescribed by the structural or bridge designer in the form of vehicle and other types of transient load types. Thus, with the exception of the type of soils used for backfill behind walls and abutments and around culverts, geotechnical engineers have only limited control over the load side of the relationship. However, on the resistance side, geotechnical engineers have the opportunity to control the extent and type of sampling and testing used to characterize a site, the procedures or models used for design, and the measures employed to monitor the construction processes.

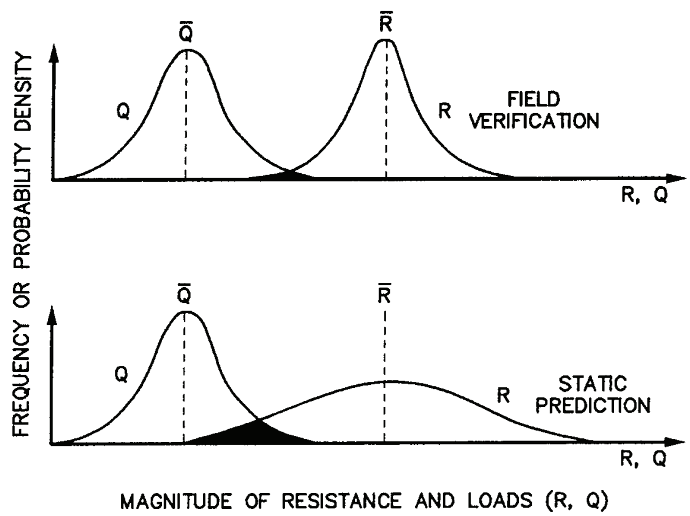

Figure 2-4
Distribution of Load and Resistance

Reliability-based design (or LRFD) is the process by which the risks and uncertainties associated with the safety of a system are defined in mathematical terms. In applying the process, the uncertainties (or distributions) of load and resistance are assumed to be independent, random variables, and the design risk is defined by the probability of failure, p_f. To evaluate p_f, a single probability density function is developed, as shown in Figure 2-5, which represent the combined distributions and uncertainties of Q and R.

The left side of Figure 2-5 shows typical distributions of load and resistance and the right side shows the combined distribution function for Q and R. For the pair of distributions to the left, failure is possible in the shaded area where the distributions overlap and the margin of safety is represented by the difference between $\overline{R}$ and $\overline{Q}$. For the combined distribution, failure is possible in the shaded area to the left of the ordinate and the margin of safety is represented by the number of standard deviations, β, of the mean value of the combined distribution to the right of the ordinate. In LRFD parlance, β is referred to as the reliability index. Depending on the shape of original distributions (i.e., both normal, both lognormal or one normal and one lognormal), the probability of failure can be mathematically related to the reliability index. For lognormal distributions, the relationship between p_f and β is presented in Table 2-1.

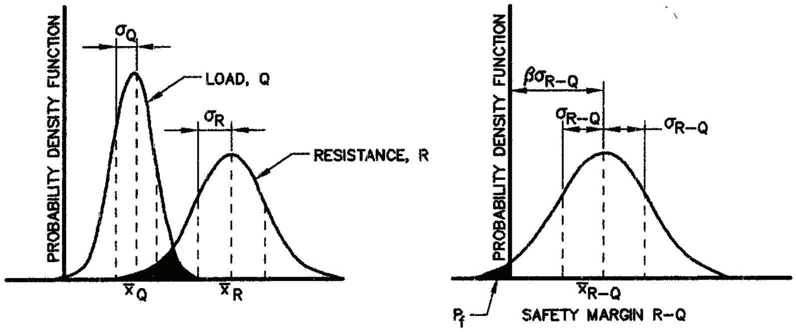

Figure 2-5
Reliability Index, β

Table 2-1
Relationship Between p_f and β for Lognormal Distribution

Reliability Index β	Probability of Failure p_f
1.96	1:10
2.50	1:100
3.03	1:1 000
3.57	1:10 000
4.10	1:100 000
4.64	1:1 000 000

To implement the LRFD concept in a design code, it is necessary to select a minimum level of safety which must be achieved. One process is to conduct cost-benefit analyses to determine the total costs (i.e., sum of initial construction cost, maintenance costs and estimated costs of failures) as a function of the probability of failure. Using this approach, the target probability of failure would be the p_f for which costs are minimized. Alternatively, the target probability of failure could be established based on the failure rates estimated from actual case histories. However, the probability of failure for constructed facilities is not solely a function of uncertainties associated with the design process, and probably is at least an order of magnitude lower than the theoretical probability of failure. For general guidance, information such as that presented in Figure 2-6 regarding empirical rates of failure for civil engineering facilities can be used. For structure foundations, the annual probability of failure ranges from about 1:100 to 1:1000. In calibrating the LRFD Specification, values of p_f range approximately within these limits, depending on the foundation type and the level of redundancy available in the system (e.g., pile foundations are usually constructed in a group such that failure of a single pile does not imply failure of the group).

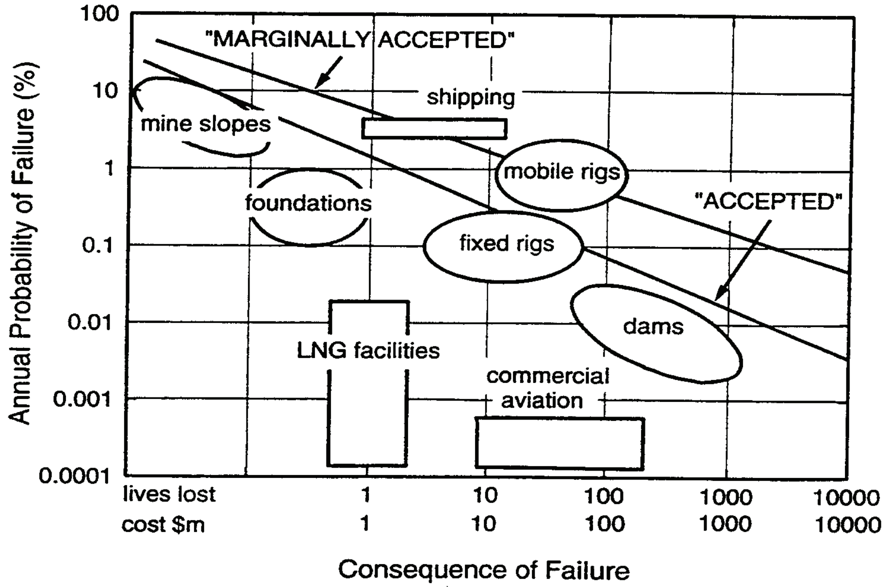

Figure 2-6
Empirical Rates of Failure for Civil Works Facilities
(Kulhawy, et al., 1995)

A limit state is a condition beyond which a structural component, such as a foundation or other bridge component, ceases to fulfill the function for which it is designed. The limit states which must be evaluated in the AASHTO LRFD Specification (AASHTO, 1997a) include:

- Service Limit State
- Strength Limit State
- Extreme Limit State
- Fatigue Limit State

The Service Limit State represents structure performance under service load conditions. Examples for substructure design include settlement of a foundation or lateral displacement of a retaining wall. Another example of a Service Limit State condition is presented in Figure 2-7 which shows the rotation of a rocker bearing on an abutment caused by instability of the earth slope which supported the abutment.

Strength Limit States involve the total or partial collapse of the structure. Examples of Strength Limit States in geotechnical engineering include bearing capacity failure, sliding, and overall slope instability such as shown in Figure 2-8.

Figure 2-7
Example Condition for Service Limit State Evaluation

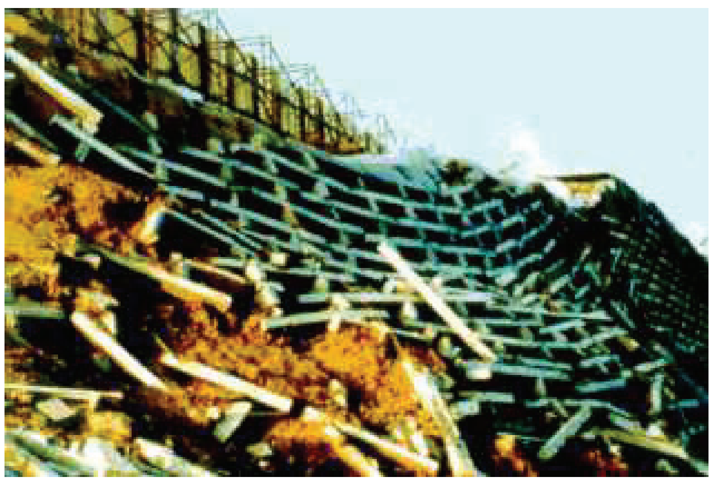

Figure 2-8
Example Condition for Strength Limit State Evaluation

2.4 LRFD Calibration

Calibration of the load and resistance factors is required to achieve the desired results when applying LRFD. Calibration procedures for selection of resistance factors are described fully in Chapters 3 and 7, but generally involve:

- Engineering judgment
- Fitting to ASD
- Reliability theory
- A combination of approaches

For the LRFD Specification, a combination of approaches was used in selecting resistance factors, ϕ, for design. In general, the resistance factors selected for most of the methods used for foundation design were developed principally using reliability-based calibration procedures where sufficient performance data were available (e.g., bearing resistance of footings and individual deep foundation elements). The value of ϕ chosen for a particular design procedure and limit state from a reliability-based calibration can take into account the:

- Variability of the soil and rock properties
- Reliability of the equations used for predicting resistance
- Quality of the construction workmanship
- Extent of soil exploration
- Consequence(s) of a failure

Where insufficient or no data were available to conduct a reliability-based calibration, resistance factors were selected primarily by fitting to ASD (e.g., eccentricity, anchored and MSE wall design) and judgment. Therefore, the principal benefit of LRFD, namely to achieve consistent levels of safety in component design, are not completely realized. The resistance factors incorporated in the LRFD Specification were checked by trial designs to confirm that the results are comparable to current ASD practice.

2.5 Summary

The incorporation of LRFD represents a significant step and major improvement in the processes of foundation, retaining wall and culvert design, as it permits design based primarily on a rational evaluation of performance reliability, rather than the judgment and experience of and individual designer (although the importance of these even in LRFD should not be minimized). As it is currently embodied in the AASHTO Specification, LRFD offers many advantages, primarily that it:

- Accounts separately for variability in load and resistance prediction

- Achieves more consistent levels of safety in structure and substructure design

- Does not require knowledge of probability or reliability theory

On the other hand, LRFD does present a challenge to the practicing engineer, in that:

- Implementation requires a change for engineers accustomed to ASD

- Resistance factors vary with design methods and are not constant

- Rigorous calibration of load and resistance factors to meet individual situations requires availability of statistical data and probabilistic design algorithms

Subsequent chapters will provide additional information regarding the development and application of LRFD for substructure design.

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CHAPTER 3
PRINCIPLES OF LIMIT STATES DESIGN

3.1 Introduction

In this chapter, comparisons between Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD) are presented. Emphasis is placed on reliability concepts on which LRFD is based and how these concepts relate to a predefined limit state. Procedures are described for calibrating and selecting resistance factors for geotechnical components which target an acceptable probability of survival. Because resistance factors are based on statistics, a brief review of statistics and probability concepts is presented.

Simply stated, safety in engineering design is assumed when the cross-sections and materials supplied exceed the demands put on them by applied loads so that:

$$\text{Supply} \geq \text{Demand}$$

Another way of stating this same principle is that the resistance of the materials must exceed the effect of the loads, so that:

$$\text{Resistance} \geq \text{Effect of Loads} \qquad \text{(Eq. 3-1)}$$

When applying this simple principle to design, it is essential that both sides of the inequality be evaluated for the same conditions. For example, if the effect of applied loads produces compressive stress on a soil, it is obvious that the load should be compared to the bearing resistance of the soil, and not some other aspect of the material (e.g., soil).

When a particular loading condition reaches its limit, failure will result. Such a condition is referred to as a limit state, and is defined as:

> A **limit state** is a condition beyond which a structural component, such as a foundation or other bridge component, ceases to fulfill the function for which it is designed.

Strength Limit States involve the total or partial collapse of the structure. Examples of Strength Limit States in geotechnical engineering include bearing capacity failure, sliding, and overall instability.

Service Limit States affect the function of the structure under regular service conditions. Service Limit States may be reached in foundations through excessive settlement, excessive lateral deflection, structural deterioration of the foundation or excessive vibration.

In this definition of a limit state, both the resistance and load sides of Eq. 3-1 are included. For example, if adequacy of bearing strength of a soil under a footing is being investigated, more than one load combination must be evaluated, especially if the footing is subjected to eccentric or inclined loads. When the bearing pressure due to the loads exceeds the bearing strength, a limit state (i.e., a Strength Limit State) is reached and failure results. Similarly, if the footing movements due to the loads exceeds the tolerable settlement, the Service Limit State is reached. In the chapters on component design (i.e., Chapters 8 -16), the various resistance or deformation conditions (e.g., bearing capacity or settlement) investigated for each component are defined as Performance Limits.

An important goal, but not the only goal of the designer is to prevent a limit state from being reached. Other goals that must be considered and balanced in the overall design are function, appearance, and economy. Because it is not economical to design a bridge so that none of its components could ever fail under the most improbable loads, an acceptable level of risk or probability of failure must be determined.

The determination of an acceptable margin of safety (i.e., how much greater the predicted magnitude of resistance should be compared to the magnitude of the loads) is not based on the opinion of one individual. The acceptable margin of safety must combine the collective experience and judgment of a qualified group of engineers and officials. Many public owners apply their experience and judgment to their own unique methods of design and they need to know what level of risk their design entails. This concern is addressed in Section 3.4 and is more fully developed in Chapter 7.

In the sections that follow, a review of current design procedures in geotechnical engineering will be addressed. A discussion of the shortcomings of current design procedures will show why a new approach, the LRFD method, is desirable.

3.2 Design Procedures

Over the years, design procedures have been developed by engineers to provide satisfactory margins of safety. These procedures were based on the engineer's confidence in predicting the magnitude of the load and the effect of the load on the strength of the materials being provided. In the next section, the existing procedure for design of foundations is discussed.

3.2.1 Allowable Stress Design (ASD)

The design of foundations has traditionally been based on ASD. ASD is different for the Service Limit State and the Strength Limit State. For the Strength Limit State, safety is achieved in the foundation element by restricting the estimated loads (or stresses) to values less than the ultimate resistance divided by a factor of safety, FS using the relationship:

$$\frac{R_n}{FS} \geq \sum Q_i \qquad \text{(Eq. 3-2)}$$

where:

R_n = Nominal resistance (e.g., ultimate bearing resistance of foundation soils)

$\sum Q_i$ = Q_n = Nominal load effect (e.g., moment produced by design vehicle)

Load effects consist of dead, live and environmental load components. Environmental loads include wind, water and earthquake forces, for example. In ASD all of these loads are assumed to have the same variability. As a result, load factors are not applied on the load combinations considered for either the strength or service limit states.

The factor of safety is a number greater than unity. The FS provides reserve strength in the event that an unusually high load occurs or in the event that the resistance is less than expected.

For the Service Limit State, unfactored loads are used to calculate deformations, and these deformations are compared to the maximum tolerable values.

The advantage of ASD is its simplicity; however, there are shortcomings of ASD.

3.2.2 Shortcomings of ASD

In ASD, no consideration is given to the fact that various types of loads have different levels of uncertainty. For example, the dead load of a bridge can be estimated with a high degree of accuracy. However, earthquake loads acting on bridge abutments and piers cannot be estimated with the same degree of accuracy and confidence. Nevertheless, dead, live, and environmental loads are all treated equally in ASD. In ASD, fixed values of design loads are selected, usually from a specification or design code. The factor of safety is applied to the resistance side of the design inequality, and the load side of the inequality is not factored.

Factors of safety in geotechnical engineering vary considerably depending on the type of problem.

- Slope Stability: $1.3 \leq FS \leq 1.5$
- Foundation Bearing Capacity: $2 \leq FS \leq 3$
- Foundation Sliding: $FS \geq 1.5$
- Foundation Overturning: $FS \geq 2.0$

Because the factor of safety chosen is based on experience and judgment, quantitative measures of risk cannot be determined for ASD.

Limitations of ASD:

- Does not adequately account for variability of loads and resistances. The FS is applied only to resistance. Loads are considered to be without variation (i.e., deterministic).

- Does not embody a reasonable measure of strength, which is a more fundamental measure of resistance than is allowable stress.

- Selection of a FS is subjective, and does not provide a measure of reliability in terms of probability of failure.

What is needed to overcome these deficiencies is a method that:

- Considers variability not only in the resistance, but also in the effect of loads
- Uses the strength of the material (e.g., soil and rock) as a basis of resistance
- Provides a measure of safety related to probability of failure

Such a method is incorporated in load and resistance factor design, LRFD.

3.2.3 Load and Resistance Factor Design (LRFD)

The American Concrete Institute (ACI) introduced a limit state design code in an appendix to the 1956 ACI building code. Initially, the code did not include any resistance factors, only load factors, so the code was known as load factor design (LFD). The load factors (and resistance factors when they were introduced) were not based on the reliability concepts used in developing the AASHTO LRFD Specification, but rather on matching with the then existing ASD ACI code. The fact that reliability theory was not used in the selection of load and resistance factors represents the greatest difference between LFD and LRFD.

In LRFD, the resistance side of Eq. 3-1 is multiplied by a statistically-based resistance factor, φ, whose value is usually less than one. As applied to the geotechnical design of substructures, φ accounts for factors such as weaker foundation soils than expected, poor construction of the foundations, and foundation materials such as concrete, steel or wood that may not completely satisfy the requirements in the specifications.

The load components on the right side of Eq. 3-1 are multiplied by their respective statistically based load factors, γ_i, whose values are usually greater than one. Because the load effect at a particular limit state involves a combination of different load types, Q_i, each of which has different degrees of predictability, the load factors differ in magnitude for the various load types. Therefore, the load effects can be represented by a summation of γ_i Q_i products. If the nominal resistance is given by R_n, then the safety criterion can be written as:

$$R_r = \phi R_n \geq \sum \eta_i \gamma_i Q_i \qquad \text{(Eq. 3-3) } (\textit{A1.3.2.1-1})$$

where:

ϕ = Statistically-based resistance factor (dim)
R_n = Nominal resistance
η_i = Load modifier to account for effects of ductility, redundancy and operational importance (dim)
γ_i = Statistically-based load factor (dim)
Q_i = Load effect

Because Eq. 3-3 involves both load factors and resistance factors, the design method is called Load and Resistance Factor Design (LRFD). For a satisfactory design, the factored nominal resistance should equal or exceed the sum of the factored load effects for a particular limit state. Load and resistance factors are chosen so that in the highly improbable event that the nominal resistance of the foundation material is overestimated, and at the same time the loads are underestimated, there is a reasonably high probability that the actual resistance of the foundation material should still be large to support the loads.

The value of ϕ chosen for a particular limit state can take into account the:

- Variability of the soil and rock properties
- Reliability of the equations used for predicting resistance
- Quality of the construction workmanship and quality control programs
- Extent of soil exploration (little versus extensive)
- Consequence(s) of a failure

In the AASHTO LRFD Specifications (1997a), not all of these features have been implemented. However, they all can be included in the LRFD format once research has been completed and the experience base has been established. The LRFD Specifications will continue to improve and the resistance factors will be adjusted as more field performance measurements are evaluated. It is important that the experience of geotechnical engineers with LRFD be shared and that the quality of the geotechnical data base be improved (e.g., load-deformation response of spread footing foundations) through the use of well planned and instrumented testing programs. As a result, our understanding of design methods and the safety margins needed for their effective and economic use in reducing the risk of failure can be improved.

Some methods of predicting the nominal resistance are empirical whereas others are based on classical theories of mechanics. Also, different methods of predicting resistances employ the use of different soil parameters [e.g., friction angle of cohesionless soils based on Standard Penetration Test (SPT) blow counts or Cone Penetration Test (CPT) tip resistance, and undrained shear strength of cohesive soils based on CPT sleeve friction]. It is generally true in ASD that a higher FS is used for empirically-based methods (e.g., bearing resistance estimated using SPT blow counts), as opposed to methods based on classical bearing resistance theories. Because different methods of predicting resistance have different degrees of reliability, different values of resistance factors are required for each method.

The load factor, γ_i, chosen for a particular load type must consider the uncertainties in the:

- Magnitude and direction of loads
- Location of application of loads
- Possible combinations of loads (i.e., dead load + live load to dead load + environmental load)

Loads and load combinations are addressed in Chapter 4.

3.2.4 Advantages and Limitations of LRFD

Before 1970 in the United States, design of both the superstructure and substructure components of highway bridges was accomplished using ASD. In the 1971 and 1972 Interims, AASHTO introduced load factor design (LFD) for the design of bridge superstructures. Design of steel and concrete structures has switched from ASD to LFD to LRFD over the years. Limit state concepts are currently used in the LFD American Concrete Institute (ACI, 1995) design code, the LRFD American Institute of Steel Construction (AISC, 1989) specifications for design of steel buildings, and the LFD AASHTO Standard Specifications (AASHTO, 1997b) for design of concrete, wood, and metal bridge superstructures. Several countries have adopted the limit states design code format for design of substructures including the Canadian Foundation Engineering Manual (1985), Ontario Highway Bridge Design Code (1991) and the Danish Code of Practice for Foundation Engineering

(1985). Limit states concepts for foundation design are also being considered in Japan and other countries in Europe. Some of the advantages and limitations of the LRFD method are:

Advantages of LRFD:

- Accounts for variability in both resistance and load.
- Achieves relatively uniform levels of safety based on the strength of soil and rock for different limit states and foundation types.
- Provides more consistent levels of safety in the superstructure and substructure as both are designed using the same loads for predicted or target probabilities of failure.

Limitations of LRFD:

- The most rigorous method for developing and adjusting resistance factors to meet individual situations requires availability of statistical data and probabilistic design algorithms.
- Resistance factors vary with design methods and are not constant.
- Implementation requires a change in design procedures for engineers accustomed to ASD.

Due to the advantages of LRFD and the prospect that the method will eventually supersede ASD in geotechnical engineering, a discussion on the basis for the derivation of resistance factors is presented in Section 3.3. This derivation is presented for background information and for understanding the process. In using the AASHTO LRFD Specifications, resistance factors are provided and no probability or statistical analysis is required.

3.3 Calibration

The process of assigning values to resistance factors and load factors is called calibration. ***A design code may be calibrated by use of (1) judgment, (2) fitting to other codes, (3) reliability theory, or (4) a combination of approaches.***

Calibration by judgment requires experience. For example, poor past performance of foundations may force a code authority to adjust the code until satisfactory results are achieved. Code parameters for structures that perform satisfactorily were accepted as correct, although this may be excessively conservative. A fundamental disadvantage of this method of calibration is that it results in non-uniform levels of conservatism.

Calibration by fitting to other codes, or simply fitting, involves using parameters (i.e., resistance factors) that would result in the same minimum permissible physical dimensions of a foundation as by ASD. Calibration by fitting does not achieve more uniform margins of safety than the ASD procedures it replaces. It does, however, make it possible to use the same loads for superstructure and foundation, and it ensures that the new code will not lead to radically different designs from the old code. Calibration by fitting with ASD can be used where there is insufficient statistical data to perform a more formal process of calibration by reliability theory.

A code can be calibrated by fitting to ASD as follows:

- Divide the LRFD equation (Eq. 3-3) with $\eta_i = 1.0$ by the ASD equation (Eq. 3-2):

$$\frac{\phi R_n}{R_n} \geq \frac{\sum \eta_i \gamma_i Q_i}{FS \times \sum Q_i}$$

from which:

$$\phi \geq \frac{\sum \gamma_i Q_i}{FS \times \sum Q_i} \qquad \text{(Eq. 3-4)}$$

- If the loads consist only of dead load Q_D and live load Q_L, then Eq. 3-4 becomes:

$$\phi = \frac{\gamma_D Q_D + \gamma_L Q_L}{FS(Q_D + Q_L)} \qquad \text{(Eq. 3-5)}$$

- Dividing both numerator and denominator by Q_L, Eq. 3-5 becomes:

$$\phi = \frac{\gamma_D (Q_D / Q_L) + \gamma_L}{FS\ (Q_D / Q_L + 1)} \qquad \text{(Eq. 3-6)}$$

From Eq. 3-6, and equivalent LRFD resistance factor can be calculated from a given ASD safety factor for known load factors and the unfactored dead to live load ratio.

Example Problem 3-1
Calculate the resistance factor, ϕ, that is equivalent to an ASD safety factor FS = 2.5 if the dead load factor γ_D = 1.25, the live load factor γ_L = 1.75, and the dead to live load ratio Q_D/Q_L = 3.0. Substituting values into Eq. 3-6:

$$\phi = \frac{\gamma_D (Q_D / Q_L) + \gamma_L}{FS\ (Q_D / Q_L + 1)} = \frac{1.25(3.0) + 1.75}{2.5\ (3.0 + 1)} = 0.55 \qquad \text{(Eq. 3-6)}$$

The calculated resistance factor indicates that to obtain an LRFD resistance equivalent to that obtained using ASD with a safety factor of 2.5, the resistance predicted by the LRFD limit state equations must be reduced by multiplying by a factor of 0.55. Values of resistance factor obtained from Eq. 3-6 for a range of safety factors and dead to live load ratios are shown in Table 3-1 for γ_D = 1.25 and γ_L = 1.75. The ratio of dead to live load depends on the construction material (steel, concrete or wood) and the span length of the bridge (the longer the span, the larger the dead to live load ratio; $Q_D/Q_L \approx$ span (m)/20). In general, Table 3-1 shows that the resistance factor decreases with increasing safety factor and the influence of Q_D/Q_L is small.

Note in Eq. 3-6 that the values of resistance factors vary with values of load factors. The load factors and load combinations for highway bridge design using the AASHTO LRFD Specification (1997a) are presented and discussed in Chapter 4.

Table 3-1
Values of Resistance Factors by Fitting
for Different Values of Safety Factor and
Dead to Live Load Ratios for γ_D = 1.25 and γ_L = 1.75

Safety Factor	Resistance Factor, ϕ			
	Q_D/Q_L = 1	Q_D/Q_L = 2	Q_D/Q_L = 3	Q_D/Q_L =4
1.5	1.00	0.94	0.92	0.90
2.0	0.75	0.71	0.69	0.68
2.5	0.60	0.57	0.55	0.54
3.0	0.50	0.47	0.46	0.45
3.5	0.43	0.40	0.39	0.39
4.0	0.38	0.35	0.34	0.34

Calibration by reliability theory involves the use of probabilistic design concepts.

There are several levels of probabilistic design. The fully probabilistic method (i.e., Level III) is the most complex and requires knowledge of the probability distributions of each random variable (i.e., loads and resistances), and correlations between the variables. Because of its complexity, Level III was not used in calibrating the AASHTO LRFD Specification (1997).

Level II and Level I probabilistic methods include the first order second moment (FOSM) method, which uses simpler statistical characteristics (i.e., mean and standard deviation) of the load and resistance variables to describe the probability distributions. In some respects, the process is similar to representing the moment of inertia of a section (i.e., second moment of the area) by its area and radius of gyration. Further, it is assumed that the load, Q, and the resistance, R, are statistically independent random variables such that events related to one are independent of the other.

The Level II method is referred to as the advanced FOSM (AFOSM) method. When the AFOSM method is used, the limit state function g( ) is linearized at the design point on the nonlinear failure surface rather than at the mean value of the random variables. An analogy would be finding the absolute maximum bending moment in a simple beam due to a passing tractor trailer. The moment curve is similar to the limit state function. The maximum moment is near midspan of the beam and is like the design point on the failure surface in the Level II method. The Level I method says the midspan value is sufficiently close and does not examine other points on either side of the midspan for the absolute moment. For the AFOSM method, an iterative procedure must be used in which an initial value of reliability index, β, is assumed and the process is repeated until the difference in calculated values of β on successive iterations is within a small tolerance. This iterative procedure is based on normal approximations to nonnormal distributions at the design point developed by Rackwitz and Fiessler (1978). The load and resistance factors used in the AASHTO LRFD Specification for foundation design were developed using the nonlinear Level II FOSM procedures (Barker, et al., 1991b). Implementing the Level II method requires the use of a computer program to efficiently perform the iterations.

The Level I method is referred to as the mean value FOSM (MVFOSM) method because the linearization occurs at the mean values of Q and R, rather than at the design point. The values of the reliability index, β, determined by the MVFOSM method are the least accurate of the methods, but have the advantage that explicit equations can be written for β. The equations for β in Chapter 3 are based on Level I approximations.

The basic procedure adopted for calibration of the AASHTO LRFD Specifications (Level I or Level II) by reliability theory employed the following steps:

Step 1 Estimate the level of reliability (which is related to the probability of success or failure) implied in the current ASD methods for analyzing foundations. (Calibration with existing ASD criteria ensures a proper design evolution and avoids drastic deviations in designs by the new procedure from existing designs. Instead of specifying a new level of reliability, the new design procedure is based on risk levels implied in current ASD criteria.)

Step 2 Observe the variation of reliability levels with different span lengths, load ratios (e.g., dead to live load and other load combinations), geometry of the foundations and methods of predicting resistance.

Step 3 Select a target "reliability index" based on the margin of safety implied in current designs.

Step 4 Calculate resistance factors consistent with the selected target reliability index. It is important to couple experience and judgment with the calibration results.

A simple example is provided to illustrate the various steps involved in the calibration process in Section 3.4. To help understand the calibration process using reliability theory, a brief primer on the basic concepts of reliability theory is given in the next section.

Use of reliability analyses is *not* necessary to apply the LRFD method in practice. For design situations or methods that are not encompassed by the AASHTO LRFD Specifications, reliability analysis *may* be used by the engineer in the calibration process to obtain appropriate resistance factors for design. Alternatively, appropriate values of the resistance factor can be established by fitting. Fitting is always used, even when reliability analyses are used, because fitting provides an effective means of checking to ensure that the results of the more complex reliability analyses are reasonable.

3.4 Review of Statistics and Probability Concepts

3.4.1 Statistical Descriptors

Statistical uncertainties can be described by the mean, $\overline{x}$, standard deviation, σ, and coefficient of variation, COV. These terms are defined in the following paragraphs.

The mean value, $\overline{x}$, of a given set of data, $x = (x_1, x_2, ..., x_N)$, is calculated as:

$$\overline{x} = \frac{\sum x_i}{N} \tag{Eq. 3-7}$$

where

N = Number of data values (dim).

The mean value is also called the expected value or the average of the data set. The standard deviation, σ, is a measure of dispersion of the data in the same units as the data, x_i, and is defined as:

$$\sigma = \sqrt{\sum (x_i - \overline{x})^2 / (N-1)} \tag{Eq. 3-8}$$

The coefficient of variation, COV, is a dimensionless measure of the variability of the data. The COV is defined as the standard deviation (σ) divided by the mean value ($\overline{x}$):

$$COV = \sigma / \overline{x} \tag{Eq. 3-9}$$

The COV expresses the magnitude of the variability as a percentage or fraction of the mean value.

Knowledge of these statistical descriptions is needed when compiling the load and resistance statistics required for calibration by reliability analyses. An example is provided below.

Example Problem 3-2:

This example illustrates the calculation of the mean, $\overline{x}$, standard deviation, σ, and coefficient of variation, COV, using Eqs. 3-7, 3-8 and 3-9, respectively. The bias factor is also defined and calculated. Analysis of the data shows that a larger sample size is needed so that a few outlying points do not distort the distribution of the data.

Several case histories of good quality static pile load tests in sands were compiled, together with the boring information available from these sites. Using the SPT blow counts from the borings, the driven pile axial soil resistance for each case was predicted using Meyerhof's (1976) procedure.

Table 3-2 summarizes the results of the predicted resistances, the measured resistances and the ratio of the measured to the predicted resistances.

Table 3-2
Summary of Measured and Predicted Driven Pile Axial Soil Resistance Using Meyerhof Procedure

Measured Resistance (tons)	Predicted Resistance (tons)	Measured Resistance/ Predicted Resistance
60	89	0.67
67	89	0.75
39	52	0.77
65	72	0.90
34	37	0.90
44	48	0.91
232	240	0.97
197	199	0.99
71	72	0.99
55	55	0.99
145	145	1.00
120	120	1.00
25	25	1.01
49	48	1.03
229	221	1.04
54	48	1.13
272	220	1.23
158	129	1.23
98	73	1.34
14	10	1.46
27	19	1.47
85	56	1.52
21	10	2.13
20	5	3.96

The mean, $\overline{x}$, standard deviation, σ, and coefficient of variation, COV, of the ratio of measured to predicted capacities are 1.22, 0.66 and 0.54, respectively. Because $\overline{x}$, of the ratio of the measured to the predicted resistance is 1.22, this implies for this example that Meyerhof's method tends to underpredict, on average, the pile resistance. Thus, there is a difference between what is predicted and what is measured. This difference is referred to as the "bias." The bias factor, λ, of Meyerhof's SPT method is defined as the ratio of the measured resistance to the predicted resistance as shown in Eq. 3-10:

$$\lambda = R_m / R_n \qquad \text{(Eq. 3-10)}$$

where:

R_m = Measured nominal resistance
R_n = Predicted nominal resistance

For the example given in Table 3-2, the average bias factor, λ, is 1.22 with a COV = 0.54. The COV is relatively high because one standard deviation represents a variation of 54 percent from the mean compared to typical values less than 20 percent. The high variation is caused by the large bias in the last two data entries in Table 3-2. If these two data points are dropped, the values of $\bar{x}$, σ, and COV are changed to 1.06, 0.23 and 0.22, respectively. These values are more reasonable. Because the data set is relatively small, dropping the number of data points from 24 to 22 by removing the two outlying points changes the statistics dramatically. The data base of load tests should be large enough and should contain high quality data, so that the statistics derived from the data base will be representative of the loads and prediction practice.

3.4.2 Probability Density Functions

A histogram showing the frequency of occurrence of the measured to predicted axial driven pile resistances from Table 3-2 is presented in Figure 3-1. The histogram was constructed by counting the number of ratios in each of the equal intervals of 0.25 (e.g., in the interval from 0.51 to 0.75 there are 2 values and in the interval from 0.76 to 1.00 there are 10 values). It is apparent by examining Figure 3-1 that the distribution of measured to predicted resistance is not symmetrical about the mean value of 1.22. Further, the histogram shows how extreme the value near 4.0 is and brings into question its validity.

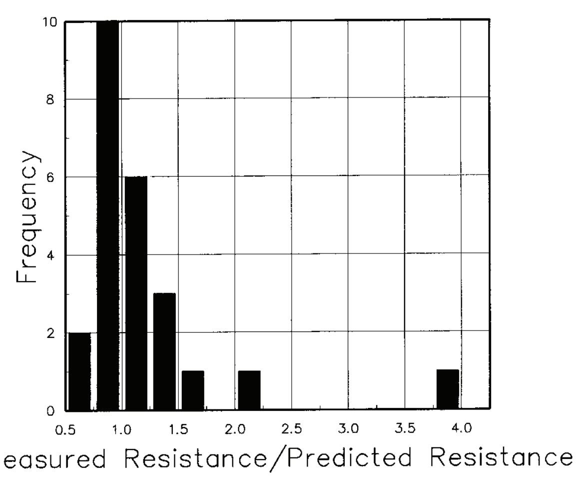


Figure 3-1
Histogram of Measured to Predicted Axial Driven Pile Resistance for Example Problem 3.2

The histogram also provides an approximate estimate of the probability of Meyerhof's method to equal or overpredict the driven pile resistance as 12/24 = 0.50 (the estimate is approximate because the data set includes only 24 values out of possibly thousands of load tests that could be used for the comparison). This ratio is analogous to flipping a coin to obtain either a heads or tails outcome; but that is not the whole story. The histogram shows that most of the data points are clustered around the mean value (1.22), and that is good. In this case, 16 of the 24 values (i.e., 67%) are within the range 0.76 to 1.25, and 22 of 24 (92%) are within the range 0.51 to 1.75. The histogram shows that there is more to interpreting data than knowing the mean value. To have confidence in measured to predicted resistance, the scatter or dispersion of the data must also be known.

To ensure that the predicted resistance is less than or equal to the measured resistance for all cases in Figure 3-1, the predicted resistance can be multiplied by 0.67, where 0.67 represents the minimum ratio of measured to predicted resistance in Table 3-2. The result of this multiplication is to shift the entire histogram to the right so that no value of measured to predicted resistance will be less than 1. The value of 0.67 is similar to a reduction or resistance factor applied to Meyerhof's method for predicting the axial resistance of driven piles.

For years, histograms have been plotted to show distributions of natural phenomena. A recurring pattern was noticed in these distributions that could be described mathematically. The derived mathematical function is referred to as a "probability density function." This function is similar to drawing a smooth curve that approximately passes through the values of the histogram in Figure 3-1. A sketch of a function *f(x)* is shown in Figure 3-2.

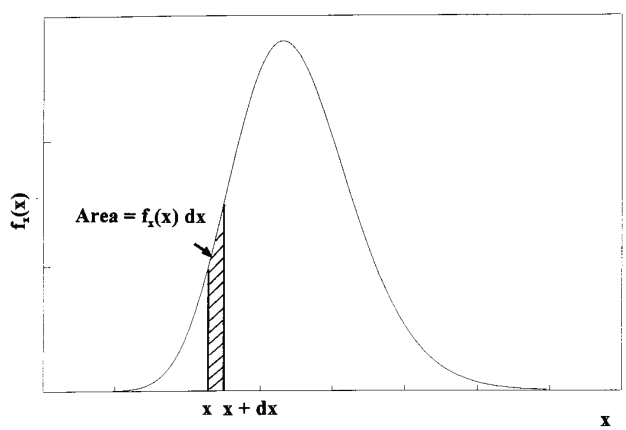


Figure 3-2
Lognormal Probability Density Function

The function f(x) in Figure 3-2 is standardized by dividing its ordinates by the total area under the curve (includes all of the data points). The total area under this normalized curve is unity or a probability of one because it includes all the data points in the theoretical total population of all possible outcomes. Thus, the area under the curve, or probability of occurrence, *P(x)*, over a small interval, *dx*, between *x* and *x* + *dx* is equal to the product of the function at *x* and *dx* (i.e., *P(x)* = *f(x)dx*).

The mathematical expression for the standard or normal form of *f(x)* is derived as an exponential function symmetrical about the mean value. Values of *f(x)* and areas (probabilities) between fixed intervals are available in textbooks. The shape of the normal distribution curve is like a bell and just two parameters can describe the function: the mean, $\overline{x}$ (*m* in Figure 3-3), and the standard deviation, σ. Changes in the shape of *f(x)* with variations in these two parameters are shown in Figure 3-3. If m changes and σ remains constant, the curve shifts to the right, but its shape does not change (i.e., Figure 3-3b). If *m* remains constant and σ changes, the position of the curve does not change, but its shape does. If σ decreases (i.e., less scatter of data), the shape becomes more compact (i.e., Figure 3-3c). If σ increases (i.e., more scatter), the shape spreads out (i.e., Figure 3-3d). In all of the cases shown in Figure 3-3, the area under the curve is unity.

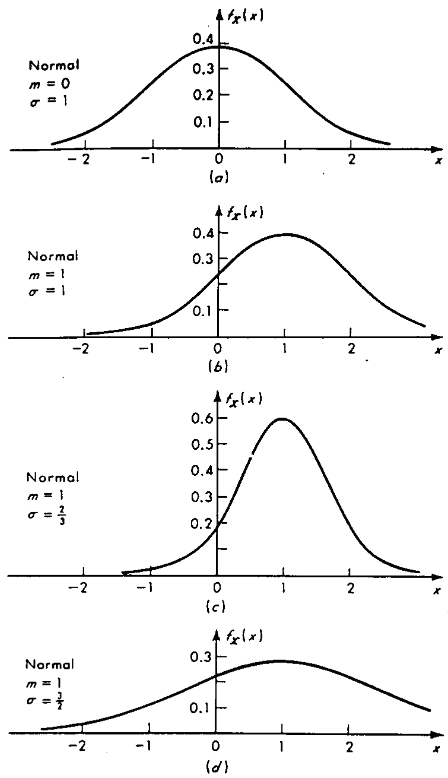


Figure 3-3
Standard Normal Density Function (Benjamin and Cornell, 1970)

When the data distribution is unsymmetric, a logarithmic normal (or simply lognormal) probability density function is often suitable. Stated mathematically, if $y = \ln(x)$ is normally distributed, then x is said to be lognormal. In calibrating the AASHTO LRFD Specification, the lognormal function was used because it appeared to better represent the observed distribution of resistance data for predicting the moment strength of bridge girders. The distribution is probably skewed because the materials supplied usually have strengths greater than the nominal values assumed in the prediction equations. Based on the distribution of data collected from weigh-in-motion studies, a normal probability density function was used to represent the observed distribution of load data. Other design methods and data bases could result in the use of different distribution functions for load and resistance (e.g., both distributions for load and resistance could be lognormal).

Lognormal probability density functions are shown in Figure 3-4 for different values of its standard deviation, ζ. Notice that as ζ increases, the lack of symmetry becomes more pronounced and the smooth curve more closely represents the histogram of Figure 3-1.

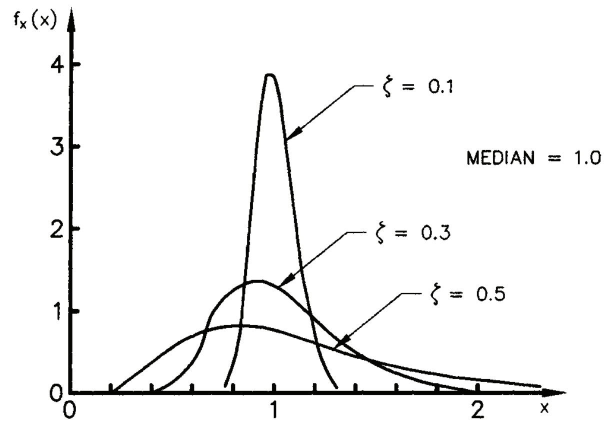


Figure 3-4
Lognormal Density Function

The lognormal mean, ξ_m, and lognormal standard deviation, ζ, can be determined using Eq. 3-11 and 3-12, respectively.

$$\xi = \ln\left[\bar{x} / \sqrt{\left(1 + COV^2\right)}\right] \qquad \text{(Eq. 3-11)}$$

and

$$\zeta = \sqrt{\ln\left(1 + COV^2\right)} \qquad \text{(Eq. 3-12)}$$

where:

$\bar{x}$ = Mean value defined by Eq. 3-7
COV = Coefficient of variation defined by Eq. 3-9
ln ( ) = Natural logarithm of the expression in parentheses

For values of COV less than 0.2, Eqs. 3-11 and 3-12 are approximately equal to the following simplified relationships:

$$\bar{\xi} = \ln \bar{x} \qquad \text{(Eq. 3-13)}$$

and

$$\zeta^2 = \ln\left(\text{COV}^2\right) \qquad \text{(Eq. 3-14)}$$

Thus, the lognormal mean and lognormal standard deviation can be calculated from the statistics obtained from the standard normal function. In Example Problem 3-2, using Eqs. 3-11 and 3-12, the lognormal mean and lognormal standard deviation are 0.071 and 0.50, respectively. The COV is too large to use the approximate equations. Taking the anti-log of 0.071 gives a comparable normal mean of 1.07 which indicates that the skew is to the left of the standard normal mean value of 1.22.

3.4.3 Probability of Failure

In the context of reliability analysis, failure is defined as the conditions where a predefined limit state is reached. Load and resistance factors are selected to insure that each possible limit state has an acceptably small probability of occurrence. The probability of failure can be determined if the mean and standard deviation of the resistance and load are known.

To illustrate the procedure, consider the probability density functions for the load, Q, and resistance, R, shown in Figure 3-5 for a particular limit state. Provided the resistance R is greater than the load Q, there is a margin of safety for the limit state under consideration.

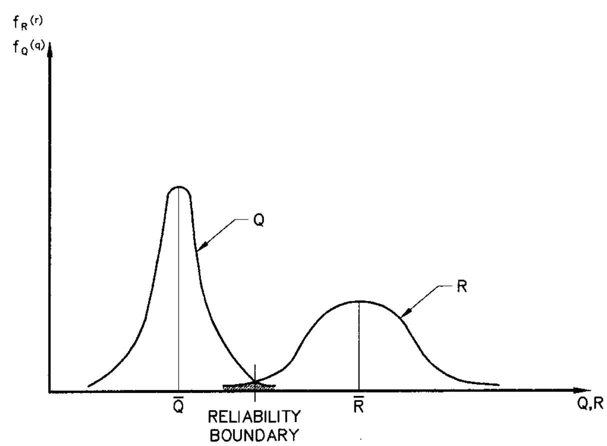


Figure 3-5
Probability Density Functions for Normally Distributed Load and Resistance

A quantitative measure of safety is the probability of survival, p_s, given by:

$$p_s = P(R > Q) \qquad \text{(Eq. 3-15)}$$

where the right side of Eq. 3-15 represents the probability, P, that R is greater than Q. Because the values of both R and Q vary, load and resistance factors are chosen so that there is a small probability that the load, Q, may exceed the resistance, R.

The complement of the probability of survival is the probability of failure, p_f, which can be expressed as:

$$p_f = 1 - p_s = P(R < Q) \qquad \text{(Eq. 3-16)}$$

where the right hand side of Eq. 3-16 represents the probability, P, that R is less than Q.

The probability density functions for R and Q in Figure 3-5 have been drawn to represent different coefficients of variation, COV_R and COV_Q. The areas under the two curves are both equal to unity, but the resistance, R, is shown with greater dispersion than the load, Q. This pattern typically occurs because the design loads are usually known with more certainty than the resistance (e.g., axial pile geotechnical resistance). The shaded area where the curves overlap in Figure 3-5 indicates the region of failure, but the shaded area is not equal to the probability of failure because it is a mixture of areas from two distributions with different ratios of standard deviation to mean value.

To evaluate the probability of failure, p_f, a single combined probability density function, g(R,Q), should be used that represents the margin of safety. This limit state function has its own unique statistics. Use of a combined probability density function, g(R,Q) is examined below to show how the probability of failure can be estimated.

If R and Q are normally distributed, the limit state function g(R,Q) can be expressed as:

$$g(R,Q) = R - Q \qquad \text{(Eq. 3-17)}$$

For lognormally distributed R and Q, the limit state function g(R,Q) shown in Figure 3-6 can be written as:

$$g(R,Q) = \ln(R) - \ln(Q) = \ln(R/Q) \qquad \text{(Eq. 3-18)}$$

In both cases, the limit state is reached when R = Q and failure occurs when g(R,Q) < 0.

To determine the probability of failure, p_f, it is not necessary to construct the function g(R,Q).

All that is required are the mean values, $\overline{R}$ and $\overline{Q}$, and the coefficients of variation, COV_R and COV_Q of the resistance, R, and load, Q, determined separately. This procedure is described in Section 3.4.4.

3.4.4 Reliability Index, β

A simple method of expressing the probability of failure is to use the "reliability index," β. Assuming the function g(R,Q) defined by Eq. 3-18 has a lognormal distribution, the frequency distribution of the function would have the shape similar to the curve shown in Figure 3-6. This curve is a single frequency distribution curve which combines the uncertainties of both R and Q. The probability of attaining a limit state (i.e., R < Q) is equal to the probability that ln(R/Q) < 0, and is represented by the shaded area in Figure 3-6.

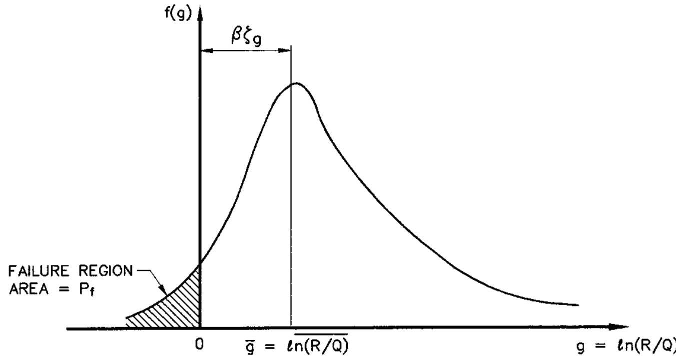


Figure 3-6
Definition of Reliability Index, β for Lognormal Distributions of R and Q

The probability of failure would be smaller, and thus safety increased, if the grouping of data about the mean, $\bar{g}$, was smaller, or if $\bar{g}$ was located further to the right in Figure 3-6. These two possibilities can be combined into one if the position of the mean from the origin is specified in terms of the standard deviation, ζ_g, of g(R,Q). Thus, the distance $\beta\,\zeta_g$ from the origin to the mean in Figure 3-6 provides a measure of safety. The number of standard deviations in this measure is known as the reliability index, β.

> **Reliability index, β,** is defined as the number of standard deviations, ζ_g, between the mean value, $\bar{g}$, and the origin (i.e., $\beta = \bar{g}/\zeta_g$).

If the resistance, R, and load, Q, are both lognormally distributed random variables and are statistically independent, it can be shown that the mean value of g(R,Q) is:

$$\bar{g} = \ln\left[\frac{\bar{R}}{\bar{Q}}\sqrt{\frac{1+COV_Q^2}{1+COV_R^2}}\right] \qquad \text{(Eq. 3-19)}$$

and its standard deviation is:

$$\zeta_g = \sqrt{\ln\left[\left(1+COV_Q^2\right)\left(1+COV_R^2\right)\right]} \qquad \text{(Eq. 3-20)}$$

where:

$\bar{R}$, $\bar{Q}$ =	Mean values
COV_R, COV_Q =	Coefficients of variation of R and Q

Using $\beta\,\zeta_g = \overline{g}$, and substituting for $\overline{g}$ and ζ_g using Eqs. 3-19 and 3-20, respectively, the relationship for the reliability index, β, can be expressed as:

$$\beta = \frac{\ln\left[\left(\overline{R}/\overline{Q}\right)\sqrt{\left(1+COV_Q^2/1+COV_R^2\right)}\right]}{\sqrt{\ln\left[\left(1+COV_Q^2\right)\left(1+COV_R^2\right)\right]}} \qquad \text{(Eq. 3-21)}$$

Eq. 3-21 is convenient because β does not depend on the distribution of the combined function g(R,Q), but only on the statistics of R and Q individually. The reliability index, β, is important because it is the basic parameter for code calibration using reliability theory.

Eq. 3-21 shows the effect of the variables that influence the value of β. While values of β determined from Eq. 3-21 (Level I probability theory) are sufficiently accurate to be useful for most purposes, more accurate procedures are available to estimate values of β (Level II probability theory). These techniques are iterative and are beyond the scope of this chapter. A description of this advanced method is presented in Appendix A of Barker, et al. (1991b). Using Level I theory gives values within about 10 percent of those obtained using Level II theory for lognormal distributions of load and resistance. The same is true for other combinations of load and resistance distributions.

A commonly accepted relationship between the reliability index, β, and the probability of failure, p_f, has been developed by Rosenblueth and Esteva (1972) for lognormally distributed values of R and Q using the relationship:

$$p_f = 460\ \exp(-4.3\beta) \quad 2 < \beta < 6 \qquad \text{(Eq. 3-22)}$$

The inverse function for this relationship is:

$$\beta = \frac{\ln(460/p_f)}{4.3} \quad 10^{-1} < p_f < 10^{-9} \qquad \text{(Eq. 3-23)}$$

Values for both of these relationships are given in Table 3-3. In general, a change of 0.5 in β results in an order of magnitude change in p_f (e.g., β = 2.5 compares to 1 chance in 100, β = 3.0 compares to 1 chance in 1000). As mentioned previously, there is no comparable relationship between the factor of safety used in ASD and probability of failure. Inclusion of the probability of failure is a major advantage of LRFD compared to ASD.

If the resistance, R, and the load, Q, are both normally distributed random variables, equations for the mean value, $\overline{x}$, standard deviation, σ, reliability index, β, and probability of failure, p_f, are presented in Appendix 3A. Equations are also presented in the appendix for a lognormally distributed resistance, R, and a normally distributed load, Q, which is the combination used in calibration of the AASHTO LRFD Specification.

For a given value of β, the probability of failure, p_f, for the lognormal distributions (Table 3-3) is generally less than for the normal distributions (Table 3A-1 in Appendix A). For example, for β =

3.5, p_f from Table 3-3 is 1.34×10^{-4} while from Table 3A-1, $p_f = 2.33 \times 10^{-4}$. If the distributions for R and Q are mixed with one lognormal and the other normal, p_f will be between the values presented in Tables 3-3 and 3A-1.

Table 3-3
Relationship Between Probability of Failure and Reliability Index for Lognormal Distribution

Reliability Index β	Probability of Failure p_f
2.5	0.99×10^{-2}
3.0	1.15×10^{-3}
3.5	1.34×10^{-4}
4.0	1.56×10^{-5}
4.5	1.82×10^{-6}
5.0	2.12×10^{-7}
5.5	2.46×10^{-8}

Probability of Failure p_f	Reliability Index β
1×10^{-1}	1.96
1×10^{-2}	2.50
1×10^{-3}	3.03
1×10^{-4}	3.57
1×10^{-5}	4.10
1×10^{-6}	4.64
1×10^{-7}	5.17

3.4.5 Resistance Statistics

In preparation for working an example illustrating the calibration process (selection of resistance factors) for driven piles, numerical values must be developed for the bias factors and coefficients of variation for resistance and load. The database for calculating these statistics can come from trends provided in professional publications, engineering reports and results from weighing vehicles trafficking across bridges. This collection of data is a significant effort in the calibration process. Numerical estimates for resistance statistics are presented in this section and load statistics are developed in Section 3.4.6.

The calculated nominal resistance, R_n, can differ from the mean value of the measured resistance, R_m. To account for the discrepancy between R_m and R_n, a bias or correction factor is introduced as follows:

$$R_m = \lambda R_n \qquad \text{(Eq. 3-24)}$$

where λ represents the bias factor. The bias factor includes the net effect of various sources of error such as the tendency of a method to underpredict pile resistance, energy losses in the equipment in obtaining SPT blow counts, and soil borings in strata not representative of the site.

The uncertainty associated with λ may be represented by its mean and COV. Assuming that the values of λ are statistically independent, the mean of the overall bias factor may be written as the product of the individual bias factors:

$$\overline{\lambda} = \overline{\lambda}_1 \times \overline{\lambda}_2 \times \ldots\ldots \times \overline{\lambda}_N \qquad \text{(Eq. 3-25)}$$

and the COV of the overall bias factor is the square root of the sum of the squares of the individual coefficients of variation:

$$COV_{\lambda} = \sqrt{COV_{\lambda 1}^2 + COV_{\lambda 2}^2 + \ldots\ldots\ldots + COV_{\lambda N}^2} \qquad \text{(Eq. 3-26)}$$

Sources of error in predicting axial geotechnical pile resistance by the SPT method include:

1. *Model error*: Error due to an overall bias in the prediction method. For example, the tendency of Meyerhof's method to underpredict, on average, the axial geotechnical resistance of driven piles.

2. *Systematic error*: Error in measurement of SPT blow count due to equipment or procedure. For example, use of an older drill rig may provide systematically higher blow counts compared to a new rig using a safety hammer.

3. *Inherent spatial variability*. The uncertainty associated with the value of SPT blow count at a point is larger than the uncertainty associated with an average value measured over a distance or volume, because of the averaging effect. For example, only a limited number of soil borings are advanced at a site to characterize the subsurface conditions for a large volume of soil and rock. If more borings are advanced, the uncertainty of spatial variations across a site will be reduced.

4. *Statistical uncertainty due to an insufficient number of tests*: The uncertainty in the predicted value of SPT blow count is inversely proportional to the square root of the number of tests performed.

5. *Error in measuring actual pile capacities in load tests*: The error due to friction in jacking systems, inaccuracy in calibration or data recording and method used to define the failure load.

The error associated with Item 4 is site specific and the error associated with item 5 is difficult to quantify. In our example, only the first three sources of error are considered. The model error associated with Meyerhof's SPT method for predicting axial pile capacity in sands was discussed in 3.4.1. (The values of bias and COV from Example 3-2 for this method are 1.22 and 0.54, respectively.) Because Items 4 and 5 have not been included in the analysis, as well as other unknown factors, engineering judgment must be used in the final selection of resistance factors applied to resistance prediction methods.

The SPT is the most commonly used method for in situ testing of soils. However, many variations of the test equipment give rise to different energy levels imparted to the soil. Such factors include the type of hammer (which affects the amount of energy delivered to the system), length of drill rods, diameter of borehole, nature of drilling fluid, type of drill bit, type of sampling spoon, rate of blow count and type of drill rods (Seed and DeAlba, 1986). Most of these effects can be minimized by standardizing the test for 60 percent efficiency of the hammer blows. Nevertheless, variability in the equipment and procedure cannot be eliminated. Orchant, et al. (1988) found that systematic error due to equipment, procedure and random effects cause variation in the measured values of N. They proposed the COV values for these effects ranging from 0.15 to 0.45. A bias factor approaching unity may be used due to averaging effects, if statistics are not available.

Vanmarke (1977) showed that the COV of a soil property is reduced when the property is averaged over a length or volume. The amount of reduction depends on the variance function V(dz), which is defined as the ratio of the COV of a soil property averaged over a length dz, $(COV_{SPT})_{dz}$, to the point COV of that soil property measured at a point in the soil, COV_{SPT}:

$$V(dz) = \frac{(COV_{SPT})dz}{COV_{SPT}} \qquad \text{(Eq. 3-27)}$$

Vanmarke (1977) defined the scale of fluctuation, S, as the distance over which the soil property shows strong correlation from point to point. He further showed that the variance function may be related to the scale of fluctuation as follows:

$$V(dz) = \sqrt{\frac{S}{dz}} \qquad \text{(Eq. 3-28)}$$

Equating Eq. 3-27 and 3-28, the COV of the soil property (in this example the SPT blow counts) over a depth dz can be written as:

$$(COV_{SPT})dz = COV_{SPT}\sqrt{\frac{S}{dz}} \qquad \text{(Eq. 3-29)}$$

where:

S = Scale of fluctuation of the soil property
COV_{SPT} = COV of the soil property at a point

Studies by Vanmarke (1977) showed that the scale of fluctuation for SPT is approximately 2.5 m. Briaud and Tucker (1984) showed that the COV_{SPT} is about 0.42. Therefore, with S = 2.5 m and COV_{SPT} = 0.42, over a depth equal to L, the COV due to inherent spatial variability is:

$$(COV_{SPT})_{dz} = COV_{SPT}\sqrt{\frac{S}{dz}} = 0.42\sqrt{\frac{2.5}{L}} = \left(\frac{0.44}{L}\right)^{0.5}$$

A summary of the mean and COV values for this example is presented in Table 3-4.

Table 3-4
Summary of Resistance Statistics

Correction	Statistics for Correction Factors	
	Bias, λ	COV
1. Model Error	1.3	0.5
2. Equipment/Procedure Used in SPT Test	1.0	0.15 - 0.45 (Use 0.3)
3. Inherent Spatial Variability	1.0	$(0.44/L)^{0.5}$

Note: L = Length of pile (m)

Using Eq. 3-25, the overall bias for resistance is:

$$\overline{\lambda} = \overline{\lambda}_1 \times \overline{\lambda}_2 \times \overline{\lambda}_3$$
$$\overline{\lambda}_R = 1.3 \times 1.0 \times 1.0 = 1.3$$

Using Eq. 3-27, the COV of the overall bias on the resistance is:

$$COV_\lambda = \sqrt{COV_{\lambda 1}^2 + COV_{\lambda 2}^2 + COV_{\lambda 3}^2}$$
$$COV_R = (0.5^2 + 0.3^2 + 0.44/L)^{0.5} = (0.34 + 0.44/L)^{0.5}$$

The resistance statistics for driven piles used in the example problem are estimated to be a bias factor, $\lambda_R = 1.3$, and coefficient of variation, $COV_R = (0.34 + 0.44/L)^{0.5}$, where L is the pile length. The length of pile represents the estimated depth of soil over which variations of resistance can be averaged. If the variations in resistance are averaged over a long pile, the COV_R will be smaller than for a short pile.

3.4.6 Load Statistics

The bias factors and coefficients of variation for load components can be developed in a fairly straightforward manner. Physical measurements can be made of various weights of materials and their statistics calculated. Vehicle live loads and their variations can be measured without interference to vehicles using weigh-in-motion instrumentation. From this load data, the load statistics can be compiled and tabulated.

The results of statistical analysis of highway dead and live loads are summarized in Table 3-5 (Nowak, 1993). The largest variation is the weight of the wearing surface placed on bridge decks. Also of interest, as indicated by the bias factor, is that the observed actual loads are greater than the specified nominal values.

Table 3-5
Statistics for Bridge Load Components

Load Component	Bias, λ	COV
Dead Load		
• Factory-made	1.03	0.08
• Cast-in-place (CIP)	1.05	0.10
• Asphaltic wearing surface	1.00	0.25
Live Load (w. dynamic load allowance)	1.10 - 1.20	0.18

In this example, the bridge is assumed to be constructed using a CIP concrete deck ($\lambda = 1.05$, COV = 0.10) on steel girders (i.e., $\lambda = 1.03$, COV = 0.08) without an asphaltic wearing surface, and subject to live load ($\lambda = 1.15$ and COV = 0.18).

Using Eq. 3-25, the overall bias for the dead load components is:

$$\overline{\lambda} = \overline{\lambda}_1 \times \overline{\lambda}_2$$
$$\overline{\lambda}_{QD} = 1.03 \times 1.05 = 1.08$$

Using Eq. 3-26, the COV of the overall bias on the dead load is:

$$COV_\lambda = \sqrt{COV_{\lambda 1}^2 + COV_{\lambda 2}^2}$$
$$COV_{QD} = (0.08^2 + 0.10^2)^{0.5} = 0.128$$

Thus, the statistics for dead load are estimated as: $\lambda_{QD} = 1.08$ and $COV_{QD} = 0.13$. From Table 3-5, the bias factor and COV for live load are taken as: $\lambda_{QL} = 1.15$ and $COV_{QL} = 0.18$.

Example Problem 3-3
The principles of limit state design are illustrated in a comprehensive numerical example that shows how resistance factors are selected (calibration) for driven piles using reliability theory. The mechanics of the calibration process are illustrated by following the four steps outlined in Section 3.3. The example used is based on the method of predicting the geotechnical resistance of an axially-loaded pile using SPT blow counts (Meyerhof, 1976).

Step 1: Estimate the Reliability Index β Using Current Design Methods
For this example, we will only consider combined dead and live load. Using the definition of bias from Eq. 3-10 we have:

$$\overline{Q} = \lambda_Q Q_n \text{ and } \overline{R} = \lambda_R R_n$$

where:

$\overline{Q}$, $\overline{R}$ = Mean values of load and resistance

Q_n, R_n = Nominal (unfactored) load and resistance
λ_Q, λ_R = Bias factors

Thus, Eq. 3-24 can be rewritten as:

$$\beta = \frac{\ln\left[\frac{\lambda_R R_n}{\lambda_Q Q_n}\sqrt{\frac{1+COV_Q^2}{1+COV_R^2}}\right]}{\sqrt{\ln\left[\left(1+COV_R^2\right)\left(1+COV_Q^2\right)\right]}}$$

R_n can now be expressed in terms of Q_n from Eq. 3-2 as $R_n = FS \times Q_n$, where FS is the factor of safety, and Q_n is composed of both dead load, Q_D, and live load, Q_L, each with their own separate bias factors, λ_{QD} and λ_{QL} so that:

$$R_n = FS\left(Q_D + Q_L\right)$$

$$\lambda_n Q_n = \lambda_{QD} Q_D + \lambda_{QL} Q_L$$

Further, it is assumed that the square of the COV of a function of multiple variables is equal to the sum of the squares of the individual COV's (Eq. 3-26), so that:

$$\beta = \frac{\ln\left[\frac{\lambda_R FS\left(Q_D + Q_L\right)}{\lambda_{QD} Q_D + \lambda_{QL} Q_L}\sqrt{\frac{1+COV_{QD}^2+COV_{QL}^2}{1+COV_R^2}}\right]}{\sqrt{\ln\left[\left(1+COV_R^2\right)\left(1+COV_{QD}^2+COV_{QL}^2\right)\right]}}$$

Finally, dividing the numerator and denominator of the ln [ ] term in the numerator of the relationship above by Q_L, we obtain:

$$\beta = \frac{\ln\left[\frac{\lambda_R FS\left(Q_D/Q_L + 1\right)}{\lambda_{QD}\, Q_D/Q_L + \lambda_{QL}}\sqrt{\frac{1+COV_{QD}^2+COV_{QL}^2}{1+COV_R^2}}\right]}{\sqrt{\ln\left[\left(1+COV_R^2\right)\left(1+COV_{QD}^2+COV_{QL}^2\right)\right]}} \qquad \text{(Eq. 3-30)}$$

where:

Q_D, Q_L =	Nominal values of the dead and live load (kN)
FS =	Factor of safety factor used in existing designs (dim)
λ_R, λ_{QD}, λ_{QL} =	Bias factors (dim)
COV_R, COV_{QD}, COV_{QL} =	Coefficients of variation of the resistance, dead load and live load, respectively (dim).

From inspection of Eq. 3-30, the reliability index, β, is a function of the:

- FS
- Q_D/Q_L
- λ_R, λ_{QD}, λ_{QL}
- COV_R, COV_{QD}, COV_{QL}

The safety factor used with Meyerhof's SPT method is usually about 3.5.

We can study the variation of the reliability index with the dead to live load ratio. The ratio of dead to live load is related to the span length of a bridge. The dead to live load ratio is smaller for bridges with shorter spans, and is approximately equal to the span in meters divided by 20.

Step 2: Observe the Variation of the Reliability Indices
A plot of reliability index, β, versus pile length can now be generated by substituting bias factors: $\lambda_R = 1.3$, $\lambda_{QD} = 1.08$, $\lambda_{QL} = 1.15$; and coefficients of variation: $COV_R = (0.34 + 0.44/L)^{0.5}$, $COV_{QD} = 0.13$, $COV_{QL} = 0.18$ from Sections 3.4.5 and 3.4.6 into Eq. 3-30 for several values of dead to live load ratio and pile length. The result of such a substitution is shown in Figure 3-7. Note that the reliability index varies between 2.4 and 2.6 and is not very sensitive to pile length (a measure of spatial variability) or dead to live load ratio (a measure of bridge span). For an abutment supported on piles, this lack of sensitivity means that the reliability appears to be independent of the size of the abutment and the span of the bridge.

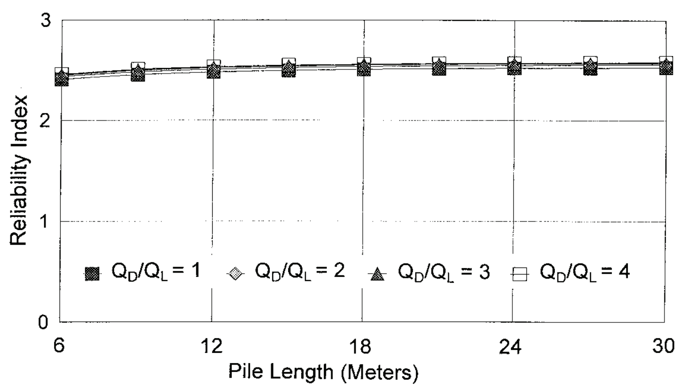


Figure 3-7
Reliability Index for Meyerhof's SPT Method

Step 3. Select a Target Reliability Index β_T
Based on the computed values of reliability index, desired or target reliability indices can now be selected. Values of β reported by others can provide guidance in this selection. Meyerhof (1970) suggested that p_f of foundations should be between 1×10^{-3} and 1×10^{-4}, corresponding to β between 3 and 3.6 (Table 3-3). Wu, et al. (1989) calculated that β for pile systems is approximately 4, corresponding to $p_f = 5 \times 10^{-5}$. Tang, et al. (1990) reported that β for offshore piles ranges from 1.4 to 3.0, corresponding to p_f between 1×10^{-1} and 1×10^{-3}. The value calculated for Meyerhof's SPT method of $\beta \approx 2.5$ is well within the range determined by others.

In addition to the SPT method, values of reliability indices for other commonly used methods (e.g., α, β, λ and CPT) of predicting the axial resistance of driven piles were calculated and were found to be between about 1.5 and 3.0 (Barker, et al., 1991a). Thus, a target reliability index between 2.5 and 3.0, corresponding to a p_f of about 1×10^{-3}, appears to be about the average value determined for single piles based all of the previous studies. However, because piles are often used in groups, failure of one pile does not necessarily imply that the pile group will fail. Due to redundancy in pile groups, it is reasonable to use a lower value of β for a single pile. If a single pile in a group has the smallest resistance and begins to fail, the load is transferred to other piles in the group with greater resistance and the foundation does not fail. Thus, a reasonable value of target reliability index, β_T, for single driven piles appears to be in the range of 2.0 to 2.5, corresponding to p_f between 1×10^{-1} and 1×10^{-2}.

Step 4: Calculate the Resistance Factor ϕ

The selected values of reliability index, β_T, provide the basis for determining values of resistance factors. An expression for the resistance factor can be derived from the LRFD safety criterion in Eq. 3-3.

$$\phi R_n \geq \sum \gamma_i Q_i \qquad \text{(Eq. 3-31)}$$

from which:

$$\phi \geq \sum \gamma_i Q_i / R_n \qquad \text{(Eq. 3-32)}$$

Replacing the nominal resistance, R_n, by the mean value divided by the bias factor, $\overline{R}/\lambda_R$, gives:

$$\phi \geq \lambda_R \sum \gamma_i Q_i / \overline{R} \qquad \text{(Eq. 3-33)}$$

For a lognormal resistance distribution, the mean value of resistance, $\overline{R}$, can be solved from Eq. 3-21:

$$\overline{R} = \frac{\overline{Q} \exp\left\{\beta\sqrt{\ln\left[\left(1+COV_R^2\right)\left(1+COV_Q^2\right)\right]}\right\}}{\sqrt{\left(1+COV_Q^2\right)/\left(1+COV_R^2\right)}} \qquad \text{(Eq. 3-34)}$$

Substituting $\overline{R}$ from Eq. 3-34 into Eq. 3-33, and replacing β with the target (desired) reliability index β_T, gives the following expression for the resistance factor:

$$\phi = \frac{\lambda_R (\Sigma\, \gamma_i\, Q_i)\sqrt{(1 + COV_Q^2)/(1 + COV_R^2)}}{\overline{Q} \exp\{\beta_T \sqrt{\ln[(1 + COV_R^2)(1 + COV_Q^2)]}\}} \qquad \text{(Eq. 3-35)}$$

When dead and live loads are considered, Eq. 3-35 can be written as:

$$\phi = \frac{\lambda_R \left(\gamma_D \frac{Q_D}{Q_L} + \gamma_L\right)\sqrt{\frac{1 + COV_{QD}^2 + COV_{QL}^2}{1 + COV_R^2}}}{\left(\lambda_{QD}\frac{Q_D}{Q_L} + \lambda_{QL}\right)\exp\left[\beta_T\sqrt{\ln(1 + COV_R^2)(1 + COV_{QD}^2 + COV_{QL}^2)}\right]} \qquad \text{(Eq. 3-36)}$$

where γ_D and γ_L are the specified load factors for dead and live load. Even though the asphaltic wearing surface of the bridge has a different load factor than the dead load, it constitutes only a small fraction of the bridge load compared to the dead load of the bridge. In our example, the load factor for the asphaltic wearing surface can be taken equal to the load factor for the dead load without significant error.

It can be seen from Eq. 3-36 that the resistance factor, ϕ, depends on the dead to live load ratio, Q_D/Q_L, load factors for dead and live loads: $\gamma_D = 1.25$ and $\gamma_L = 1.75$; bias factors: $\lambda_R = 1.3$, $\lambda_{QD} = 1.08$, $\lambda_{QL} = 1.15$; and coefficients of variation: $COV_R = (0.34 + 0.44/L)^{0.5}$, $COV_{QD} = 0.13$, $COV_{QL} = 0.18$ from Sections 3.4.5 and 3.4.6; and the target reliability index, β_T. Values of resistance factors for Meyerhof's SPT method corresponding to β_T of 2.0 and 2.5 are shown in Figure 3-8, for different dead to live load ratios (measure of bridge span) and pile lengths (measure of spatial variability).

The values of resistance factors vary from 0.39 to 0.46 for $\beta_T = 2$. For $\beta_T = 2.5$, the resistance factor varies between 0.28 and 0.33. *The resistance factors, similar to the reliability index, are relatively insensitive to pile length and dead to live load ratios. These results, tempered by judgment and fitting with ASD experience, can be used to select an appropriate value of φ for use with Meyerhof's SPT method of estimating pile resistance. A discussion of the issues involved in selecting φ for use in a design specification is provided in Section 3.4.7.*

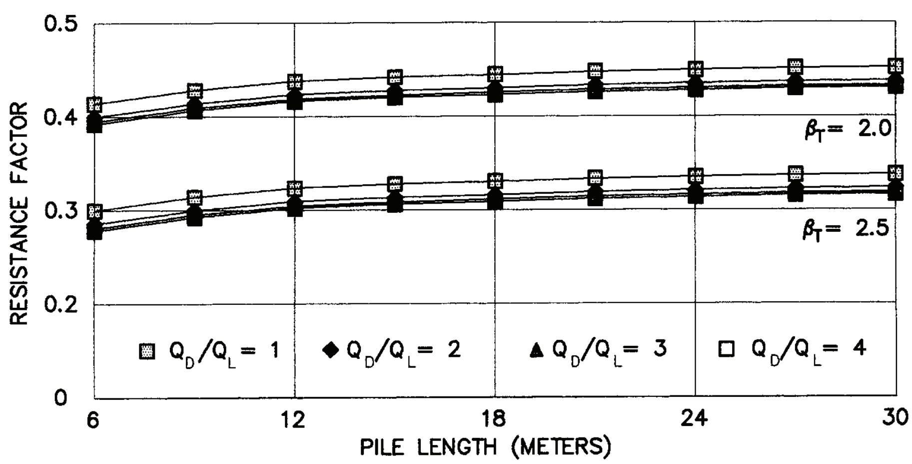


Figure 3-8
Resistance Factors for Meyerhof's SPT Method for β_T = 2.0 and 2.5

3.4.7 Calibration by a Combined Approach

The values of the resistance factors, φ, given in the AASHTO LRFD Specifications for foundations were chosen by a combination of reliability theory, fitting with ASD, and judgment. Table 3-6 from Barker, et al. (1991a) illustrates the approach that was used for driven piles. ***At the time when these studies were performed, $\gamma_D = 1.3$ and $\gamma_L = 2.17$. If the studies were repeated using the present load factors in the AASHTO specifications ($\gamma_D = 1.25$, $\gamma_L = 1.75$), the values of φ determined by the Level II reliability analysis would be approximately 5 percent to 15 percent lower, depending on the bridge span (ratio of Q_D/Q_L).*** For example, consider Eq. 3-35 applied to a relatively short span bridge of 18 m ($Q_D/Q_L \approx 1$) with bias factors, $\lambda_{QD} = \lambda_{QL} = 1$, while the other variables remain constant, then φ is proportional to the weighted average load factor, γ_{avg}:

$$\gamma_{avg} = \frac{\gamma_D (Q_D / Q_L) + \gamma_L}{\lambda_{QD} (Q_D / Q_L) + \lambda_{QL}}$$

Therefore, if the weighted average load factor reduces from:

$$\gamma_{avg} = \frac{1.3(1.0) + 2.17}{1.0(1.0) + 1.0} = 1.74$$

to:

$$\gamma_{avg} = \frac{1.25(1.0) + 1.75}{1.0(1.0) + 1.0} = 1.50$$

the resistance factor will be lower by the ratio of 1.50/1.74 = 0.86 or a reduction of 14 percent. For a longer span of 74 m ($Q_D/Q_L \approx 3.7$), γ_{avg}, drops from 1.49 to 1.36 representing a reduction of 9 percent. Applying a ratio of 0.89, which is between 0.86 and 0.91, to the reliability determined resistance factors in Table 3-6, the SPT values become 0.43 and 0.45 while the CPT values become 0.53 and 0.55. ***This relatively small drop in the calculated resistance factor does not suggest a need to revise the corresponding values prescribed in the AASHTO LRFD Specifications (1997a) of φ = 0.45 for SPT and φ = 0.55 for CPT.***

Table 3-6
Resistance Factors for Axially Loaded Driven Piles in Sand, In-Situ Methods

Soil Test	Pile Length (m)	Factor of Safety	Target Reliability Index β_T (1)	Resistance Factor, φ		
				Reliability Analysis(2)	Fitting with ASD(3)	Selected
SPT data	10	3.5	2.0	0.48	0.39	0.45
SPT data	30	3.5	2.0	0.51	0.39	0.45
CPT data	10	2.5	2.0	0.59	0.54	0.55
CPT data	30	2.5	2.0	0.62	0.54	0.55

(1) β_T for a single pile

[2] Level II reliability analysis with $Q_D/Q_L = 3.7$ for $\gamma_D = 1.3$ and $\gamma_L = 2.17$
[3] Eq. 3-6 with $Q_D/Q_L = 3.7$ for $\gamma_D = 1.25$ and $\gamma_L = 1.75$

When the resistance factors determined by reliability theory and fitting with ASD are close to one another, such as with the CPT data in Table 3-6, it is not difficult to select the final value of $\phi = 0.55$. However, when they are quite different, as with the SPT data, engineering judgment is required to select an appropriate value. In this case, the reliability based value was given more weight because the ASD safety factor for SPT data was judged to be larger than necessary, and the selected $\phi = 0.45$.

In a similar manner, values of resistance factors were selected by Barker, et al. (1991a) for bearing resistance of footings and sliding of footings on sand and clay, bearing resistance of axially loaded single piles and pile groups, and bearing resistance of axially loaded single drilled shafts and groups of drilled shafts. Recognizing the uncertainties in the selection of the φ factors, it was decided to select values that varied by increments of 0.05, because it was felt that use of additional significant figures was not justified.

Application of these resistance factors to the design of bridge foundations are illustrated in the design manuals prepared by Barker, et al. (1991b). Tables summarizing the resistance factors from the AASHTO LRFD Specifications are presented in the design chapters.

3.5 Student Exercise

1. Define the term “limit state”. (Refer to Section 3.1)

2. What is the fundamental equation governing ASD? (Refer to Section 3.2.1)

3. What is the fundamental equation governing LRFD? (Refer to Section 3.2.3)

4. What are the advantages and disadvantages of both LRFD and ASD? (Refer to Sections 3.2.4 and 3.2.2)

5. What are the methods by which resistance factors can be calibrated? (Refer to Section 3.3)

6. Using Eq. 3-6 in Section 3.3, determine the appropriate resistance factor through calibration with ASD for the following parameters:

FS	= 3.0
γ_D	= 1.25
γ_L	= 1.75
Q_D/Q_L	= 2.0

7. Define the mean, standard deviation and coefficient of variation of a set of normally distributed data. (Refer to Section 3.4.1)

8. Define the bias, λ, of a set of data. (Refer to Section 3.4.1)

9. Define the lognormal mean and lognormal standard deviation. (Refer to Section 3.4.2)

10. Define the reliability index, β. (Refer to Section 3.4.4)

11. For lognormal distributions of load and resistance, recall from Section 3.4.4 that the reliability index, β, can be expressed as follows:

$$\beta = \frac{\ln\left[(\bar{R}/\bar{Q})\sqrt{(1+COV_Q^2)/(1+COV_R^2)}\right]}{\sqrt{\ln[(1+COV_R^2)(1+COV_Q^2)]}} \qquad \text{(Eq. 3-21)}$$

and that the probability of failure can be estimated as:

$$p_f = 460 \exp(-4.3\beta) \quad 2 < \beta < 6 \qquad \text{(Eq. 3-22)}$$

A subsurface exploration and laboratory testing program including field vane shear tests (VST) and unconfined compression (UC) tests results in a mean ultimate pile capacity prediction, $\bar{R}$ = 1000 kN as shown below. The mean unfactored load on this pile, $\bar{Q}$ = 333 kN. The reliability of each test method is somewhat different, with COV_R = 0.25 for the VST and a COV_R = 0.35 for the UC tests.

For these load and resistance characteristics, determine the ASD factor of safety and the LRFD-based probability of failure based on VST and UC testing.

$\bar{R}$ = 1000 kN
$\bar{Q}$ = 333 kN
COV_Q = 0.15
COV_R (VST) = 0.25
COV_R (UC) = 0.35

3.6 Appendix 3A

Both R and Q Normally Distributed

Mean Value, $\bar{g}$:

$$\bar{g} = \bar{R} - \bar{Q} \qquad \text{(Eq. 3A-1)}$$

Standard Distribution, σ_g:

$$\sigma_g = \sqrt{\sigma_R^2 + \sigma_Q^2} \qquad \text{(Eq. 3A-2)}$$

where $\bar{R}$ and $\bar{Q}$ are mean values, σ_g and σ_g are standard deviations of the resistance, R, and the load, Q.

Reliability Index, β:

$$\beta = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_Q^2 + \sigma_R^2}} \qquad \text{(Eq. 3A-3)}$$

Probability of Failure, p_f:

$$p_f = 1 - F_u(\beta) \qquad \text{(Eq. 3A-4)}$$

where $F_u(\ )$ is the standard normal cumulative distribution function. There is no simple expression for F_u, but it has been evaluated numerically and tabulated. Relating the reliability index, β to the probability of failure, p_f:

$$\beta = F_u^{-1}(1 - p_f) \qquad \text{(Eq. 3A-5)}$$

where $F_u^{-1}(\ )$ is the inverse standard cumulative distribution function. Values for the relationships in Eq. 3A-4 and Eq. 3A-5 are presented in Table 3A-1.

Table 3A-1
Relationship Between Probability of Failure
and Reliability Index for Normal Distribution

Reliability Index β	Probability of Failure p_f
2.5	0.62×10^{-2}
3.0	1.35×10^{-3}
3.5	2.33×10^{-4}
4.0	3.17×10^{-5}
4.5	3.40×10^{-6}
5.0	2.90×10^{-7}
5.5	1.90×10^{-8}

Probability of Failure p_f	Reliability Index β
1×10^{-2}	2.32
1×10^{-3}	3.09
1×10^{-4}	3.72
1×10^{-5}	4.27
1×10^{-6}	4.75
1×10^{-7}	5.20
1×10^{-8}	5.61

Resistance Factor, ϕ

$$\phi = \frac{\lambda_R\,(\Sigma\,\gamma_i\,Q_i)}{\overline{Q} + \beta_T\sqrt{\sigma_R^2 + \sigma_Q^2}} \qquad \text{(Eq. 3A-6)}$$

$$\phi = \frac{\lambda_R\,(\gamma_D\,Q_D + \gamma_L\,Q_L)}{(\lambda_{QD}\,Q_D + \lambda_{QL}\,Q_L) + \beta_{Tsqrt}\,\sigma_R^2 + \sigma_Q^2} \qquad \text{(Eq. 3A-7)}$$

Lognormally Distributed R and Normally Distributed Q

Mean Value $\overline{g}$

$$\overline{g} \approx \overline{R}\,[1 - k(COV_R)]\ \{1 - 1n\,[1 - k(COV_R)]\} - \overline{Q} \qquad \text{(Eq. 3A-8)}$$

Standard Deviation, σ

$$\sigma \approx \sqrt{\{\sigma_R\,[1 - k(COV_R)]\}^2 + \sigma_Q^2} \qquad \text{(Eq. 3A-9)}$$

where $\overline{R}$ and $\overline{Q}$ are mean values, σ_R and σ_Q are standard deviations, COV_R and COV_Q are coefficients of variation of the resistance R and the load Q. The parameter k is comparable to the number of standard deviations from the mean value. As an initial guess, k is often taken as 2.

Reliability Index, β

$$\beta \approx \frac{\overline{R}\,[1 - k(COV_R)]\ \{1 - 1n\,[1 - k(COV_R)]\} - \overline{Q}}{\sqrt{\{\sigma_R\,[1 - k(COV_R)]\}^2 + \sigma_Q^2}} \qquad \text{(Eq. 3A-10)}$$

Probability of Failure

For this mixed case, p_f cannot be explicitly defined. However, for a given value of β, it is reasonable to assume that p_f is between the values given in Table 3-3 and those in Table A-1.

Resistance Factor, ϕ

$$\phi \approx \frac{\gamma_R\,(\sum \gamma_1 Q_1)[1 - k(COV_R)]\,\{1 - 1n\,[1 - k(COV_R)]\}}{\overline{Q} + \beta_T \sqrt{\{\sigma_R\,[1 - k(COV_R)]\}^2 + \sigma_Q^2}} \qquad \text{(Eq. 3A-11)}$$

$$\phi \approx \frac{\gamma_R\,(\gamma_D Q_D + \gamma_L Q_L)[1 - k(COV_R)]\,\{1 - 1n\,[1 - k(COV_R)]\}}{(\gamma_{QDQ_D} + \gamma_{QL} Q_L) + \beta_T \sqrt{\{\sigma_R\,[1 - k(COV_R)]\}^2 + \sigma_Q^2}} \qquad \text{(Eq. 3A-12)}$$

CHAPTER 4
LOADS

4.1 Introduction

The fundamental principals of LRFD as embodied in the AASHTO LRFD Specification (AASHTO, 1997a), and comparisons between working stress and limit state design methods were presented in Chapter 3. Recall the LRFD equation:

$$\Sigma \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 4-1) } (\textit{A1.3.2.1-1})$$

where:

η_i = Factors to account for ductility, redundancy and operational importance (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)

In using this approach, the designer must (1) estimate the loads and load combinations which could be imposed on the structure, and (2) estimate the nominal (ultimate) resistance of the structure component or ground. This chapter describes general methods which can be used to estimate loads, Q_i, and load factors, γ_i, required for the design of foundations, walls and culverts using LRFD. Chapters 8 through 16 will present general methods which can be used to estimate nominal resistance, R_n, and resistance factors, ϕ, needed for substructure design.

LRFD requires that loads on and from structures be estimated for the geotechnical design of substructures. This process includes estimating permanent loads resulting from the dead weight of structure components and burial below grade, and transient loads such as those due to traffic, water, wind, ice, and vessel impact. With the exception of earth loads, the loads required for substructure design are usually provided by the structural engineer. In this regard, there is no difference between ASD and LRFD. The development of loads and factored loads is covered in Section 3 of the AASHTO LRFD Specification.

This chapter:

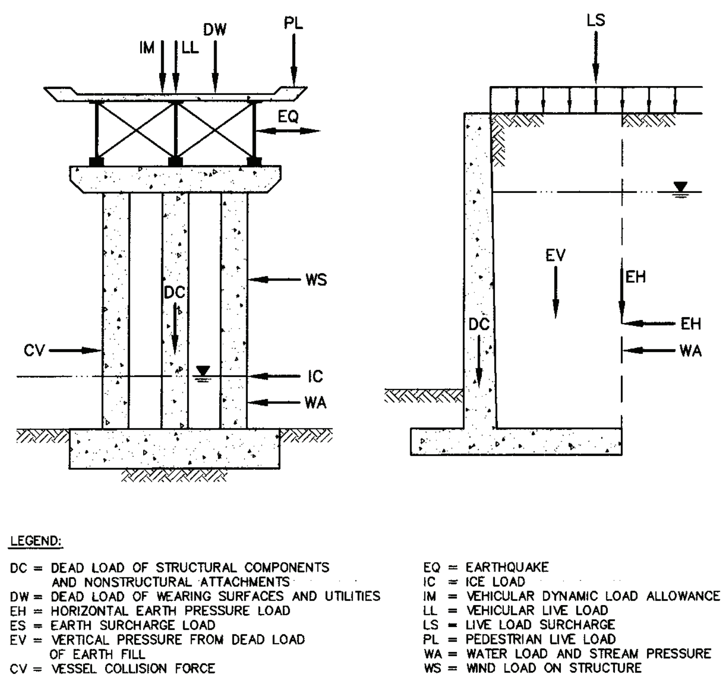
- Identifies the primary loads needed for substructure design
- Illustrates methods used by structural engineers to estimate certain loads on superstructures

- Presents examples of load calculations used for the design of foundations, walls and culverts

The chapter does not describe in detail methods prescribed in the AASHTO LRFD Specifications to estimate loads familiar to geotechnical engineers (e.g., earth loads).

4.2 Load Considerations for Geotechnical Design of Substructures

Substructures must be designed to support load combinations which include permanent and transient loads from a bridge superstructure and from externally applied forces (e.g., impact loading from a vessel, earth loading by structure backfills and downdrag loading on deep foundations extending through a compressible fill). *Figure 4-1 illustrates the types of loads which need to be considered for the design of a bridge pier and a cantilevered retaining wall.*



(a) Bridge Pier (b) Cantilever Retaining Wall

Figure 4-1
Typical Loading for Substructure Design

With the possible exception of earth, surcharge and downdrag loads which are often developed by the foundation engineer, the selection of applicable loads and load combinations for structure design are made by the structural engineer for superstructure design. Although the foundation engineer may never have to develop the loads needed for substructure design, some general familiarity with the types and methods used to develop design loads is useful in understanding the selection of limit states and load combinations used in LRFD. The following sections describe the permanent and transient loads which may be required for substructure design.

4.2.1 Permanent Loads

Permanent loads on substructures include dead loads from structure components, earth loads from embedment of walls and burial of culverts, and downdrag loads on piles or drilled shafts.

4.2.1.1 Dead Loads - DC, DW and EV (*A3.5.1*)

Dead loads include the weight of factory-made (e.g., structural steel and prestressed concrete) and cast-in-place (e.g., deck slabs, abutments and footings) structure components, asphalt wearing surfaces, future overlays and planned widenings, miscellaneous items (e.g., scuppers, railings and supported utility services), and the weight of earth cover.

4.2.1.2 Downdrag Load - DD (*A3.11.8*)

When a driven pile or drilled shaft penetrates a soft layer subject to settlement (e.g., where an overlying embankment may cause settlement of the layer), the force effects of downdrag or negative loading on the foundations must be evaluated. These force effects are illustrated in Figure 4-2, and are fully mobilized at relative movements of about 3 to 12 mm (Reese and O'Neill, 1988). Downdrag acts as a permanent additional axial load on the pile or shaft. If the force is of sufficient magnitude, structural failure of the foundation element or a bearing capacity failure at the tip is possible. At smaller magnitudes of downdrag, the foundation may experience additional settlement.

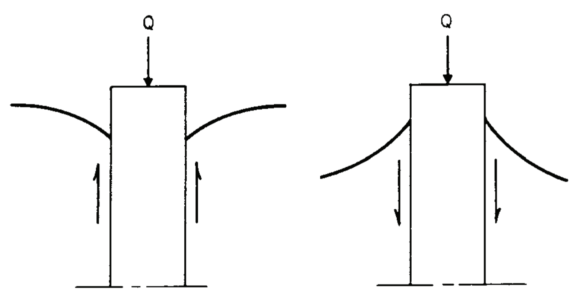


Figure 4-2
Downdrag Loading
(Hunt, 1986)

For driven piles or drilled shafts that derive their resistance mostly from end bearing, the structural resistance of the foundation section must be adequate to support the factored structure and downdrag loads. Battered piles should be avoided at sites where downdrag loading is possible due to the potential for bending of the pile section. Details and example problems showing the analysis of deep foundations subjected to downdrag loading are presented in GRL (1996), Reese and O'Neill (1988) and Briaud and Tucker (1994).

4.2.1.3 Earth Pressure Load- EH (*A3.11.5*)

The force effects of lateral earth pressures due to partial or full embedment into soil must be considered during substructure design. The lateral earth pressures described in this chapter and course are principally those resulting from static load effects.

The magnitude of lateral earth pressure loads on a substructure are a function of:

- Structure type (i.e., gravity, cantilever, anchored or mechanically-stabilized earth wall, or flexible or rigid buried structure)
- Type, unit weight and shear strength of the retained earth
- Anticipated or permissible magnitude and direction of lateral substructure movement
- Compaction effort used during placement of soil backfill
- Location of the ground water table within the retained soil
- Location, magnitude and distribution of surcharge loads on the retained earth mass

The stiffness of the structure and the characteristics of the retained earth are the most significant factors in the development of lateral earth pressure distributions. Structures which can tilt, move laterally or deflect structurally away from the retained soil (i.e., most retaining walls and abutments) can mobilize an active state of stress in the retained soil mass. These structures should be designed using an active (i.e., minimum) earth pressure distribution. Structures which are restrained against movement (e.g., integral abutments or walls for which lateral ground movement of the backfill could adversely affect nearby facilities within a distance of less than about one-half the wall height behind the wall) should be designed to resist an at-rest earth pressure distribution. Walls which are forced to deflect laterally toward the retained soil should be designed to resist the passive earth pressure.

For practical purposes, the passive state of stress occurs most commonly as a result of lateral deflection of the embedded portions of retaining walls into the supporting soil. In the AASHTO LRFD Specification, passive earth pressure is treated as a resistance rather than a load.

The basic earth pressure (p) is defined by:

$$p = k_h \gamma_s z \qquad \text{(Eq. 4-2) } (\mathit{A3.11.5\text{-}1})$$

where:

p = Basic earth pressure (kPa)

k_h = Lateral earth pressure coefficient (dim)
γ_s = Unit weight of soil (kN/m^3)
z = Depth from ground surface (m)

The value of k depends on the stress history of the soil (i.e., whether the soil is normally consolidated [NC] or over consolidated [OC]) and the displacement of the structure (i.e., whether the structure is flexible or stiff and whether soil loading is active or passive). The initial or at-rest value of k_h (i.e., k_o) ranges between about 0.4 and 0.6 for NC soils, and can exceed 1.0 for heavily OC soils. Structure movement will increase or decrease the value of k_h from k_o such that movement away from the soil will cause the value of k to decrease below k_o and movement toward the soil will cause the value of k to increase above k_o. Minimum and maximum values of k_h (i.e., k_a, and k_p) are achieved when the shear strength of the soil is completely mobilized. For conventional walls (i.e., gravity, semi-gravity and cantilever), the lateral movement required to develop the minimum active earth pressure or maximum passive earth pressure is a function of the type of soil retained as shown in Table 4-1.

Table 4-1
Relative Movements Needed to Achieve Active or Passive Earth Pressure Conditions
Clough and O'Rourke (1991)

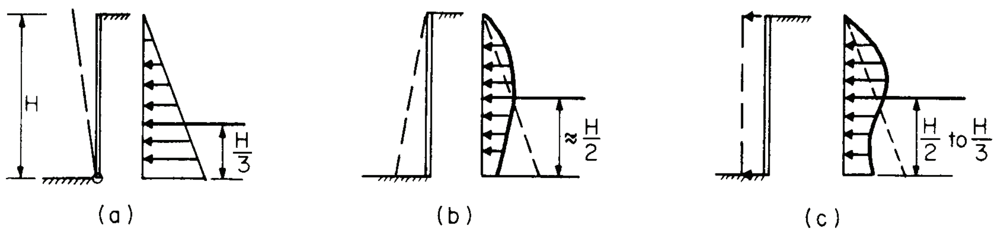
Backfill Type	Values of Δ/H	
	Active	Passive
Dense Sand	0.001	0.01
Medium Dense Sand	0.002	0.02
Loose Sand	0.004	0.04

where:

Δ = Lateral movement (i.e., rotation and/or translation) required for development of active or passive earth pressure (m)
H = Wall height (m)

Nearly all cantilever retaining walls of typical proportions deflect sufficiently to permit mobilization of active earth pressures. Gravity and semi-gravity walls designed with a sufficient mass to support only active earth pressures will tilt and/or translate in response to more severe loading conditions (e.g., at-rest earth pressures) until stresses in the retained soil are relieved sufficiently to permit development of an active stress state in the retained soil. However, at-rest earth pressures could develop on the stem of cantilevered retaining walls where a rigid stem-to-base connection may prevent lateral deflection of the stem with respect to the base. For such a condition, excessive lateral earth pressures on the stem could conceivably cause structural failure of the stem or stem-to-base connection.

As shown in Figure 4-3a, the resultant force from a linearly increasing pressure distribution is located at the centroid of the pressure diagram at H/3 from the base of a wall which tilts about its base. However, if the wall tilts about its top or translates laterally as shown in Figures 4-3b and 4-3c, the location of the resultant force is higher than traditionally assumed for design. Location of the resultant force above the centroid of the pressure diagram occurs because as a wall deflects in response to lateral earth loading, the backfill must slide down along the back of the wall for the retained soil mass to achieve an active state of stress. This movement causes arching of the backfill against the upper portion of the wall which causes an upward shift in the location at which the resultant of the lateral earth load is transferred to the wall. In the AASHTO LRFD Specification (*A3.11.5.1*), it is assumed that the resultant earth pressure for active loading is located at 0.4H from the base of the unsupported wall section for conventional concrete gravity and reinforced concrete cantilever walls.



Tilting About Base Tilting About Top Translation

Figure 4-3
Location of Resultant Lateral Earth Pressure
(Hunt, 1986)

For other wall types such as nongravity cantilever retaining walls or other flexible walls which tilt or deform laterally in response to lateral loading, significant arching of the backfill against the wall does not occur, and the resultant lateral load due to active and other pressure distributions can be assumed to act at H/3 above the base of the wall.

Development of a horizontal earth pressure for anchored wall design must consider the method and sequence of wall construction and anchor installation, the characteristics and overall stability of the ground mass to be supported, the acceptability of lateral wall deflections and settlements behind the wall, the anchor locations and spacing, and the level of anchor prestressing. A number of earth pressure distribution diagrams are in common use for the design of anchored retaining walls (e.g., Cheney, 1988; NAVFAC, 1986b). Some of the distributions are based on deflection and load measurements in braced excavations (Terzaghi and Peck, 1967) whereas others are based on the results of analytical and scale model studies. While the various studies result in a variety of different design earth pressure distributions, all suggest a relatively uniform pressure distribution with depth.

For anchored walls constructed from the top down, the design lateral earth pressure can be assumed to be constant with depth as shown in Figure 4-4, so that the earth pressure, p_a is:

$$p_a = 0.65\, k_a \gamma'_s\, H \qquad \text{(Eq. 4-3) } (\textit{A3.11.5.6-1})$$

where:

p_a = Earth pressure (kPa)
k_a = Active earth pressure coefficient (dim) = $\tan^2 (45° - \phi_f/2)$
γ'_s = Effective unit weight of soil backfill (kN/m^3)
H = Total height of retained earth (m)

The lateral earth pressure on a mechanically-stabilized earth (MSE) retaining wall is assumed to increase linearly with depth, as shown in Figure 4-5 and 4-6, for level and sloping backfill surfaces, respectively. The earth pressure resultant is assumed to act at a height of h/3 above the base of the wall for evaluation of external wall stability. The magnitude of the lateral earth pressure resultant is taken as:

$$P_a = 0.5\, k_a\, \gamma'_s\, h^2 \qquad \text{(Eq. 4-4) } (\mathit{A3.11.5.7\text{-}1})$$

where:

P_a = Force resultant per unit wall length (kN/m)
k_a = Active earth pressure coefficient (dim)
γ'_s = Effective unit weight of soil backfill (kN/m^3)
h = Nominal height of horizontal earth pressure as shown in Figures 4-5 and 4-6 (m)

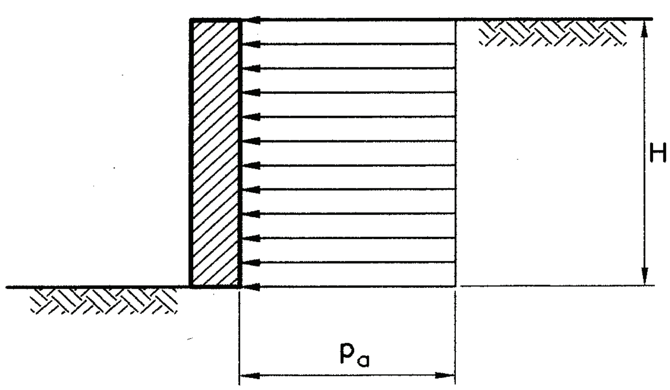


Figure 4-4 (*A3.11.5.6-1*)
Earth Pressure Distribution for Anchored Wall Design
(AASHTO, 1997a)

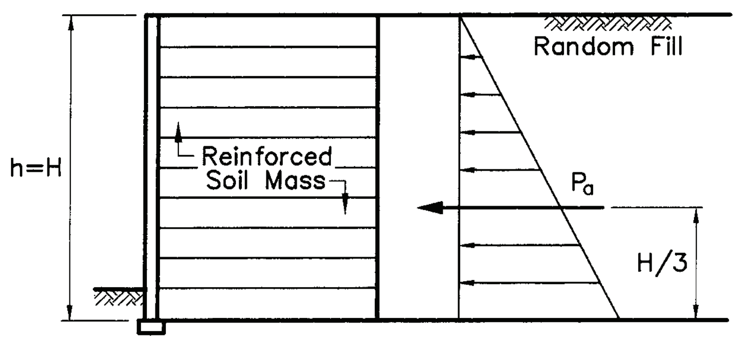


Figure 4-5 (*A3.11.5.7-1*)
Earth Pressure Distribution for MSE Wall with Level Backfill Surface
(AASHTO, 1997a)

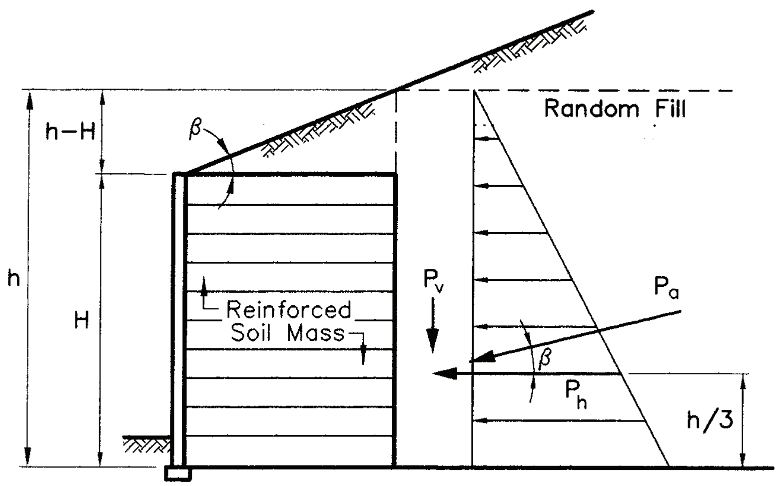


Figure 4-6 (*A3.11.5.7-2*)
Earth Pressure Distribution for MSE Wall with Sloping Backfill Surface
(AASHTO, 1997a)

4.2.1.4 Earth Surcharge Load - ES (*A3.11.6.1*)

The force effects of earth surcharge loads on backfills must be considered for the design of walls and abutments. Where a uniform surcharge is applied over a retained earth surface, the additional lateral earth pressure due to the surcharge is assumed to remain constant with depth and has a magnitude, Δp, of:

$$\Delta p = k_s q_s \qquad (4\text{-}5)\ (A3.11.6.1\text{-}1)$$

where:

$\Delta p =$ Constant horizontal earth pressure due to uniform surcharge (kPa)

k_s = Coefficient of earth pressure due to surcharge (dim)
q_s = Uniform surcharge applied to the upper surface of the active earth wedge (kPa)

For active earth pressure conditions, k_s, is taken as k_a; for at-rest conditions, k_s is taken as k_o.

4.2.2 Transient Loads

The transient loads that may need to be developed for structure design include vehicular traffic, earthquake, water and stream pressure, ice, wind, temperature, and vehicle impact. The methods used in the AASHTO LRFD Specification to estimate these transient loads are described below.

4.2.2.1 Vehicular Live Load - LL (*A3.6.1*)

The force effects of vehicle loads are modeled using a design vehicle to represent typical variations in axle loads and spacing. The live load model in the AASHTO LRFD Specification is based on evaluation of various vehicle configurations (NCHRP, 1990). The group of vehicles and loadings considered in the report were analyzed to determine their force effect on simply-supported and two-span continuous structures. The results of the analyses demonstrated that the combination of the design tandem and uniform lane load and the combination of the HS20 design truck with the design lane load provide an adequate basis for design. The configuration and loading from the HS20 truck in the AASHTO LRFD Specification is shown in Figure 4-7. The design tandem consists of a pair of 110 kN axles spaced at 1.20 m apart with a transverse spacing of 1.80 m. The design lane load consists of a load, uniformly distributed in the longitudinal direction equal to 9.3 kN/m.

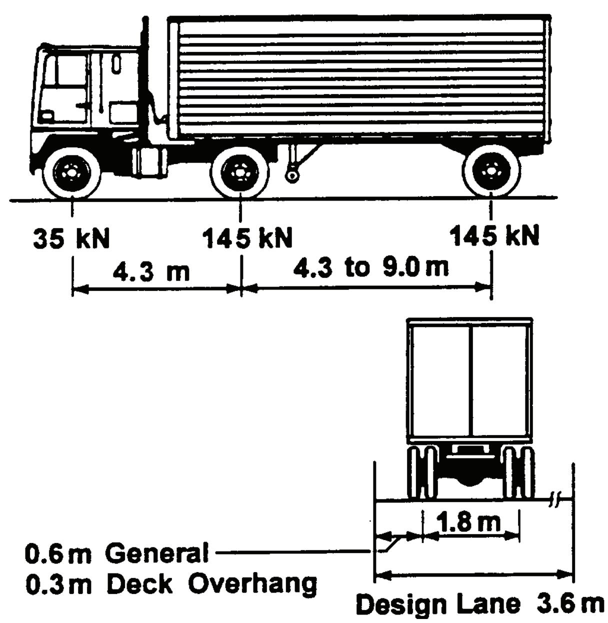


Figure 4-7 (*A3.6.1.2.2-1*)
HS20 Truck Configuration
(AASHTO, 1997a)

Additional guidance regarding the application and distribution of vehicle loads is presented in Section 4.4 where vehicle load effects are discussed for culvert design.

4.2.2.2 Vehicular Dynamic Load Allowance - IM (*A3.6.2*)

The force effects of dynamic vehicle loading on structures is reflected by applying a dynamic allowance factor to the design load. This factor accounts for the effects of:

- Bridge characteristics (e.g., span length and type)
- Vehicle speed
- Gross weight and number of axles
- Roadway roughness

For foundations and abutments supporting bridges, these force effects are incorporated in the loads used for superstructure design. For buried substructures (e.g., culverts and embedded wall elements), the effects of dynamic loading dissipate with depth such that below a depth of about 3 m, the effects are negligible.

4.2.2.3 Vehicular Centrifugal Force - CE (*A3.6.3*)

Vehicles traveling on a superstructure location on a horizontal curve generate a centrifugal force effect that must be considered in design. In the AASHTO LRFD Specification, the centrifugal force is estimated as the product of the design axle load of the design truck or tandem and the factor C which is determined as:

$$C = \frac{4\,v^2}{3\,g\,R} \qquad \text{(Eq. 4-6) } (\textit{A3.6.3-1})$$

where:

C = Centrifugal force factor (dim)
v = Highway design speed (m/s)
g = Gravitational acceleration: (9.81 m/s^2)
R = Radius of curvature of traffic lane (m)

In the AASHTO LRFD Specification, centrifugal forces are applied horizontally at a distance of 1.8 m above the roadway surface. In computing the centrifugal force, the lane load is neglected because the spacing of vehicles at high speed is assumed to be large so that the density of vehicles is small. For substructure design, centrifugal forces would be developed by the structural engineer and would represent a horizontal force effect.

4.2.2.4 Vehicular Braking Force - BR (*A3.6.4*)

Vehicle braking represents a horizontal force effect along the length of a bridge which must be resisted by the structure foundations. In the AASHTO LRFD Specification, these forces are taken as 25 percent of the design axle load of the design truck or tandem in each lane of traffic headed in the same direction. As with centrifugal force effects, braking forces are applied at a distance of 1.8 m above the roadway surface.

4.2.2.5 Pedestrian Live Load - PL (*A3.6.1.6*)

Vehicular bridges supporting pedestrian sidewalks and bridges used for pedestrian and/or bicycle traffic must be designed for the loads imposed by this use. The AASHTO LRFD Specification requires a pedestrian load of 3.61 kPa for bridges with sidewalks wider than 0.6 m and a load of 4.1 kPa for bridges used solely for pedestrian and bicycle traffic.

4.2.2.6 Live Load Surcharge - LS (*A3.11.6.2*)

The force effects of surcharge and traffic loads on backfills must be considered for the design of walls and abutments. As shown in Figure 4-8, live load surcharge effects produce a lateral pressure component on a wall in addition to lateral earth loads. The magnitude of the lateral pressure due to the surcharge load is a function of the type (i.e., point or uniformly distributed), magnitude and proximity of surface loading, the strength of the backfill, and the stiffness of the wall relative to wall displacement. The distribution of horizontal pressure produced by surcharge loads is usually estimated using a Boussinesq distribution and assuming the soil backfill is elastic.

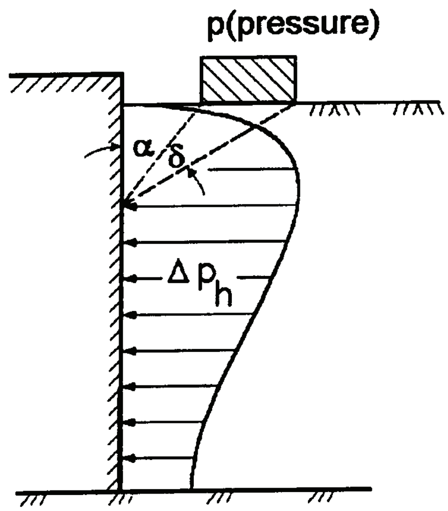


Figure 4-8 (*A3.11.6.1-1*)
Horizontal Pressure on Wall Due to Uniformly-Loaded Strip Pressure
(AASHTO, 1997a)

If traffic is expected within a distance behind a wall equal to about the wall height, the live load traffic surcharge is assumed to act on the retained earth surface. The uniform increase in lateral earth pressure due to live load surcharge is typically estimated as:

$$\Delta p = k_s \gamma'_s h_{eq} \qquad \text{(Eq. 4-7) } (A3.11.6.2\text{-}1)$$

where:

p = Constant lateral earth pressure due to uniform traffic surcharge (kPa)
γ'_s = Effective unit weight of soil (kN/m^3)
k_s = Coefficient of earth pressure (dim)
h_{eq} = Equivalent height of soil for the design live load (m)

For active earth pressure conditions, k_s is taken as k_a, and for at-rest conditions, k_s is taken as k_o.

Traditionally, h_{eq} has been assumed to be constant for walls of any height (e.g., 0.61 m in AASHTO ASD). However, in the AASHTO LRFD Specification, h_{eq} is currently applied as a function of wall height as shown in Table 4-2.

Table 4-2 (*A3.11.6.2-1*)
Equivalent Height of Soil for Vehicular Loading
(AASHTO, 1997a)

Wall Height (m)	h_{eq} (m)
< 1.5	1.70
3.0	1.20
6.0	0.76
> 9.0	0.61

The values of h_{eq} in the table were determined based on evaluation of horizontal pressure distributions produced on retaining walls by the vehicular live loads prescribed in AASHTO LRFD Specification using a Boussinesq elastic half-space solution and a Poisson's Ratio of 0.5. For other live load distributions, the live load surcharge, Δp_h, can be determined as shown in Figure 4-8 using:

$$\Delta p_h = 2\,p/\,\pi\,(\alpha - \sin\alpha\,\cos(\alpha + 2\delta)) \qquad \text{(Eq. 4-8) (A3.11.6.1-2)}$$

where:

p = Live load intensity from Figure 4-8 (kPa)

4.2.2.7 Water Load and Stream Pressure Force- WA (*A3.7*)

Force effects on structures due to water loading include static pressure, buoyancy and stream pressure. Static water pressure loading needs to be considered whenever differential water loads develop on a structure. The effects of buoyancy need to be considered whenever substructures are constructed below a temporary or permanent ground water level. Some typical conditions in which buoyancy must be considered during design include a culvert constructed below the ground water table, and a spread footing or pile cap located below the seasonal high-water elevation. Examples of the effects of stream pressure include floating debris, waves and stream currents, and scour.

4.2.2.8 Wind Load - WS and WL (*A3.8*)

The principal wind loads which need to be considered by the structural engineer include horizontal wind pressure effects on the structure, WS, and vehicles, WL. The effects of vertical wind pressure on the underside of bridges due to an interruption of the horizontal flow of air, and the effects of aeroelastic instability represent special load conditions for long-span bridges. For small and/or low structures, wind loading does not usually govern the design. However, for large and/or tall bridges, noise walls, overhead signs and light standards, wind loading can govern the design and should be investigated. Where wind loading is important, the wind pressure should be evaluated from two or more different directions to determine the windward (facing the wind), leeward (facing away from the wind) and side pressures which produce the most critical loads on the structure.

The design wind velocity is a function of the height of the structure and the aerodynamic characteristics of the near-surface environment in the upstream wind direction. In the AASHTO LRFD Specification, the wind velocity for bridges more than 10 m above the water or low ground, V_{DZ}, can be determined using:

$$V_{DZ} = 2.5\, V_0 \left[\frac{V_{10}}{V_B}\right] \ln\left[\frac{Z}{Z_0}\right] \qquad \text{(Eq. 4-9) } (\textit{A3.8.1.1-1})$$

where:

V_{DZ} = Design wind velocity at design elevation, z (km/hr)
V_{10} = Wind velocity at 10 m above low ground or design water level (km/hr)
V_B = Base wind velocity of 160 km/hr
Z = Structure height above low ground or design water level (m)
V_0 = Friction velocity from Table 4-3 (km/hr)
Z_0 = Friction length of upstream fetch from Table 4-3 (m)

Table 4-3 (*A3.8.1.1-1*)
Values of V_0 and Z_0 for Various Upstream Surface Conditions
(AASHTO, 1997a)

Condition	Open Country	Suburban	City
V_0 (km/hr)	13.2	15.2	19.4
Z_0 (m)	0.07	0.30	0.80

Using the design velocity V_{DZ}, the wind pressure on structures, WS is determined as:

$$P_D = P_B\left[\frac{V_{DZ}}{V_B}\right]^2 = P_B\,\frac{V_{DZ}^2}{25\,600} \qquad \text{(Eq. 4-10) } (\textit{A3.8.1.2-1})$$

where:

$P_B =$ Base wind pressure from Table 4-4 (kPa)

Table 4-4 (*A3.8.1.2-1*)
Base Pressure, P_B, for V_a = 160 km/hr
(AASHTO, 1997a)

Structural Component	Windward Load, kPa	Leeward Load, kPa
Trusses, Columns and Arches	2.4	1.2
Beams	2.4	N/A
Large Flat Surfaces	1.9	N/A

In the AASHTO LRFD Specification, the wind pressure on vehicles, WL, is applied as a moving force of 1.46 kN/m acting normal to the structure at a height of 1.8 m above the roadway surface. This load is based on a long row of randomly sequenced passenger cars, commercial vans and trucks exposed to a 90 km/hr design wind.

When the wind direction is not normal to the exposed surface area of the structure, the components of wind pressure are:

$$P_N = P_D \cos^2 \theta \quad \text{(Eq. 4-11) (}\mathit{A3.8.1.4\text{-}1}\text{)}$$

and

$$P_P = P_D \cos\theta \sin\theta \quad \text{(Eq. 4-12) (}\mathit{A3.8.1.4\text{-}2}\text{)}$$

where:

$P_N =$ Wind pressure normal to the structure component (kPa)
$P_p =$ Wind pressure parallel to the structure (kPa)
$P_D =$ Design wind pressure (kPa)
$\theta =$ Angle between the wind direction and the normal to the structure component (deg)

4.2.2.9 Friction Force - FR (*A3.13*)
Frictional force effects due to sliding between structural members (e.g., bearings) must be considered for superstructure design. In the AASHTO LRFD Specification, these forces are established based on the extreme values of the friction coefficient between the surfaces. The effects of friction are not considered for substructure design unless the forces represent a load applied on the substructure.

4.2.2.10 Force Effects Due to Superimposed Deformations (Uniform Temperature - TU; Temperature Gradient - TG; Creep - CR; Shrinkage - SH; and Settlement - SE) (*A3.12*)
Internal forces are developed on structure components as a result of temperature changes, creep and shrinkage of materials and settlement of structures. While these force effects do not often need to be considered for substructure design and analysis, they should be considered for:

- Design of an integral bridge abutment where the passive resistance of the abutment backfill must be evaluated for the temperature gradient used for superstructure design; or

- Analysis after the construction of a structure of the force effects of greater than tolerable foundation movements (e.g., foundation settlement due to soft ground or scour, or longitudinal differential settlement along a culvert) on the performance of structure components.

Thus, consideration of force effects due to superimposed deformations are of limited concern during most substructure design.

4.2.2.11 Earthquake Force - EQ (*A3.10*)
Earthquake force effects are predominately horizontal and act through the center of mass of the structure. Because most of the weight of a bridge is in the superstructure, seismic loads are assumed to act through the bridge deck as shown in Figure 4-9. These loads are due to inertial effects and therefore are proportional to the weight and acceleration of the superstructure. The effects of vertical components of earthquake ground motions are typically small and are usually neglected except for complex bridges.

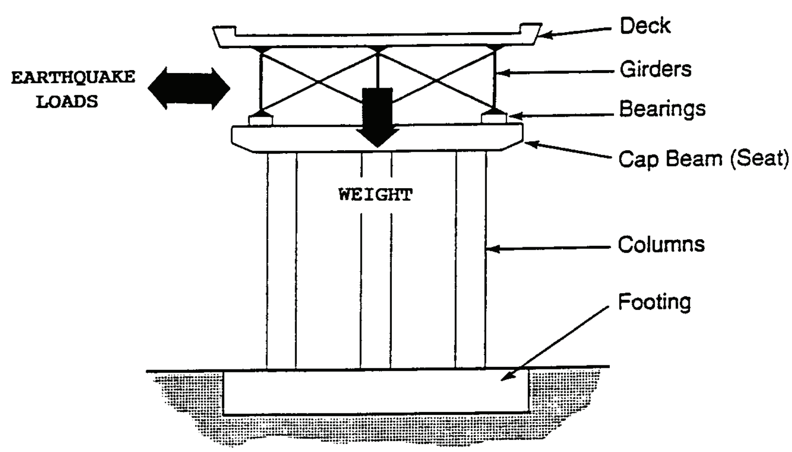


Figure 4-9
Earthquake Loads on a Bridge
(Buckle, 1994)

The inertial loads in the superstructure must be adequately transferred to the ground through the connections and bearings to the columns, piers and abutments, and then through the foundations. Because there is usually little redundancy in this load path, failure of any one can lead to partial or complete collapse of a bridge. The importance of load-path integrity has been demonstrated repeatedly from recent earthquakes. In some cases, the failure of a small component (e.g., a bolt) has led to progressive failures of other components and the collapse of major bridge components. The effects of soil liquefaction and slope movements resulting from earthquake events must be evaluated separately from the effects of seismic loading on structure components.

Earthquake design loads are horizontal forces determined from the product of the elastic seismic response coefficient, C_{sm}, the equivalent weight of the superstructure, and a response modification factor. For most sites and bridges with span lengths less than 150 m, the elastic seismic response coefficient can be determined as:

$$C_{sm} = \frac{1.2\,AS}{T_m^{2/3}} \leq 2.5\,A \qquad \text{(Eq. 4-13) } (\textit{A3.10.6.1-1})$$

where:

C_{sm} = Elastic seismic response coefficient for m^{th} mode of vibration (dim)
A = Acceleration coefficient (dim)
S = Site coefficient from Table 4-6 (dim)
T_m = Period of vibration in the m^{th} mode from dynamic structural analysis (sec)

Exceptions to using Eq. 4-13 for determining C_{sm} include:

- For Soil Profiles III and IV (Table 4-7) and in areas where $A \geq 0.3$, C_{sm} need not exceed 2.0A

- For Soil Profiles III and IV (Table 4-7), and for modes other than the fundamental mode which have periods less than 0.3 sec, the value of C_{sm} can be determined as:

$$C_{sm} = A\,(0.8 + 4\,T_m) \qquad \text{(Eq. 4-14) } (\textit{A3.10.6.2-1})$$

- If the period of vibration exceeds 4 sec, the value of C_{sm} can be determined as:

$$C_{sm} = 3\,AST_m^{3/4} \qquad \text{(Eq. 4-15) } (\textit{A3.10.6.2-2})$$

Except where sites are located near an active fault, the acceleration coefficient, A, can be determined using the contour maps presented in the AASHTO LRFD Specification (*A3.10.2*). In areas of active faulting, the value of A should be determined by an experienced seismologist.

The AASHTO LRFD Specification uses Seismic Performance Zones to define the minimum requirements which must be followed with regard to methods of analysis, minimum support lengths, column design details and foundation and abutment design procedures for various seismic regions. The Seismic Performance Zones are presented in Table 4-5.

Table 4-5 (*A3.10.4-1*)
Seismic Performance Zones
(modified after AASHTO, 1997a)

Acceleration Coefficient (dim)	Seismic Zone
A $\leq$ 0.09	1
0.09 < A $\leq$ 0.19	2
0.19 < A $\leq$ 0.29	3
0.29 < A	4

The effect of subsurface conditions at a site on the response of a bridge or other structure to earthquake loading is accounted for using a site coefficient, S. As used in the AASHTO LRFD Specification, the value of S depends on the soil profile at the site as shown in Table 4-6. The soil conditions corresponding to each soil profile type are presented in Table 4-7. In general, sites that have softer soils have a higher site coefficient to account for the effects of amplification of bedrock ground motions through soil.

Table 4-6 (*A3.10.5-1*)
Site Coefficients
(AASHTO, 1997a)

Site Coefficient	Soil Profile Type			
	I	II	III	IV
S	1.0	1.2	1.5	2.0

Because it is uneconomical to design a bridge to resist large earthquake forces elastically (i.e., without damage), most bridges are designed to perform in a ductile manner during large earthquakes. The seismic force effects on substructures and connections between parts of structures are, therefore, limited due to displacement and/or yielding. Accordingly, the force for seismic design can be determined by dividing the force from an elastic analysis by a response modification factor, R, from either Table 4-8 or 4-9.

Table 4-7
Classification of Soil Profile Types
(after AASHTO, 1997a)

Soil Profile Type	Classification
I	Shale-like or crystalline rock; stable deposits of gravel, sand or stiff clays with the depth to rock < 60 m; or soils with shear wave velocity > 750 m/sec.
II	Stiff cohesive or deep cohesionless soils comprised of stable deposits of gravel, sand or stiff clays with depth to rock > 60 m.
III	Soft to medium-stiff clays with or without intervening layers of sand or other cohesionless soils with thickness $\geq$ 9 m.
IV	Soft clays or silts > 12 m in depth; or loose natural deposits or man-made, non-engineered fill; or materials with shear wave velocity < 150 m/sec

Table 4-8 (*A3.10.7.1-1*)
Response Modification Factors for Substructure Design
(AASHTO, 1997a)

Substructure	Importance Category		
	Critical	Essential	Other
Wall-type piers - larger dimension	1.5	1.5	2.0
Reinforced concrete pile bents • Vertical piles only • With batter piles	 1.5 1.5	 2.0 1.5	 3.0 2.0
Single columns	1.5	2.0	3.0
Steel or composite steel and concrete pile bents • Vertical piles only • With batter piles	 1.5 1.5	 3.5 2.0	 5.0 3.0
Multiple column bents	1.5	3.5	5.0

In the AASHTO LRFD Specification, essential bridges are structures which must remain open to emergency vehicles for security or defense purposes after the design earthquake (i.e., event with a 475-year return period). Critical bridges are defined as structures which must remain open to emergency vehicles for security or defense purposes after a large earthquake (i.e., event with a minimum return period of 2,500 years).

Table 4-9 (*A3.10.7.1-2*)
Response Modification Factors for Connector Design
(AASHTO, 1997a)

Connection	All Importance Categories
Superstructure to abutment	0.8
Expansion joints within a span of the superstructure	0.8
Columns, piers, or pile bents to cap beam or superstructure	1.0
Columns or piers to foundations	1.0

R factors for connections are smaller than those for substructure members to preserve the integrity of the bridge under extreme loads.

Lateral earth pressures on retaining structures are amplified during earthquake loading due to horizontal acceleration of the retained earth mass. The Mononobe-Okabe method of analysis is a pseudo-static method which is often used to develop an equivalent static fluid pressure to model seismic earth pressure on walls. The method is applicable when:

- The wall is unrestrained and capable of deflecting sufficiently to mobilize the active earth pressure in the retained soil
- The backfill is cohesionless and unsaturated
- The failure surface defining the active wedge of soil loading the wall is planar
- Accelerations are uniform throughout the retained soil mass

Equilibrium of the retained soil wedge is defined in Figure 4-10.

The combined static and seismic earth pressure is:

$$P_{ae} = 0.5 \text{ x } \gamma_s H^2 (1 - k_v) K_{ae} \qquad \text{(Eq. 4-16) } (\textit{AA.11.1.1.1-1})$$

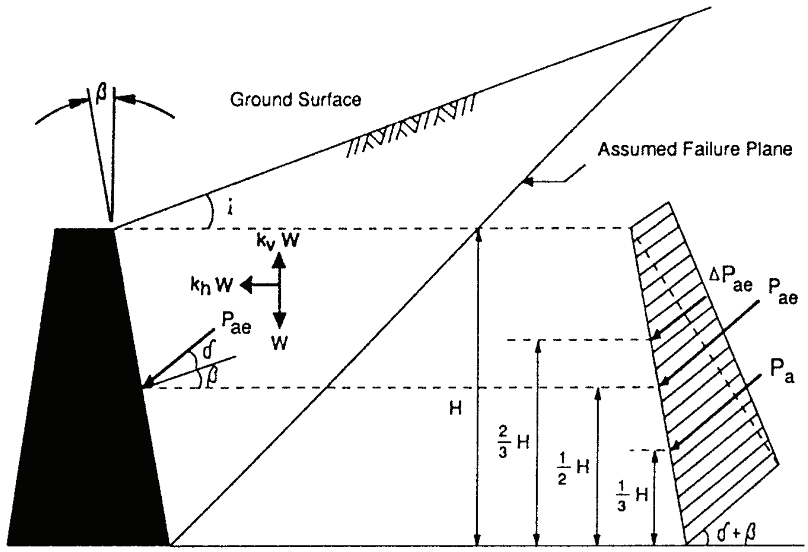


Figure 4-10
Definition Sketch for Mononobe-Okabe Analysis
(Barker, et al., 1991)

where the seismic active earth pressure coefficient, K_{ae} (*AA.11.1.1.1-2*) is:

$$K_{ae} = \frac{\cos^2(\phi - \theta - \beta)}{\cos\theta \cos^2\beta \cos(\delta + \beta + \theta)} \left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \theta - i)}{\cos(\delta + \beta + \theta)\cos(i - \beta)}} \right]^{-2} \qquad \text{(Eq. 4-17)}$$

where:

- γ_s = Unit weight of soil (kN/m^3)
- H = Height of the retained earth mass behind the wall (m)
- k_v = Vertical acceleration coefficient (dim)
- k_h = Horizontal acceleration coefficient (dim)
- ϕ = Internal friction angle (deg)
- θ = Arc tan $[k_h/(1 - k_v)]$ (deg)
- β = Slope of wall backface (deg)
- δ = Angle of friction between the soil and wall (deg)
- I = Slope of backfill behind wall (deg)

The equivalent expression for the passive force mobilized if a wall is pushed into soil is:

$$P_{pe} = 0.5\ \gamma_s H^2 (1 - k_v) K_{pe} \qquad \text{(Eq. 4-18) } (AA.11.1.1.1\text{-}3)$$

where the seismic passive earth pressure coefficient, K_{pe} (*AA.11.1.1.1-4*) is:

$$K_{pe} = \frac{\cos^2(\phi - \theta + \beta)}{\cos\theta \cos^2\beta \cos(\delta - \beta + \theta)} \left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \theta + i)}{\cos(\delta - \beta + \theta)\cos(i - \beta)}}\right]^{-2} \qquad \text{(Eq. 4-19)}$$

To estimate lateral earth pressures for earthquake loading, the vertical acceleration coefficient, k_v, is often assumed equal to zero and the horizontal acceleration coefficient, k_h, is taken as:

$$k_h = 0.5\,\alpha \qquad \text{(Eq. 4-20)}$$

for walls permitted to move horizontally up to about 250 α mm (e.g., gravity, cantilever and MSE walls, and

$$k_h = 1.5\,\alpha \qquad \text{(Eq. 4-21)}$$

for walls for which essentially zero horizontal displacement is permitted (e.g., integral abutments and anchored walls).

where:

α = A/100
A = Horizontal earthquake acceleration coefficient (% of g)

The resultant of the dynamic component of the earth pressure, ΔP_{ae}, is located significantly higher on the wall than the static active earth pressure resultant, $P_{a,}$ as shown in Figure 4-10. Typically, the combined static and seismic lateral earth pressure are assumed to be uniformly distributed with a resultant, P_{ae}, acting at the midheight of the wall.

4.2.2.12 Ice Load - IC (*A3.9*)
Ice force effects on piers are a function of the size of ice flow, the strength and thickness of ice, and geometry of the pier. In determining the ice force, site conditions and the modes of ice flow must be considered. These factors include:

- Dynamic pressure due to moving sheets or flows of ice transported by stream flow, currents or wind
- Static pressure due to thermal movements of ice sheets
- Pressure from hanging dams or jams of ice
- Static uplift or vertical load from adhering ice in waters or fluctuating level

Ice failure modes include:

- Crushing by local failure across the width of the pier
- Bending against piers with an inclined nose
- Splitting when small flows strike a pier
- Impact of small flows across width of the pier
- Bending of large flows across wide piers

The effective ice crushing strength, p, can range from about 380 kPa for ice near melting and substantially disintegrated to about 1530 kPa where ice break up occurs where the ice temperature is measurably below the melting point.

For piers of standard dimensions on larger water bodies, crushing and bending ice failure usually control the dynamic ice force for design. On smaller streams which cannot transport large ice flows, impact failure usually controls the dynamic ice force for design.

For ice failure by crushing or bending, the horizontal force acting on a pier resulting from moving ice, F, is affected by the ratio of the pier width to ice thickness, w/t, such that:

$$F = F_c \text{ or } F_b\text{, whichever is less} \qquad \text{(Eq. 4-22)}$$

if w/t $\leq$ 6.0, and

$$F = F_c \qquad \text{(Eq. 4-23)}$$

if w/t > 6.0, for which:

$$F_c = C_a\, p\, t\, w \qquad \text{(Eq. 4-24) } (\mathit{A3.9.2.2\text{-}1})$$

$$F_b = C_n\, p\, t^2 \qquad \text{(Eq. 4-25) } (\mathit{A3.9.2.2\text{-}2})$$

$$C_a = \left[\frac{5\,t}{w} + 1\right]^{0.5} \qquad \text{(Eq. 4-26) } (\mathit{A3.9.2.2\text{-}3})$$

$$C_n = \frac{0.5}{\tan(\alpha - 15)} \qquad \text{(Eq. 4-27) } (\mathit{A3.9.2.2\text{-}4})$$

where:

t = Ice thickness (m)
α = Inclination of pier nose to the vertical; (deg)
p = Effective ice crushing strength (kPa)

w = Pier width at ice level (m)
F_c = Horizontal force caused by ice flows which fail by crushing (kN)
F_b = Horizontal force caused by ice flows which fail by bending (kN)
C_a = Coefficient for effect of pier width to ice thickness ratio where flow fails by crushing (dim)
C_n = Coefficient for horizontal inclination of pier nose to a vertical line (dim)

The ice force used for design can be modified additionally to account for the effects of stream size, pier orientation and shape, and ice adhesion to the pier.

4.2.2.13 Vehicular Collision Force - CT (*A3.6.5*)

Structures not protected from collisions by roadway and rail vehicles must be designed for the force effects of vehicle collisions. The AASHTO LRFD Specification requires that abutments and piers located within a distance of 9 m to the edge of a roadway, or within a distance of 15 m to the centerline of a railway track be designed to resist an equivalent static force of 1800 kN acting in any direction in a horizontal plane at a height of 1.2 m above ground. This equivalent static force was developed from analysis of full-scale crash tests. For column piers, the force should be applied as a point load. For other structures, the load can be applied as a point force or distributed over an area deemed suitable for the size of structure and anticipated impacting vehicle. Measures deemed suitable for structure protection from vehicle collisions include embankments, and structurally-independent, crashworthy, ground-mounted barriers of suitable height.

4.2.2.14 Vessel Collision Force - CV (*A3.14*)

Due to their proximity to navigation channels, bridge substructures may be subjected to vessel collision by ships and barges. The principal factors affecting the risk and consequences of vessel collisions with waterway substructures are related to vessel, waterway and bridge characteristics. The consequences of vessels colliding with a structure are a function of the size, type, loading condition and direction of the vessel. The width and depth of the navigation channel, stream velocity, channel alignment, cross-section geometry, water elevation and hydraulic conditions represent the primary characteristics of waterways. The bridge characteristics which need to be considered include bridge layout, geometry and strength. While bridge piers should be located outside the waterway, economic and engineering constraints limit the span of bridges and their vertical clearance. In general, bridge piers can be designed or protected from vessel collisions. However, it is usually not economically feasible to design bridge superstructures for vessel collisions.

The impact force from a head-on vessel collision with a pier is:

$$P_s = 120\,V\sqrt{DWT} \qquad \text{(Eq. 4-28) } (\textit{A3.14.8-1})$$

where:

P_s = Equivalent static vessel impact force (kN)

DWT = Deadweight tonnage of vessel (metric ton)
V = Vessel impact velocity (m/s)

The impact force from a head-on collision of a barge with a pier is:

$$P_B = 60\,000\, a_B \qquad \text{(Eq. 4-29) } (\mathit{A3.14.11\text{-}1})$$

if $a_B < 0.1$ m, and

$$P_B = 6000 + 1600\, a_B \qquad \text{(Eq. 4-30) } (\mathit{A3.14.11\text{-}2})$$

if $a_B \geq 0.1$ m, where:

P_B = Equivalent static impact force for a standard inland hopper barge (kN)
a_B = Barge bow damage length (m)

The barge bow damage length for a standard hopper barge is:

$$a_B = 3.1\left(\sqrt{1 + 1.3 \times 10^{-7}\, KE} - 1\right) \qquad \text{(Eq. 4-31) } (\mathit{A3.14.12\text{-}1})$$

where:

a_B = Barge bow damage length (m)
KE = Vessel collision energy (joule)

The energy of a moving vessel to be absorbed during a noneccentric collision with a bridge pier is:

$$KE = 500\, C_H\, M\, V^2 \qquad \text{(Eq. 4-32) } (\mathit{A3.14.7\text{-}1})$$

where:

KE = Vessel collision energy (joule)
M = Vessel displacement tonnage (metric ton)
C_H = Hydrodynamic mass coefficient equal to 1.05 if the underkeel clearance exceeds 0.5 times the draft, and 1.25 if the underkeel clearance is less than 0.1 times the draft (dim)
V = Vessel impact velocity (m/s)

4.3 Load Factors and Load Combinations

When using ASD, there is no distinction between the loads (i.e., either in magnitude or combination of load types) used to evaluate the ultimate load capacity or the deformation potential of the ground. Thus, the same load magnitudes are used to estimate the ultimate bearing capacity and the settlement of a foundation for ASD, despite the inherent uncertainty in loads applied from a structure to its substructure components. LRFD incorporates this uncertainty by application of load factors for various permanent and transient load types using the LRFD equation:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 4-33) (\textit{A1.3.2.1-1})}$$

where:

- η_i = Load modifier to account for ductility, redundancy and operational importance (dim)
- γ_i = Load factor for permanent and transient loads (dim)
- Q_i = Load (kN)
- ϕ = Resistance factor (dim)
- R_n = Nominal (ultimate) resistance (kN)
- R_r = Factored resistance (kN)

Selection of the load factor(s) to be used is a function of the type of load and limit state being evaluated. (Recall from Chapter 3 that a limit state is a condition beyond which a foundation or structure component ceases to fulfill its intended function.) This section addresses load factor and load modifier selection. The selection of a resistance factor(s) and estimation of the nominal resistance is presented in Chapters 8 through 10 for design of foundations, Chapters 11 through 14 for design of walls, and in Chapters 15 and 16 for design of culverts.

For the AASHTO LRFD Specification, the limit states, load factors and load combinations which must be investigated are presented in Tables 4-10 and 4-11.

Structure and foundation design by LRFD requires that:

- All applicable limit states be evaluated
- Each load for each limit state be modified by a prescribed load factor, γ
- Factored loads for each limit state be combined in a prescribed manner

Table 4-10 (*A3.4.1-1*)
Load Combinations and Load Factors
(AASHTO, 1997a)

LOAD COMBINATION LIMIT STATE	DC DD DW EH EV ES	LL IM CE BR PL LS	WA	WS	WL	FR	(1) TU CR SH EL	TG	SE	Use one of these at a time			
										EQ	IC	CT	CV
STRENGTH-I (unless noted)	γ_p	1.75	1.00	-	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-II	γ_p	1.35	1.00	-	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-III	γ_p	-	1.00	1.40	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-IV EH, EV, ES, DW DC ONLY	γ_p 1.50	-	1.00	-	-	1.00	0.50/ 1.20	-	-	-	-	-	-
STRENGTH-V	γ_p	1.35	1.00	0.40	0.40	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
EXTREME EVENT-I	γ_p	γ_{EQ}	1.00	-	-	1.00	-	-	-	1.00	-	-	-
EXTREME EVENT-II	γ_p	0.50	1.00	-	-	1.00	-	-	-	-	1.00	1.00	1.00
SERVICE-I	1.00	1.00	1.00	0.30	0.30	1.00	1.00/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
SERVICE-II	1.00	1.30	1.00	-	-	1.00	1.00/ 1.20	-	-	-	-	-	-
SERVICE-III	1.00	0.80	1.00	-	-	1.00	1.00/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
FATIGUE-LL, IM & CE ONLY	-	0.75	-	-	-	-	-	-	-	-	-	-	-
CONSTRUCTION	1.25	1.50	1.00	1.25	1.25	1.25	1.00	1.00	1.00	-	-	-	-

(1) The reduced values of γ are used when calculating force effects other than displacements of joints and bearings (*A14.4.1*).

Table 4-11 (*A3.4.1-2*)
Load Factors for Permanent Loads, γ_P
(AASHTO, 1997a)

Type of Load	Load Factor	
	Maximum	Minimum
DC: Component and Attachments	1.25	0.90
DD: Downdrag	1.80	0.45
DW: Wearing Surfaces and Utilities	1.50	0.65
EH: Horizontal Earth Pressure		
• Active	1.50	0.90
• At-Rest	1.35	0.90
EV: Vertical Earth Pressure		
• Retaining Structure	1.35	1.00
• Rigid Buried Structure	1.30	0.90
• Rigid Frames	1.35	0.90
• Flexible Buried Structures	1.95	0.90
• Flexible Metal Box Culverts	1.50	0.90
ES: Earth Surcharge	1.50	0.75

From the Table 4-10, the limit states which must be investigated include:

- Strength Limit State
- Extreme Event Limit State
- Service Limit State
- Fatigue Limit State
- Construction Limit State

The limit states are further subdivided based on consideration of applicable load combinations as follows:

- Strength I - Basic load combination related to the normal vehicular use of the bridge without wind.

- Strength II - Load combination relating to the use of the bridge by Owner-specified special design vehicles and/or evaluation permit vehicles, without wind.

- Strength III - Load combination relating to the bridge exposed to wind velocity exceeding 90 km/hr without live loads.

- Strength IV - Load combination relating to very high dead load to live load force effect ratios exceeding about 7.0 (e.g., for spans greater than 75 m).

- Strength V - Load combination relating to normal vehicular use of the bridge with wind velocity of 90 km/hr.

- Extreme Event I - Load combination including earthquake.

- Extreme Event II - Load combination relating to ice load or collision by vessels and vehicles.

- Service I - Load combination relating to the normal operational use of the bridge with 90 km/hr wind.

- Service II - Load Combination intended to control yielding of steel structures and slip of slip-critical connections due to vehicular live load.

- Service III - Load combination relating only to tension in prestressed concrete structures with the objective of crack control.

- Fatigue - Fatigue and fracture load combination relating to repetitive gravitational vehicular live load and dynamic responses under a single design truck.

- Construction - Load combination relating to construction equipment live load during structure installation/erection.

As will become evident in this and later chapters, most substructure designs will require evaluation of foundation and structure performance at the Strength I and Service I Limit States. These limit states are generally analogous to evaluations of ultimate capacity and deformation behavior in ASD, respectively.

Load factors used in the AASHTO LRFD Specification for permanent loads and labeled as γ_p in Table 4-10, are presented in Table 4-11 as maximum and minimum values.

In reviewing Tables 4-10 and 4-11, the load factors vary for different load categories and limit states to reflect either the certainty with which the load can be estimated or the importance of each load category for a particular limit state. Some general comments about magnitude and relationship between various load factors are highlighted below:

- A load factor of 1.00 is used for all permanent and most transient loads for Service I.

- The live load factor for Strength I is greater than that for Strength II (i.e., 1.75 versus 1.35) because variability of live load is greater for normal vehicular traffic than for a permit vehicle.

- The live load factor for Strength I is greater than that for Strength V (i.e., 1.75 versus 1.35) because variability of live load is greater for normal vehicular use without wind than for a bridge subjected to a wind of 90 km/hr, and because less traffic is anticipated during design wind conditions.

- The load factor for wind load on structures for Strength III is greater than for Strength V (i.e., 1.40 versus 0.40) because the wind load represents the primary load for Strength III where structures are subjected to a wind velocity greater than 90 km/hr compared to Strength V where wind velocity of 90 km/hr represents just a component of all loads on the structure.

- The live load factor for Strength III is zero because vehicular traffic is considered unstable and, therefore unlikely, under extreme wind conditions.

- The load factors for wind load for Strength V are less than 1.00 (i.e., 0.40) consistent with the common practice in ASD of allowing somewhat lower factors of safety for structures subjected to both normal vehicle and wind loads.

- The maximum and minimum load factor for downdrag loads represent the extreme values of γ_{pmax} and γ_{pmin} due to the uncertainty in accurately estimating downdrag loads on piles.

The AASHTO LRFD Specification requires that certain permanent loads, including earth loads, be factored using maximum and minimum load factors as shown in Table 4-11. Criteria for their application require that:

- Load factors be selected to produce the total extreme factored force effect, and for each combination, both maximum and minimum extremes be investigated

- For load combinations where one force effect decreases the effect of another force, the minimum value shall be applied to the load that reduces the force effect;

- For permanent force effects, the load factor which produces the more critical combination shall be selected from Table 4-11; and

- If a permanent load increases the stability or load carrying capacity of a structure component (e.g., load from soil backfill on the heel of a wall), the minimum value for that permanent load also be investigated.

In the AASHTO LRFD Specification, each factored load is adjusted by a load modifier, η_i, to account for the combined effects of ductility, η_D, redundancy, η_R, and operational importance, η_I. The η_i factors (*A1.3.2.1*) represent a first attempt at codifying the influence of ductility, redundancy and operational importance on structure performance. For loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \, \eta_R \, \eta_I \geq 0.95 \qquad \text{(Eq. 4-34) } (\textit{A1.3.2.1-2})$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \, \eta_R \, \eta_I} \leq 1.00 \qquad \text{(Eq. 4-35) } (\textit{A1.3.2.1-3})$$

Due to a lack of precise information, the effect of these modifiers has been judged to range between $\pm$ 5 percent when accumulated geometrically. With time, it is hoped that improved quantification of ductility, redundancy and operational importance, and their interaction and system synergy, can be attained to better account for these factors in design.

For design at the Strength Limit State, values of η_i range as follows:

- Ductility - η_D
  - $\eta_D \leq$ 1.05 for non-ductile components and connections
  - $\eta_D =$ 1.00 for conventional designs and details
  - $\eta_D \geq$ 0.95 for components and connections for which additional ductility-enhancing measured are specified

- Redundancy - η_R
  - $\eta_R \leq$ 1.05 for non-redundant components and connections
  - $\eta_R =$ 1.00 for conventional levels of redundancy
  - $\eta_R \geq$ 0.95 for exceptional levels of redundancy

- Operational Importance - η_I
  - $\eta_I \leq$ 1.05 for important structures
  - $\eta_I =$ 1.00 for typical structures
  - $\eta_R \geq$ 0.95 for relatively less important structures

Classification of operational Importance should be based on social, survival and/or security or defense requirements. With respect to seismic design, bridges classified as critical or essential as described in Table 4-8, should be considered operationally important structures.

For design at the Service Limit State, $\eta_D = \eta_R = \eta_I = 1.0$.

4.4 Loads on Foundations and Retaining Walls

The methods used to estimate loads for the design of foundations and walls using LRFD are fundamentally no different than procedures used in the past for ASD. Thus, procedures commonly used by geotechnical engineers to estimate earth pressures, negative loads on piles, seepage pressures due to hydraulic gradients, or surcharge load effects on earth pressure, for example, can be used for LRFD. What has changed however is the manner in which loads are considered for evaluations of foundation/structure stability (e.g., bearing and sliding resistance of spread footing foundations) and foundation movement (e.g., axial and lateral displacement of a pile-supported pier). This section highlights some of the areas in which considerations of force effects for the design of foundations and walls differ between LRFD and ASD.

The design of foundations supporting bridge piers or abutments should consider all limit states loading conditions applicable to the structure being designed. From a review of the Limit States loading descriptions in Section 4.3 and the load factors in Tables 4-10 and 4-11, it can be seen that a variety of Strength Limit States may control the design of a bridge pier or abutment foundation.

- The Strength I Limit State will control for very high live to dead load ratios
- The Strength III or V Limit States may control for structures subjected to high wind loads
- The Strength IV Limit State will control for high dead to live load ratios

The Extreme Limit States may control the design of foundations in a seismically active area (i.e., Extreme Limit I) or for foundations for piers which may be exposed to vehicle or vessel impact (i.e., Extreme Limit II). With respect to deformation, (i.e., lateral deflection or settlement), the Service I Limit State will control. The Service II and Service III Limit States are used to evaluate specific critical structural components and are not generally applicable to foundation design.

For a typical retaining wall, dead weight, earth pressure and live load surcharge will be the predominant loads influencing wall stability. ***From a review of the Limit States loading descriptions in Section 4.3 and Tables 4-10 and 4-11, it can be seen that the Strength I and IV Limit States, for which the largest dead, earth and live load factors apply, will control the design of a retaining wall with respect to stability. With respect to deformation, the Service II and Service III Limit States only apply to special structures such that only the Service I Limit State applies to retaining wall design.***

Applying the criteria in Section 4.3 to evaluate the resistance of walls and foundations at the Strength Limit State, some general observations can be made. The stability of conventional (i.e., gravity, semi-gravity and cantilever) retaining walls must be evaluated for sliding, bearing and overturning. When applying the AASHTO LRFD Specification, these stability cases are discussed below for a typical analysis of a cantilever wall at the Strength I Limit State.

Figure 4-11 shows that the vertical earth load, EV, on the rear of a wall is factored by γ_{pmin} (1.00) and the structure weight, DC, is factored by γ_{pmin} (0.90) because these forces result in an increase in the contact stress (and shear strength) at the base of the foundation. The horizontal earth load, EH, is factored by γ_{pmax} (1.50) for an active earth pressure distribution because the force results in a more critical sliding force at the base of the wall. If EH has both horizontal and vertical components (e.g., a wall supporting a sloping backfill) both components are factored by 1.50.

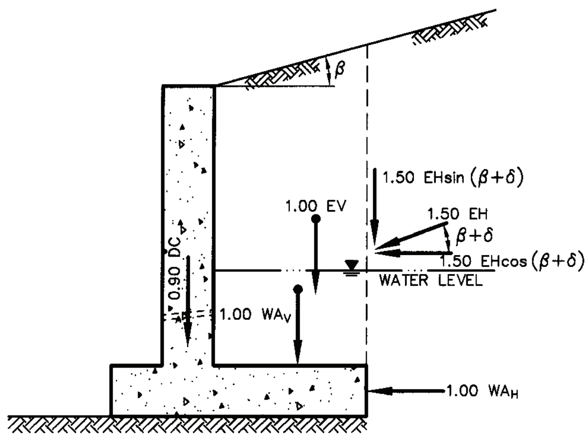
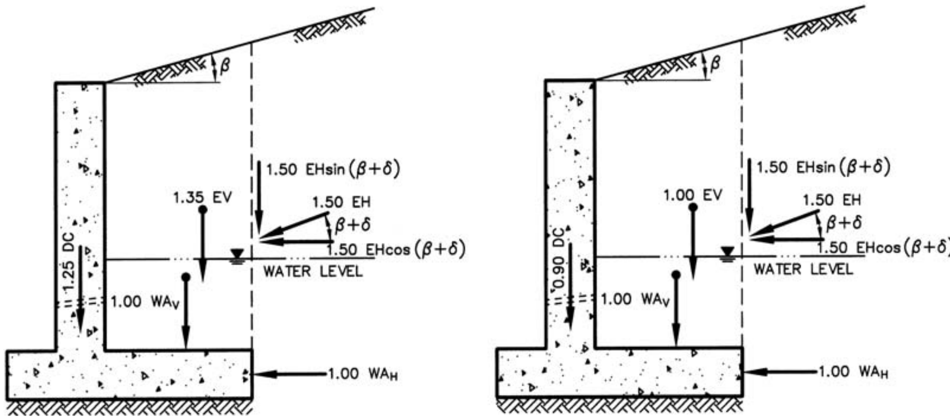


Figure 4-11
Load Combination for Evaluation of Sliding Resistance of a Cantilever Retaining Wall Supported on a Spread Footing

Figure 4-12a shows that the critical load combination for an analysis of bearing resistance of the wall foundation often, but not always, occurs when the structure weight is factored by γ_{pmax} (1.25), the vertical earth load is factored by γ_{pmax} (1.35), and the horizontal active earth pressure is factored by γ_{pmax} (1.50). For the case of a wall supporting sloping backfill, both the horizontal and vertical components of EH are factored by 1.50.

Figure 4-12b shows that the critical load combination for an analysis of eccentricity (or overturning) of the wall foundation occurs when the structure weight is factored by γ_{pmin} (0.90), the vertical earth load is factored by γ_{pmin} (1.00), and the horizontal active earth pressure is factored by γ_{pmax} (1.50). For a wall supporting a sloping backfill, both the horizontal and vertical components of EH are factored by 1.50. (This case may also control for analysis of bearing resistance due to the decrease in effective foundation bearing width associated with increasing eccentricity.)



(a) Bearing Resistance (b) Eccentricity (Overturning)

Figure 4-12
Load Combination for Evaluation of Bearing Resistance and Eccentricity of a Retaining Wall Supported on a Spread Footing

Using the retaining wall examples in Figures 4-11 and 4-12, the vertical and horizontal components of differential water pressure forces, WA, on the wall stem is factored by 1.00.

Because Figures 4-11 and 4-12 only show some of the typical loads applied to a retaining wall without consideration of magnitude, the comments above regarding factored load combinations which could control design represent conditions which may control a majority of cases. For an actual problem, the designer must investigate all viable load combinations and limit states to complete a design using the AASHTO LRFD Specification.

Application of the AASHTO LRFD Specification for design of a driven pile foundation subjected to downdrag loading is presented in Figure 4-13. The neutral plane is located at the elevation where the settlement of the pile equals the settlement of the soil. Above the neutral plane, load in the pile continues to increase with depth due to downdrag and the factored downdrag load adds to the initial factored load in the pile, as indicated by the path A-B in Figure 4-13. Below the neutral plane, skin friction begins to support the pile, and initially offsets the accumulated downdrag load. Along the path B-C in Figure 4-13 skin friction is considered to offset downdrag and is, therefore, regarded as a negative factored load. In the example idealized in Figure 4-13, the skin friction is sufficient to offset all of the downdrag when the load path reaches Point C. Along the path C-D, the resistance of the pile accumulates for a total equal to the factored resistance from skin friction along the path C-D plus the factored tip resistance.

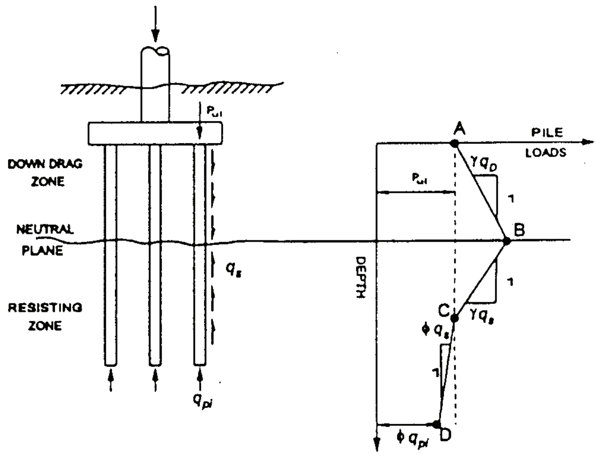


Figure 4-13 (*AC 10.7.1.4-2*)
Schematic Representation of Factored Loads on Driven Piles Subjected to Downdrag
(AASHTO, 1997a)

4.5 Loads on Culverts

Culverts must be designed for the force effects from vertical earth and unbalanced horizontal earth pressures, the weight of pavement layers, and vehicle live loads. For large flexible structures, loads due to compaction of soil backfill may be important. In addition, water buoyancy loads should be evaluated for structures with inverts below the water table. Because culverts are constructed below grade, loads considered important for other types of structures (e.g., wind, temperature, vehicle braking and structure dead load), are insignificant compared to the force effects due to vertical earth pressure. Likewise, fatigue has not been shown to be a factor in long-term performance of culverts. Similarly, the effects of earthquakes can generally be neglected for typical culverts, except where the potential for ground instability or for unbalanced loading exists. This section focuses on the determination of force effects on culverts due to vertical earth pressures and vehicular live loads transmitted through earth fill. Design aspects for flexible and rigid culverts are presented in Chapters 15 and 16, respectively.

The force effects of external loads on culverts depend on a number of factors including cover height, backfill density, culvert size and shape, culvert stiffness relative to the adjacent side fill and foundation stiffness. Moreover, the side fill soil adjacent to the culvert can represent either a load or resistance, depending on its mode of interaction (i.e., either active or passive). Therefore, the force effects on culverts can be variable and their determination depends on the accuracy of the assumptions used in their development.

4.5.1 Earth Loads

Both the AASHTO LRFD Specification (1997a) and the AASHTO ASD Specification (1997b) calculate the force effects of vertical earth pressure on a culvert on the basis of a prism load. The prism load is the weight of the rectangular prism of soil above the culvert, or the free field geostatic stress at the crown of the culvert multiplied by the culvert diameter. If a culvert deflects vertically, contracts circumferentially or settles into the foundation more than the adjacent side fill, the load supported by the culvert is reduced (i.e., positive arching occurs). Conversely, negative arching and an increase in the load supported by a culvert occurs when the side fill settles less than the culvert can deflect vertically, or when the culvert contracts circumferentially or settles more than the foundation material can settle. The condition of equal settlement between the culvert and adjacent side fill is termed neutral arching. The approximate boundaries of soil backfill mobilized by positive, neutral and negative arching are illustrated in Figure 4-14.

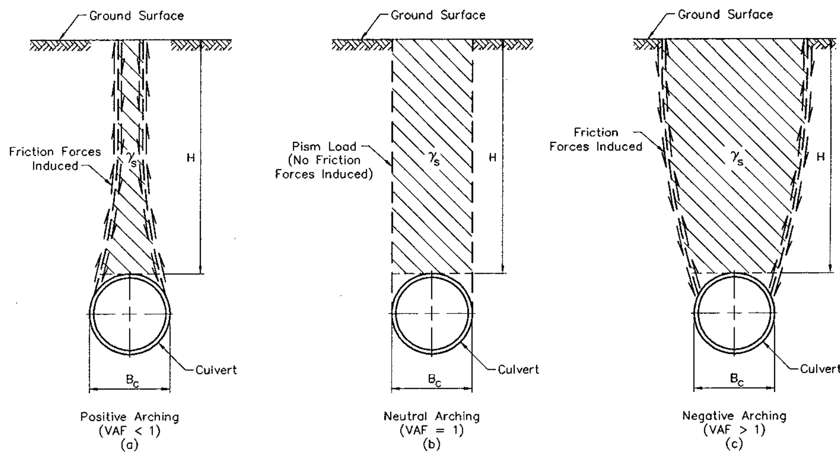


Figure 4-14
Soil Blocks Mobilized by Arching Over Culverts

The prism load is represented in Figure 4-14(b) for the neutral arching case. Positive and negative arching can be accounted for by modifying the prism load for soil-structure interaction using the following relationship:

$$W_E = F_e \gamma_s B_c H \qquad \text{(Eq. 4-35) } (\mathit{A12.10.2.1\text{-}1})$$

where:

W_E = Unit load on culvert due to vertical earth pressure (kN/m)
F_e = Soil-structure interaction factor = VAF (dim)
VAF = Arching factor (dim)
<1.0 for positive arching
= 1.0 for neutral arching
> 1.0 for negative arching
γ_s = Unit weight of soil above culvert (kN/m^3)
H = Height of backfill above culvert crown (m)
B_c = Diameter of culvert, outside diameter for wall profiles with a smooth outside surface, or wall centerline diameter for profiles with a corrugated outside surface (m)

Current design practice limits the arching factors to a value of 1.0 for flexible culverts and 1.5 for rigid culverts. Different methodologies for determining arching factors have evolved between flexible and rigid culverts. A more complete treatment of these various approximate methods specific to culvert type is given in Chapter 15 for flexible culverts and Chapter 16 for rigid culverts.

4.5.2 Vehicular Live Loads

The force effects due to vehicular live loads on culverts can be estimated by closed-form elastic solutions, numerical finite element methods, and approximate methods as used in the AASHTO specifications. ***The AASHTO ASD Specification (1997b) specifies use of a surface point load that is spread through the underlying soil over an area having sides equal to 1.75 times the depth of cover. This distribution is applicable only for cover depths greater than 0.6 m.***

The AASHTO LRFD Specification (1997a) assumes that the contact pressure from a prescribed tire footprint is distributed through the soil backfill in a manner similar to the 60 degree (from horizontal) spreading rule found in many textbooks on soil mechanics. In this case, the spreading is to take effect for cover depths greater than 0.6 m. For depths less than 0.6 m, the area of the tire footprint itself is to be used to determine pressure below the surface due to live loads. At 1 m of cover, this modification has the effect of increasing design pressures by 70% over the method in the AASHTO ASD Specification.

As with other approximate methods for determining vertical earth pressures, the AASHTO procedures for spreading live loads through earth fills are intended to obtain force effects averaged across the culvert diameter. These procedures are used to calculate the average wall thrust due to vehicle live loads, but are not appropriate for determining concentrated force effects from live loads, such as bending stresses or localized deflections, because actual wheel loads do not distribute uniformly through the soil. Rather, wheel loads distribute more as predicted by elastic theory as shown in Figure 4-15. As indicated on Figure 4-15, live loads produce much higher localized effects under shallow covers than predicted by the approximate average pressure models. However, the peak pressures attenuate rapidly with depth. Thus, consideration of localized bending effects due to live loads generally are of concern only for culverts under shallow covers or those subjected to larger than typical concentrated live loads.

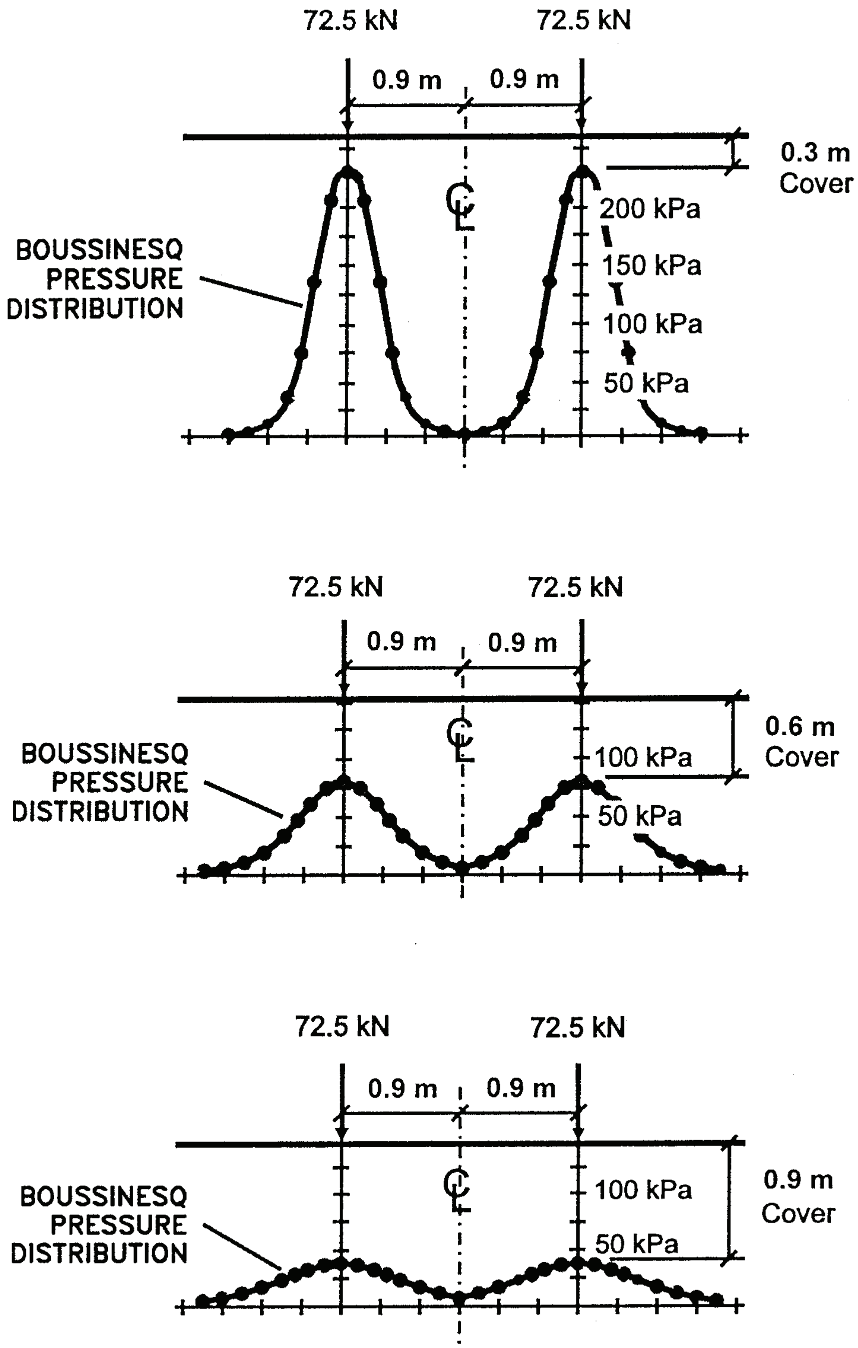


Figure 4-15
Design Truck Pressure Distribution From Elastic Theory

Table 4-12 (*A12.6.6.3-1*)
Minimum Soil Cover for Culverts
(after AASHTO, 1997a)

Type	Condition	Minimum Cover
Corrugated Metal Pipe	-	$S/8 \geq 0.3$ m
Spiral Rib Metal Pipe	Steel Conduit	$S/4 \geq 0.3$ m
	Aluminum pipe for $S \leq 1.2$ m	$S/2 \geq 0.3$ m
	Aluminum pipe for $S > 1.2$ m	$S/2.75 \geq 0.6$ m
Reinforced Concrete Pipe	Unpaved areas and under flexible pavement	$B_c/8$ or $B'_c/8$, whichever is greater, ≥ 0.3 m
	Compacted granular fill under rigid pavement	0.23 m
Thermoplastic Pipe	-	$ID/8 \geq 0.3$ m

where:
- S = Diameter or span (m);
- B_c = Outside diameter or width of the structure (m);
- B'_c = The out-to-out vertical rise of pipe (m); and
- ID = Inside diameter (m).

Other factors regarding vehicle loads that have been modified in the AASHTO LRFD Specification include load factors, live load impact factors (now termed dynamic load allowance factors), multiple presence factors and the design live load model itself. The AASHTO LRFD Specification has reduced the live load factor from 2.0 to 1.75, a reduction of 12%. However, both the magnitude and the effective depth of live load impacts have been increased. At 1 m of cover, the modification to the impact factor increases design pressures by over 125%. Also, the multiple lane presence factor has been increased by 20% for one lane contributions, although it was left unchanged for two lane contributions.

The net effect on design due to all of the factors described above is illustrated in Figure 4-16 for one lane's contribution from the design truck. As can be seen, the changes significantly increase the conservatism of the specification for live load design. At 1 m of cover the required design pressure is about 100% greater under the AASHTO LRFD Specification than under the AASHTO ASD Specification.

While considerable effort was expended in developing the live load model in the AASHTO LRFD Specification, the appropriateness of these changes for below-ground structures was not evaluated. Further study is needed to refine the new LRFD live load model for the design of culverts.

Currently, utilization of the AASHTO LRFD provisions for distributing vehicle live loads to culverts involves:

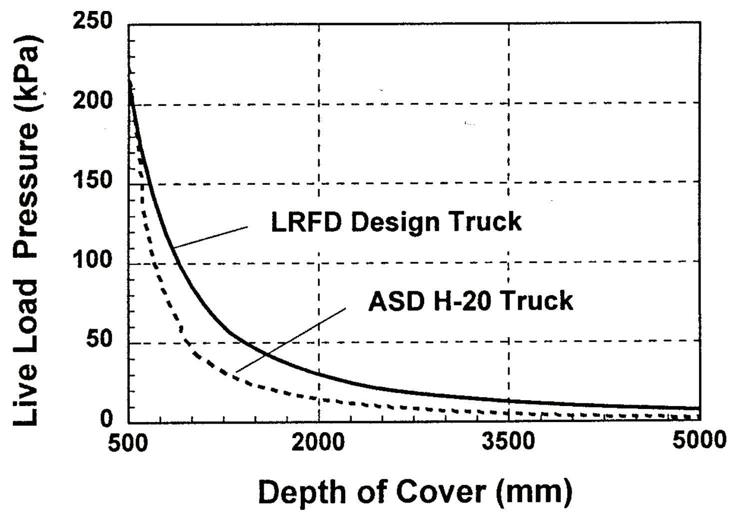


Figure 4-16
Comparison of AASHTO Live Load Pressures Through Earth Fills

- Calculating the distributed pressure at the crown of the culvert due to tire contact pressures at the road surface from alternative design vehicles

- Calculating the lane load pressure at the crown of the culvert

- Determining which combination of alternative design vehicle plus lane load that produces the largest distributed pressure at the crown of the culvert

To perform these steps, the following independent and dependent variables, and geometric constants must be determined or evaluated:

- Independent Variables - Depth of cover (D_E), group axle load (AL), load factor (γ), soil spreading factor (S_E), number of lanes (N) and capacity reduction factor (R)

- Dependent Variables - Tire footprint length (ℓ), dynamic load allowance (IM), distributed truck/tandem pressure (DTP), multiple presence factor (m), wheel load (P) and distributed length (L_D)

- Geometric Constants - Tire footprint width, w, (0.51 m), truck width (1.80 m), tandem spacing (1.20 m), lane width (3.60 m) and lane clearance (0.60 m)

The general relationship for distributing design vehicle live loads to culverts is:

$$DTP = \frac{m\,R\,\gamma\,AL\left(1+\frac{IM}{100}\right)}{A_D} \qquad \text{(Eq. 4-36)}$$

where:

DTP =	Distributed truck or tandem pressure at depth D_E (kPa)
m =	Multiple presence factor (dim)
R =	Reduction factor, based on ADTT (dim)
ADTT =	Number of trucks per day in one direction averaged over design life of structure (vehicles/day)
γ =	Live load factor for the limit state evaluated (dim)
AL =	Group axle load = 145 kN for design truck and 220 kN for design tandem
IM =	Dynamic load allowance at depth D_E (%)
A_D =	Sum of individual distributed wheel pressure areas from total axle group or net area defined by perimeter of overlapping distributed wheel areas $(m)^2$

The general relationship for calculating the distributed lane load, DLP, on a culvert is calculated as:

$$DLP = \frac{9.3mR\gamma}{3.00} \qquad \text{(Eq. 4-37)}$$

where:

DLP = Distributed lane pressure (kPa)

The vehicle lane load is distributed at the surface and includes the multiple presence factor, m, and ADTT reduction factor, R (*from A3.6.11.2*), based on the number of lanes that govern the design vehicle, but does not include the dynamic load allowance factor, IM.

Characteristics of the design truck and design tandem were described in Section 4.2.2.1. The extreme effect of this load combination (designated HL-93 in the AASHTO LRFD Specification), is determined by considering each possible combination of loaded lanes, the potential for multiple presence of vehicles in each loaded lane, and the average one-way daily truck traffic. Other factors which can affect determination of the design live load include:

- Culvert diameter and height of cover
- Dimensions and contact area of the tire footprint at the ground surface
- Dynamic load allowance factor
- Number of contributing lanes
- Reduction factor based on average one-way daily truck traffic (ADTT)

- Distributed area of pressure contribution at the culvert crown depth

For the design vehicle, the surface tire contact area equals the width of each wheel (i.e., 0.51 m) times the wheel length given by:

$$\ell = 0.00228\,\gamma\left(1+\frac{IM}{100}\right)P \qquad \text{(Eq. 4-38) } (\textit{A3.6.1.2.5-1})$$

where:

ℓ = Length of tire contact in direction of travel (m)
γ = Live load factor for the limit state evaluated (dim)
P = Wheel load = 72.5 kN for design truck and 55 kN for design tandem
IM = Dynamic load allowance at the road surface (%)

The value of IM is determined as:

$$IM = 40\,(1.0 - 0.41\,D_E) \geq 0\% \qquad \text{(Eq. 4-39).}(\textit{A3.6.2.2-2})$$

where:

D_E = Depth below road surface (m); $D_E = 0$ for calculation of tire contact area at road surface.

For calculation of pressures due to dynamic loads, D_E = depth of cover from surface to culvert crown. The effects of dynamic load allowance can be safely neglected for depths of cover greater than about 2.4 m.

The maximum live load force effects are to be determined by considering each possible combination of number of loaded lanes multiplied by the multiple presence factors given in Table 4-13. The factors in Table 4-13 are based on average one-way daily truck traffic (ADTT) of 5000 vehicles. For sites having significantly less truck traffic, a reduction factor (R) may be applied to the multiple presence factors of Table 4-13. Values for (R) are based on ADTT as follows:

- For $100 \leq ADTT \leq 1000$, R = 0.95
- For ADTT < 100, R = 0.90

The dimensions of the distributed area at depth is equal to the dimensions of the tire footprint area at the surface plus the depth, D_E, times the soil spreading factor, S_E. In the AASHTO LRFD Specification, the distribution of surface live load pressure at depth is spread through the soil cover as follows (*A3.6.1.2.6*):

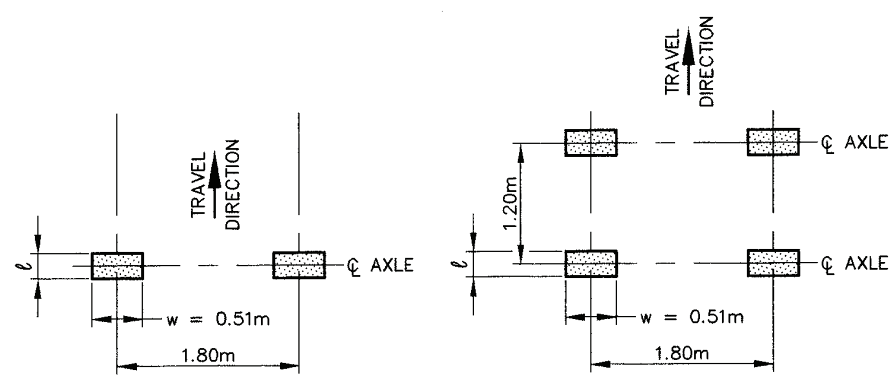
- For average backfill, S_E = 1.00 (i.e., 63° spreading); and

- For select, well-compacted backfill, $S_E = 1.15$ (i.e., 60° spreading).

Table 4-13 (*A3.6.1.1.2-1*)
Multiple Presence Factors
(After AASHTO, 1997a)

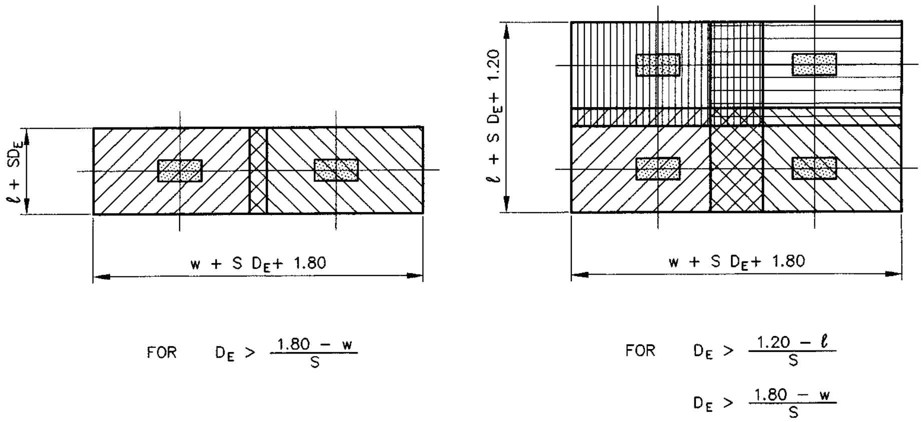
Number of Loaded Lanes	Multiple Presence Factor, m
1	1.20
2	1.00
3	0.85
> 3	0.65

The distributed area calculation is more complicated than merely adding the product ($S_E D_E$) to the surface footprint dimensions because as depth increases, distributed areas begin to overlap, either from adjacent wheel loads, adjacent axle loads, or at greater depths, from adjacent lanes. The concept of overlapping distributed area is shown in Figures 4-17 and 4-18. Figure 4-17 illustrates the tire footprint areas at the surface for both the design truck and the design tandem vehicles, whereas Figure 4-18 illustrates the distributed area at depth when both wheel and axle loads have overlapped.



(a) Design Truck (b) Design Tandem

Figure 4-17
Surface Loading Pattern



(a) Design Truck (b) Design Tandem

Figure 4-18
Distributed Area At Depth

The transition depths at which overlap occurs varies with load factor, γ, magnitude of wheel load, P, dynamic load allowance, IM, and the soil spreading factor, S_E. Several transition depths occur due to varying geometry between truck width (1.80 m), tandem spacing (1.20 m), and lane width (3.60 m). Figure 4-18 provides dimensions of the distributed areas for the case of a single lane with full overlap for both the design truck and the design tandem vehicles.

The distributed length, L_D, due to the single lane contribution is calculated as:

For the design truck:

$$L_D = \ell + S_E D_E \qquad \text{(Eq. 4-40)}$$

where:

L_D = Length dimension in direction of travel for distributed area at depth D_E (m)

- For the design tandem where $D_E \leq \frac{1.20 - \ell}{S_E}$:

$$L_D = 2(\ell + S_E D_E) \qquad \text{(Eq. 4-41)}$$

- For the design tandem where $D_E > \frac{1.20 - \ell}{S_E}$:

$$L_D = \ell + S_E D_E + 1.20 \qquad \text{(Eq. 4-42)}$$

The distributed width, W_D, due to the single lane contribution for both the design truck and design tandem is:

- For $D_E \leq \dfrac{1.80 - w}{S_E}$:

$$W_D = 2\,(w + S_E\,D_E) \qquad \text{(Eq. 4-43)}$$

where:

W_D = Width dimension perpendicular to the direction of travel for distributed area at depth D_E (m)

w = Width dimension perpendicular to direction of travel of surface tire contact area (m); for design truck and design tandem, w = 0.51 m

- For $D_E > \dfrac{1.80 - w}{S_E}$:

$$W_D = w + S_E\,D_E + 1.80 \qquad \text{(Eq. 4-44)}$$

The distributed area, A_D, is then determined by:

$$A_D = L_D\,W_D \qquad \text{(Eq. 4-45)}$$

The calculation of distributed area and length can be simplified with reasonable accuracy using only one transition depth using the following relationships:

- Design truck, one lane contribution:

$$A_D = 4.056\,(S_E\,D_E)^{1.33} \qquad \text{(Eq. 4-46)}$$

$$L_D = 0.231\gamma + S_E\,D_E \qquad \text{(Eq. 4-47)}$$

- Design tandem, one lane contribution:

$$A_D = 0.675\,(S_E\,D_E)^3 - 3.53\,(S_E\,D_E)^2 + 13.40\,(S_E\,D_E) - 3.18 \qquad \text{(Eq. 4-48)}$$

For $D_E \leq \dfrac{1.20 - 0.176\gamma}{S_E}$:

$$L_D = 0.176\gamma + S_E D_E \qquad \text{(Eq. 4-49)}$$

For $D_E > \dfrac{1.20 - 0.176\gamma}{S_E}$:

$$L_D = 0.176\gamma + S_E D_E + 1.20 \qquad \text{(Eq. 4-50)}$$

The effect of multiple lanes also can be investigated using Eq. 4-36. The results are presented in Figure 4-19 for the design truck and Figure 4-20 for the design tandem. Note that the single lane contribution could be used conservatively in lieu of considering multiple lane effects for depths of cover less than 2.4 m. Also, at a depth of 2.4 m, the distributed pressures from the design truck and design tandem are nearly equal. At shallower depths, the design truck appears to govern the design for all multiple lane loading combinations.

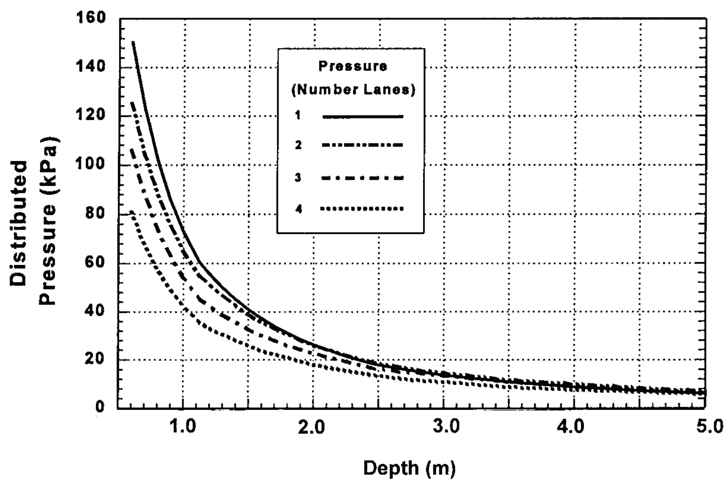


Figure 4-19
Live Load Distribution Through Soil for Design Truck

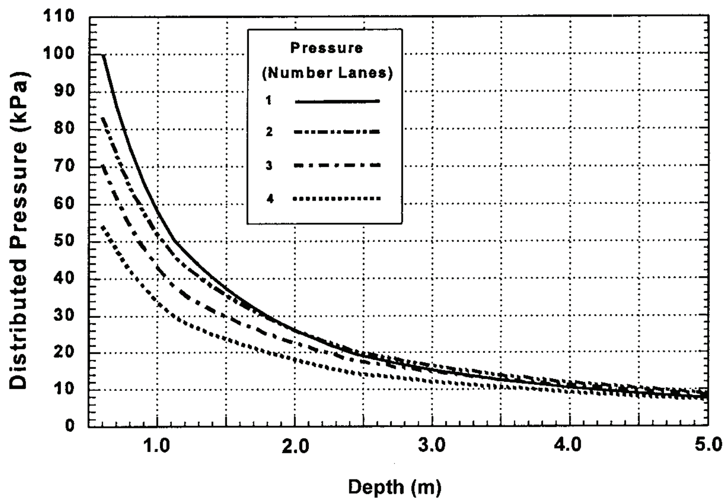
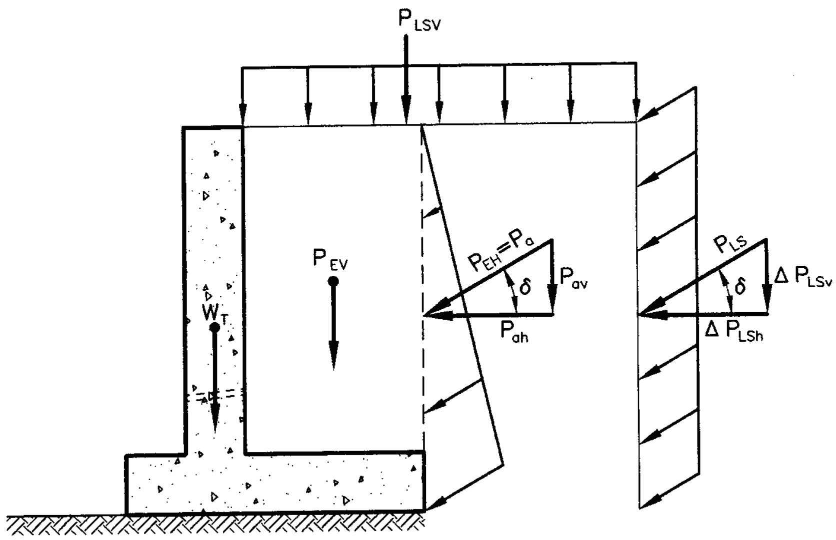


Figure 4-20
Live Load Distribution Through Soil for Design Tandem

This evaluation demonstrates that accounting for multiple presence factors for buried culverts can lead to unrealistic and unconservative results for more than two lanes. At 1 m of cover, a 35 percent reduction in pressure is permitted for structures loaded by 4 or more lanes (Table 4-13). Clearly, vehicles that are three lanes away should not be considered either as adding to or subtracting from the pressure distribution under shallow cover. Further modification of the AASHTO LRFD Specification is needed to better define the influence of multiple lane loadings for culvert design.

4.6 Student Exercise

Problem: Select load factors for the Strength I and Service I Limit States from Tables 4-10 and 4-11 for the problem illustrated below.



Loading Diagram for Student Exercise

For this exercise, complete the following table with the appropriate load factors for the critical load combinations for the performance limits indicated.

Load Effect	Limit State and Performance Limit			
	Strength I			Service I
	Sliding	Bearing	Overturning	Settlement
Vertical Live Load Surcharge, LS_V				
Horizontal Live Load Surcharge , LS_H				
Horizontal Earth Load, EH				
Vertical Earth Load, EV				
Concrete Dead Load, DC				

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4.7 Student Problem: Load Combinations for Retaining Wall

Problem: The cantilever retaining wall in Figure 4-21 is being considered for a grade separation between roadway lanes in a non-seismic area. The wall will be backfilled with a free draining granular fill such that the seasonal high water table will be below the bottom of the footing. The vehicular live load surcharge, LS, on the backfill will be applied as shown in the figure.

Objective: You need to develop unfactored and factored loads and moments needed for the geotechnical design of the cantilever retaining wall.

Approach: You will perform the evaluation using the following steps:

- Calculate the unfactored loads and resulting moments due to structure components, earth pressures and live load surcharge
- Select the load factors and load combinations controlling geotechnical design
- Calculate the factored loads and moments by multiplying the unfactored loads and moments by the appropriate load factors and load combinations

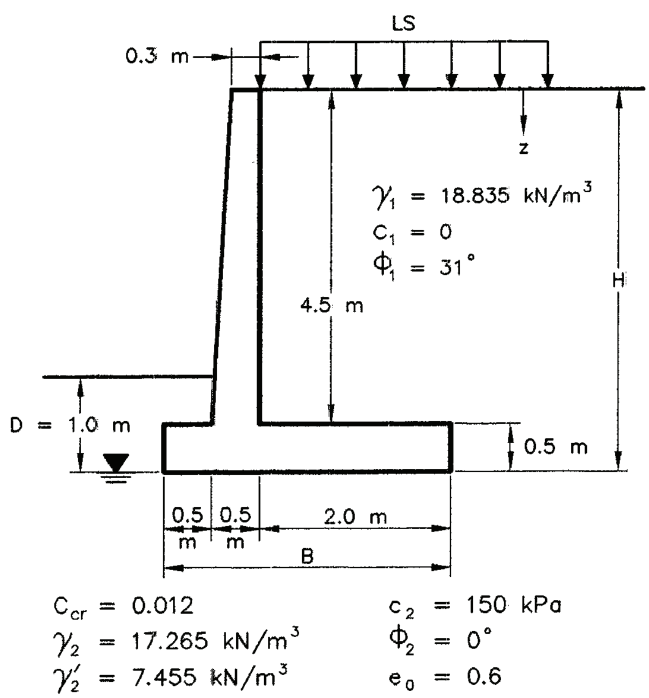


Figure 4-21
Schematic of Student Problem

Step 1: Calculate the Unfactored Loads

(A) Dead Load of Structural Components and Nonstructural Attachments (DC)
Referring to Figure 4-22 and assuming a unit weight of concrete, γ_C, equal to 23.544 kN/m^3:

$\mathbf{W_1} = B_1\, H_1\, \gamma_c = (0.3\text{ m})(4.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{31.8\ kN/m}$

$\mathbf{W_2} = 1/2\, B_2\, H_1\, \gamma_c = (0.5)(0.2\text{ m})(4.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{10.6\ kN/m}$

$\mathbf{W_3} = B\, H_2\, \gamma_c = (3.0\text{ m})(0.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{35.3\ kN/m}$

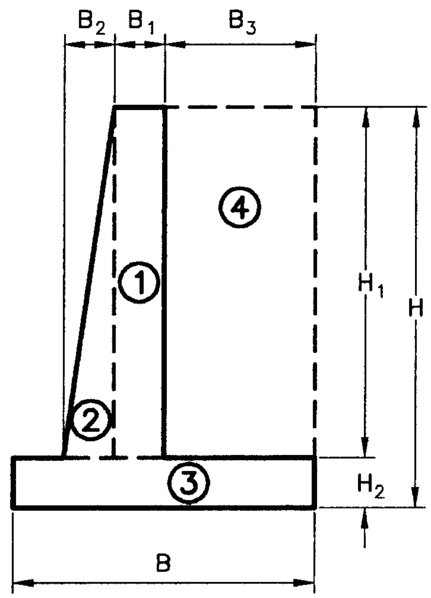


Figure 4-22
Retaining Wall Area Designation for Weight of Concrete

(B) Vertical Earth Pressure (EV)
Unit Weight of Soil $\gamma_1 = 18.835$ kN/m^3
Weight of Soil on Footing

$\mathbf{P_{EV}} = W_4 = B_3\, H_1\, \gamma_1 = (2.0\text{ m})(4.5\text{ m})(18.835\text{ kN/m}^3) = \mathbf{169.5\ kN/m}$

(C) Live Load Surcharge (LS)
A live load surcharge is applied when vehicle loads will be supported on the backfill within a distance equal to H. The live load surcharge is applied as an equivalent height of soil for the design vehicle loading (h_{eq}) using Table 4-2, and a wall height of 5 m.

By interpolation, $\mathbf{h_{eq}} = \mathbf{0.907\ m}$

Using the unit weight of the soil backfill (i.e., γ_1 = 18.835 kN/m^3), the unit vertical live load surcharge, LS, over the heel of the wall is:

$p_{LSV} = \gamma_1 \, h_{eq}$ = (18.835 kN/m^3)(0.907 m) = 17.1 kPa (kN/m^2)

For a heel width B_3 of 2 m:

P_{LSV} = $p_{LSV} \, B_3$ = (17.1 kN/m^2)(2 m) = **34.2 kN/m of wall length**

The active earth pressure coefficient k_a for a wall friction angle, $\delta = \phi_f = 31^o$, and a horizontal backslope is, from AASHTO (1997a) *Table 3.11.5.3-1*:

$k = k_a = 0.29$

From Eq. 4.7, the lateral earth pressure due to the live load surcharge is:

$\Delta p = k_s \, \gamma'_s \, h_{eq} = k_a \, \gamma'_1 \, h_{eq}$= (0.29)(18.835 kN/m^2)(0.907 m) = 4.95 kPa

Using a rectangular distribution, the live load lateral earth pressure resultant is:

$P_{LS} = \Delta p \, H$ = (4.95 kN/m^2) (5 m) = 24.8 kN/m of wall length

The horizontal and vertical components of the live load lateral earth pressure are:

ΔP_{LSh} = P_{LS} coṣ = (24.8 kN/m)(cos 31^o) = **21.3 kN/m**
ΔPLS_v = P_{LS} siṇ = (24.8 kN/m)(sin 31^o) = **12.8 kN/m**

(D) Lateral Earth Pressure (EH)
The lateral earth pressure is assumed to vary linearly with the depth of soil backfill as given by:

$p = k_h \, \gamma_s \, z$ (Eq. 4-2)

where $k_h = k_a = 0.29$

At the base of the footing (i.e., @ z = H):

p = (0.29)(18.835 kN/m^3)(5.00 m) = 27.3 kPa (kN/m^2)

The resultant of the basic lateral earth pressure (triangular distribution) acting on the wall is:

$P_{EH} = P_a = 0.5 \, pH$ = (0.5)(27.3 kN/m^2)(5.00 m) = 68.3 kN/m length of wall

The horizontal and vertical components of the lateral earth pressure are:

P_{ah} = P_a cosδ = (68.3 kN/m)(cos 31°) = **58.6 kN/m**

P_{av} = P_a sinδ = (68.3 kN/m)(sin 31°) = **35.2 kN/m**

(E) Summary of Unfactored Loads

Table 4-14
Unfactored Vertical Loads and Resisting Moments

Item	V kN/m	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
W_1			
W_2			
W_3			
P_{EV}			
P_{LSV}			
ΔP_{LSv}			
P_{av}			
TOTAL			

Table 4-15
Unfactored Horizontal Loads and Overturning Moments

Item	H (kN/m)	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
ΔP_{LSh}			
P_{ah}			

Step 2: Determine the Appropriate Load Factors
In theory, structures could be evaluated for each of the limit states identified in Section 4.3. However, depending on the particular loading conditions and performance characteristics of a structure, only certain limit states need to be evaluated. For the classroom example problem, each limit state will be qualitatively assessed below relative to that limit state is applicable for the design problem:

- Strength I - Basic load combination related to the normal vehicular use of the bridge without wind. ***(Applicable as a standard load case)***.

- Strength II - Load combination relating to the use of the bridge by Owner-specified special design vehicles and/or evaluation permit vehicles, without wind. ***(Not applicable because special vehicle loading is not specified)***.

- Strength III - Load combination relating to the bridge exposed to wind velocity exceeding 90 km/hr without live loads. ***(Not applicable because wall is not subjected to other than standard wind loading)***.

- Strength IV - Load combination relating to very high dead load to live load force effect ratios exceeding about 7.0 (e.g., for spans greater than 75 m). ***(Applicable because dead loads predominate)***.

- Strength V - Load combination relating to normal vehicular use of the bridge with wind velocity of 90 km/hr ***(Not applicable because wind load not a design consideration)***.

- Extreme Event I - Load combination including earthquake. ***(Not applicable because problem does not include earthquake loading)***.

- Extreme Event II - Load combination relating to ice load or collision by vessels and vehicles. ***(Not applicable because problem does not include ice or collision loading)***.

- Service I - Load combination relating to the normal operational use of the bridge with 90 km/hr wind***.*** ***(Applicable for design loading)***.

- Service II - Load Combination intended to control yielding of steel structures and slip of slip-critical connections due to vehicular live load. ***(Not applicable due to structure type.)***

- Service III - Load combination relating only to tension in prestressed concrete structures with the objective of crack control. ***(Not applicable due to structure type.)***

- Fatigue - Fatigue and fracture load combination relating to repetitive gravitational vehicular live load and dynamic responses under a single design truck. ***(Not applicable due to structure type.)***

Consequently, only the Strength I, Strength IV and Service I Limit States apply to the retaining wall design. Therefore, from Tables 4-10 and 4-11, select the applicable load factors and combinations and present them in Table 4-16. ***(Note: Strength I-a and I-b represent the Strength I Limit State using minimum and maximum load factors, respectively, from Table 4-11.)***

Table 4-16
Load Factors

Group	γ_{DC}	γ_{EV}	γ_{LS}	γ_{EH} (active)	Probable Use
Strength I-a					
Strength I-b					
Strength IV					
Service I					

Notes: BC - Bearing Capacity; EC - Eccentricity; SL - Sliding

By inspection:

- Strength I-a (minimum vertical loads and maximum horizontal loads) will govern for the case of sliding and eccentricity (overturning)
- For the case of bearing capacity, maximum vertical loads will govern, and the factored loads must be compared for Strength I-b and Strength IV

Step 3: Calculate the Factored Loads and Factored Moments

Table 4-17
Factored Vertical Loads

Group/ Item Units	W_1 kN/m	W_2 kN/m	W_3 kN/m	P_{EV} kN/m	P_{LSV} kN/m	ΔP_{LSv} kN/m	P_{av} kN/m	V_{TOT} kN/m
V (Unf.)	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4
Strength I-a								
Strength I-b								
Strength IV								
Service I								

Table 4-18
Factored Horizontal Loads

Group/Item Units	ΔP_{LSh} kN/m	P_{ah} kN/m	H_{TOT} kN/m
H (Unf.)	21.3	58.6	79.9
Strength I-a			
Strength I-b			
Strength IV			
Service I			

Table 4-19
Factored Moments from Vertical Forces (M_v)

Group/ Item Units	W_1 kN-m/m	W_2 kN-m/m	W_3 kN-m/m	P_{EV} kN-m/m	P_{LSV} kN-m/m	ΔP_{LSv} kN-m/m	P_{av} kN-m/m	M_{vTOT} kN-m/m
M_v (Unf.)	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0
Strength I-a								
Strength I-b								
Strength IV								
Service I								

Table 4-20
Factored Moments from Horizontal Forces (M_h)

Group/Item Units	ΔP_{LSh} kN-m/m	P_{ah} kN-m/m	M_{hTOT} kN-m/m
M_h (Unf.)	53.3	97.9	151.2
Strength I-a			
Strength I-b			
Strength IV			
Service I			

Summary

This example illustrates:

- ***Selection of critical limit states (load combinations), load factors***
- ***Development of factored loads for geotechnical design of a reinforced cantilever retaining wall***
- ***For cantilever retaining walls, dead, earth and live load surcharge are the predominate loads***
- ***Because the load factor for active horizontal (or lateral) earth pressure in Table 4-11 is the same for all limit states, the controlling limit states for a typical cantilever retaining wall are generally those for which dead load and live load surcharge load factors in Table 4-11 are the greatest (i.e., Strength IV and Strength I, respectively)***
- ***Minimum load factors typically control for sliding and eccentricity criteria, because the lower factored soil and concrete dead weights provide less resistance to sliding and overturning***

- ***Maximum load factors typically control for bearing as the higher factored soil and concrete dead weights exert a higher bearing pressure***

The geotechnical (foundation) design for the retaining wall in this example is presented in the Classroom Example in Chapter 8, Section 8.4.

4.8 Design Example:
Load Combinations for Bridge Pier Foundation

Problem: This example illustrates the development of unfactored and factored loads and load combinations needed for the geotechnical and structural design of a bridge pier foundation. The bridge being designed is a multi-span, two-lane bridge supported on reinforced concrete, hammerhead piers. The pier in this example is located on the riverbank above the design high water level and is skewed 10 degrees. The pier has a design height of 9.5 m above the top of footing or pile cap, and the roadway surface is 2.213 m above the top of the pier.

Approach: To define the critical load combinations for geotechnical and structural design of a bridge pier foundation, the following steps are taken:

- The loads and resulting moments developed by the structural engineer for design of the bridge pier are reviewed and tabulated
- The factored loads and moments for limit states applicable to design of the pier, as determined by the structural engineer, are tabulated
- The critical limit states and load combinations for design of the pier foundation are established based on a review of the tabulation of factored design loads and moments from the pier design, and the factored loads and moments are adjusted to account for the footing or pier cap and overlying soil

The geotechnical design of spread footing-, pile- and drilled shaft-supported foundations for the pier are presented in Chapters 8, 9 and 10, respectively.

Step 1: Summarize Unfactored Loads and Moments
The loads which the structural engineer has determined to be applicable to the pier design are described briefly below, followed by the structural engineer's tabulation of unfactored loads and moments:

(DC) *Dead Load of Structural Components and Non-Structural Attachments*
This load includes the weight of the bridge superstructure and the pier cap and stem, and results in a longitudinal moment and axial load at the base of the pier stem.

(DW) *Dead Load of Wearing Surfaces and Utilities*
This load consists of the weight of pavement on the bridge deck, and results in a longitudinal moment and axial load at the base of the pier stem.

(LL) (IM) *Vehicular Live Load with Dynamic Load Allowance*
For design of the pier, the structural designer has determined that the maximum force effect is produced with 90% of two design trucks and 90% of the design lane load. The pier designer has determined that four live load conditions apply, as follows:

LL_1 Two lanes fully loaded on both spans with traffic offset to one side of bridge (transverse loading)

LL_2 Two lanes fully loaded on one span and offset to one side of bridge (biaxial loading)

LL_3 One lane fully loaded on both spans and offset to outside of bridge (transverse loading)

LL_4 One lane fully loaded on one span and offset to outside of bridge (biaxial loading)

Whereas transverse loading results only in a transverse moment and axial load on the pier stem, biaxial loading also results in a longitudinal moment on the stem.

(BR) *Vehicular Braking Force*
Braking forces for one (BR_1) and two (BR_2) lanes of traffic are considered. Due to the 10° skew of the pier with respect to the bridge, the braking forces result in longitudinal and transverse horizontal loads and moments on the bridge pier stem.

(CE) *Vehicular Centrifugal Force*
The structural designer has determined that vehicular centrifugal forces are negligible.

(WS) (WL) *Wind on Structure and Live Load*
This load includes the effects of wind on the bridge superstructure and piers, and on vehicular traffic, respectively. The structural engineer has determined that the critical wind load cases with respect to loading on the pier stem are for wind perpendicular to the superstructure (Case 1) and wind at 45° to the superstructure (Case 2). A vertical wind pressure is also included on the deck for wind perpendicular to the superstructure.

(H) *Horizontal Thrust*
The structural engineer has incorporated an additional horizontal thrust force and moment on top of the pier associated with loads on the pier cap related to thermal expansion of the bridge deck support beams. Due to the skew of the pier, this thrust results in longitudinal and transverse loads and moments.

The unfactored loads and moments acting at the centroid of the base of the pier stem are summarized in Table 4-21.

Table 4-21
Unfactored Pier Loads and Moments

Load Designation	Axial Load (kN)	Horizontal Load (kN)		Moment (kN-m)	
		Long.	Trans.	Long.	Trans.
Dead Loads:					
DC	6129	0	0	167	0
DW	450	0	0	16	0
Braking Forces:					
BR_1	0	95	24	1279	319
BR_2	0	158	39	2131	531
Wind Case 1:					
WS_H	0	32	180	304	1723
WS_V	-294	0	0	0	772
WL	0	7	41	98	554
Wind Case 2:					
WS_H	0	150	105	1266	886
WL	0	24	17	326	228
Thrust:					
H_u	0	50	9	477	85
M_u	0	0	0	53	10
Live Load					
LL_1	1412	0	0	0	2504
LL_2	1044	0	0	345	1851
LL_3	848	0	0	0	3027
LL_4	627	0	0	207	2238

Step 2: Summarize Factored Loads and Moments from Structural Design of Pier Stem
The structural designer has determined that the following limit states are applicable for design of the bridge pier:

- Strength I: Normal vehicular use w/o wind
- Strength III: High wind velocity (143 km/h) w/o vehicular live load
- Strength V: Normal vehicular use w/ 90 km/h wind load
- Service I: Normal vehicular use w/ 90 km/h wind load

The structural designer has also defined eight applicable load combinations corresponding to the various possible combinations of live load, braking force and wind load as follows:

- Load Case A: LL_1, BR_2, Wind Case 1
- Load Case B: LL_1, BR_2, Wind Case 2
- Load Case C: LL_2, BR_2, Wind Case 1
- Load Case D: LL_2, BR_2, Wind Case 2
- Load Case E: LL_3, BR_1, Wind Case 1
- Load Case F: LL_3, BR_1, Wind Case 2
- Load Case G: LL_4, BR_1, Wind Case 1

- Load Case H: LL_4, BR_1, Wind Case 2

From Eq. 4-1, the factored load effect for each limit state and load combination is determined using:

$$Q = \sum \eta_i \gamma_i Q_i \qquad \text{(Eq. 4-1)}$$

where:

η_i = Factor related to operations importance, ductility and redundancy
γ_i = Applicable load factor for load Q_i

As the pier is deemed to be of significant operational importance, values of $\eta_i = \eta_I = 1.05$ and $\eta_i = 1/\eta_I = 1/1.05 = 0.95$ have been established for maximum and minimum load factors, respectively, using Eq. 4-34 and 4-35. Applicable load factors for the limit states and load combinations defined above are highlighted in Tables 4-22 and 4-23.

Table 4-22 (*A3.4.1-1*)
Load Combinations and Load Factors
(Modified after AASHTO, 1997a)

LOAD COMBINATION LIMIT STATE	DC DD DW EH EV ES	LL IM CE BR PL LS	WA	WS	WL	FR	(1) TU CR SH EL	TG	SE	Use one of these at a time			
										EQ	IC	CT	CV
STRENGTH-I (unless noted)	γ_p	1.75	1.00	-	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-II	γ_p	1.35	1.00	-	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-III	γ_p	-	1.00	1.40	-	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
STRENGTH-IV EH, EV, ES, DW DC ONLY	γ_p 1.50	-	1.00	-	-	1.00	0.50/ 1.20	-	-	-	-	-	-
STRENGTH-V	γ_p	1.35	1.00	0.40	0.40	1.00	0.50/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
EXTREME EVENT-I	γ_p	γ_{EQ}	1.00	-	-	1.00	-	-	-	1.00	-	-	-
EXTREME EVENT-II	γ_p	0.50	1.00	-	-	1.00	-	-	-	-	1.00	1.00	1.00
SERVICE-I	1.00	1.00	1.00	0.30	0.30	1.00	1.00/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
SERVICE-II	1.00	1.30	1.00	-	-	1.00	1.00/ 1.20	-	-	-	-	-	-
SERVICE-III	1.00	0.80	1.00	-	-	1.00	1.00/ 1.20	γ_{TG}	γ_{SE}	-	-	-	-
FATIGUE-LL, IM & CE ONLY	-	0.75	-	-	-	-	-	-	-	-	-	-	-

(1) The reduced values of γ are used when calculating force effects other than displacements.

Shaded areas represent Limit States evaluated in example problem.

Table 4-23 (*A3.4.1-2*)
Load Factors For Permanent Loads, γ_p
(Modified after AASHTO, 1997a)

Type of Load	Load Factor	
	Maximum	Minimum
DC: Component and Attachments	1.25	0.90
DD: Downdrag	1.80	0.45
DW: Wearing Surfaces and Utilities	1.50	0.65
EH: Horizontal Earth Pressure		
• Active	1.50	0.90
• At-Rest	1.35	0.90
EV: Vertical Earth Pressure		
• Retaining Structure	1.35	1.00
• Rigid Buried Structure	1.30	0.90
• Rigid Frames	1.35	0.90
• Flexible Buried Structures	1.95	0.90
• Flexible Metal Box Culverts	1.50	0.90
ES: Earth Surcharge	1.50	0.75

As the thrust, H_u, (transmitted through shear deformation of the elastometric bearings which support beams on the piers) is used to compute force effects other than joint or bearing displacements, the lower load factors in Table 4-22 for TU apply.

The thrust moment, M_u, is a factored moment resulting from moment transfer due to rotational deformation superimposed on the pier by temperature changes in the superstructure. Because M_u is already factored, a load factor of 1.0 for TU is applied for design at the Strength Limit State. M_u is not applied for design at the Service Limit State.

In consideration of the potential that deflections due to bending of the pier about its weak (transverse) axis may result in a magnification of longitudinal moments on the pier, the structural designer has computed longitudinal moment magnification factors for each load combination and Strength Limit State based on the factored loads and pier stiffness. The Moment Magnification Factors summarized in Table 4-24 were developed using the approximate method presented in the AASHTO LRFD Specification (*A4.5.3.2.2b*).

Table 4-24
Moment Magnification Factors, γ

Load Comb.	A		B		C		D		E		F		G		H	
Limit State	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
STR. I	1.07	1.05	1.07	1.05	1.06	1.05	1.06	1.05	1.06	1.05	1.06	1.05	1.06	1.04	1.06	1.04
STR. II	1.05	1.03	1.05	1.04	1.05	1.03	1.05	1.04	1.05	1.03	1.05	1.04	1.05	1.03	1.05	1.04
STR. V	1.06	1.05	1.06	1.05	1.06	1.04	1.06	1.04	1.06	1.04	1.06	1.04	1.06	1.04	1.06	1.04

The factored load and moment effects for the various load combinations and limit states applicable to design of the pier stem are summarized in Table 4-25. Example calculations for the Load Case A Strength I and Service I Limit States follow.

Table 4-25
Summary of Factored Loads and Moments in Pier Stem

LOAD CASE		MAXIMUM LOADS					MINIMUM LOADS				
		Moments		Axial Load (kN)	Horiz. Load		Moment		Axial Load (kN)	Horiz. Load	
		Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)	Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)
A	STR I	4772	5632	11 348	317	77	4171	5096	7866	287	70
	STR III	1046	3723	8321	74	270	862	3369	5128	67	245
	STR V	3982	5314	10 755	267	153	3475	4808	7330	242	139
	SER I	2912	3803	7991	220	115	2912	3803	7991	220	115
B	STR I	4772	5632	11 348	317	77	4177	5097	7866	287	70
	STR III	2537	1358	8753	247	159	2189	1229	5519	224	144
	STR V	4514	4825	10 755	324	112	3948	4366	7330	294	102
	SER I	3269	3454	7991	261	85	3269	3454	7991	261	85
C	STR I	5427	4432	10 671	317	77	4761	4010	7254	287	70
	STR III	1046	3723	8321	74	270	862	3369	5128	67	245
	STR V	4488	4388	10 233	267	153	3926	3970	6858	242	139
	SER I	3257	3150	7623	220	115	3257	3150	7623	220	115
D	STR I	5427	4432	10 671	317	77	4761	4010	7254	287	70
	STR III	2537	1358	8753	247	159	2189	1229	5519	224	144
	STR V	5019	3899	10 233	324	112	4398	3528	6858	294	102
	SER I	3614	2801	7623	261	85	3614	2801	7623	261	85
E	STR I	3080	6203	10 311	201	49	2671	5613	6928	182	45
	STR III	1046	3723	8321	74	270	862	3369	5128	67	245
	STR V	2683	5754	9955	178	132	2318	5214	6606	161	120
	SER I	2060	4114	7427	157	100	2060	4114	7427	157	100
F	STR I	3080	6203	10 311	201	49	2671	5613	6928	182	45
	STR III	2537	1358	8753	247	159	2189	1229	5519	224	144
	STR V	3212	5266	9955	234	90	2790	4765	6606	212	82
	SER I	2417	3765	7427	198	70	2417	3765	7427	198	70
G	STR I	3475	4754	9905	201	49	3024	4302	6561	182	45
	STR III	1046	3723	8321	74	270	862	3369	5128	67	245
	STR V	2988	4636	9642	178	132	2591	4195	6323	161	120
	SER I	2267	3325	7206	157	100	2267	3325	7206	157	100
H	STR I	3475	4754	9905	201	49	3024	4302	6561	182	45
	STR III	2537	1358	8753	247	159	2189	1229	5519	224	144
	STR V	3517	4148	9642	234	90	3061	3753	6323	212	82
	SER I	2624	2976	7206	198	70	2624	2976	7206	198	70

[1] Note: Tabulated and calculated values may differ slightly due to round-off.

- ***Strength I; Axial Load:***

  $$\mathbf{Q = \eta\ [(\gamma_{DC}\ DC) + (\gamma_{DW}\ DW) + (\gamma_{LL}\ LL_1)}$$

  - For Maximum Load Factors:
    Q = 1.05[(1.25)(6129 kN)+(1.50)(450 kN)+(1.75)(1412 kN)] = **11 348 kN**

  - For Minimum Load Factors:
    Q = 0.95[(0.90)(6129 kN)+(0.65)(450 kN)+(1.75)(1412 kN)] = **7866 kN**

- ***Strength I; Longitudinal Moment:***

  $$\mathbf{M = \eta\ \delta\ [(\gamma_{DC}) + (\gamma_{DW}) + (\gamma_{BR} BR_2) + (\gamma_{TU} H_u) + M_u]}$$

  - For Maximum Load Factors:
    M = (1.05)(1.07)[(1.25)(167 kN-m) + (1.50)(16 kN-m) +
    (1.75)(2131 kN-m) + (0.5)(477 kN-m) + 53 kN-m] = **4779 kN-m**

  - For Minimum Load Factors:
    M = (0.95)(1.05)[(0.90)(167 kN-m) + (0.65)(16 kN-m) +
    (1.75)(2131 kN-m) + (0.5)(477 kN-m) + 53 kN-m] = **4171 kN-m**

- ***Service I; Axial Load:***

  $$\mathbf{Q = \gamma_{DC}\ DC + \gamma_{DW}\ DW + \gamma_{LL}\ LL_1}$$
  Q = (1.0) (6129 kN) + (1.0) (450 kN) + (1.0) (1412 kN) = **7991 kN**

Note: WS_v neglected due to its negative value. WS_v would only be included if investigating overturning of the bridge.

- ***Service I; Longitudinal Moment:***

  $$\mathbf{M = \gamma_{DC}DC + \gamma_{DW}DW + \gamma_{BR}BR_2 + \gamma_{WS}WS_H + \gamma_{WL}WL + \gamma_{TU}H_U}$$
  M = (1.0)(167 kN-m) + (1.0)(16 kN-m) + (1.0)(2131 kN-m) +
  (0.3)(304 kN-m) +(0.3)(98 kN-m) + (1.0)(477 kN-m)
  M = 2912 kN-m

Step 3: Develop Factored Load Combinations for Bridge Pier Foundation

Critical load combinations for design of the bridge pier foundation will generally be as follows:

(A) Bearing Resistance and Settlement

The critical load cases will be those resulting in the maximum factored axial load and moment, and the maximum average bearing pressure over the effective bearing area, determined as:

$q = Q/A'$

where the net effective bearing area, A′, is computed as:

$A' = [(B - 2\ e_B)(L - 2\ e_L)] = [(1.5\ m - 2\ e_B)(3.5\ m - 2\ e_L)]$

Values of the factored axial load and factored average bearing pressure at the base of the pier stem (for B=1.5 m and L=3.5 m) are summarized in Table 4-26. The eccentricities are computed as:

e_B = Longitudinal Moment/Axial Load
e_L = Transverse Moment/Axial Load

Table 4-26
Summary of Factored Axial Loads and Average Bearing Pressures at Base of Pier Stem

LOAD CASE		MAXIMUM LOAD							MINIMUM LOAD		
		Axial Load (kN)	e_B (m)	e_L (m)	A′ (m^2)	q (kPa)	Axial Load (kN)	e_B (m)	e_L (m)	A′ (m^2)	q (kPa)
A	STR. I	11 348	0.421	0.496	1.650	6876	7866	0.531	0.648	0.965	8148
	STR. III	8321	0.126	0447	3.252	2559	5128	0.168	0.657	2.545	2015
	STR. V	10 755	0.370	0.494	1.909	5633	7330	0.474	0.656	1.208	6068
	SER. I	7991	0.364	0.476	1.967	4062	7991	0.364	0.476	1.967	4062
B	STR. I	11 348	0.421	0.496	1.650	6876	7866	0.531	0.648	0.965	8148
	STR. III	8753	0.290	0.155	2.935	2982	5519	0.397	0.223	2.156	2560
	STR. V	10 755	0.420	0.449	1.717	6263	7330	0.539	0.596	0.974	6808
	SER. I	7991	0.409	0.432	1.798	4445	7991	0.409	0.432	1.798	4445
C	STR. I	10 671	0.509	0.415	1.287	8292	7254	0.656	0.553	0.450	16 117
	STR. III	8321	0.126	0.447	3.252	2559	5128	0.168	0.657	2.544	2015
	STR. V	10 233	0.439	0.429	1.643	6227	6858	0.573	0.579	0.829	8271
	SER. I	7623	0.427	0.413	1.727	4413	7623	0.427	0.413	1.727	4413
D	STR. I	10 671	0.509	0.415	1.287	8292	7254	0.656	0.553	0.450	16 117
	STR. III	8753	0.290	0.155	2.935	2982	5519	0.397	0.223	2.156	2560
	STR. V	10 233	0.490	0.381	1.424	7187	6858	0.641	0.514	0.539	12 725
	SER. I	7623	0.474	0.367	1.527	4993	7623	0.474	0.367	1.527	4993
E	STR. I	10 311	0.299	0.602	2.071	4979	6928	0.386	0.810	1.369	5062
	STR. III	8321	0.126	0.447	3.252	2559	5128	0.168	0.657	2.545	2015
	STR. V	9955	0.270	0.578	2.250	4424	6606	0.351	0.788	1.535	4302
	SER. I	7427	0.277	0.554	2.263	3282	7427	0.277	0.554	2.263	3282
F	STR. I	10 311	0.299	0.602	2.071	4979	6928	0.386	0.810	1.369	5062
	STR. III	8753	0.290	0.155	2.935	2982	5519	0.397	0.223	2.156	2560
	STR. V	9955	0.323	0.529	2.085	4774	6606	0.422	0.721	1.350	4893
	SER. I	7427	0.325	0.507	2.113	3515	7427	0.325	0.507	2.113	3515
G	STR. I	9905	0.351	0.480	2.027	4887	6561	0.461	0.656	1.265	5188

	STR. III	8321	0.126	0.447	3.252	2559	5128	0.168	0.657	2.545	2015
	STR. V	9642	0.310	0.481	2.233	4317	6323	0.410	0.663	1.478	4277
	SER. I	7206	0.315	0.461	2.243	3213	7206	0.315	0.461	2.243	3213
H	STR. I	9905	0.351	0.480	2.027	4887	6561	0.461	0.656	1.265	5188
	STR. III	8753	0.290	0.155	2.935	2982	5519	0.397	0.223	2.156	2560
	STR. V	9642	0.365	0.430	2.033	4743	6323	0.484	0.594	1.230	5140
	SER. I	7206	0.364	0.413	2.064	3491	7206	0.364	0.413	2.064	3491

(I) Deep Foundations:

The critical loading conditions for evaluation of the bearing resistance and settlement of a group of piles or drilled shafts are combinations of axial load and moment which produce the maximum axial stresses in the piles or shafts.

For design at the Strength Limit State, the critical combinations from Table 4-25 are:

- Case A, Strength I Limit State, Max Loads - produces maximum combination of axial load (11 348 kN) and moment (5632 kN-m) in the transverse direction.

- Case D, Strength I Limit State, Max Loads - produces maximum combination of axial load (10 671 kN) and moment (5427 kN-m) in the longitudinal direction.

For design at the Service Limit State, the critical combinations from Table 4-25 are:

- Case A, Service I Limit State & Case E, Service I Limit State produce maximum combinations of axial load and moment in the transverse direction, respectively:
  Case A, Service I: Q = 7991 kN; M_T = 3803 kN-m
  Case E, Service I: Q = 7427 kN; M_T = 4114 kN-m

- Case B, Service I Limit State & Case D, Service I Limit State produce maximum combinations of axial load and moment in the longitudinal direction, respectively:
  Case B, Service I: Q = 7991 kN; M_L = 3269 kN-m
  Case E, Service I: Q = 7623 kN; M_L = 3614 kN-m

(II) Spread Footing Foundations:

The critical loading conditions for evaluation of bearing resistance and settlement of spread footing foundations are those which produce the maximum factored and unfactored bearing pressures.

For design at the Strength Limit State, the critical load combination from Table 4-26 is:

- Case C/D, Strength I, Minimum Loads
  q = 16 117 kPa

For design at the Service Limit State, the critical load combination from Table 4-26 is:

- Case D, Service I Limit State
  q = 4993 kPa

(B) Overturning:

(I) Deep Foundations:
Overturning failure is not typically evaluated for deep foundation systems for bridge piers. Overturning would be considered only for structures subjected to extreme uplift and/or horizontal loads which could result in net tension loading of deep foundation elements.

(II) Spread Footing Foundations
The critical loading condition for spread footing foundations are those which produce the maximum base pressure resultant eccentricity. This failure mode is checked only for the Strength Limit State.

The critical loading conditions, from Table 4-26, are:

- Case C/D, Strength I, Minimum Loads - produces maximum eccentricity in the longitudinal direction:
  e_B = 0.656 m

- Case E/F, Strength I, Minimum Loads - produces maximum eccentricity in the transverse direction:
  e_L = 0.810 m

(C) Lateral Loading/Sliding and Lateral Deflection

(I) Deep Foundations
The critical loading conditions for lateral loading of deep foundation groups are generally combinations of maximum horizontal load, moment and axial load which produce the greatest foundation element stresses and lateral deflections.

For Strength Limit State Design, the critical load combinations from Table 4-25 are:

- Cases A/C, Strength III, Maximum Loads - produce most severe loading in the transverse direction:
  Horizontal load = 270 kN

- Cases C/D, Strength I, Maximum Loads - produce most severe loading in the longitudinal direction:
  Horizontal load = 317 kN

For Service Limit State Design, the critical load combinations from Table 4-25 are:

- Cases A/E, Service I - produce most severe loading in the transverse direction:
  Case A, Service I: Horizontal load = 115 kN; M_T = 3803 kN-m
  Case E, Service I: Horizontal load = 100 kN; M_T = 4114 kN-m

- Case D, Service I - produces most severe loading in the longitudinal direction:
  Case D, Service I: Horizontal load = 261 kN; M_L = 3614 kN-m

(II) Spread Footing Foundations

The critical loading conditions for sliding of spread footing foundations are those which produce the greatest horizontal loads and horizontal to vertical load ratios. This failure mode is checked only for the Strength Limit State. The critical load combinations for sliding of spread footing foundations, from Table 4-25, are:

- Case A/C, Strength III, Minimum Loads - produces maximum horizontal load (245 kN) and horizontal to vertical load ratio (0.05) in the transverse direction

- Case D, Strength V, Minimum Loads - produces maximum horizontal load (294 kN) and horizontal to vertical load ratio (0.04) in the longitudinal direction

(E) Summary of Critical Loading Combinations:

Table 4-27
Critical Load Combinations for Deep Foundations

Critical Load Combination	Evaluation Criteria
Cases A/D, Strength I, Max Loads	Bearing Resistance
Cases A/C, Strength III, Max Loads Cases C/D, Strength I, Max Loads	Lateral Load Resistance
Cases A/B/D/E, Service I	Settlement and Lateral Deflection

Table 4-28
Critical Load Combinations for Spread Footing Foundations

Critical Load Combinations	Evaluation Criteria
Cases C/D, Strength I, Min Loads	Bearing Resistance
Cases C/D/E/F, Strength I, Min Loads	Overturning (Eccentricity)
Cases A/C, Strength III, Min Loads Case D, Strength V, Min Loads	Sliding
Case D, Service I	Settlement

Factored Foundation Design Loads
The factored loads and moments in Table 4-25 represent values at the base of the pier stem (i.e., at the top of the footing or pile cap) resulting from the dead load of and external loads on the bridge superstructure and pier. For geotechnical and structural design of the pier foundation, the loads must be adjusted to include the effects of the footing weight, the weight of soil above the foundation, and any lateral loads associated with the overlying soil.

For this example, the ground surface at the pier location is essentially horizontal, such that adjustments are required only for the weight of the footing (or pile cap) and overlying soil. Assume a footing or pile cap having plan dimensions (B_F X L_F) of 5.2 m by 5.2 m and a thickness of 1.2 m overlain by 0.5 m of soil having a density, = 18.835 kN/m^3.

(I) Footing/Pile Cap Weight
The unfactored dead load of the footing is:

$DC_F = 765$ kN

The factored dead load of the footing is:

$Q_F = \eta\ (\gamma_{DC}\ DC_F)$

For an importance factor (η_i) of 1.05 and a maximum dead load factor (γ_{DC}) of 1.25

$Q_F = 1.05\ (1.25)\ 765$ kN = **1004 kN**

For an importance factor (η_i) of 0.95 and a minimum dead load factor (γ_{DC}) of 0.90:

$Q_F = 0.95\ (0.90)\ 765$ kN = **654 kN**

For design at the Service Limit State, η =1.0 and γ_{DC} = 1.0, such that:

Q_F = **765 kN**

(II) Soil Pressure on Footing
The unfactored vertical earth pressure on top of the footing is:

EV = 206 kN

- The factored earth pressure on the footing is:

$Q_S = \eta\ (\gamma_{EV}\ EV)$

For an importance factor η_I = 1.05 and a maximum earth load factor, γ_{EV} = 1.35:

Q_S = 1.05(1.35)206 kN = **292 kN**

For an importance factor (η) of 0.95 and a minimum load factor of 0.90:

Q_S = 0.95(0.90)206 kN = **176 kN**

For design at the Service Limit State:

Q_S = **206 kN**

(III) Summary of Factored Loads for Possible Critical Loading Combinations

For this example, it is assumed that the pier stem is centered on the footing. Therefore, the footing and overlying soil weight impart no unbalanced loading such that they only increase the vertical load at the base of the footing. The factored loads from Table 4-25 for the possible critical loading combinations in Tables 4-27 and 4-28, adjusted for footing weight and embedment, are summarized in Table 4-29.

Table 4-29
Summary of Factored Loads for Critical Foundation Design Load Combinations

LOAD CASE		MAXIMUM LOADS					MINIMUM LOADS				
		Moments		Axial Load (kN)	Horiz. Load		Moment		Axial Load (kN)	Horiz. Load	
		Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)	Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)
A	STR I	4772	5632	12 644	317	77	---	---	---	---	---
	STR III	1046	3723	9617	74	270	862	3369	5958	67	245
	SER I	2912	3803	8962	220	115	---	---	---	---	---
B	STR I	4772	5632	12 644	317	77	---	---	---	---	--
	SER I	3269	3454	8960	261	85	---	---	---	---	---
C	STR I	5427	4432	11 967	317	77	4761	4010	8084	287	70
	STR III	1046	3723	9617	74	270	862	3369	5958	67	245
D	STR I	5427	4432	11 967	317	77	4761	4010	8084	287	70
	STR V	---	---	---	---	---	4398	3528	7688	294	102
	SER I	3614	2801	8594	261	85	---	---	---	---	---
E	STR I	---	---	---	---	---	2671	5613	7758	182	45
	SER I	2060	4114	8398	157	100	---	---	---	---	---
F	STR I	---	---	---	---	---	2671	5613	7758	182	45

Shaded areas duplicate another Load/Limit State Case

The design of a pile foundation to support the pier in this example is described in Chapter 9, Design Example 1, Section 9.6.

CHAPTER 5
GEOTECHNICAL SITE CHARACTERIZATION

5.1 Introduction

Geotechnical characterization of sites for bridge and other highway structures is an integral part of the design process. Proper site characterization has always been needed for the design of highway substructures, and the evaluation of prospective structure sites should be performed as a collaborative effort between the structure designer, geotechnical engineer, and for structures near rivers and streams, the hydraulic engineer. The LRFD concepts presented herein for the design of foundations and retaining walls were developed and calibrated based on the use of specific types of information to define the subsurface conditions and engineering properties of soil and rock materials and the use of resistance factors consistent with the reliability of the information. This chapter provides some of the information essential to any refinement of this calibration process along with a brief review of the exploration and testing methods which can be used to develop the geotechnical information needed for design.

The following sections of this chapter:

- Describe the process and basis for planning exploration and testing programs for substructure design using LRFD
- Describe the sources of variability in exploration and testing that lead to uncertainty in estimated material properties
- Identify the statistical parameters necessary for development or calibration of a resistance factor

5.2 Planning Exploration and Testing Programs

5.2.1 General

Subsurface exploration and testing are required for highway structures to:

- Aid in the preliminary selection of substructure types which are viable for a particular site, and for bridges, the type of superstructure to be supported;
- Provide a basis for selecting soil and rock properties needed for substructure design; and
- Identify special subsurface conditions requiring special provisions to supplement standard construction specifications

Examples of the latter include the presence of elevated ground water levels which could affect the stability of excavations during construction, and the presence of corrosive materials which could affect the service life of a structure.

For most highway projects, subsurface data are acquired during the planning stages when the feasibility of particular routes and structure types are being evaluated, and during the design stage when localized site-specific data are required. Additional data regarding subsurface conditions are not usually required after the design stage unless the need arises to resolve questions and/or install construction-phase instrumentation and monitoring. For most projects, the following type of subsurface information is needed for the selection, design and construction of highway substructures:

- Definition of stratum boundaries
- Variation of ground water level
- Location of foundation bearing level
- Magnitude of structure settlement or heave
- Potential instability of slopes and excavation bottoms
- Lateral earth pressure and excavation support
- Construction dewatering requirements
- Use of excavated material

Because it is not possible to develop criteria for selecting the type, location and frequency of subsurface exploration and testing applicable for all sites and types of planned construction, it is important that the project team include geotechnical engineers, engineering geologists and geologists so that their perspective is available to the team.

Geotechnical load and resistance factor design requires an initial "calibration" of the relationship between the applied loads and available resistance. The predictive model(s) and accompanying in-situ or laboratory tests used to estimate the resistance of soil and rock are inherent in the determination of an appropriate resistance factor in this calibration process. For the present state of practice, this calibration process has used available data bases covering the types of exploration and testing programs usually performed. Consequently, the resistance factors recommended by the AASHTO LRFD Specifications are not related in a precise manner to variables such as the number of borings, exploration depth, or boring spacing, but instead reflect the standard of care representative of each data set considered. To a large extent it will never be possible to rationally specify a single set of requirements for exploration programs due to the range of subsurface conditions which may be encountered. In the future, additional research utilizing comprehensive exploration and testing programs may allow evaluations of the best means to optimize and quantify exploration program requirements.

5.2.2 Soil and Rock Variability

In developing exploration and testing programs, the geotechnical engineer should qualitatively assess the effects of variables such as the expected type and importance of the structure, magnitude and rate of loading, and viability of foundation alternatives relative to technical, economic and constructibility considerations. From the planning stage, information regarding land use and topographic, surficial soil and geologic conditions can be used to define guidelines for developing subsurface exploration and testing programs. Sources of information available for many sites include:

- Topographic Maps - Landforms, ground slopes and shapes, and stream locations

- Aerial Photographs - Information on landforms, soil types, rock structure and stream types

- Agricultural Soil Maps - Landform, soil associations, soil descriptions and approximate engineering characteristics for surficial soils

- Well Drilling Logs - General description of soil and rock, and ground water levels at the time of drilling

- Existing Borings - Information from subsurface explorations in the vicinity of a structure.

For example, given the project information that a heavily-loaded bridge pier is to be located adjacent to an old river with low velocity might suggest the need to use some type of deep foundation systems due to the likely presence of relatively soft, fine-grained alluvial soils. With this information, the geotechnical engineer can determine the types and number of in-situ and laboratory tests needed for the expected geology, foundation type(s) to be used, load conditions to be evaluated (i.e., undrained versus drained loading) and stress history of the foundation soils. A drained analysis should be based on effective stress parameters and an undrained analysis should be based on total stress parameters. The stress history of the foundation soils is also an important factor in the design process. For example, whereas the stability of an embankment constructed on a lightly overconsolidated clay will be controlled by undrained behavior representative of the short-term case, a similar embankment built over a deposit of heavily over consolidated clay will probably be controlled by drained behavior representative of the long term case. Accordingly, selection of an appropriate in-situ test method must reflect these considerations so that the test program will yield the necessary information about the subsurface conditions. For example, whereas a SPT can be expected to provide reasonable information about the effective stress friction angle for a clean sand, it has almost no value in evaluating the undrained shear strength of a clay. Likewise, a CPT will not provide meaningful information in a deposit of coarse gravel.

At present, the variability of subsurface conditions and the level of subsurface exploration (i.e., number of borings) or testing (i.e., number of in-situ or laboratory tests) are not explicitly related to resistance factors used in the AASHTO LRFD Specification. Instead, planning subsurface exploration and testing programs are based on guidelines such as those recommended by FHWA (1988) in Table 5-1, the availability of information from previous explorations in the vicinity of the site, and/or engineering experience and judgment.

A simple quantitative measure of the variability of data or an engineering property is the coefficient of variation, COV, which is defined as:

$$COV = \sigma / \bar{x} \qquad \text{(Eq. 5-1)}$$

where:

COV = Coefficient of variation (dim);
σ = Standard deviation of the data; and
$\bar{x}$ = Mean value of data.

Table 5-1 - Guideline Minimum Boring and Sampling Criteria
(Modified After FHWA, 1988)

Geotechnical Feature	Minimum Number of Borings	Minimum Depth of Borings
Structure Foundation	1 per substructure unit for width $\leq$ 30 m 2 per substructure unit for width > 30 m	Advance borings: (1) through unsuitable foundation soils into competent material of suitable bearing capacity and; (2) to a depth where $\Delta\sigma_v <$ 10% of existing effective soil overburden stress or; (3) a minimum of 3 m into bedrock if bedrock is encountered at shallower depth.
Retaining Walls	Borings alternatively spaced every 30 to 60 m in front of and behind wall.	Extend borings to depth of 2 times wall height or a minimum of 3 m into bedrock.
Culverts	Two borings depending on length.	See structure foundations.
Bridge Approach Embankments Over Soft Ground	For approach embankments placed over soft ground, one boring at each embankment to evaluate embankment stability and foundation settlement. (Note: Borings for approach embankments usually located at proposed abutment locations to serve a dual function.)	See structure foundations. Shallow explorations at approach embankment locations are an economical means to determine depth of unsuitable surface soils.
Cuts and Embankments	Borings typically spaced every 60 m (erratic conditions) to 150 m (uniform conditions) with at least one boring taken in each separate landform. For high cuts and fills, 2 borings along a line perpendicular to centerline or planned slope face to establish geologic cross-section for analysis.	Cut: 1) In stable materials, extend borings a minimum of 3 to 5 m below cut grade. 2) In weak soils, extend borings below cut grade to firm materials, or to depth of cut below grade whichever occurs first. Embankment: Extend borings to firm material or to depth of twice the embankment height.

The greater the value of COV, the less reliable the data. An example of how the COV can be used as a guide in planning a subsurface exploration program is illustrated in Figure 5-1.

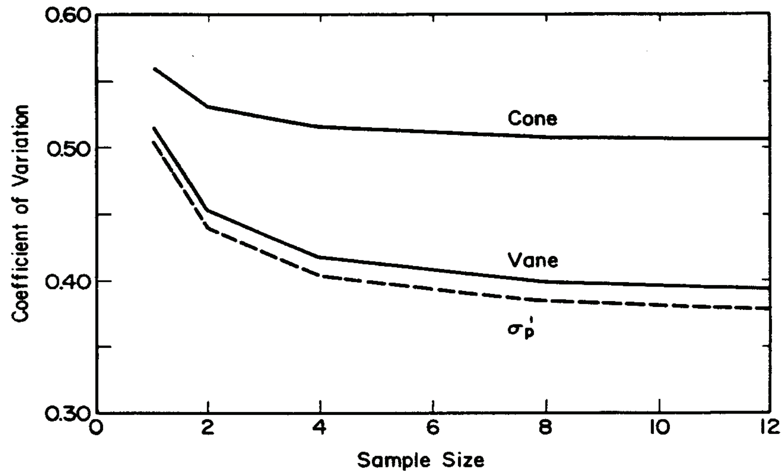


Figure 5-1
Reliability Variation with Sample Size for Indirect Testing for Mobilized Undrained Shear Strength
(after Teng, et al., 1992)

Figure 5-1 relates the COV of the mobilized undrained shear strength, S_u, estimated using cone penetrometer (CPT) and field vane shear (VST) in-situ tests with the number of samples tested for a slope stability problem (Teng, et al., 1992). These in-situ tests are compared with a correlation of S_u with the preconsolidation stress, σ'_p, as determined by laboratory consolidation tests. As expected, the COV decreases with increasing numbers of samples until a limiting value is reached. The figure also shows that the field VST is a more reliable method for estimating S_u than the CPT, and that the field VST and laboratory test methods provide comparable results. In the future, it is conceivable that relationships such as those shown in Figure 5-1 could be used in designing site exploration programs with the type and number of tests selected to achieve a specified β_T.

In addition to considerations regarding the type and extent of exploration, another important factor in planning any subsurface exploration is the cost-benefit relationship of the exploration program relative to construction cost. In general, as the amount and quality of subsurface exploration increases, the uncertainties and resulting conservatisms in the design process decrease. Conversely, an inadequate geotechnical exploration program can result in significant project cost overruns during construction as shown in the Table 5-2 which summarizes a recent study of 58 highway projects in the United Kingdom.

Table 5-2
Comparison of Ground Investigation Cost to Project Cost Overruns
(Modified after Whyte, 1995)

Ground Investigation Cost/ Total Project Cost (%)	Mean Cost Increase Due to Geotechnical Origins (% of Total Project Cost)
$\leq$ 1.5	14
1.5 - 2.0	8
> 2.0	4

The process of optimization of subsurface exploration with respect to the project foundation costs is illustrated from a conceptual perspective in Figure 5-2.

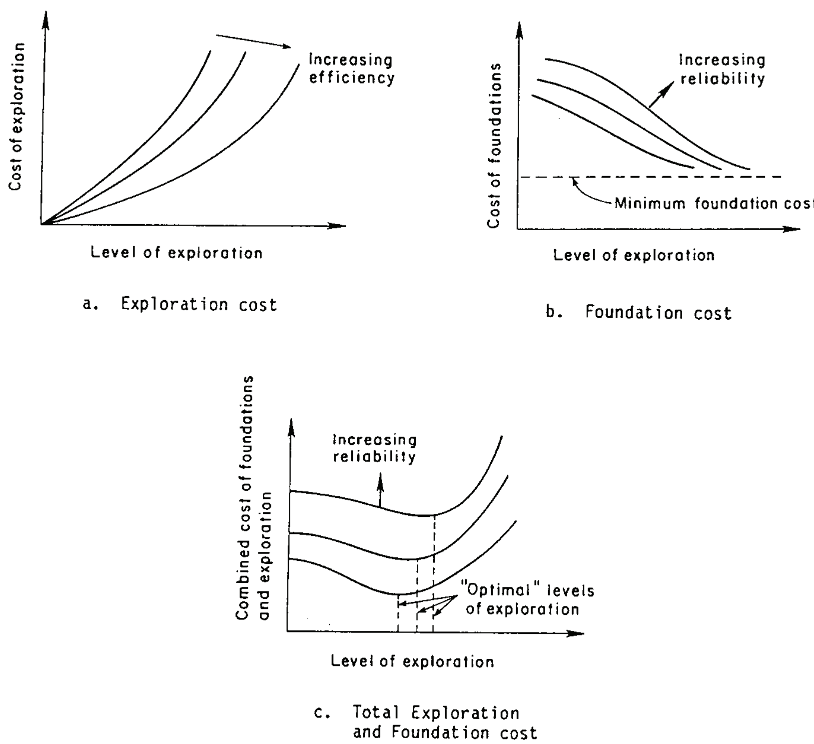


Figure 5-2
Optimization of Foundation and Exploration Cost
(after Kulhawy, et al., 1983)

Figure 5-2a shows the efficiencies of scale which may be realized by an expanded exploration program, and Figure 5-2b shows the expected trend of decreasing foundation cost with the increasing reliability associated with greater exploration. These two concepts are combined in Figure 5-2c which shows the optimization of the combined costs of exploration and foundations. While it may never be possible to quantify the combined costs to establish an optimal level of exploration, the concepts of reliability-based design provide a more rational framework for developing economical subsurface exploration programs than is currently practiced.

Optimization of the exploration program should also consider the reliability of the different methods available for engineering property assessment of soil and rock. The three primary sources of error which contribute to the uncertainty of material resistance estimates were identified in Chapter 3 as:

- *Inherent Spatial Variability* represented by the uncertainty in using point measurements compared to measurements reflecting a larger volumetric extent

- *Measurement Error* due to equipment and testing procedures

- *Model Error* reflected by the uncertainty of the predictive method

To develop a resistance factor, ϕ for a particular design approach (e.g., bearing resistance of a spread footing on sand using SPT) or subsurface conditions, these sources of uncertainty must be combined with uncertainties in load and the level of safety required. As described in Section 3.4.6 of Chapter 3, these factors can be combined to develop a ϕ-factor for design using:

$$\phi = \frac{\lambda_R \left(\gamma_D \dfrac{Q_D}{Q_L} + \gamma_L\right)\sqrt{\dfrac{1 + COV_{QD}^2 + COV_{QL}^2}{1 + COV_R^2}}}{\left(\lambda_{QD}\dfrac{Q_D}{Q_L} + \lambda_{QL}\right)\exp\left[\beta_T\sqrt{\ln(1 + COV_R^2)(1 + COV_{QD}^2 + COV_{QL}^2)}\right]} \qquad \text{(Eq. 5-2)}$$

where loads and uncertainty in loads are represented by:

γ_D, γ_L =	Dead load factor and live load factor, respectively (dim)
Q_D/Q_L =	Ratio of dead load to live load (dim)
λ_{QD}, λ_{QL} =	Bias on dead load and live load, respectively (dim)
COV_{QD} =	Coefficient of variation of the overall bias on dead load (dim)
COV_{QL} =	Coefficient of variation of the overall bias on live load (dim)

The desired level of safety is represented by:

β_T = Reliability index (dim)

and the uncertainty of material resistance is represented by:

λ_R = Overall bias on the resistance (dim); and
COV_R = Coefficient of variation of the overall bias on resistance (dim)

The value of COV_R, is determined using:

$$COV_R = \sqrt{COV^2_{MODEL} + COV^2_{MEASUREMENT} + COV^2_{INHERENT}} \qquad \text{(Eq. 5-3)}$$

where:

COV_{MODEL} = Coefficient of variation of predictive model (dim)
$COV_{MEASUREMENT}$ = Coefficient of variation of property measurement (dim)
$COV_{INHERENT}$ = Coefficient of variation of inherent soil variability (dim)

Figure 5-3 shows the relationship between ϕ and COV_R for $Q_D/Q_L = 2.0$ (i.e., characteristic of a medium-span structure) using Eq. 5-2. Representative values of the other variables are presented on the figure and were developed from information presented in Chapter 3.

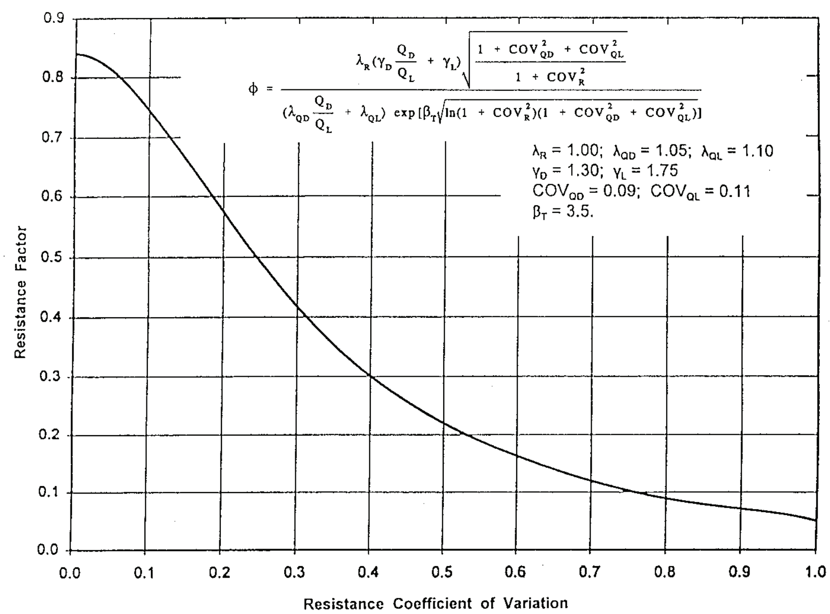


Figure 5-3
Relationship Between ϕ and COV_R for $Q_D/Q_L = 2.0$ Using Eq. 5-2

The following section provides information about the inherent variability of important soil and rock properties and measurement errors resulting from various in-situ and laboratory test methods. Model error is discussed in Chapter 6.

5.3 Field Test Methods

A variety of methods are available for the design of foundations and walls based on the results of field tests. The results of these tests can be used to estimate engineering properties (e.g., strength or compressibility) needed for analysis or design, or used directly in semi-empirical design methods for design (e.g., ultimate axial bearing resistance of a driven pile). The most commonly used in-situ test methods for geotechnical site characterization and foundation design include:

- Standard Penetration Test (SPT)
- Cone Penetration Test using a mechanical (MCPT) or electrical (ECPT) device
- Field Vane Shear Test (VST)
- Pressuremeter Test (PMT)

This section summarizes the devices and procedures used for each method and discusses the relative reliability and sources of error in each.

The resistance factors presented in the chapters for foundation design take into account different in-situ test methods and reflect the inherent reliability of each test type. Table 5-3 shows the coefficient of variation (COV) of the three sources of error for each test method, total COV range for the test method and the most likely range in COV for each test method when reasonable care is exercised in performance of the test. As shown in this table, the test results of the SPT are likely to be less reliable than those of the VST or ECPT due to the larger inherent variability in the SPT. This expectation is well reflected in the resistance factors presented in later sections on foundations, which recommend using $\phi = 0.35$ for bearing capacity based on SPT derived friction angles and $\phi = 0.45$ for CPT derived friction angles. The data shown in Table 5-3 can be used to calibrate models not presented herein.

Table 5-3
Estimates of In-Situ Test Variability
(after Orchant, et al., 1988)

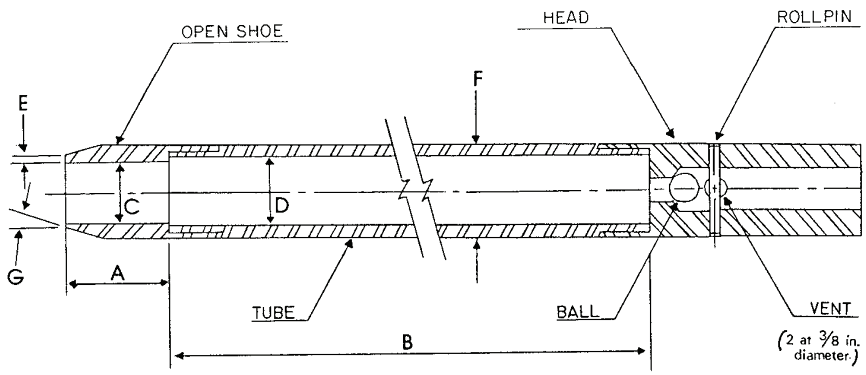
Test	COV Equipment	COV Procedure	COV Random	COV (1) Total	COV (2) Range
SPT	0.05 (3) - 0.75 (4)	0.05 - 0.075	0.12 - 0.15	0.14 - 1.00	0.15 - 0.45
MCPT	0.05	0.10 (5) - 0.15 (6)	0.10 - 0.15	0.15 - 0.22	0.15 - 0.25
ECPT	0.03	0.05	0.05 (5) - 0.10 (6)	0.07 - 0.12	0.05 - 0.15
VST	0.05	0.08	0.10	0.14	0.10 - 0.20
PMT	0.05	0.12	0.10	0.16	0.10 - 0.20 (7)

[1] COV (Total) = [COV (Equipment)^2 + COV (Procedure)^2 + COV (Random)^2]$^{1/2}$; [2] Due to limited data and judgment used to estimate COV, ranges represent probable magnitudes of test measurement error when reasonable care is exercised in performing test; [3] Best case scenario for SPT test conditions; [4] Worst case scenario for SPT test conditions; [5] Tip resistance CPT measurements; [6] Side resistance CPT measurements; [7] Results may differ for p_o, p_f, and p_L, but data are insufficient to clarify this issue

The following sections present a brief description of each of these test methods, as well as a more detailed discussion of the potential sources of measurement error and variability for each method. This discussion of each test is prefaced by more general comments regarding the applicability of the tests.

5.3.1 Split-Barrel Sampling

In the U.S., split-barrel (or split-spoon) sampling is the most common method of in-situ testing and disturbed sampling of soils. As shown in Figure 5-4, the steel split-barrel sampler consists of a 51 mm O.D. (38 mm I.D.) by 457 mm long tube which is split longitudinally to permit opening of the sampler and removal of soil retained within the sampler.



A = 25 to 50 mm; B = 0.457 to 0.762 m; C = 34.93 $\pm$ 0.13 mm; D = 38.1 $\pm$ 1.3 - 0.0 mm; E = 2.54 $\pm$ 0.25 mm; F =50.8 $\pm$ 1.3 - 0.0 mm; G = 16.0° to 23.0°

Figure 5-4
Split-Barrel Sampler
(ASTM, 1997)

The sampler is advanced into the ground by impacting the drill rods with a 620 Newton (N) weight falling from a height of 760 mm. Samples can be obtained continuously, although samples taken at intervals of 0.9 to 1.5 m are common. At shallow depths (e.g., 1.5 to 3 m), samples can be obtained by driving the sampler into the open borehole created by the sampler. For greater depths, samples are usually obtained by advancing the borehole using hollow-stem flight augers which provide access for the sampler and drill rods through the hole in the augers. In some parts of the U.S., the borehole is advanced by solid-flight augers and drilling mud to maintain the stability of the borehole.

When samples are obtained, the sampler is driven into the ground and the number of blows required to drive the sampler are recorded in three 150 mm intervals. The sum of blow counts for the last two 150 mm intervals is referred to as the Standard Penetration Test (SPT) blow count or "N"-value. The blow count is used to estimate the in-situ relative density of cohesionless soils and as a direct input to various design methods (e.g., for estimating footing settlement on sandy soils and axial capacity of driven piles). The standard method for the SPT is described in ASTM D 1586 (1997). Because the soil retained in the sampler is disturbed, the samples are used only for index laboratory tests such as moisture contents, Atterberg limits, and particle-size analyses. The soil samples are not for tests to estimate in-situ soil shear strength or compressibility.

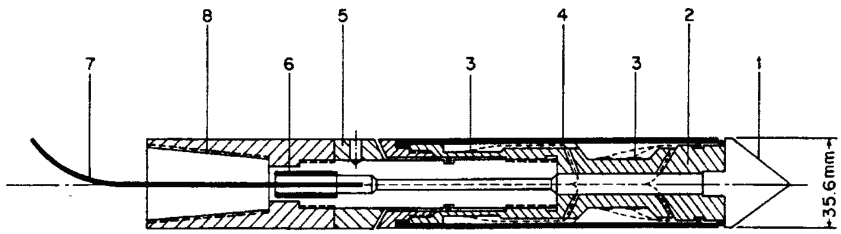
Several of the major sources of error that contribute to the variability in results due to equipment, procedure, and random variability are listed in Table 5-4. The ASTM standard for Standard Penetration Testing suggests that variations in N-values greater than 100 percent have been reported for separate drillers and different equipment working closely in the same soil layer, but current opinion indicates a COV of about 10 percent for a single apparatus and driller.

Table 5-4
Selected Major Sources of Error in the Standard Penetration Test
(after Kulhawy and Mayne, 1990)

Cause	Effect	Influence on N Value
Failure to maintain adequate head of water in borehole	Borehole bottom may become quick	Decreases
Hammer weight inaccurate	Hammer energy varies (5 to 7% common)	Increases or Decreases
Lack of hammer free fall	Hammer energy reduced	Increases
Sampler driven above casing bottom	Sample driven in disturbed, artificially densified soil	Increases greatly
Careless blow count	Inaccurate results	Increases or Decreases
Coarse gravel, cobbles, shells or cemented sands	Sampler becomes clogged or impeded	Increases

5.3.2 Cone Penetration Testing

The cone penetration test (CPT) involves pushing a cone-tipped device similar to that shown in Figure 5-5 into soil at a rate of 10 to 20 mm per second. Due to the need to push the device at a continuous rate, a special load apparatus is required, although it is possible to use a drill rig adapted with special attachments and operated by a skilled work crew. Cone penetration development began with a mechanical device which has largely been replaced with electrical devices such as that shown in Figure 5-5. For electrical cones, the maximum force is measured using a digital gage or strip chart attached by leads to strain-gaged sensors located in the tip and friction sleeve. Recent developments in CPT equipment permits measurement of pore water pressure, temperature, shear wave velocity, and some types of soil and ground water contaminants. The standard method for the CPT is fully described in ASTM D 3441 (1997). Additional details regarding application of CPT results for foundation design are presented in Riaund and Miran (1991).



1 Conical point (10 cm^2)
2 Load cell
3 Strain gages
4 Friction sleeve (150 cm^2)
5 Adjustment ring
6 Waterproof bushing
7 Cable
8 Connection with rods

Figure 5-5
Electric Friction-Cone Penetrometer Tip
(ASTM, 1997)

The results of the CPT can be used to continuously log and classify soil strata, estimate soil strength and compressibility, and to directly design shallow and deep foundations. Because no sample is retrieved from a CPT, some engineers do not use the test. Although the device is relatively rugged, CPT testing is difficult in very dense cohesionless soils, and in very stiff to hard cohesive soils due to the large reaction load needed to advance the cone.

The ASTM standard for cone penetration testing indicates that a standard deviation of 5 percent in tip resistance and 10 percent in side resistance results for electrical cones is typical when CPTs are conducted by experienced personnel. Some of the major sources of variability associated with the cone penetration test are shown in Table 5-5.

At present, the AASHTO LRFD Specification resistance factors have not been refined to the state where effects on testing such as those listed in Table 5-5 can be quantified in selection of an appropriate resistance factor. Some of the resistance factors for cone penetration testing in the AASHTO LRFD Specification are statistically based and include consideration of a range of potential reliability of the CPT, but other resistance factors are still simply a product of correlations with current ASD practice. In the future, compilation and analysis of large, detailed data sets may allow more rational selection of resistance factors and may incorporate the effects of more variables on testing procedure.

Table 5-5
Major Sources of Error in the Cone Penetration Test
(after Kulhawy and Mayne, 1990)

Cause	Effect	Influence on Results
Gravel, cobbles, shells or cemented sands	Impedes penetration of cone tip Causes deviations in instrument verticality	Increases q_c greatly Increase or decrease q_c and f_s
Worn penetrometer tip	Tip may become dull and/or surface roughness may become greater or lesser than standard	Increases or decreases q_c and f_s slightly.
Leaky water seal	Electrical transducers may become corroded	Increases or decreases q_c and f_s
Improper calibration	Inaccurate measurements	Increases or decreases q_c and f_s
Partial saturation of porous stone	Inaccurate measurements	Decreases pore pressure magnitude

5.3.3 Vane Shear Testing

The field vane shear test (VST) is performed by inserting a four-bladed vane in undisturbed soil at the base of a borehole, rotating the device from the surface while measuring the torque used in rotation, and then converting the torque to a unit shearing resistance along the cylindrical failure surface sheared by the vane. Figure 5-6 shows the field vane geometries allowed under the ASTM D 2573 (1997). After the peak strength is measured, the vane is rotated repeatedly through at least 10 revolutions and the remolded strength is measured. The ratio of the peak strength to the remolded strength is defined as the sensitivity and provides useful information about the structure and potential strain softening for a given deposit.

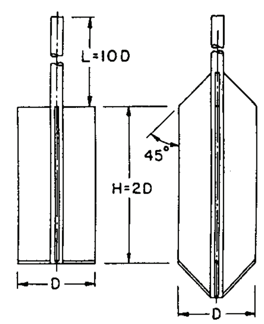


Figure 5-6
Field Vane Geometries and Sizes
(ASTM, 1997)

When performed correctly, the VST can provide one of the best measures of the in-situ undrained shear strength for soft or sensitive clays which are difficult to sample. While ASTM does not provide any commentary about the reliability of this test procedure, Orchant, et al. (1988) suggests a total COV of 0.14 for the VST. Table 5-6 presents some of the major sources of error in the test.

Table 5-6
Major Sources of Error in the Vane Shear Test
(after Kulhawy and Mayne, 1990)

Cause	Effect	Influence on Strength Measurement
Friction between torque rods and soil or casing	Measured torque includes spurious component of resistance	Increases
Poorly calibrated torque measurement	Inaccurate torque	Increases or decreases
Vane rotated too quickly	Soil sheared too rapidly	Increases
Test performed in disturbed soil	Soil structure broken down	Decreases
Damaged vane	Disturb soil excessively or shear only partial surface	Decreases
Unknown sand/silt/shell lenses	Drainage during test	Increases
Isolated gravel/cemented nodules	Measured torque includes spurious component of resistance	Increases

For tests performed in a cased hole, it is very important to limit the stress relief at the bottom of the hole by maintaining an adequate head of water or drilling mud to prevent disturbance near the hole bottom.

Again, many of the AASHTO LRFD resistance factors applicable to the vane shear test do not have a firm statistical basis. However, it is likely that sufficient data on the test itself exists to provide this basis and simply needs to be collected and analyzed.

5.3.4 Pressuremeter Test

The pressuremeter test is conducted by advancing a cylindrical probe into the ground and expanding the probe radially while measuring the changing probe volume and the pressure necessary for expansion. The probe can be inserted into a pre-bored hole as shown in Figure 5-7, or advanced by a self-boring mechanism which is an integral part of the device. ASTM D 4719 (1997) describes the testing process in detail. Once the probe is installed to the test depth, the PMT is performed by expanding the guard cells against the sides of the borehole and incrementally increasing the pressure in the measuring cell and measuring the change in volume or lateral displacement of the cell. Although numerous measurements of pressure and volume change are made during the test, key measurements are made at the beginning and end of the linear portion of the volume versus pressure curve as well as the limit pressure asymptote which is approached at the end of the test.

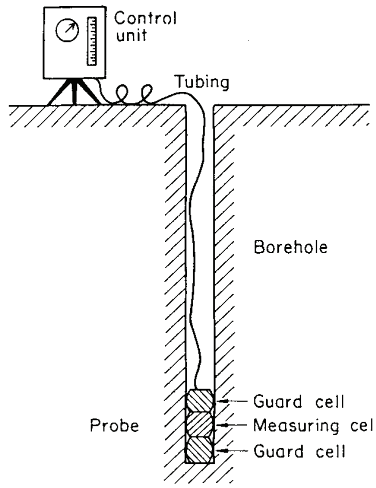


Figure 5-7
Menard Pressuremeter Equipment
(after Kulhawy and Mayne, 1990)

The PMT gives information which can be used to estimate the in-situ stress state, strength and stiffness of the soil, as well as provide information used in direct correlations with foundation capacity and settlement. Table 5-7 lists some of the major sources of error associated with the PMT. While the ASTM standard does not provide an evaluation of the test precision, it does emphasize the importance and difficulty of preparing a good borehole for each test.

Table 5-7
Major Sources of Error for the Pressuremeter Test
(after Kulhawy and Mayne, 1990)

Variable	Relative Effect on Test Results
Expansion of Tubing	Minor to moderate
Probe dimensions	Minor to moderate
Membrane aging	Minor
Method of drilling and borehole preparation	Significant
Rate of probe inflation	Minor to moderate

5.3.5 Other Methods
Other methods available for in-situ characterization include undisturbed sampling, rock coring and ground water level measurement. Although there are no quantitative measures of the reliability of these methods, they are discussed herein because the reliability of laboratory methods rely on the quality of soil and rock samples used for testing and accurate measurement of the ground water level is an important aspect of substructure design.

5.3.5.1 Undisturbed Sampling

If the strength, compressibility or permeability of in-situ cohesive soils of soft to stiff consistency is required, undisturbed samples are usually obtained by pushing a thin-walled tube into the soil. The typical thin-walled tube sampler shown in Figure 5-8 has a 76 mm O.D., a wall thickness of 1.7 mm, and a length of 900 mm. Larger diameter samplers are available for obtaining higher quality undisturbed samples of soft to medium stiff cohesive soils. One end of the tube has a machined taper to facilitate advancement of the sampler into the ground and minimize soil disturbance, and the other has four mounting holes for attachment to a head which threads to the drill rods used to push the sampler into the ground. For very soft to soft cohesive soils, it is sometimes necessary to attach a piston to the sampler to minimize sample disturbance and facilitate retention of soil in the tube.

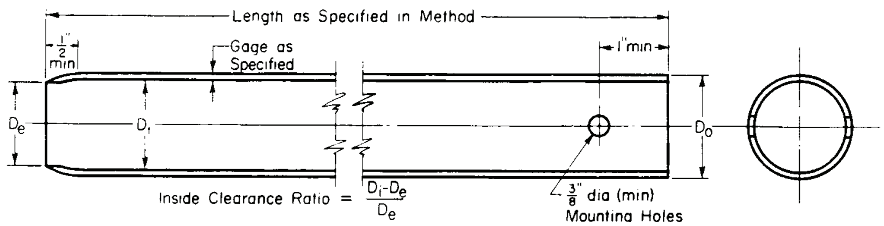


Figure 5-8
Thin-Walled Tube Sampler
(Modified after ASTM, 1997)

Thin-walled tube sampling does not produce a numerical result, and therefore cannot be described statistically. However, the quality of the sample taken can have a significant effect on the results of any subsequent testing of the sample. For soils particularly sensitive to disturbance effects, it may be desirable to evaluate the sample quality (e.g., by X-raying) of each tube.

5.3.5.2 Rock Coring

Rock core drilling using single-, double-, or triple-tube core barrels with diamond end bits is used to advance borings in rock when the resistance to driving a split-barrel sampler is greater than about 50 blows per 150 mm. A variety of core barrel sizes is available, but the most common size used is the NW-size core barrel which has an O.D. of 75.4 mm and an I.D. of 54.8 mm. The length of a core barrel is either 1.5 or 3.0 m. Double-tube core barrels such as shown in Figure 5-9 are most commonly used, although triple-tube core barrels are sometimes used when weak rock is drilled and high quality rock core is required for testing. A single-tube core barrel should not be used due to the poor quality of rock that is often recovered.

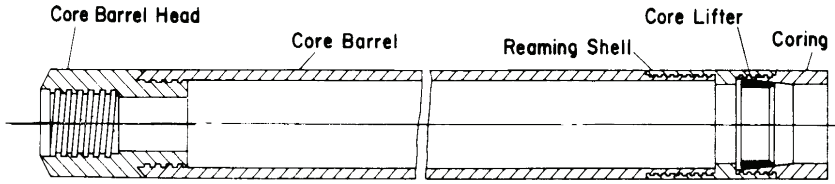


Figure 5-9
Double-Tube Core Barrel
(Fang, 1991)

The core barrel is attached to a string of drill rods, and the rods are attached to the kelly bar of a drill rig. Rock cutting is accomplished by a shoe or bit which is attached to the end of the core barrel and containing embedded industrial diamonds. Rock cuttings are removed from the borehole and the diamond bit is cooled by water which is flushed through the drill string, into the core barrel and up the sides of the borehole to the ground surface. Torque and thrust to the core barrel are provided by a drill rig. Coring is accomplished by inserting the core barrel and drill string to the rock surface within hollow-stem augers or casing sealed into the top of rock surface. Core is retained in the core barrel using a spring-like retainer inside the sampler. Special core barrels known as wireline samplers permit retrieval of the rock core from the barrel without removal of the drill rods from the boring by use of an inner liner of the core barrel which is lifted through the special drill rod using a steel wire line. When the full or partial length of coring is completed, the barrel and drill string are removed from the borehole and the sample is removed and usually placed in a core box. The standard method for diamond core drilling is fully described in ASTM D 2113 (1997).

5.3.5.3 Ground Water Location

Definition of ground water highs and lows over time is critical for the design process due to the importance of ground water during construction and its effect on the shear strength of the foundation medium. The design ground water level should reflect any possible seasonal variations and its selection should include consideration of direct ground water readings taken from borings and piezometers, as well as information inferred from soil coloration changes (e.g. mottling) observed during drilling and any other available records (e.g., adjacent river level fluctuations).

5.4 Laboratory Test Methods

5.4.1 Soil Index Testing

Soil classification is accomplished by a combination of visual identification and selective laboratory index testing to confirm the visual classification. The most commonly used method for classifying soils is the Unified Soil Classification System (USCS) as presented in ASTM D 2488 (1997) for visual identification and ASTM D 2487 (1997) for classification based on the results of laboratory index testing. To develop a soil classification using the USCS with laboratory tests, the particle-size distribution and Atterberg (i.e., liquid and plastic) limits of the soil are needed. Particle-size analyses provide the maximum size of particle and the size distribution of soil particles in the sample, and the liquid and plastic limits are used to classify the character of the fine soil fraction which is less than 0.425 mm in size. In addition to their usefulness in classifying soils, these tests can be used with the results of other index tests (i.e., moisture content, specific gravity and unit weight) to provide preliminary estimates of the possible engineering behavior of soils (e.g., Carter and Bentley, 1991). For example, the Atterberg limits and natural moisture content can be used as a guide to the consolidation and undrained shear strength behavior of cohesive soils, and the particle-size distribution of cohesionless soils can be used to estimate permeability. Because index tests are inexpensive and can be completed relatively quickly, it is useful to obtain and evaluate the results of these tests at the initial stages of the testing program to aid in selecting samples for engineering property testing.

Table 5-8 presents the inherent variability and measurement variability for several of the most commonly performed index tests for soil.

5.4.2 Rock Index Testing

The most commonly used index tests for characterizing rock behavior include the compressional wave velocity, V_p, Schmidt hammer rebound, R, and the point load test, I_s. These measurements are commonly used in correlations with mechanical properties such as tensile strength, T_o, unconfined compressive strength, C_o, and the Young's modulus, E. Due to the need to consider discrete lithological units, anisotropy, the sufficiency of the available data set for statistical analysis and distribution best suited to model the data, few studies present useful statistically summaries of the variability of these index tests. Grasso, et al. (1992) present the results of a study on a homogeneous, slightly anisotropic, moderately strong calcareous mudstone with more than 50 measurements each of V_p, R and I_s. The results of this study are summarized in Table 5-9 and serve as an illustration of the magnitudes of variability that might be expected for the relatively homogeneous material tested.

Table 5-8
Summary of Inherent Soil Variability and Measurement Variability for Index Tests
(after Phoon, et al, 1995)

Property[(1)]	Soil Type	Inherent Soil Variability Mean COV	Measurement Variability Mean COV	ASTM Precision Estimate COV
w_n	Fine-grained	0.18	0.08	----
w_l	Fine-grained	0.18	0.07	0.05
w_p	Fine-grained	0.16	0.1	0.17
PI	Fine-grained	0.29	0.24	----
LI	Clay, Silt	0.74	----	----
γ	Fine-grained	0.09	0.01	----
γ_d	Fine-grained	0.07	----	----
D_r	Sand	0.19[(2)]	----	----
	Sand	0.61[(3)]	----	----

[(1)] w_n = Natural water content; w_L = Liquid Limit; w_P = Plastic Limit; PI = Plasticity Index
LI = Liquidity Index; γ = Total unit weight; γ_d = Dry unit weight; D_r = Relative density
[(2)] Total variability for direct method of determination
[(3)] Total variability for indirect determination using SPT values

Table 5-9
Index Test and Inherent Variability of Calcareous Mudstone
(after Grasso, et al., 1992)

Index Test	Approximate COV
Compression Wave Velocity, V_p	0.09
Schmidt Hammer Rebound, R	0.19
Point Load Strength, I_s	0.36

While resistance factors are presented in the following sections for use in foundation design in rock, the high degree of potential variability between rock types suggests that it is prudent and may also be economically attractive to modify those resistance factors for large projects to reflect the site specific rock parameters.

5.4.3 Engineering Property Testing of Soil

Engineering property testing of soils provides a basis for directly defining the soil material properties needed for design and analysis. This type of laboratory testing is performed using undisturbed soil samples, and the duration and cost of testing is greater than for index tests. It is important to carefully plan the subsurface exploration program to include an adequate number and quality of samples for testing. Engineering property testing of soils includes:

- *Consolidation* - Define the stress-deformation and deformation-time behavior of soft to stiff cohesive soils (consolidation time estimates from laboratory tests are usually greater than measured in the field).

- *Shear Strength* - Define the shear strength and shear stress-deformation behavior of soil subjected to various loading conditions representative of field loading cases (unconfined and unconsolidated-undrained triaxial tests for short-term loading of cohesive soils; consolidated-undrained triaxial tests with pore pressure measurements for effective shear strength and pore pressure parameters of cohesive soils; consolidated-drained triaxial tests for long-term loading of soils; and direct shear tests for peak and residual drained shear strength of soil).

In addition to these two major categories of laboratory testing, tests may be conducted to assess swell and collapse potential, permeability, moisture-density relationships and dynamic soil properties and behavior.

At the strength limit state, the shear strength of the soil is typically the most important parameter in the evaluation of the resistance. Table 5-10 presents a summary of the inherent soil variability and measurement variability for some of the various strength tests described previously. Note that no information is currently available concerning the statistical reliability of consolidation test data. In contrast to some of the relatively large COV values in this table, Nowak (1991) reports a bias factor of 1.095 and a COV = 0.075 for the moment capacity of steel, non-compact sections, and a bias factor of 1.12 and COV = 0.12 for lightly reinforced concrete T-beams.

5.4.4 Engineering Property Testing of Rock

Engineering property testing of rocks provides a partial basis for directly defining the rock material properties needed for design and analysis. In addition to the intact rock properties, it is essential to incorporate the effect of discontinuities and structure on the overall behavior of the rock mass. This section focuses on laboratory testing of intact rock specimens. For weak, moisture-sensitive rock types (e.g., clay shales), it is important that testing only be conducted using rock core which has been carefully preserved to minimize loss of moisture.

Table 5-10
Summary of Inherent Soil Variability
and Measurement Variability for Strength Tests
(after Phoon, et al., 1995)

Property[(1)]	Soil Type	Inherent Soil Variability Mean COV	Measurement Variability Mean COV
S_u (UC)	Fine-grained	0.33	----
S_u (UU)	Clay, Silt	0.22	----
S_u (CIUC)	Clay	0.32	0.19
ϕ_f[2)]	Sand Clay, Silt	0.09 0.21	---- ----
tan ϕ_f (TC)	Clay, Silt Sand, Silt	0.20 ----	---- 0.08
tan ϕ_f (DS)	Clay, Silt Clay	0.23 ----	---- 0.14
tan ϕ_f[(2)]	Sand	0.09	----

(1) UC = Unconfined compression; UU = Unconsolidated-undrained triaxial compression
CIUC = Isotropically consolidated undrained triaxial compression;
TC = Triaxial compression; and DS = Direct shear
(2) Laboratory test type not reported

Engineering property testing of rocks includes the following types of tests:

- Density (Unit Weight)
- Water Content
- Uniaxial Compressive Strength - Define uniaxial compressive strength, elastic modulus and Poisson's Ratio (test most commonly used to estimate rock strength for highway engineering purposes)
- Porosity and Water Adsorption - Usually conducted to evaluate durability characteristics for aggregate and rock rip rap
- Tensile Strength Determination

The engineering behavior and properties of intact rock can be highly variable, and the rock mass perhaps even more so. Table 5-11 presents the variability of Young's modulus for a number of different rock groups, with the COV for any given group ranging from a low of 0.14 to a high of 1.02 (in this table different rock types are from different sites, e.g., Pikes Peak Granite and Barre Granite). For the example loading conditions and load variability described in Section 5.2, this range of COV would correspond to resistance factors ranging from 0.82 to 0.05. This extreme variability clearly illustrates the necessity for careful, site specific consideration of the intact rock and rock matrix in the selection of an appropriate resistance factor.

Table 5-11
Selected Variability of Rock Modulus Values
(after Kulhawy, 1978)

Rock Group	No. Values	No. Rock Types	COV
Granite	26	26	0.46
Diabase	7	7	0.14
Basalt	12	12	0.32
Marble	14	13	0.40
Gneiss	13	13	0.26
Slate	11	2	0.69
Schist	12	12	0.64
Phyllite	3	3	0.33
Sandstone	27	19	0.56
Siltstone	5	5	0.69
Shale	30	14	1.02
Limestone	30	30	0.65
Dolostone	17	16	0.81
Mean	16	13	0.54

Figure 5-10 shows histograms of COV for both the compressive strength and tensile strength for a number of rock types. Again, it is important to distinguish between the measured properties of the intact rock as shown in this figure and the actual properties of the entire rock matrix including discontinuites, anisotropy and differential weathering.

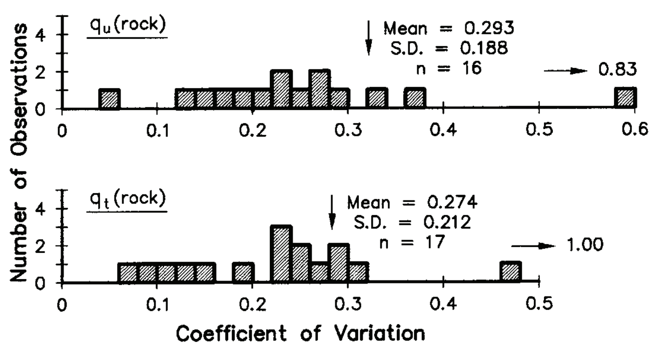


Figure 5-10
Histograms of Intact Rock Strength Properties
(after Kulhawy, et al., 1991)

5.5 Local Geologic Problem Conditions

No specification for substructure design can envision all possible geologic conditions or provide sufficient guidance regarding their evaluation in the design process. The variability of soil and rock materials and the reliability of field and laboratory test methods to estimate their engineering behavior as described in the previous sections are representative of typical subsurface conditions. Accordingly, it might be necessary to modify resistance factors used for LRFD when unusual or highly variable soil and rock conditions are encountered, or when very uniform or well defined soil and rock conditions exist. Modification of resistance factors would be analogous to modifying the factor of safety in ASD when critical situations or geologic conditions are present (e.g., proximity to deformation-sensitive structures or heterogeneous foundation soils) and which require special attention to design and construction details (e.g., presence of moisture-sensitive soils). Special geologic conditions which can be encountered in the U.S. are summarized in Table 5-12.

Table 5-12
Problem Conditions Requiring Special Consideration
(AASHTO, 1991)

Problem	Description	Comments
Soil	Organic Soil; Highly Plastic Clay Sensitive Clay Micaceous Soil	Low strength and high compressibility Potentially large strength loss upon large straining Potentially highly compressible (saprolite)
	Expansive Clay/Silt; Expansive Slag Liquefiable Soil	Potentially large expansion upon wetting Complete strength loss and high deformations due to earthquake loading
	Collapsible Soil Pyritic Soil	Potentially large deformations upon wetting (Caliche; Loess) Potentially large expansion upon oxidation
Rock	Laminated Rock Expansive Shale	Low strength when loaded parallel to bedding Potentially large expansion upon wetting; degrades readily upon exposure to air/water
	Pyritic Shale	Expands upon exposure to air/water
	Soluble Rock	Soluble in flowing and standing water (Limestone, Limerock, Gypsum)
	Cretaceous Shale Weak Claystone (Red Beds) Gneissic and Schistose Rock	Indicates potentially corrosive groundwater Low strength and readily degradable upon exposure to air/water Highly distorted with irregular weathering profiles and steep discontinuities
	Subsidence	Typical in areas of underground mining or high ground water extraction
	Sinkholes/Solutioning	Karst topography; typical of areas underlain by carbonate rock strata
Condition	Negative Friction/Expansion Loading Corrosive Environments Permafrost/Frost Capillary Water	Added compressive/uplift load on deep foundations due to settlement/uplift of soil Acid mine drainage; degradation of certain soil/rock types Typical in northern climates Rise of water level in silts and fine sands leading to strength loss

5.6 Example Problem
Develop Site Specific Resistance Factors from Field and Laboratory Test Data

Problem: Table 5-13 summarizes data from eight laboratory CIUC triaxial tests and two sets of field vane shear test data collected on a soft, near normally consolidated clay for design of an anchored sheetpile wall along a river. The project required the wall to provide a stable parking area to be constructed through placement of about 5 to 7 m of additional fill. The strength characterization for this project used both raw field vane shear test data as well as data normalized by the in-situ vertical effective stress and preconsolidation level. The "corrected" undrained shear strengths, S_u, in the table below have already been modified based on plasticity index as recommended by Azzouz, et al. (1983).

The first phase of exploration and preliminary design included field vane shear tests which are identified herein as Series 1. As shown in Table 5-13, the average S_u for the Series 1 testing was 13.8 kPa, which led several designers to preliminarily conclude that an anchored sheetpile wall was not a feasible solution to the problem. However, it was suspected that, a full head of water was not always maintained in the augers during the Series 1 field vane testing and that the data was, therefore, suspect.

Subsequently, a contractor and designer, believing that the anchored sheetpile wall was a viable solution, performed the Series 2 field vane shear testing as well as the CIUC tests, the results of which are also presented in Table 5-13. The Series 2 field vane shear testing was performed using the same drilling rig and testing equipment as the Series 1 testing, but was performed using a full head of water in the hollow stem augers for each test. Both the Series 1 and 2 field vane testing were distributed relatively evenly over the thickness of the soft clay. The CIUC testing data was subsequently performed on undisturbed samples primarily to assess the effective stress strength parameters of the soft clay, but also as another characterization of the total stress strength parameters for the clay. The CIUC testing was performed at effective confining pressures representing the range of in-situ conditions.

Objective: Demonstrate statistical interpretation of testing as related to LRFD and development of site specific resistance factors.

Solution: First complete the data tabulation by calculating the statistical parameters at the bottom of the table. Recall that the standard deviation, σ, is calculated from Chapter 3 as:

$$\sigma = \sqrt{\Sigma\,(x_i - \bar{x})^2 \,/\, (N - 1)} \qquad \text{(Eq. 3-8)}$$

where:

x_i = Individual data point
$\bar{x}$ = Average of all data points
N = Total number of data points

and the COV from Chapter 3 is:

COV = $\sigma/\bar{x}$ (Eq. 3-9)

Table 5-13
Field and Laboratory Shear Strength Test Data for Example Problem

N	FVT Series 1 Corrected S_u, (kPa)	FVT Series 2 Corrected S_u, (kPa)	CIUC Series S_u, (kPa)
1	3.7	11.8	22.4
2	10.1	42.7	37.6
3	11.2	30.1	65.9
4	12.0	54.1	30.4
5	28.0	17.5	61.4
6	13.1	17.0	17.6
7	5.6	33.5	29.6
8	16.4	38.2	41.4
9	26.2	22.2	
10	13.4	25.9	
11	7.5	26.0	
12	18.7	17.7	
13	19.1	19.1	
14	14.9	41.7	
15	13.1	30.4	
16	7.5	17.3	
17		29.5	
18		13.6	
19		20.4	
20		31.2	
21		22.3	
22		11.9	
23		29.7	
N	***16***	***23***	***8***
$\bar{x}$	***13.8***	***26.2***	***38.3***
σ	***6.8***	***10.6***	***17.4***
COV	***0.49***	***0.40***	***0.45***
λ_R	***1.86***	***1.00***	***0.67***

Step 1: Select the nominal (ultimate) resistance strength value for use in design

For this problem the design strength value is selected as follows:

Design S_u: = *26.2 kPa* (from the Series 2 FVTs)

Rationale for selection of ultimate S_u:

1. The Series 1 FVTs may be biased due to a lack of water head in the borehole or other reasons. Therefore, this data is discarded.

2. The CIUC test provides a total stress strength parameter which is unconservative for use in a situation in which multiple shear modes including simple shear and extension may be active. Therefore, this test should not be used unless this bias is directly quantified and incorporated in the resistance factor. Since the AASHTO resistance factors are currently based on a simple calibration with ASD and have no statistical basis, it is important to select an appropriate test method without this bias.

3. The FVT has been shown to provide one of the best measurements of undrained shear strength for use in global stability types of problems.

Step 2: Estimate the Bias Factor, λ_r, For Each Data Series From Chapter 3 as:

$$\lambda_R = R_m/R_n \qquad \text{(Eq. 3-10)}$$

where:

R_m = Measured resistance
R_n = Predicted resistance

The values of λ_R in Table 5-13 represent a "relative bias" calculated as the ratio of the average results of the Series 1 and CIUC testing (predicted values) to the average from the Series 2 testing (assumed measured value). It is assumed that the Series 2 testing provides the correct, unbiased undrained shear strength. During the course of construction, this assumption was largely validated when a contractor placed too much fill behind the wall in one area and nearly precipitated a failure of the system. An inclinometer installed directly behind the wall at this location showed rapidly increasing deformations before the load was removed and stability analyses including the excess fill suggested that the Series 2 strength characterization was consistent with the imminent failure observed with the inclinometer.

Step 3: Develop Resistance Factors For Each Data Set
As discussed in Section 5.2.2, it is theoretically conceivable to develop a site specific or material specific resistance factor using Eq. 5-2 and Eq. 5-3 if the target reliability index, β_T, can be satisfied. However, Eq. 5-2 requires that the COV and bias of the loads also be known. Chapter 7 presents an approximate method to evaluate resistance factors which assumes that the standard deviations for the load and resistance do not differ by more than a factor of 3. Using Eq. 7-6, the resistance factor for lognormally distributed R and Q is approximated as:

$$\phi = \lambda_R \exp\left(-\alpha\, \beta_T\, COV_R\right) \qquad \text{(Eq. 7-6)}$$

where:

λ_R = Bias of the resistance (Table 5-13)
α = Correction coefficient with a typical value of 0.87
COV_R = Coefficient of variation of the resistance (Table 5-13)
β_T = Target reliability index

Using Eq. 7-6, the following resistance factors, ϕ, are obtained:

β_T	*Resistance Factors, ϕ*		
	FVT Series 1 $\lambda_R = 1.86$ $COV_R = 0.49$	*FVT Series 2* $\lambda_R = 1.00$ $COV_R = 0.40$	*CIUC Tests* $\lambda_R = 0.67$ $COV_R = 0.45$
3.5	***0.42***	***0.30***	***0.17***
3.0	***0.51***	***0.35***	***0.21***
2.5	***0.64***	***0.42***	***0.25***
2.0	***0.79***	***0.50***	***0.31***

Summary:
This example illustrates the development of site specific resistance factors using field and laboratory test data. As discussed in Section 5.2.2, the development of resistance factors, ϕ, must consider uncertainty related to:

- *Inherent Spatial Variability represented by the uncertainty in using point measurements compared to measurements reflecting a larger of volumetric extent*

- *Measurement Error due to equipment and testing procedures*
- *Model Error reflected by the uncertainty of the predictive method*

as well as uncertainties in load estimates.

The ϕ-factors developed in this example consider spatial variability and consider only relative differences (i.e., relative bias) in measurement error. The tabulated ϕ-factors do not reflect the model uncertainty of any particular predictive method and make a conservative assumption relative to load estimate variability. *The COV_R values for the three data sets shown are essentially equal. However, if the data were normalized with respect to stress history and effective confinement, the data for FVT Series 2 and the CIUC testing would exhibit significantly lower COV_R values (and therefore higher resistance factors) than the FVT Series 1 test data.*

This example illustrates two important points relative to the impact of testing procedures on the development of resistance factors:

(1) Conservative test methods which consistently under-predict resistance (as

indicated by a high bias factor) will tend to have a higher resistance factor than less conservative test methods for similar coefficients of variation and target reliability indexes

(2) Any applicable test method may be used to predict resistance as long as the statistical parameters ($\bar{x}$, σ, COV_R and λ_R) describing the inherent spatial variability, measurement error and model error are accurately defined

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CHAPTER 6
GEOTECHNICAL DESIGN PARAMETER SELECTION

6.1 Introduction

Chapter 3 introduced inherent soil variability, measurement uncertainty and transformation model uncertainty as the three primary factors which contribute to the overall uncertainty in estimating soil properties. The contribution of these factors in the property estimation process is illustrated in Figure 6-1. Details regarding inherent soil variability and measurement uncertainty for commonly used index and strength properties developed from various in-situ and laboratory measurement tests were presented in Chapter 5. This chapter addresses the transformation model used for estimating engineering properties of soils from in-situ and laboratory tests.

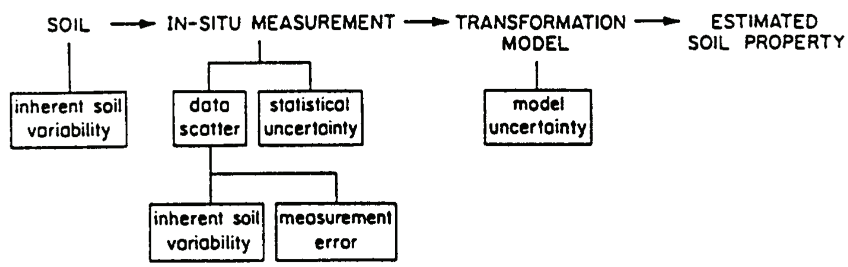


Figure 6-1
Uncertainty in Soil Property Estimates
(after Kulhawy, 1990)

In applying LRFD for substructure design, it is important to use resistance factors which have been calibrated to account for each of these three components of uncertainty, and the uncertainty involved in using the estimated soil properties in a given predictive model. For example, when using the α-method to determine skin friction capacity of a pile or drilled shaft in cohesive soils, it is important that the undrained shear strength be characterized in a manner consistent with the resistance factor calibration. The following sections of this chapter:

- Discuss the suitability of various field and laboratory test methods for estimating soil properties or foundation performance

- Describe the reliability and variability of design parameters estimated based on various field and laboratory tests

6.2 Test Method Selection

Selection of a particular field or laboratory test method for estimating a geotechnical parameter for design depends on the technical acceptability and relative cost and benefit of the method. To be technically acceptable, the test must be directly applicable to the design requirements for the project.

For example, using standard penetration tests (SPTs) to estimate the shear strength of granular soils is appropriate, whereas using SPTs to estimate the shear strength of cohesive soils is not appropriate.

LRFD concepts can be applied directly to evaluate the cost effectiveness of various exploration and testing programs. Although this process can be demonstrated by many example problems presented in later chapters, a problem in Chapter 9 provides a particularly good example of the approach. The problem involves the design of a pile group in medium dense sand where a subsurface exploration was conducted by SPT and cone penetration test (CPT) methods. In the problem, the SPT-based design requires 21 piles whereas the CPT-based design requires 16 piles. Although significant regional variability exists in construction costs, this 5-pile difference represents an approximate savings of at least $7,500 (Means, 1996) using the CPT versus SPT approach. Regional variations in construction costs and project specific variables may either increase or decrease this cost difference. The remainder of this chapter focuses on the technical applicability of a given test type to estimate a particular soil property and to address specific design needs.

6.2.1 Field Test Methods

As will be highlighted in later chapters, the results of in-situ testing can be used in one of two ways for substructure design. One approach is to use in-situ measurements (e.g., N, q_c or f_s) directly to estimate foundation performance such as bearing resistance or settlement. Tables 6-1 and 6-2 identify some of these relationships for cohesive and cohesionless soils, respectively. Note that a set of calibrated resistance factors has been prepared (AASHTO, 1997a) for several of the methods listed in these tables.

Alternatively, in-situ measurements can be used to estimate engineering properties (e.g., ϕ'_f, S_u or E) which can then be incorporated into a predictive model to evaluate foundation performance. Some of the correlations which are available to estimate engineering properties are presented in Table 6-3 for cohesive soils and in Table 6-4 for cohesionless soils.

Based on the combined expectations of subsurface conditions and foundation types, it is possible to use tables like those shown above in the preliminary selection of the types of in-situ tests which are applicable to a given project.

6.2.1.1 Shear Strength

The effective stress shear strength of cohesionless soils, ϕ'_f, can be estimated from the results of SPTs, CPTs and pressuremeter tests (PMTs). Selection of a particular test method should be based on regional availability and expertise, cost and the reliability of each test in predicting the effective stress friction angle, ϕ'_f. The reliability of these various in-situ test methods is described in Section 6.3.1.1.1.

Table 6-1
Correlation of In-Situ Test with Foundation Performance for Cohesive Soils

Foundation Type	Behavior Component	In-Situ Test Method		
		SPT	CPT	PMT
Driven Pile	Side Resistance	--	--	✔
	Tip Resistance	--	--	✔
	Settlement	--	--	✔
Drilled Shaft	Side Resistance	--	--	✔
	Tip Resistance	--	--	✔
	Settlement	--	--	✔
Spread Footing	Bearing Capacity	--	✔(1)	✔
	Settlement	--	--	✔

(1) Resistance factors have been developed for at least one procedure.

Table 6-2
Correlation of In-Situ Test with Foundation Performance for Cohesionless Soils

Foundation Type	Behavior Component	In-Situ Test Method		
		SPT	CPT	PMT
Driven Pile	Side Resistance	✔(1)	✔(1)	✔
	Tip Resistance	✔(1)	✔(1)	✔
	Settlement	✔	✔	✔
Drilled Shaft	Side Resistance	✔(1)	--	✔
	Tip Resistance	✔(1)	--	✔
	Settlement	--	--	✔
Spread Footing	Bearing Capacity	✔(1)	✔(1)	✔
	Settlement	--	--	✔

(1) Resistance factors have been developed for at least one procedure.

Table 6-3
Correlations of In-Situ Tests with Engineering Behavior for Cohesive Soils

Property Category	Soil Property	Field Test Correlation				
		SPT	CPT	CPTU	PMT	VST
In-Situ Stress	Preconsolidation Stress, σ_p	✔	✔	✔	✔	✔
	Overconsolidation Ratio, OCR	✔	--	✔	--	✔
	Coefficient of Horizontal Stress, K_o	✔	✔	✔	✔	--
Strength	Effective Stress Friction Angle, φ'_f	--	--	--	--	--
	Undrained Shear Strength, S_u	✔	✔[1]	✔	✔	✔[1]
Deformability	Poisson's Ratio, v	--	--	--	--	--
	Young's Modulus, E	--	--	--	✔	--
	Constrained Modulus, M	✔	--	✔	--	--
	Coefficient of Consolidation, c_v	--	--	✔	--	--
Permeability	Coefficient of Permeability, k	--	--	✔	--	--

[1] Resistance factors have been developed for at least one procedure.

Table 6-4
Correlations of In-Situ Tests with Engineering Behavior for Cohesionless Soils

Property Category	Soil Property	Field Test Correlation			
		SPT	CPT	CPTU	PMT
In-Situ Stress	Coef. of Horizontal Stress, K_o	--	✔	--	✔
Strength	Effective Stress Friction Angle, ϕ'_f	✔[1]	✔[1]	--	✔
Deformability	Poisson's Ratio, v	--	--	--	--
	Young's Modulus, E	✔	--	--	✔
	Constrained Modulus, M	--	✔	--	--
Permeability	Coefficient of Permeability, k	--	--	--	--
Liquefaction	Cyclic Stress Ratio, $\tau_a\surd\sigma_{vo}$	✔	✔	--	--

[1] Resistance factors have been developed for at least one procedure.

The total stress (i.e., undrained) shear strength of cohesive soils, S_u, can be estimated most reliably by the vane shear test (VST); however where a site-specific calibration is available based on previous testing, PMTs, CPTs and SPTs can also be used. The reliability of these tests for estimating S_u is discussed in Section 6.3.1.1.2. In general, the SPT should not be used to estimate S_u of cohesive soils. Again, selection of a particular test should be based on local availability and experience, cost and the reliability of the test method.

6.2.1.2 Compressibility

The drained modulus of cohesionless soils can be estimated using SPTs, CPTs, CPTs with pore pressure measurements (CPTUs) or PMTs. The undrained modulus for cohesive soils can be estimated using the CPTU and SPT. The CPTU has a unique capability in that it is the only in-situ test which can provide information on pore pressure dissipation for estimating the rate of consolidation of cohesive soils. This engineering property can be developed when cone advancement is stopped for a period of time to permit excess pore pressures around the cone tip to dissipate.

6.2.2 Laboratory Tests

6.2.2.1 Shear Strength

When selecting a laboratory strength test for substructure design, the type and rate of structure loading and the drainage characteristics of the foundation soils must be defined as they dictate the type of test to be used and whether total and/or effective stress strength parameters will control design. For example, a consolidated-undrained triaxial compression test with pore pressure measurements will provide both the total and effective stress properties of a cohesive soil whereas a drained test will provide only effective stress parameters. Also, the boundary conditions which characterize the test and those expected to represent actual field conditions must also be recognized. Figure 6-2 shows some typical failure modes for common engineering problems and the corresponding laboratory tests which most closely model the shearing modes in these problems. While it is important to consider these shearing modes in both total and effective stress conditions, the total stress strength parameter selection is especially sensitive to the type and rate of loading.

6.2.2.2 Compressibility

Soil compressibility depends on the rate of load application and the drainage characteristics of the soil. Due to the high permeability of cohesionless soils, deformation is nearly immediate as load is applied, such as the rate of loading effects are not significant (except for dynamic loads) and the drained soil modulus controls compressibility. With the possible exception of very loose soils, the time-dependent deformation of cohesionless soils is insignificant. For cohesive soils however, soil compressibility can occur during both undrained and drained loading, with immediate settlements occurring during undrained loading, and time-dependant settlements (e.g., consolidation and secondary compression) occurring during drained loading. The compressibility characteristics of soils can be determined in the laboratory using oedometer or consolidation tests in which the sample is laterally confined within a rigid ring, or by unconfined or triaxial compression tests.

6.2.2.3 Index Tests

Correlations between engineering properties and index tests (e.g., particle-size analyses, unit weight, water content and Atterberg limits) provide a relatively inexpensive means for making broad initial estimates of soil properties during the preliminary phases of a project, and serve as indicators of general soil behavior. Some possible correlations of test data with soil properties are identified in Table 6-5. Because most laboratory test programs include water content, particle-size analyses and Atterberg limits determinations, the results of these tests are generally available for use as a guide in estimating material properties such as shear strength, compressibility and permeability.

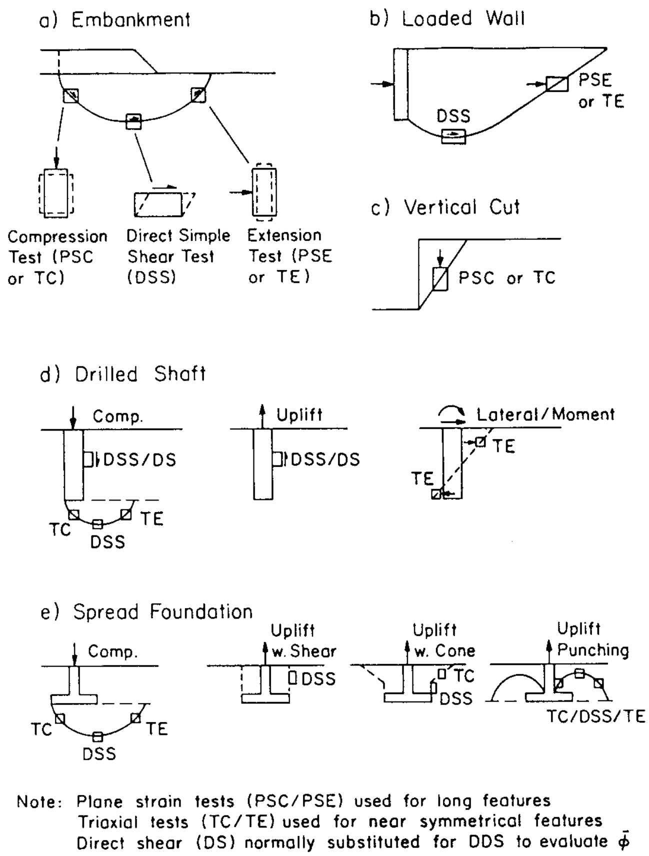


Figure 6-2
Comparison of Field and Laboratory Shearing Modes
(after Kulhawy and Mayne, 1990)

Table 6-5
Soil Property Correlations with Index Test Data

Index Test	Possible Correlation
SPT and CPT	Shear strength, relative density, and compressibility
Water Content	Shear strength and compression index of clay
Particle-Size Distribution	Permeability, strength and drainability of cohesionless soils
Atterberg Limits	Compressibility, shrink-swell, drained strength of cohesive soils
Void Ratio, Density	Compressibility and shear strength
Relative Density	Strength and compressibility of cohesionless soil

6.2.3 Parameter Evaluation

Following the completion of field and laboratory testing programs, the test results should be evaluated for reasonableness and variability in selecting material parameters for use in design. When possible, test results should be compared with data from prior testing in the vicinity of the planned construction site, and performance of adjacent structure foundations should be considered using information from foundation submissions and maintenance records. When highway construction occurs in areas not previously developed or when records on past construction are not available or are incomplete, selection of soil and rock properties for design can be aided using project test results and reported correlations between index parameters and soil and rock properties for strength, compressibility and permeability (e.g., Barker, et al., 1991a; Carter and Bentley, 1991; Kulhawy and Mayne, 1990; and NAVFAC, 1986).

6.3 Reliability of Tests in Estimating Design Parameters

As discussed in Section 6.1 and illustrated in Figure 6-1, a transformation model must be used to manipulate the test results to arrive at either a direct prediction of foundation performance or an engineering property estimate. This section focuses on the uncertainty of a variety of transformation models which have been developed based on the results of field and laboratory tests. The uncertainty associated with the measurement process and inherent soil variability was discussed in Chapter 5.

6.3.1 Field Test Methods

Several field test methods were identified in Section 6.2.1 for estimating the drained strength of cohesionless and cohesive soils, and the undrained strength of cohesive soils. These tests include:

- SPT
- CPT using either a mechanical (MCPT) or electrical (ECPT) device
- Field VST
- PMT

The reliability of these tests for estimating the strength and compressibility of soils is discussed in Sections 6.3.1.1 and 6.3.1.2, respectively.

6.3.1.1 Shear Strength

6.3.1.1.1 Effective Stress Strength

Standard Penetration Test

The SPT has been and remains the mainstay of in-situ testing in the United States, and has perhaps been used most often to estimate the effective stress friction angle of cohesionless soils. Numerous relationships exist between SPT blow count, N, and effective stress angle of friction, ϕ'_f (e.g., Meyerhof, 1956; Peck, Hanson, Thornburn, 1974); however, there is no direct, statistically-based correlation between these parameters.

Cone Penetration Test

Correlations exist between the CPT tip resistance, q_c, and N as a function of mean particle diameter, D_{50}, as shown in Figure 6-3, and as a function of the percentage fines content as shown in Figure 6-4. For this database, the predictive equation shown in Figure 6-3 has a COV of 0.19 to 0.32. Similarly, Figure 6-4 represents a database containing a wide range of fines content. For this database, the predictive equation shown in Figure 6-4 has a COV of 0.21. In these figures, the statistical description of each correlation is given in terms of the sample size, n, standard deviation, σ (or S.D.), and the coefficient of determination, r^2.

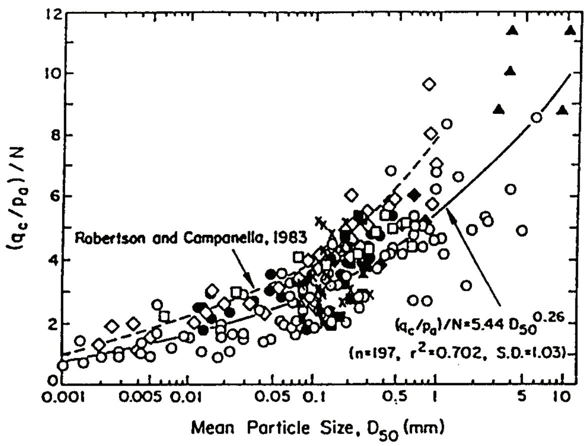


Figure 6-3
Correlation of CPT Tip Resistance, SPT N-Value and Mean Particle Size
(after Kulhawy and Mayne, 1990)

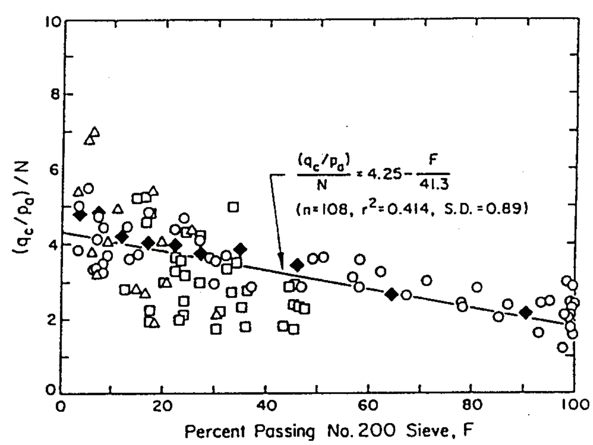


Figure 6-4
Correlation of CPT Tip Resistance, SPT N-Value and Fines Content
(after Kulhawy and Mayne, 1990)

Figure 6-5 shows a statistical correlation of the normalized CPT tip resistance, $(q_c/p_a)/(\sigma'_{vo}/p_a)$ with the triaxial compression effective stress friction angle, ϕ'_{tc}. The CPT tip resistance, q_c, is normalized with respect to atmospheric pressure, p_a and the in-situ vertical effective stress, σ'_{vo}. The correlation is based on 633 tests. The data set from which Figure 6-5 is derived consists of 24 sets of calibration chamber data on reconstituted, recently-deposited sands with a percentage of fines from 0 to 6 percent, D_{50} from 0.16 to 1.0 mm, D_{10} from 0.10 to 0.70 mm and a uniformity coefficient from 1.10 to 2.60. For the range of friction angles reported in Figure 6-5, 0.05 < COV < 0.10 with a much tighter spread of about 0.07 < COV < 0.08 for $35^o < \phi'_{tc} < 40^o$.

6.3.1.1.2 Total Stress Strength

Standard Penetration Test

Although correlations have been developed between N and the undrained shear strength, S_u, of cohesive soils, the correlations do not account for an N-value standardized to a given energy value, any particular reference strength for S_u, or the sensitivity of the clay. Recall that different drilling rigs, hammer arrangements and general test practices can impart significantly different amounts of energy to the split spoon and result in very different N-values for the same soil. Therefore, unless calibrated for a given geologic setting, energy level and reference strength, the SPT does not provide a useful quantitative correlation with undrained shear strength.

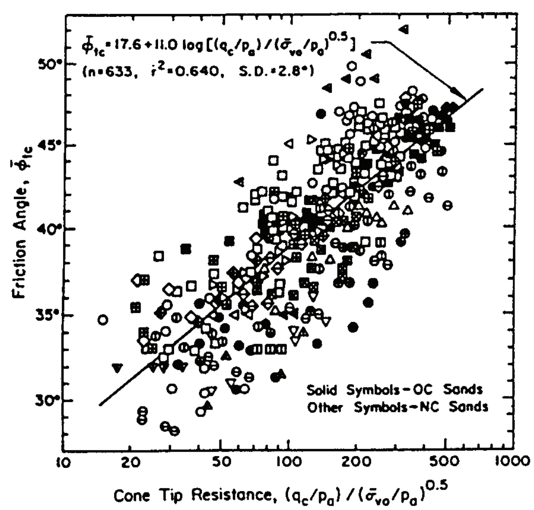


Figure 6-5
Correlation of Triaxial Compression Effective Stress Friction Angle with Cone Penetration Tip Resistance
(after Kulhawy and Mayne, 1990)

Cone Penetration Test

The cone tip resistance has been correlated with undrained shear strength, S_u, using the relationship:

$$q_c = N_k S_u + \sigma_{vo} \qquad \text{(Eq. 6-1)}$$

where:

q_c = Cone tip resistance (kPa)
σ_{vo} = Total overburden stress (kPa)
N_k = Cone bearing factor (dim)

Theoretical predictions of N_k range from about 7 to 18, and empirical correlation for specific sites range from about 5 to 75. The reliability of Eq. 6-1 can be improved by substituting a corrected tip resistance accounting for pore pressure effects when using a piezocone (CPTU) by:

$$q_t = q_c + (1 - a)\, u_{bt} \qquad \text{(Eq. 6-2)}$$

where:

q_t = Corrected cone tip resistance (kPa)
a = Net area ratio (dim). Defined as the ratio of the square of the cone diameter within the pore pressure sensing filter to the square of the cone diameter at the base of the conical point
u_{bt} = Pore pressure measured behind the cone tip (kPa). Note Eq. 6-2 applies only to piezocones in which the pore pressure transducer is located immediately behind the tip of the cone.

Robertson, et al. (1986) have concluded that no unique relationship exists between CPTU data and S_u for any soil type. The CPTU still remains a potentially valuable tool for characterizing S_u due to its relatively economic rate of data collection. In practice then, the use of the CPT or CPTU to predict S_u should include site specific calibration of the N_k factor through use of some reference strength such as might be determined with the field VST.

Field Vane Shear Test
The shear strength measured with the VST should not be used directly in design without prior correction to account for strain rate and anisotropy effects. A correction factor, μ, was first proposed by Bjerrum (1972) based on a two-dimensional, plane strain back analyses of 14 case histories of embankment failures. This correction factor has since been modified by Azzouz, et al. (1983) as shown in Figure 6-6 to include three-dimensional end effects of these failures using a database of 18 case histories.

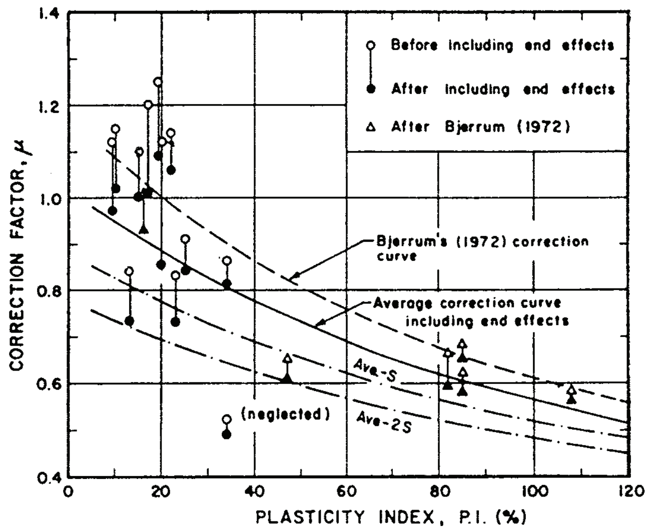


Figure 6-6
Field Vane Shear Correction Factor
(after Azzouz, et al., 1983)

6.3.1.2 Compressibility

Pressuremeter Test
The PMT provides a direct measure of a modulus value which is typically assumed to be equal to an undrained modulus, E_u, in cohesive soils and Young's modulus, E, in cohesionless soils.

Standard Penetration Test
Prior to the introduction of probability based design concepts, numerous non-statistical correlations between the SPT N-value and the drained modulus for cohesionless soils were proposed (e.g. D'Appolonia, et al., 1970, Mitchell and Gardner, 1975). Due to the lack of a quantitative estimate of the reliability associated with these early methods, only some more recent correlations are described herein. Figure 6-7 shows a correlation between the uncorrected SPT N value and a normalized pressuremeter modulus, E_{PMT}/p_a.

The wide spread in data about the trend line in Figure 6-7 illustrates the high degree of uncertainty associated with trying to predict the deformation properties of a cohesionless soil based on SPTs. The COV for this model ranges from a low of about 1.5 to greater than 2.0. A similar relationship is shown in Figure 6-8 for cohesive soils, for which the COV ranges from approximately 1.5 to 2.5. Considering the uncertainty associated with the relationships in Figures 6-6 and 6-7, SPTs should only be used for preliminary estimation of the modulus of soils.

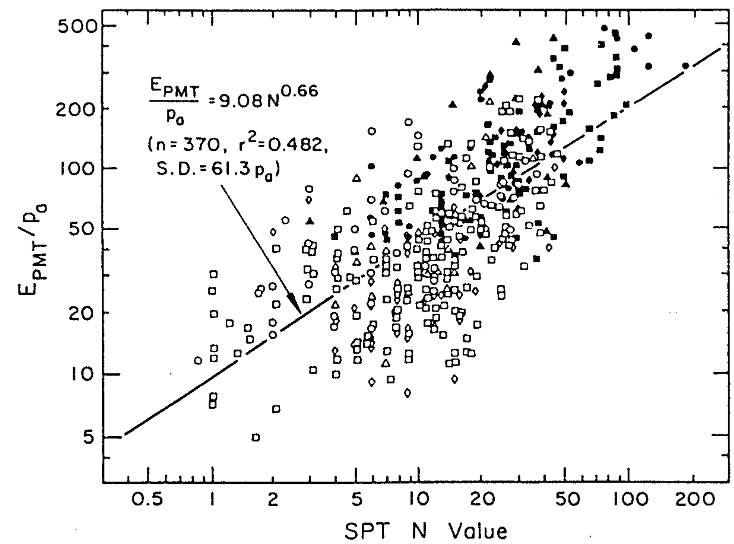


Figure 6-7
Pressuremeter Modulus versus N Value for Cohesionless Soil
(after Kulhawy and Mayne, 1990)

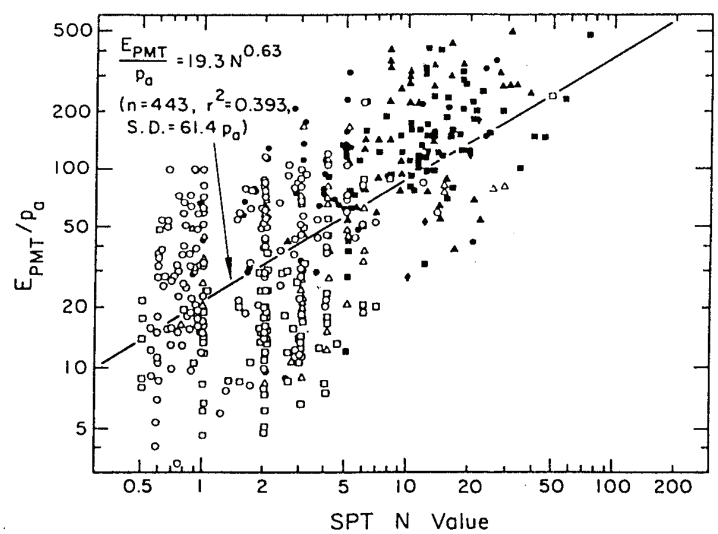


Figure 6-8
Pressuremeter Modulus versus N Value for Cohesive Soils
(after Kulhawy and Mayne, 1990)

Cone Penetration Test

Calibration chamber testing has provided a correlation between the tangent constrained drained modulus, M_{dt}, relative density, D_r, and the CPT tip resistance, q_c. Correlations for NC and OC sands are shown in Figures 6-9 and 6-10, respectively.

Figures 6-9 and 6-10 illustrate that the relative uncertainty (0.05 <COV< 0.63) associated with modulus predictions is significantly smaller using the CPT than using the SPT. However, it is important to note that the relationships shown in Figures 6-9 and 6-10 are based on a data set consisting of calibration chamber tests which do not account for effects such as aging and the wider range of in-situ variability encountered in practice.

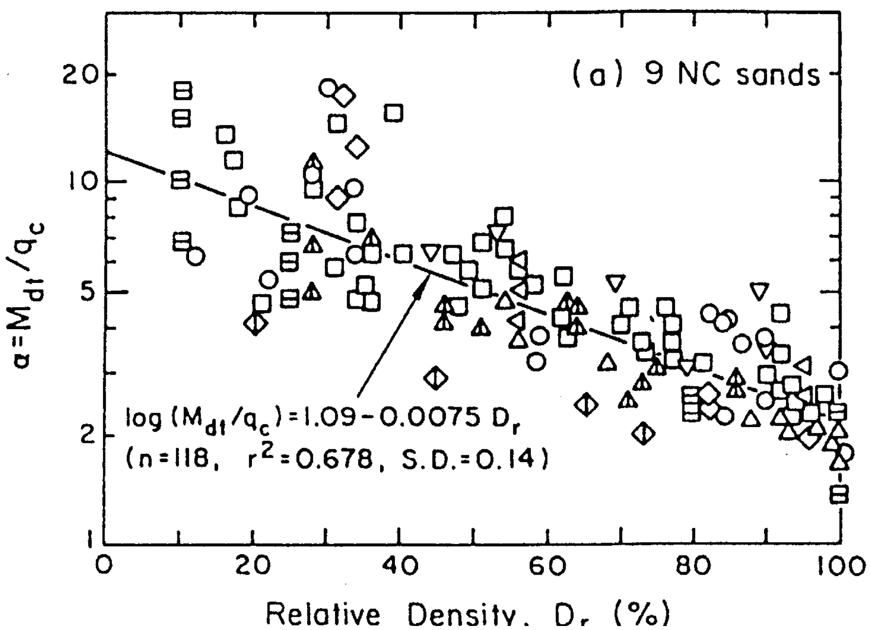


Figure 6-9
Correlation of CPT Tip Resistance with Constrained Modulus and Relative Density for NC Sands
(after Kulhawy and Mayne, 1990)

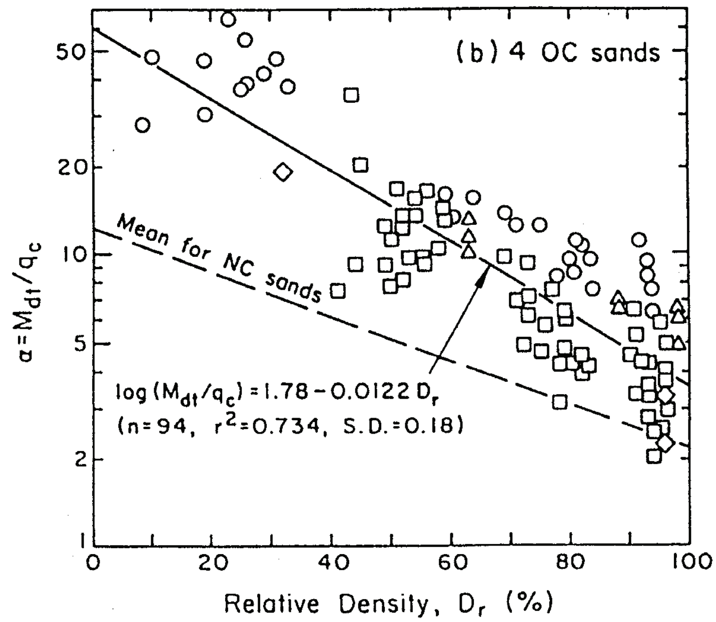


Figure 6-10
Correlation of CPT Tip Resistance with Constrained Modulus and Relative Density for OC Sands
(after Kulhawy and Mayne, 1990)

6.3.2 Laboratory Tests

The primary laboratory strength and compressibility tests consist of the triaxial compression test and the consolidation test. Results from each of these tests exhibit some measurement error and error due to inherent variability of the sample tested. As each test provides a direct measure of a particular engineering property, a transformation model is generally not required to utilize the test results. However, test results may be correlated to other parameters where the potential failure modes in the field do not match the laboratory boundary conditions in the triaxial test (e.g. plane strain vs. triaxial failure modes).

Laboratory index tests can be used to provide cost-effective preliminary estimates of some common engineering properties. For cohesive soils, Atterberg limits and their relationship to the in-situ water content are most commonly used for such correlations. Figure 6-11 shows one correlation between the Liquidity Index, LI, and the ratio of undrained shear strength to vertical effective stress level in triaxial compression, S_u/σ'_{vo}.

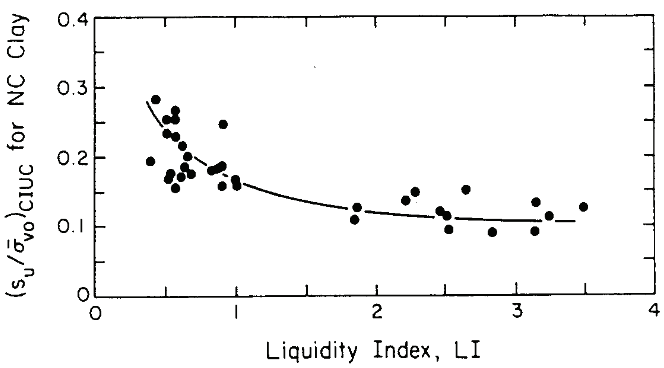
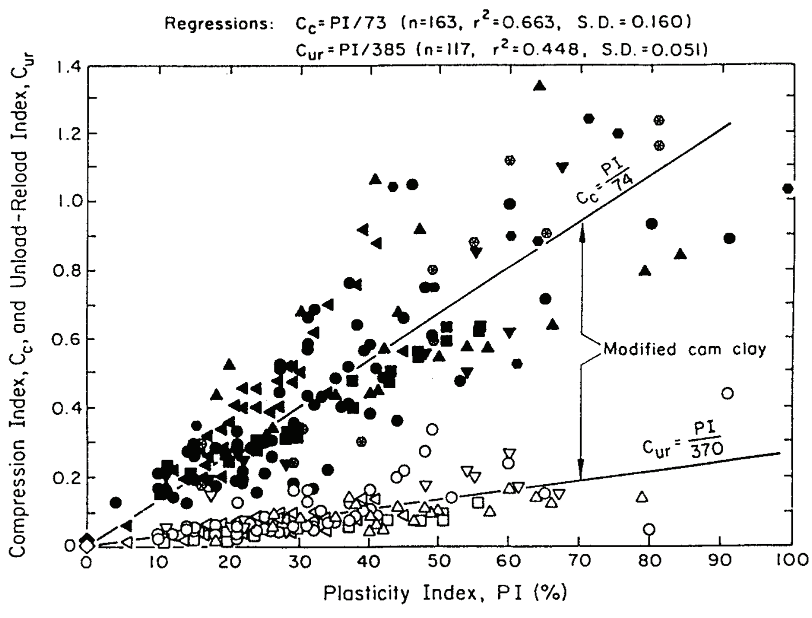


Figure 6-11
Correlation Between Normalized Undrained Shear Strength and Liquidity Index for NC Clays
(after Kulhawy and Mayne, 1990)

Correlations of index tests with some deformation properties are also available, as shown in Figure 6-12 which depicts relationships between the Compression Index, C_c, and the Plasticity Index, PI, and between the Recompression Index, C_{ur}, and the PI, for cohesive soils.



Compression and Unload-Reload Indices versus Plasticity Index
(after Kulhawy and Mayne, 1990)

6.4 Design Parameter Selection and Resistance Factors

As described in Section 6.2, a number of field and laboratory test methods are available for characterizing the engineering parameters of soils required for geotechnical design. Selection of an appropriate method must consider the effects of loading rate and soil drainage characteristics, that is whether total- or effective-stress parameters should be used. Effective-stress strength parameters should be used to evaluate problems controlled by drained or long-term conditions, whereas total-stress parameters are suitable for undrained or short-term conditions. In addition, it is important to consider the relationship between the boundary conditions for in-situ failure modes and the boundary conditions which prevail in the field or laboratory tests. Boundary conditions are particularly important in evaluating undrained shear strength. For example, the undrained shear strength determined in triaxial compression is often more than twice that determined in triaxial extension. Even after the appropriate design parameter has been determined, there are still usually several methods available to estimate the numerical value of that parameter. As a result, it is useful to compare the reliability of the different characterization methods available.

The reliability of a design parameter estimate is a function of the degree of uncertainty with respect to the inherent soil variability, measurement error, number of samples, and the reliability of the transformation model. Chapter 5 focused on uncertainty related to the inherent soil variability and measurement error, whereas this chapter covers the reliability of selected transformation models in parameter selection. In developing a resistance factor for a particular soil property characterization method and predictive model, it is necessary to consider each of the factors which contribute to the overall uncertainty. Table 6-6 presents some guideline design ranges of the coefficient of variation for some of these parameters. The COVs in Table 6-6 represent the variability or repeatability of test results for a single sample (i.e., point COV) or a number of samples (i.e., spatial average COV).

While the COV is a measure of the expected variation of results for a given test type, the appropriateness of a particular test method to define the value of the required soil property for design must also be considered. The boundary conditions affecting a specific test can have a significant effect on the resulting property measurement. This effect was discussed in 6-2 and is illustrated in Figure 6-13 (Chen and Kulhawy, 1994) which shows the relationship between CIUC and UU laboratory strengths. As indicated in Figure 6-13, the value of S_u measured in a CIUC test is significantly higher than the value measure in a UU test for a normally to lightly overconsolidated clay.

Table 6-6
Approximate Guidelines for Design Property Variability
(after Phoon, et al., 1995)

Design Property	Test	Soil Type	Point COV (%)	Spatial Avg. COV (%)
S_u (UC)	Direct (lab)	Clay	20-55	10-40
S_u (UU)	Direct (lab)	Clay	10-35	7-25
S_u (CIUC)	Direct (lab)	Clay	20-45	10-30
S_u (field)	VST	Clay	15-50	15-50
S_u (UU)	q_T	Clay	30-40	30-35
S_u (CIUC)	q_T	Clay	35-50	35-40
S_u (UU)	N	Clay	40-60	40-55
S_u (field)	PI	Clay	30-55	--
ϕ_f'	Direct (lab)	Clay, Sand	7-20	6-20
ϕ_f' (TC)	q_T	Sand	10-15	10
E_{PMT}	Direct (PMT)	Sand	20-70	15-70
E_{PMT}	N	Clay	85-95	85-95
E_D	N	Silt	40-60	35-55

S_u = Undrained shear strength; UC = Unconfined compression test; UU = Unconsolidated-undrained triaxial compression test; CIUC = Consolidated isotropic undrained triaxial compression test; S_u (field) = Corrected S_u from vane shear test; ϕ_f' = Effective stress friction angle; TC = Triaxial compression test; q_T = Corrected cone tip resistance; E_{PMT} = Pressuremeter modulus; E_D = Dilatometer modulus averaged over a depth interval of 5 m

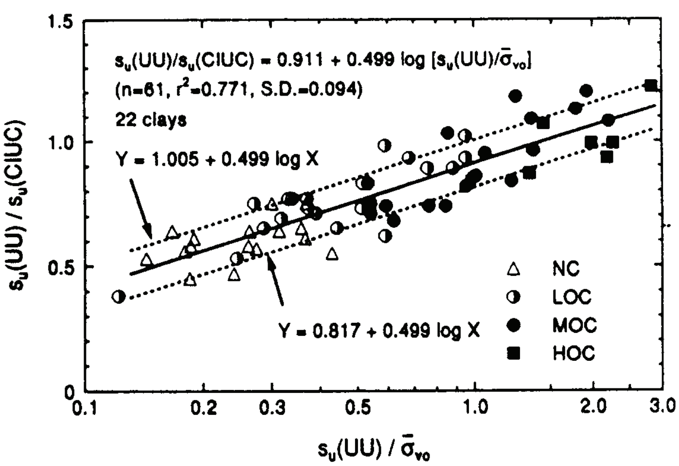


Figure 6-13
Normalized Undrained Shear Strength
Versus Undrained Strength Ratio from CIUC and UU Tests
(Chen and Kulhawy, 1993)

It is important to note that the resistance factors presented in the AASHTO LRFD Specifications are developed using data bases which include a variety of property characterization methods. Further development and/or refinement of resistance factors should explicitly consider both the suitability and variability of the property characterization. Specifically, resistance factor development should consider the overall bias of the resistance and the coefficient of variation of the bias. For example, if this development is done in a rational manner it would be possible to use a test which overpredicts the available soil strength because this bias would be accounted for in the resistance factor determination. However, in general it is preferable to utilize a testing procedure which provides the best measurement of the property of interest without a significant bias.

6.5 Example Problem: Characterization Planning

Problem: Design a single-phase, cost-effective field exploration and laboratory testing program using LRFD concepts to provide information for design of the pier foundation, wall and embankment systems which might be considered for the site shown in Figure 6-14. Table 6-7 provides some of the regional costs associated with typical laboratory and field tests, and provides spaces for developing the scope and cost of the exploration program. The selected borings may be located directly on Figure 6-14 and Table 6-7.

Objectives: Demonstrate general LRFD concepts as they pertain to:

- Exploration program design
- Optimization of exploration foundation costs
- Reliability of different site characterization methods

Background: Figure 6-14 shows a site plan for one side of a river crossing. The proposed grading and construction plans consist of a single pier, an anchored sheet pile wall for shoreline protection and the embankment approach to the bridge. A preliminary review of the site topography, geologic setting and local knowledge suggest that the soil profile at the pier site consists of a 12-m thick deposit of medium stiff clay underlain by a deep deposit of medium dense to dense sand. The sand deposit is known to become very dense at depths exceeding about 25 m. At other sites along this river valley, past exploration programs have disclosed the existence of sand seams within the upper clay layer. The pre-construction elevation along the alignment of the anchored wall anchored wall is El. 417 m. Therefore, 4 m of fill will be needed behind the wall.

From a preliminary design of the bridge, the Strength I Limit State factored loads at the base of the pier are estimated to be 17 500 kN. The combined effects of this relatively large load and the medium stiff clay layer underlain by a dense sand suggest a pile foundation system as a likely support for the pier. Because a pile foundation would be expected to achieve most of its capacity through tip resistance, information concerning the engineering behavior of the sand deposit is of primary interest for design of the pier foundations. For the estimated pier foundation load, a simple preliminary design estimate might suggest the need for about 20 steel piles at an installed unit cost of approximately \$1,250 (Means, 1996). The wall construction and the approach embankment construction raise issues of slope stability and deformation, which will be dictated by the strength of the upper clay layer as well as the potential magnitude and rate of consolidation settlement beneath the approach embankment . The exploration program should provide sufficient data to permit consideration of viable foundation options for the pier, evaluation of the stability and settlement of the embankment, and design of the wall.

Key geotechnical issues which must be addressed for the site include definition of the water table for all structures and the following specific issues for various aspects of the proposed construction:

Approach Embankment:

- Total and effective stress strength of the top 12 m of medium stiff clay
- Magnitude of consolidation settlement of clay
- Rate of consolidation of clay and possible presence and impact of sand seams within clay

West Pier:

- Pile support potential of underlying sand

Anchored Sheet Pile Wall:

- Strength of the medium stiff clay
- Strength of sand for pile embedment and anchor pullout

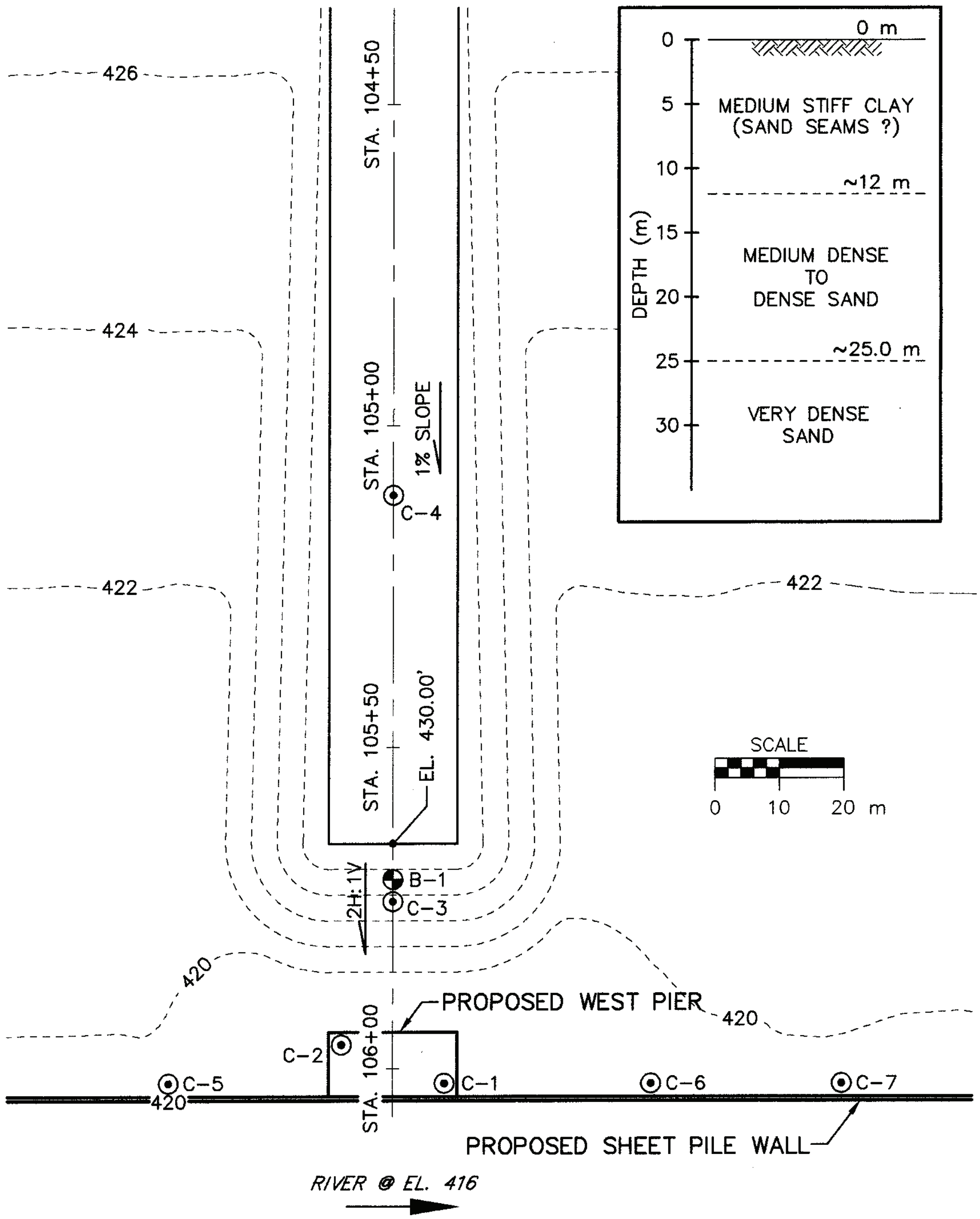


Figure 6-14
Example Problem
Site Exploration Plan for Western Approach

Table 6-7
Example Problem Work Sheet

	Unit Price ($/Unit)	Number of Tests	Total Price ($)	Reason
Laboratory Tests				
Water Content	$5/ea	***12***	***$60***	***Index info. for clay***
Atterberg Limits	$75/ea	***6***	***$450***	***Index for FVT correction***
Unit Weight	$35/ea	***2***	***$70***	***Index info. for clay***
Grain Size Distribution	$60/ea	***6***	***$360***	***Index info. for clay***
3 Pt. CIUC w/ Pore Pressure Measurement	$1000/ea	***1***	***$1,000***	***Effective stress shear strength for clay***
Consolidation	$350/ea	***2***	***$700***	***Consolidation of clay***
		Subtotal	***$2,640***	
In-Situ Tests (1 rig performs all work)				
4 1/4" I.D. HSA w/ SPT @ 3/ o.c.	$55/m	***25 m***	***$1,375***	***Obtain clay samples & N values in sand***
Undisturbed Sampling	$50/ea	***4***	***$200***	***Clay samples for lab testing***
Drill Rig Mob/Demob	$400	***1***	***$400***	***---***
		Subtotal	***$1,975***	
CPTU w/ Dissipation	$40/m	***133***	***$5,320***	***-Eval. Su for clay -Eval. pile tip capacity in sand -Explore for presence of sand lenses in clay -Eval. c_v for settle. rate***
CPTU Mob. / Demob	$750	***1***	***$750***	***--***
		Subtotal	***$6,070***	
FVT	$30/ea	***12***	***$360***	***Base evaluation of S_u***
FVT Mob. / Demob	$250	***1***	***$250***	***---***
		Subtotal	***$610***	
		Total	***$11,295***	

Composition of and Rationale for Proposed Field Exploration Program

The proposed field exploration and testing would be conducted at the eight locations shown in Figure 6-14, and consists of the following:

Boring B-1 Activities

- *Perform field vane shear testing at 1 m intervals in the clay layer. At the location of each FVT, collect a grab sample with the split-spoon for laboratory index testing*

- *Collect 4 undisturbed samples (i.e. shelby tubes) in the clay between FVT locations for strength and consolidation testing*

- *Perform SPTs at 1 m intervals in the sand to a depth of 25 m to estimate pile capacity in sand*

CPTU Soundings at C-1 and C-2 (West Pier), C-3 (Embankment Toe) and C-5, C-6 and C-7 (Sheet Pile Wall)

- *Perform CPTUs to a depth of approximately 25 meters with measurements of tip resistance, side resistance and pore pressure to evaluate S_u for clay, estimate pile tip capacity and anchor pullout resistance in sand and identify presence of sand seams in clay.*

CPTU Sounding at C-4 (Embankment)

- *Perform CPTUs to the top of the sand deposit to evaluate S_u of clay and to identify presence of sand seams in clay.*

Rationale for Proposed Exploration and Testing Program:

Approach Embankment:

The two primary design issues of concern associated with the 10-m high approach embankment are slope stability and consolidation settlement. The stability of the embankment is a possible problem due to the presence of about 12 m of medium stiff clay. FVTs conducted in Boring B-1 represent reliable (i.e. highest resistance factor) means of characterizing the undrained shear strength of a cohesive soil deposit. However, the cost of performing FVTs at a 1 m spacing and Atterberg limits on half of the samples to correct S_u from the FVTs, in addition to the cost of augering and collecting a split-spoon grab sample for laboratory testing is, in this example, \$102.50 / m. By performing a CPTU sounding adjacent to a boring with FVT's, a site specific calibration of S_u from CPTU can be developed and supplemented by additional strength characterization of the clay for only \$40.00/m. Note that the preconsolidation stress estimated using the consolidation test can also be used in estimating the design undrained shear strength for overall stability. The effective stress strength *parameters will be evaluated with the 3-point, CIUC with pore pressure measurements.*

The consolidation test will provide data allowing an estimate of the magnitude and rate of consolidation settlement for the approach embankment. In addition, the CPTUs can provide an estimate of the coefficient of consolidation by measuring pore pressure dissipation. More importantly, the CPTU soundings will provide a nearly continuous profile which will assist in identifying the presence of sand seams in the clay. Because the rate of consolidation is proportional to the square of the drainage path length, the presence of continuous sand seams within this clay layer might conceivably decrease the predicted time for consolidation by an order of magnitude or more.

West Pier:
As suggested previously, a pile foundation system might be a good foundation solution for the proposed West Pier due to its heavy loading and the combined presence of a medium stiff clay in the top 12 meters and the medium dense to dense sand below the clay. End bearing steel pipe piles should provide a cost effective foundation scheme for these conditions. For the expected subsurface conditions, it is likely that the LRFD design of a pile foundation system may be economically performed using in-situ data comprised of SPT or CPT testing, or may be based on the results of a dynamic load test. While the design could also be based on the results of a static load test, for this problem it is unlikely that this will be economical and is therefore not considered herein. The resistance factors associated with each of these methods from the AASHTO LRFD Specification (1996) (A10.5.4) are:

Method for Estimating Skin Friction and Tip Capacity in Sand	*AASHTO LRFD Specification Resistance Factor* ϕ
SPT	*0.45*
CPT	*0.55*
PDA	*0.70*

Whereas the pile driving analysis (PDA) method has the highest φ-factor for this situation, the cost of performing a single test of this type during the design phase may be $10,000 to $15,000. A PDA will provide additional information about hammer selection and drivability. In contrast, the CPT method provides a resistance φ-factor about 20 percent better than the SPT method and costs less than the SPT. The CPT therefore appears to be the most cost effective exploration tool from the standpoint of minimizing the combined exploration and foundation construction costs for this single pier. However, depending on the number of additional piers in similar soil conditions which might be needed to span the river in this problem, it may well be cost effective to mobilize a pile driving rig for pile driving analysis in the design phase of the problem.

Sheet Pile Wall:
A sheet pile wall is planned to stabilize the shoreline due to river dredging and to facilitate construction of the approach embankment. Design concerns for the anchored sheet pile wall include estimation of earth pressures, pullout resistance of permanent anchors and wall embedment. CPTUs at C-1, C-5, C-6 and C-7 can be used to estimate earth pressures and embedment requirements for the wall and the anchor resistance in sand can be estimated using CPTUs in the sand.

Summary:

- ***An exploration program was designed to address the technical issues associated with construction of the approach embankment, west pier and sheetpile wall.***

- ***The reliability concepts of LRFD were utilized in selection of exploration and design tools concurrently with consideration of the exploration and foundation costs. For example:***

  - ***The FVT was chosen as a reliable means to develop base total stress strength parameters, but the CPT was selected as a cost effective, reliable tool to gather data on the variation of total stress strength parameters across the site.***

  - ***The resistance factors and relative costs of SPT- CPT- and PDA-based pile design were evaluated and the CPT was judged to provide the best combination of reliability and economy for this example.***

Note: As discussed in Chapter 9, the 1997 Interims of the AASHTO LRFD Specifications (1997a) include an initial attempt to incorporate the specified method and extent of field pile construction control and capacity verification into the determination of the resistance factors for axially loaded piles (Table A10.5.5-2). Additional consideration and future revision of these requirements are anticipated. The resistance factors cited in this example (from AASHTO, 1996) were obtained from the original reliability-based calibration (Barker, et al., 1991a) and do not consider construction control or field capacity verification. The resistance factors in this example, therefore, reflect only the relative reliability of the SPT and CPT based static capacity prediction methods. The resistance factor for PDA is based on direct calibration to ASD.

CHAPTER 7
CALIBRATION AS PART OF THE DESIGN PROCESS

7.1 Introduction

Development of the AASHTO LRFD Specifications (1997a) involved evaluating resistance factors for the various design methods included in the Specifications. Even though the Specifications contain values of resistance factors for a considerable number of design methods and conditions, designers may need to develop resistance factors for methods or conditions that are not included in the AASHTO LRFD Specifications (1997a). For example:

- A designer may want to use a method that is not covered in the LRFD Specifications, because its previous use under ASD has shown that it is an effective method, with a successful record of use in the soil conditions in the designer's area.

- A designer's experience with local soils may indicate that the resistance factors in the LRFD Specifications are unsuitable for some of the soils in the designer's area.

In either of these cases, it is appropriate for the designer to evaluate the resistance factors needed to apply the LRFD design procedures. Fortunately, this can be done simply, and without great expenditure of time, using common sense, engineering judgment, and a few simple formulas.

7.2 Reliability Theory or Fitting with ASD?

Two approaches for calibration are available:

(1) ***Use of reliability equations, together with statistics on soil strength variations and load variations.*** *The objective of this approach is to compute resistance factors that will result in approximately the same reliability (or probability of satisfactory performance) as achieved, on average, using ASD; or*

(2) ***Fitting to ASD specifications.*** *The object of this approach is to choose resistance factors that will, on average, result on the same factors of safety as would result from design using ASD.*

The information regarding the reliability of ASD methods needed for the approach in (1) has been determined through research studies by Barker, et al. (1991a) and others.

Neither of these approaches need be cumbersome. The best procedure is to use both because they serve to check each other. Small differences in values determined by reliability and by fitting with ASD are to be expected. In such cases, the designer is justified in concluding that either value, or one between them, is suitable.

Large differences between values of resistance factors determined by these two approaches may indicate (a) a blunder in the computations, (b) use of inappropriate statistics on strength, or (c) a mismatch between the probability of failure used in the reliability method and the factor of safety used in the ASD fitting computations.

The statistics for strength that are needed for reliability calculations are the average value and the standard deviation.

When the standard deviation for strength is not known and cannot be estimated with confidence, the only alternative is fitting with ASD. In most cases, however, the designer will be able to use simple rules-of-thumb to estimate the statistics required for the reliability method.

7.3 Evaluating Resistance Factors Using Reliability Method

Although the reliability method has an extensive theoretical background, in application the method can be accomplished by evaluating a single equation. The equation is complicated, but the computation can be automated (for example, in a spreadsheet), and an engineer need only input the values of the parameters involved.

Most methods of estimating foundation capacity involve calculating the products and ratios of a number of variables. The results of these types of calculations are fitted more closely by lognormal distributions than by normal distributions (Barker et al., 1991b). In this chapter, only methods of calculating resistance factor that are based on lognormal distributions will be discussed, because this is the more practical choice.

The expression for the resistance factor, φ, based on lognormal distributions of load and resistance is given by Eq. 3-35 in Chapter 3, which for convenience is repeated as Eq. 7-1.

$$\phi = \frac{\dfrac{\lambda_R(\gamma_D Q_D / Q_L + \gamma_L)}{\lambda_{QD} Q_D / Q_L + \lambda_{QL}} \sqrt{\dfrac{1 + COV_{QD}^2 + COV_{QL}^2}{1 + COV_R^2}}}{\exp\left\{\beta_T \sqrt{\ln[(1 + COV_{QD}^2 + COV_{QL}^2)(1 + COV_R^2)]}\right\}} \qquad \text{(Eq. 7-1)}$$

The parameters needed on the right-hand side of the equation are:

λ_R = Bias factor for resistance; λ_R can be estimated if a calculated value is not available (dim)

λ_{QD} = Bias factor for dead load; λ_{QD} varies with the material (e.g., steel, concrete, asphalt as shown in Table 7-2 and equals the product of the bias factors for the materials making up the dead load (dim)

λ_{QL} = Bias factor for live load; λ_{QL} = 1.15 from field studies used to develop live load model in AASHTO LRFD Specifications (dim)

COV_R = Coefficient of variation for resistance; COV_R can be estimated if statistics are not available as shown in Section 7.3.1 (dim)

COV_{QD} = Coefficient of variation for dead load; COV_{QD} varies with material type (See Table 7-2) and equals the value of the square root of the sum of the squares of the COV's for the materials making up the dead load (dim);

COV_{QL} = Coefficient of variation for live load; COV_{QL} = 0.18 field studies used to develop live load mode in AASHTO LRFD Specifications (dim);

γ_D = Load factor for materials making up the dead load; γ_D = 1.25 in the AASHTO LRFD Specifications; (dim)

γ_L = Load factor for live load; γ_L = 1.75 in the AASHTO LRFD Specifications; (dim)

Q_D/Q_L = Ratio of dead load divided by live load; Q_D/Q_L varies depending on the structure, and can be computed when the loads are known or estimated based on the span length of the bridge (dim)

β_T = Target value of reliability index (dim)

The bias factor, λ_R, is a measure of the inherent conservatism of the method being calibrated. A value of λ_R larger than 1.00 indicates that the method tends to result in conservatively low values of calculated resistance; a value smaller than 1.00 indicates that the method is inherently unconservative (Barker et al. 1991a).

The reliability index, β_T, is related to the probability of failure that underlies the design. Probability of failure means the probability that the actual resistance may be smaller than the actual load. Studies by Barker et al. (1991a) have shown that ASD methods, on which foundation designs have traditionally been based, typically result in probabilities of failure in the range from 0.01 to 0.0001. These values of probability of failure correspond to values of β_T ranging from 2.5 to 3.5. The same studies have shown that probabilities of failure of piles designed by ASD methods are larger, ranging from 0.1 to 0.01. These higher probabilities of failure of piles, which correspond to values of β_T ranging from 2.0 to 2.5, are acceptable because piles are used in groups. Failure of one pile in a group would not necessarily cause the entire group to fail. The probability of failure of the group is therefore smaller than the probability of failure of a single pile within the group.

Simple methods of estimating λ_R, λ_{QD}, COV_R, COV_{QD}, and Q_D/Q_L are discussed in the following sections, together with examples of values of β_T for various types of structures.

7.3.1 Estimating Resistance Bias λ_R and Coefficient of Variation COV_R

As discussed in Chapter 3, the value of the bias factor on resistance (λ_R) represents the net effect of various sources of error, and COV_R represents the usual variability of the results. Values of λ_R and COV_R for various types of tests on soil are presented in Table 7-1.

Table 7-1
Values of λ_R and COV_R

Test	λ_R	COV_R
SPT (Standard Penetration Test)	1.3	0.60 to 0.80
CPT (Cone Penetration Test)	1.0	0.40
Angle of internal friction (φ' or φ)	1.0	0.10
Cohesion (c' or c)	1.0	0.40
Wall friction angle (δ)	1.0	0.20
Earth pressure coefficient (k)	1.0	0.15

For measures of soil or rock resistance or strength not listed, the designer can use judgment to estimate a value of λ_R, and a simple rule-of-thumb to estimate a value of COV_R (Demsky, 1996). Estimates of values of λ_R can be based on the observation as illustrated by the values in Table 7-1. The table shows that λ_R is likely to be 1.00 for methods of measurement that are based on principles of mechanics, while values of λ_R may differ significantly from 1.00 for methods that rely on empirical rules to relate test results to strength values.

It can be seen in Table 7-1 that the CPT, the triaxial test, and the direct shear test, which are interpreted using principles of mechanics, all have values of $\lambda_R = 1.00$. For any other method that is based on the principles of mechanics and rational interpretation of results, the value of λ_R can reasonably be estimated to be equal to 1.00.

The SPT, which can only be interpreted using empirical rules, has a value of $\lambda_R = 1.3$. The value of λ_R for other methods that are interpreted using empirical rules can only be evaluated accurately using data from field load tests, which provide a body of data for comparing calculated and measured load capacities.

The value of COV_R can be estimated using a rule-of-thumb, known as the "six sigma" rule. Use of the "six sigma" rule involves three simple steps using strength values that can be estimated by any experienced engineer:

(1) Estimate the most likely value of the property (V_{est}), the lowest conceivable value (V_{min}), and the largest conceivable value (V_{max}).

(2) Use the "six sigma rule" (the following empirical formula) to estimate the value of the standard deviation (σ):

$$\sigma = \frac{V_{max} - V_{min}}{6} \qquad \text{(Eq. 7-2)}$$

(3) Calculate the coefficient of variation (COV_R):

$$COV_R = \frac{\sigma}{V_{est}} \qquad \text{(Eq. 7-3)}$$

7.3.2 Example Problem - Estimating λ_R and COV_R for Friction Angle

For example, consider how this simple way of estimating COV_R relates to the value of COV_R for the friction angle of sand. An experienced designer might estimate the following values for a medium dense sand, with an SPT blow count of 15:

V_{min} = 25° (smallest conceivable value)
V_{est} = 35° (most likely value)
V_{max} = 45° (largest conceivable value)

Using these values, $\sigma = (45° - 25°)/6 = 3.3°$, and $COV_R = 3.3°/35° = 0.10$. It can be noted that this result agrees with the value of COV_R in Table 7-1.

7.3.3 Example Problem - Estimating λ_R and COV_R for Undrained Strength

As another example, an engineer with experience in a particular area might know that the average undrained strength of a deposit of clay based on UU triaxial testing on undisturbed samples is $S_u = c = 20$ kPa. The maximum and minimum values of S_u are estimated to be about 25 kPa and 10 kPa, respectively. Then:

σ = (25 kPa - 10 kPa)/6 = 2.5 kPa; and
COV_R = 2.5 kPa/20 kPa = 0.13.

This value is smaller than the value of COV_R shown in Table 7-1 for cohesion. The estimate based on local experience with the particular clay should be more reasonable for the local conditions, and would therefore be preferred to the value in Table 7-1.

7.3.4 Values of Dead Load Bias λ_{QD} and Coefficient of Variation COV_{QD}

Values of λ_{QD} and COV_{QD} are listed in Table 7-2.

Table 7-2
Values of λ_{QD} and COV_{QD}
(Nowak, 1993)

Component of Dead Load	λ_{QD}	COV_{QD}
Factory-made components	1.03	0.08
Cast-in-place components	1.05	0.10
Asphaltic wearing surface	1.00	0.25

When the dead load has more than one component, the bias factor and COV are determined from equations presented in Chapter 3, that is, the combined bias factor is the product of the individual bias factors,

$$\lambda = \lambda_1 \times \lambda_2 \times \ldots \times \lambda_N \qquad \text{(Eq. 7-4)}$$

and the combined COV is the square root of the sum of the squares of the individual COV's:

$$COV = \sqrt{COV_1^2 + COV_2^2 + \ldots + COV_N^2} \qquad \text{(Eq. 7-5)}$$

7.3.5 Example Problem - Estimating Values of λ_{QD} and COV_{QD} for a Bridge

Estimate the dead load bias factor and COV for a steel girder bridge with a cast-in-place concrete deck, using values in Table 7-2. Assuming that the asphaltic wearing surface contribution to the total dead load is small and that the majority of the uncertainty in dead load is due to the steel girders and concrete deck, Eqs. 7-4 and Eq. 7-5 yield:

$$\lambda_{QD} = 1.03 \times 1.05 = 1.08$$

$$COV_{QD} = \sqrt{(0.08)^2 + (0.10)^2} = 0.13$$

7.3.6 Values of Load Factors γ_P for Permanent (Dead) Loads

Values of γ_P are given in Table 7-3. The load factors associated with earth pressure quantities are greater than those applied to material weights because of greater uncertainty and more significant variations in their actual values.

Minimum and maximum values of load factors are given in Table 7-3. The value to be used in a given calculation is the one that leads to the more conservative result. For example, $\gamma_P = 1.80$ would be used in calculating downdrag loads in compression piles, and $\gamma_P = 0.45$ would be used in calculating uplift capacity of friction piles. Similarly, $\gamma_P = 1.30$ would be used to calculate earth loads on buried rigid structures where bending moment increases with increasing earth pressure. However, if the bending moment of interest decreases with increasing earth pressure, $\gamma_P = 0.45$ would be used.

7.3.7 Values of Q_D/Q_L

Hansell and Viest (1971) showed that the dead load-to-live load ratio (Q_D/Q_L) varies with span length, L. Their study used the AASHTO ASD Specifications (i.e., Article 3.8.2.1), and the corresponding Impact Factor, I, which varies with L as shown in Table 7-4. The Impact Factor is an allowance for the increased live load effect that results from vehicle impact loads on the bridge deck. Using the Impact Factors for ASD shown in Table 7-4, Hansell and Viest determined that the ratio Q_D/Q_L could be expressed by the empirical formula $Q_D/Q_L = (1 + I)(0.0132L)$, where L is expressed in feet.

Table 7-3
Load Factors for Permanent Loads, γ_P
(AASHTO, 1994)

Type of Load	Load Factor	
	Maximum	Minimum
DC: Component and Attachments	1.25	0.90
DD: Downdrag	1.80	0.45
DW: Wearing Surfaces and Utilities	1.50	0.65
EH: Horizontal Earth Pressure		
• Active	1.50	0.90
• At-Rest	1.35	0.90
EV: Vertical Earth Pressure		
• Overall Stability	1.35	N/A
• Retaining Structure	1.35	1.00
• Rigid Buried Structure	1.30	0.90
• Rigid Frames	1.35	0.90
• Flexible Buried Structures	1.95	0.90
• Flexible Metal Box Culverts	1.50	0.90
ES: Earth Surcharge	1.50	0.75

In the AASHTO LRFD Specifications, the factor corresponding to I is called the Dynamic Load Allowance Factor, IM. The value of IM is independent of span length. As shown in Table 7-4, IM = 0.33 for all span lengths. The values of Q_D/Q_L shown in rightmost column of Table 7-4 were calculated using Hansell and Viest's empirical formula, with IM replacing I. The values in this column provide reasonable estimates of Q_D/Q_L as a function of span, consistent with the provisions of the AASHTO LRFD Specifications (1997a).

Table 7-4
Values of Q_D/Q_L

Span Length, L (m)	ASD Values		LRFD Values	
	Impact, I	Q_D/Q_L	Dynamic Load Allowance, IM	Q_D/Q_L
9	0.30	0.50	0.33	0.52
18	0.27	0.99	0.33	1.04
27	0.23	1.44	0.33	1.56
36	0.20	1.90	0.33	2.07
45	0.18	2.30	0.33	2.59
60	0.15	2.99	0.33	3.46
75	0.13	3.67	0.33	4.32

7.3.8 Values of β_T

Values of target reliability index, β_T, appropriate for geotechnical design of various types of foundation elements have been determined by Barker, et al. (1991a) through analysis of conventional design practices. These values are listed in Table 7-5. The values of β_T for piles are smaller than those used for other types of foundations because piles are used in groups, and there is considerable redundancy in the capacity of pile groups that contain several piles. The probability of failure of a group of piles - that is, the probability that all of the piles in the group will fail - is smaller than the probability of failure of one pile in the group. As a result, acceptable values of β_T for capacity of a single pile are smaller than for other types of foundations, because it is presumed here that a single pile would not be used to support structural loads. If a single pile was to be used to support a structure load, the target value of β should be higher, in the range from 2.5 to 3.5, such as for drilled shafts and spread footings which are often used singly.

Table 7-5
Values of Target Reliability Index, β_T
(Barker, et al., 1991a)

Foundation Type	β_T
Spread Footings	3.0 to 3.5
Drilled Shafts	2.5 to 3.0
Driven Piles (group)	2.0 to 2.5

7.3.9 Example Problem - Resistance Factors for the Nordlund Method

The Nordlund Method (Nordlund, 1963) is used to estimate the axial load capacities of piles in cohesionless sands, silts and gravels. The method includes the effects of pile taper and the volume of soil displaced by the pile. The method has been discussed by GRL (1996), Broms (1963), Thurman (1964), Low (1964), Agarwal (1964), Nordlund (1964), Bowles (1968), Vanikar (1986) and the U.S. Army Corps of Engineers (1993).

Resistance factors for the Nordlund method were not included in the AASHTO LRFD Specifications for design of bridge foundations. However, Eq. 7-1 can be used to calculate resistance factors for this method.

Nordlund (1963) presents data comparing measured pile capacities with values calculated using his proposed method. These data can be used to establish values of the bias factor, λ_{RN} = average value of measured capacity divided by computed capacity and the coefficient of variation, COV_{RN} for the method. Using the data in Table 2 of Nordlund's paper, $\lambda_{RN} = 1.04$, and $COV_{RN} = 0.17$. These measures of the uncertainty in Nordlund's method represent a simplification of a more complex relationship. The complexity occurs because Nordlund's prediction equation for capacity has two independent terms: one for point bearing and one for skin friction. Each of these terms will have their own statistics which could be combined according to Eq. 7-4 and Eq. 7-5 to obtain better values of the bias and COV if the necessary data are available. However, for the purpose of this example, the combined measures of uncertainty in the prediction equation will be used because data showing the separate contributions of end bearing and skin friction are not available.

In applying Nordlund's method a value must be selected for the angle of internal friction of the supporting soil. Statistics for the uncertainty of determining the angle of internal friction from SPT data have been tabulated by Barker, et al. (1991a) as $\lambda_{RA} = 1.0$ and $COV_{RA} = 0.25$.

Combining the resistance statistics using Eq. 7-4 and Eq. 7-5, λ_{RA} and COV_{RA} can be determined as follows:

$$\lambda_R = \lambda_{RN} + \lambda_{RA} = 1.04 \times 1.00 = 1.04$$

and

$$COV_R = \sqrt{COV_{RN}^2 + COV_{RA}^2} = \sqrt{(0.17)^2 + (0.25)^2} = 0.30$$

Other values needed to evaluate the resistance factor for Nordlund's method are:

λ_{QD} =	Bias factor for dead load = 1.08
λ_{QL} =	Bias factor for live load = 1.15
COV_{QD} =	Coefficient of variation for dead load = 0.13
COV_{QL} =	Coefficient of variation for live load = 0.18
γ_D =	Load factor for dead load = 1.25
γ_L =	Load factor for live load = 1.75

As shown by Table 7-4, the value of Q_D/Q_L varies with span length. Because Q_D/Q_L appears on the right side of Eq. 7-1, different values of resistance factor, ϕ, will be applicable to different span lengths. Two approaches are possible:

(1) Determine values of ϕ for a range of span lengths and <u>select a single value to be used for any span length</u> based on the results. This is the approach used in developing resistance factors for the AASHTO LRFD Specifications for bridge foundations.

(2) Use a specified span length and corresponding Q_D/Q_L ratio for a specific project in Eq. 7-1, to <u>determine a value of φ for use on a specific project</u>.

For purposes of illustration in this example, values of ϕ will be determined for values of $Q_D/Q_L = 1.0$ and $Q_D/Q_L = 3.0$, corresponding to span lengths in the range from 15 m to 50 m, as shown in Table 7-4.

It is also necessary to select a value of target reliability index to calculate a value of ϕ using Eq. 7-1. In keeping with conventional practice for driven piles, values of $\beta_T = 2.0$ and $\beta_T = 2.5$ will be used in this example. These values of β_T correspond to values of probability of failure of a single pile ranging approximately from 0.1 to 0.01.

The value of ϕ for the Nordlund method can be calculated by substituting values of these parameters into Eq. 7-1.

$$\phi = \frac{\dfrac{\lambda_R (\gamma_D Q_D / Q_L + \gamma_L)}{\lambda_{QD} Q_D / Q_L + \lambda_{QL}} \sqrt{\dfrac{1 + COV_{QD}^2 + COV_{QL}^2}{1 + COV_R^2}}}{\exp\left\{\beta_T \sqrt{\ln[(1 + COV_{QD}^2 + COV_{QL}^2)(1 + COV_R^2)]}\right\}} \qquad \text{(Eq. 7-1)}$$

Substituting in Eq. 7-1 for $Q_D/Q_L = 1.0$, $\beta_T = 2.0$, and the values of λ_{QD}, λ_{QL}, COV_{QL}, γ_D, and γ_L listed above, the value of ϕ is:

$$\phi = \frac{\dfrac{1.04\,(1.25 \text{ x } 1.00 + 1.75)}{(1.08 \text{ x } 1.00 + 1.15)} \sqrt{\dfrac{1 + (0.13)^2 + (0.18)^2}{1 + (0.30)^2}}}{\exp\left\{2.0 \sqrt{\ln[(1 + (0.13)^2 + (0.18)^2)(1 + (0.30)^2)]}\right\}} = 0.66$$

Using Eq. 7-1, values of ϕ for the Nordlund method were calculated for $Q_D/Q_L = 1.0$ and 3.0, and $\beta_T = 2.0$ and 2.5. The results are shown in Table 7-6. Based on these results and the desired level of conservatism, values of ϕ ranging from about 0.50 to 0.65 would be appropriate. The average value is about 0.58.

Table 7-6
Values of φ for Nordlund's Method Calculated Using Reliability Theory

Value of Q_D/Q_L	$\beta_T = 2.0$	$\beta_T = 2.5$
1.0	0.66	0.55
3.0	0.61	0.51

7.4 Simplified Reliability Method to Evaluate Resistance Factors

As shown in the preceding sections, the value of ϕ depends on the variability of loads as well as the variability of strength. However, it is possible to derive <u>approximate</u> expressions for ϕ that are independent of the variability of the loads, as described in Appendix 7A. The required assumption is that the values of the standard deviations for load and resistance do not differ by more than a factor of 3, which is usually the case. The other approximations are numerical. The resulting expression for φ is accurate within about $\pm$ 10%.

As derived in Appendix 7A, the resistance factor for lognormally distributed R and Q is approximated by:

$$\phi = \lambda_R \exp(-\alpha\, \beta_T\, COV_R) \qquad \text{(Eq. 7-6)}$$

7.4.1 Example Problem - Estimating Resistance Factors for the Nordlund Method

By using the resistance statistics for the Nordlund method determined previously ($\lambda_R = 1.04$, $COV_R = 0.30$) and setting $\alpha = 0.87$ in Eq. 7-6, approximate values of φ can be calculated for $\beta_T = 2.0$ and 2.5. These values of ϕ are presented in Table 7-7.

Table 7-7
Values of ϕ for Nordlund's Method
Calculated Using the Simplified Approximate Reliability Method

Target Reliability Index, β_T	Resistance Factor, ϕ
2.0	0.62
2.5	0.54

It is interesting to note the close agreement between the resistance factors in Table 7-6 with the values in Table 7-7 which are calculated independent of the load statistics. Based on the results shown in Table 7-7, a reasonable value of ϕ would be about 0.55 to 0.60. The average value is 0.58. This agreement is depends on using $\alpha = 0.87$ as a means of adjusting for the effects of the approximations involved in the method.

7.5 Fitting with ASD to Evaluate Resistance Factors

Values of ϕ can also be determined by "fitting" the value of ϕ to the conventional factor of safety that would be used in allowable stress design. The procedure for fitting is derived in Appendix 7B from the basic forms of the design equations used in allowable stress design (ASD) and load and resistance factor design (LRFD).

As derived in Appendix 7B, the resistance factor estimated by fitting with ASD can be calculated from

$$\phi = \frac{\gamma_D (Q_D / Q_L) + \gamma_L}{FS(1 + Q_D /QsubL)} \qquad \text{(Eq. 7-7)}$$

Substituting $\gamma_D = 1.25$ and $\gamma_L = 1.75$:

$$\phi = \frac{1.25(Q_D / Q_L) + 1.75}{(FS)(1 + Q_D / Q_L)} \qquad \text{(Eq. 7-8)}$$

7.5.1 Example Problem - Estimating Resistance Factors for the Nordlund Method

The value of ϕ calculated using this fitting equation depends on the ratio of Q_D/Q_L, and the factor of safety that would be used in ASD. Using values of $Q_D/Q_L = 1.0$ and 3.0 as used in Example 7.4, and values of FS = 2.25 and 2.75, which are reasonable values for the Nordlund method, the following results are obtained:

It can be seen that these values do not differ much from the values of ϕ calculated using reliability theory, which are listed in Tables 7-6 and 7-7. This indicates that FS = 2.25 and FS = 2.75 correspond closely with values of $\beta_T = 2.0$ and $\beta_T = 2.5$ in this case.

Based on the results shown in Table 7-8, it would be logical to select a value of in the range from 0.55 to 0.60. The average value of ϕ by the fitting method is 0.58, the same as the average value calculated using reliability theory.

Table 7-8
Values of ϕ for Nordlund's Method
Calculated Using Fitting with ASD

Values of Q_D/Q_L	FS = 2.25	FS = 2.75
1.0	0.67	0.55
3.0	0.61	0.50

7.6 Summary and Closing Remarks

In this chapter, straightforward relationships have been presented for calculating LRFD resistance factors. These relationships are expressed by equations that can be used in three different situations.

- If both load and resistance statistics (bias factor λ and coefficient of variation COV) are known, or can be estimated, Eq. 7-1 should be used

- If only resistance statistics are available, then Eq. 7-6 is recommended

- If no statistics on resistance are available, but the factor of safety FS used for ASD design is known, Eq. 7-7 can be used

It is recommended that before a decision is made on the selection of φ, as many of these relationships as possible should be used to calculate φ. The results of reliability-based Eq. 7-1 and Eq. 7-6 should not stand alone, but should be tempered by comparison with the value from Eq. 7-7, which represents ASD practice. In most cases, the reliability-based calculations should carry more weight, but not always. There is no substitute for engineering judgment based on experience.

In the examples on calculating ϕ for Nordlund's method, all three methods -- the basic and the simplified reliability calculations, and the method of fitting with ASD -- suggest that a reasonable value of ϕ would be 0.55 to 0.60. Thus, in this particular case, there is no conflict between the results of the three methods of evaluating φ, because the values of β_T (2.0 and 2.5) are compatible with the values of safety factor (2.25 and 2.75) that were used in the calculations.

Even though all three methods result in essentially the same values of ϕ, it is important to consider whether the results of the calculations are reasonable. One way of judging the reasonableness of the result is by comparing the ϕ-factor calculated for the Nordlund method with the ϕ-factors for other methods for evaluating the capacities of piles in cohesionless soils -- SPT-method, $\phi = 0.45$, CPT-method, $\phi = 0.55$. As compared with these values, does a value of $\varphi = 0.55$ to 0.60 for the Nordlund method seem reasonable? A more reliable method should have a larger value of φ. Is Nordlund's method more reliable than the SPT and CPT methods? Does Nordlund's provide a more reliable estimate of pile capacity? Does the calculated ϕ-factor seem appropriate? Would it be more appropriate if it was increased or decreased? These questions need to be answered before a final selection of the resistance factor for the use of Nordlund's method in a particular region or for a particular site is made.

In this chapter, a number of trends relating to the resistance factor have been observed. When β_T or FS is increased, the value of ϕ decreases. This trend is expected because both β_T and FS are measures of the safety margin. If the safety margin is to be increased, a smaller fraction of the calculated resistance must be used for design, which would require use of a smaller value of ϕ.

Another trend observed is that an increase in Q_D/Q_L results in a decrease in the value of ϕ. This trend occurs because the larger proportions of dead load with a smaller load factor than live load results in a smaller effective weighted load factor. In the basic design equation -- Eq. 7A-6 -- if the load factor is smaller, the ϕ-factor must be smaller to preserve the equality.

Another observation is the direct relationship between λ_R and ϕ. This can be seen in Eq. 7-6 and also in Eq. 7-1. If λ_R increases by 10 percent, ϕ increases by 10 percent. This one-to-one increase makes sense, because if a resistance model consistently under predicts the resistance, the φ-factor should increase. This increase in the value of ϕ reflects increased confidence that this model will underestimate the capacity.

Finally, because of the way bias factors and coefficients of variation are combined in Eq. 7-4 and Eq. 7-5, the more we know about the resistance, the less our confidence in it. This is ironic, but true. If we assume that only two variables influence the reliability of a resistance prediction, we combine their uncertainties using Eq. 7-5 and calculate a value for COV_R. If we discover that a third variable also contributes uncertainty and put its COV into Eq. 7-5, the resulting COV_R is larger. The increase in knowledge has increased our estimate of uncertainty. As we add more and more variables, the calculated φ-factor continues to decrease to reflect the increased uncertainty. Some sense can be brought to this dilemma by always calculating a φ-factor based on fitting to ASD and selecting a value that does not differ greatly from the value determined by fitting.

7.7 Student Exercise

1. Name and briefly describe the two general methods of calibrating resistance factors (Refer to Section 7.2)?

2. The "six sigma rule" can be used to provide rule-of-thumb estimates of the standard deviation, σ, and coefficient of variation, COV_R. Describe the "six sigma rule" (Refer to Section 7.3.1).

Appendix 7A
Derivation of Simplified Reliability Calculations of Resistance Factors

It is assumed R and Q are lognormally distributed and statistically independent, the safety index β, is given by Eq. 3-20 in Chapter 3 as:

$$\beta = \frac{\ln\left(\overline{R}/\overline{Q}\right)\sqrt{\left(1+COV_Q^2\right)/\left(1+COV_R^2\right)}}{\ln\left[\left(1+COV_R^2\right)\left(1+COV_Q^2\right)\right]} \qquad \text{(Eq. 7A-1)}$$

where $\overline{R}$ and $\overline{Q}$ are the mean values, COV_R and COV_Q are the coefficients of variation of resistance, R, and load, Q, respectively.

If it is assumed that COV_R and COV_Q are less than about 0.30, Eq. 7A-1 can be greatly simplified. The quotient under the radical in the numerator will be close to unity, and the function under the radical in the denominator can be expressed as the sum of two logarithmic functions, that is:

$$\ln\left[(1 + COV_Q^2)\,(1 + COV_R^2)\right] = \ln(1 + COV_Q^2) + \ln(1 + COV_R^2) \qquad \text{(Eq. 7A-2)}$$

Writing the natural logarithmic function as an infinite series:

$$\ln(1 + x^2) = x^2 - \frac{1}{2}x^4 + \frac{1}{3}x^6 - \frac{1}{4}x^8 + \ldots \qquad \text{(Eq. 7A-3)}$$

For an infinite series with alternating signs, the error is no more than the first neglected term. For COV's less than 0.30, the first term of the infinite series represents the function in Eq. 7A-3 within five percent, so that Eq. 7A-2 can be written as

$$\ln\left[(1 + COV_Q^2)\,(1 + COV_R^2)\right] \approx COV_Q^2 + COV_R^2 \qquad \text{(Eq. 7A-4)}$$

Because there are two terms in Eq. 7A-4, each with an error of approximation of about five percent, the combined error is about ten percent. The overall inaccuracy in this approximate method can be minimized by judicious selection of the value of the factor α, which is discussed below in connection with Eq. 7A-6.

Using Eq. 7A-4, the expression for β given by Eq. 7A-1 can be expressed as:

$$\beta \approx \frac{\ln(\overline{R}/\overline{Q})}{\sqrt{COV_Q^2 + COVsub\,R^2}} = \frac{\ln\overline{R} - \ln\overline{Q}}{\sqrt{COV_Q^2 + COV_R^2}} \qquad \text{(Eq. 7A-5)}$$

To separate the effects of R and Q, the linear approximation for the square root term suggested by Lind (1971) is used, that is:

$$\sqrt{COV_R^2 + COV_Q^2} \approx \alpha\,(COV_R + COV_Q) \qquad \text{(Eq. 7A-6)}$$

where α is a constant whose value can be used to achieve the best agreement with the results of Eq. 7-1. Trial and error has shown that a value of $\alpha = 0.87$ is best for the ranges of values of statistics for load and resistance involved in foundation design.

Using the relationship given by Eq. 7-6, Eq. 7A-5 can be expressed as:

$$\ln(\overline{R} / \overline{Q}) = \alpha\,\beta(COV_Q + COV_R) \qquad \text{(Eq. 7A-7)}$$

By taking the anti-logarithm of both sides:

$$\overline{R} / \overline{Q} = \exp[\alpha\,\beta\,(COV_Q + COV_R)] = \exp(\alpha\,\beta\,COV_Q)\exp(\alpha\,\beta\,COV_R) \qquad \text{(Eq. 7A-8)}$$

which can be separated into:

$$\overline{R}\exp(-\alpha\,\beta\,COV_R) = \overline{Q}\exp(\alpha\,\beta\,COV_Q) \qquad \text{(Eq. 7A-9)}$$

or using the definitions of the bias factor, and setting $\beta = \beta_T$, we get:

$$\lambda_R\,R_n\exp(-\alpha\,\beta_T\,COV_R) = \lambda_Q\,\text{Qsubn}\exp(\alpha\,\beta_T\,COV_Q) \qquad \text{(Eq. 7A-10)}$$

By comparing Eq. 7A-10 with the basic design equation Eq. 7A-11:

$$\phi\,R_n = \gamma\,Q_n \qquad \text{(Eq. 7A-11)}$$

the resistance factor for lognormally distributed R and Q is approximated by:

$$\phi = \lambda_R\exp(-\alpha\,\beta_T\,COV_R) \qquad \text{(Eq. 7A-12)}$$

This approximate resistance factor is expressed only in terms of its own statistics and some fraction of a target safety index. It does not involve the load statistics. The optimum value of the fitting factor has been found to be about 0.87, as noted previously.

Appendix 7B
Derivation for Fitting with ASD

The procedure for fitting is derived from the basic forms of the design equations used in ASD and LRFD.

For ASD:

$$\frac{R}{FS} \geq \sum Q_i \qquad \text{(Eq. 7B-1)}$$

where:

R =	Resistance (nominal resistance)
FS =	Factor of safety
Q_i =	Load (Σ Q_i = sum of all loads)

For LRFD:

$$\phi R \geq \sum \gamma_i Q_i \qquad \text{(Eq. 7B-2)}$$

where:

ϕ =	Resistance factor
γ_i =	Load factor for load type I

If both equations are used to derive expressions for resistance, these are:

$$R \geq (FS) \sum Q_i \qquad \text{(Eq. 7B-3)}$$

and

$$R \geq \frac{\sum \gamma_i Q_i}{\phi} \qquad \text{(Eq. 7B-4)}$$

When these expressions for R are set equal, the result is:

$$(FS) \sum Q_i = \frac{\sum \gamma_i Q_i}{\phi} \qquad \text{(Eq. 7B-5)}$$

which can be rearranged to give the following expression for φ:

$$\phi = \frac{\sum \gamma_i \, Q_i}{(FS)\, \Sigma\, Q_i} \qquad \text{(Eq. 7B-6)}$$

This equation can be expanded to

$$\phi = \frac{\gamma_D \, Q_D + \gamma_L \, Q_L}{(FS)(Q_D + QsubL)} \qquad \text{(Eq. 7B-7)}$$

Dividing both numerator and denominator on the right side by Q_L:

$$\phi = \frac{\gamma_D \,(Q_D / Q_L) \; + \; \gamma_L}{(FS)(1 \; + \; QsubD \, / \, Q_L)} \qquad \text{(Eq. 7B-8)}$$

CHAPTER 8
SPREAD FOOTING DESIGN

8.1 Introduction

For both Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD), the design of spread footing foundations requires consideration of geotechnical capacity, overall stability and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between spread footing design by LRFD and ASD with respect to the comparison of loads and deformations to resistance and tolerable deformations

- Identifies the strength and serviceability performance limits which must be considered for spread footing design by LRFD

- Briefly summarizes methods commonly used for estimating the geotechnical capacity and load-deflection behavior of spread footings, and for evaluating overall stability of spread footings on slopes

- Presents an example of spread footing foundation design by ASD and LRFD methods

8.2 Design Methods

With few exceptions, the design procedure for spread footing foundations using LRFD (A10.6) is identical to the procedure followed using ASD. Generally, the ultimate bearing capacity, overall stability and settlement are checked for spread footings subjected to vertical loads. Where spread footings are subjected to lateral or inclined loads, the effect of inclined loading on the ultimate bearing capacity is checked, and the foundation is checked for resistance to sliding. The following sections summarize the general design processes for spread footing foundation design using the ASD and LRFD approaches.

8.2.1 ASD Summary

Existing practice for geotechnical design of spread footing foundations follows the ASD approach, wherein all uncertainty in the variation of applied loads transferred to the foundation(s) and the ultimate geotechnical capacity of the soil and rock to support the loads are incorporated in a factor of safety, FS. As a result, loads used for design, Q, consist of those actual forces estimated to be applied directly to the structure. In LRFD terminology, this process is equivalent to applying a load factor of 1.0 to the estimated forces. In ASD, six primary performance and failure conditions are evaluated in the design of a spread footing foundation:

- Settlement
- Bearing Capacity

- Sliding Resistance
- Load Eccentricity (Overturning)
- Overall Stability
- Structural Capacity

Settlement

The design of spread footings by ASD is often controlled by deformation or settlement considerations. Thus, the design of footings by ASD requires estimation of foundation settlements under the applied loads, and comparison of estimated settlements with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 8-1)}$$

where:

δ_i = Estimated displacement or differential displacement (mm)
δ_n = Tolerable displacement or differential displacement established by the designer (mm)

Tolerable movement criteria are usually a function of the type of structure, and for bridges depend primarily on the span length and whether the superstructure has fixed- or simply-supported spans. In the case of abutments and other earth retaining structures, designs may be governed by criteria for lateral deflection or tilt, as described in Chapter 11.

Bearing Capacity and Sliding

Following selection of a preliminary footing size based on settlement considerations, the ultimate bearing capacity and sliding resistance, R_n, are estimated by available theoretical or semi-empirical procedures. The suitability of the design with respect to bearing capacity and sliding are then evaluated by determining the allowable vertical and horizontal load components, Q_{all}, using:

$$Q \leq Q_{all} = R_n / FS \qquad \text{(Eq. 8-2)}$$

where:

Q = Design load (kN)
Q_{all} = Allowable design load (kN)
R_n = Ultimate geotechnical capacity of a footing (kN)
FS = Factor of safety (dim)

The required FS with respect to bearing capacity and sliding are generally specified by the governing agency, and may be constant or variable. The AASHTO ASD FS (AASHTO, 1997b) for bearing capacity and sliding are constant, as shown in Table 8-1. The AASHTO FS values assume that designs are based on soil and rock properties determined through appropriate field and laboratory testing and not presumptive properties. The relatively high FS for bearing capacity reflects the acknowledgment that significant downward footing movements are required to completely mobilize the shearing resistance within the soil below the footing.

Table 8-1
Factors of Safety on Ultimate Geotechnical Capacity of Spread Footings for Bearing Capacity and Sliding Failure
(AASHTO, 1997b)

Failure Condition	Required Minimum Factor of Safety (FS)
Bearing Capacity of Footing on Soil or Rock	3.0
Sliding Resistance of Footing on Soil or Rock	1.5

Other sources provide guidelines for variable FS values for bearing capacity which consider the likelihood that maximum design loads will occur, the extent of subsurface exploration and the consequences of failure. FHWA (Cheney and Chassie, 1993) suggest a variable factor of safety for bearing capacity for footings on soil (Table 8-2) depending on the basis of soil strength estimates. Another example of a suggested system of variable FS values, with a FS ranging from 2.0 to 4.0, is shown in Table 8-3.

Table 8-2
Factors of Safety on Ultimate Bearing Capacity of Spread Footings on Soil
(Cheney and Chassie, 1993)

Basis for Soil Strength Estimate	Suggested Minimum Factor of Safety (FS)
Standard Penetration Tests	3.0
Laboratory/Field Strength Tests	2.5

Table 8-3
Variable Factors of Safety on Ultimate Bearing Capacity of Spread Footings
(after Vesic, 1975)

Category	Typical Structures	Category Characteristics	Required Minimum Factor of Safety (FS)			
			Permanent Structures		Temporary Structures	
			Complete Soil Exploration	Limited Soil Exploration	Complete Soil Exploration	Limited Soil Exploration
A	Railway bridges Warehouses Blast furnaces Hydraulic Retaining walls Silos	Max. design load likely to occur often; consequences of failure disastrous	3.0	4.0	2.3	3.0
B	Highway bridges Light industrial and public buildings	Max. design load may occur occasionally; consequences of failure serious	2.5	3.5	2.0	2.6
C	Apartment and office buildings	Max. design load unlikely to occur	2.0	3.0	2.0	2.3

Resultant Eccentricity (Overturning)
In addition to bearing capacity and sliding failure, spread footings are checked for stability against overturning based on an evaluation of the bearing pressure resultant location with respect to the centroid of the footing. In AASHTO ASD, the location of the bearing pressure resultant must be maintained:

- Within B/6 of the center of the footing for footings on soil
- Within B/4 of the center of the footing for footings on rock

Overall Slope Stability
For spread footings constructed on or near a slope, the overall stability of the slope must also be evaluated. Slope stability is generally evaluated using limiting equilibrium methods of analysis applicable to circular-arc and sliding-block type failures. The suitability of a design based on consideration of overall slope stability is evaluated by comparing the resisting force of soil friction and cohesion along the potential failure surface to the total loading or driving force acting along the potential failure surface as a result of soil weight and surcharge loadings, using:

$$FS = \frac{\Sigma\ \text{Soil Shear Resistance}}{\Sigma\ \text{Net Driving Forces}} \qquad \text{(Eq. 8-3)}$$

which can be rewritten as:

$$\Sigma\ \text{Net Driving Forces} \leq \frac{\Sigma\ \text{Soil Shear Resistance}}{FS} \qquad \text{(Eq. 8-4)}$$

The minimum required FS with respect to overall slope stability is generally specified by the governing agency and should reflect uncertainty related to:

- Method of analysis
- Method of soil shear strength determination
- Reliability and extent of subsurface information
- Potential for variation over time in subsurface conditions from those analyzed
- Consequences of a failure

Required or recommended FS values vary significantly. FHWA (Cheney and Chassie, 1993) suggests the minimum FS values presented in Table 8-4, modified for individual projects based on the considerations listed above.

Table 8-4
Suggested Minimum Factors of Safety
for Overall Slope Stability From FHWA
(Cheney and Chassie, 1988)

Condition	Recommended Minimum Factor of Safety (FS)
Highway Embankment Side Slopes	1.25
Slopes Affecting Significant Structures (e.g., bridge abutments, major walls)	1.30

AASHTO ASD (AASHTO, 1997b) provides specific requirements for minimum FS values considering the detail of subsurface exploration and potential consequences of failure, as indicated in Table 8-5.

Table 8-5
Required Minimum Factors of Safety
for Overall Slope Stability From AASHTO ASD
(after AASHTO, 1997b)

Condition	Required Minimum Factor of Safety (FS)	
	Detailed Exploration[1]	Limited Exploration
Highway Embankment Slopes and Retaining Walls	1.3	1.5
Slopes Supporting Abutments or Abutments Above Retaining Walls	1.5	1.8

[1] Soil and rock parameters and ground water levels based on in-situ and/or laboratory tests.

Structural Resistance

Finally, after all geotechnical deformation and capacity criteria are met and a suitable spread footing size is determined, the structural design of the footing is performed using service loads and allowable stresses (AASHTO, 1997b). Because the structural design of the footing is somewhat independent of the geotechnical design, the structural design is sometimes performed by LFD or LRFD to be consistent with the superstructure design.

8.2.2 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. ***When properly developed and applied, the LRFD approach provides a consistent level of safety for the design of all structure components.*** Thus, the probability that a structure component will fail or perform unacceptably is no different than any other component. As described in Section 4.3, the resistance and deformation of supporting soil and rock materials and structure components must satisfy the LRFD equations below. For the Strength Limit States:

$$\Sigma \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 8-5)}$$

For the, Service Limit States:

$$\Sigma\ \eta_i\ \gamma_i\ \delta_i \leq \phi\ \delta_n \qquad \text{(Eq. 8-6)}$$

where:

η_i = Factors to account for effects of ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)
δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement (mm)

Relative to bearing capacity and sliding of a spread footing, the suitability of a spread footing with respect to the geotechnical resistance can be obtained using Eq. 8-6, rewritten as:

$$\sum \eta_i\ \gamma_i\ Q_i \leq \phi\ Q_{ult} = Q_R \qquad \text{(Eq. 8-7)}$$

where:

$\sum \eta_i\ \gamma_i\ Q_i$ = Factored load effect (kN)
ϕ = Resistance factor (dim)
Q_{ult} = Ultimate geotechnical resistance of a spread footing (kN)
Q_R = Factored geotechnical resistance of a spread footing (kN)

The load factors and load factor combinations used for design were presented in Chapter 4. In general, values of $\gamma_i > 1.0$ are used to evaluate ultimate ground or structure capacity at the Strength Limit States, whereas the deformation performance of structures is evaluated at the Service I Limit State using $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads). In ASD (AASHTO, 1997b), values of $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads) are used to evaluate structures for both strength (allowable stress) and serviceability (deflection). ***Therefore, analysis of foundation deformations (e.g., settlement or lateral displacement) by LRFD and ASD are identical.***

When using Eq. 8-5 for spread footing foundation design at the Strength Limit States, the following values of η can normally be used:

- $\eta_D = \eta_R = 1.00$
- $\eta_I = 1.05$ for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures.

Determination of the operational importance of a structure (such as a bridge) is made by the facility owner, as described in Chapter 4. The appropriate value of η_I, is then applied throughout the superstructure design by the bridge engineer. The value of η_I selected by the superstructure designer should then be applied in the foundation design. For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

When using Eq. 8-6 to evaluate a spread footing at any Service Limit State, η_D, η_R, and $\eta_I = 1.0$.

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4 and Section 8.2.2.1, and loads should be developed as described in Chapter 4. The ultimate resistance, R_n, should be determined for each type of resistance (e.g., bearing criteria, sliding, or overall stability) and the footing should be checked for overturning based on load eccentricity criteria as described in Section 8.3. Values of $\varphi \leq 1.0$ are applied when evaluating spread footing resistance for any strength limit state using Eq. 8-5. Currently, the value of $\varphi = 1.0$ is applied when evaluating a spread footing for any service limit state using Eq. 8-6. Selection and modification of resistance factors, φ, are described in Sections 8.2.2.2 through 8.2.2.4.

Structural design of footings is somewhat independent of the geotechnical design. Structural design is performed using existing LRFD procedures using factored loads and the factored bearing pressure distribution.

8.2.2.1 Limit States (*A10.5*)

In general, the design of spread footing foundations using LRFD requires evaluation of footing suitability at various Limit States (i.e., applicable Strength Limit States and the Service I Limit State). The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations or performance limits which must be evaluated for footings designed at the Strength and Service I Limit States are summarized in Table 8-6. As conditions warrant, it may also be necessary to evaluate foundation performance at other limit states (e.g., Extreme Event I for loading from earthquakes).

Table 8-6
Limit States for Design of Spread Footing Foundations

Performance Limit	Strength Limit State(s)	Service I Limit State
Settlement		✔
Bearing Resistance	✔	
Sliding Resistance	✔	
Overturning (Eccentricity of Base Pressure Resultant)	✔	
Overall Stability		✔
Structural Capacity	✔	

More detailed discussion of procedures used to evaluate the various Performance Limit States is provided in Section 8.3.

8.2.2.2 Resistance Factors (*A10.5.5*)

Resistance factors for geotechnical design of spread footings using LRFD with respect to bearing capacity and sliding resistance were developed using the reliability-based calibration procedure described in Chapter 3. The procedure involved:

- Estimating the level of reliability inherent in various methods for predicting spread footing capacity
- Observing the variation in reliability levels with different span lengths, dead- to live-load ratios, foundation geometry, methods used to estimate ultimate resistance and load combinations
- Selecting a target reliability, β_T, based on the margin of safety used for ASD
- Calculating resistance factors, ϕ, consistent with the target reliability index coupled with experience and judgment

As an example of this process, Table 8-7 summarizes the results of analyses performed to develop φ values for estimating the bearing capacity of spread footings on sand. For these analyses, the variables included:

- Footing sizes and load levels corresponding to span lengths of approximately 10 m and 50 m
- A dead to live load ratio of 3.0 which corresponds to a bridge with a span length of 60 m
- A target reliability index, β_T equal to 3.5
- Method used to estimate Q_{ult} (i.e., semi-empirical methods for estimating bearing capacity directly from the results of CPT and SPT in-situ testing)

Although the CPT is known to be less variable than the SPT, the lower β values associated with the CPT-based procedure reflect the use of a lower factor of safety in the CPT-based procedure compared to that used in the procedure based on SPT data. The lower factor of safety is employed partly because the CPT data is obtained continuously through the boring depth, whereas SPTs are performed intermittently (typically at a 1 meter center to center spacing) through the boring depth. The selected values of resistance factor reflect adjustment of the reliability-based values to be more consistent with ASD experience.

Table 8-7
Resistance Factors for Semi-Empirical Evaluation of Bearing Capacity for Spread Footings on Sand Using Reliability-Based Calibration
(modified after Barker, et al., 1991b)

Estimation Method	Factor of Safety FS	Average Reliability Index β	Target Reliability Index β_T	Span (m)	Resistance Factor		
					Fitting with ASD	Reliability Based	Selected ϕ
SPT	4.0	4.2	3.5	10	0.37	0.49	0.45
				50	0.37	0.53	0.45
CPT	2.5	3.2	3.5	10	0.60	0.52	0.55
				50	0.60	0.57	0.55

Similar to ASD, LRFD requires an evaluation of footing safety against overturning based on the location of the factored bearing pressure resultant on the base of the footing with respect to the footing centroid. As described in Chapter 11, the eccentricity criteria for AASHTO LRFD (*A10.6.3*) were calibrated directly with existing AASHTO ASD criteria through analysis of cantilever and gravity retaining walls and abutments covering a wide range of load conditions, design heights and dead to live load ratios (Barker, et al., 1991b). Variations in these parameters were found to have little influence on the resultant location. Based on the calibration study, the location of the factored bearing pressure resultant in AASHTO LRFD must be maintained:

- Within B/4 of the center of footing for footings on soil (*A10.6.3.1.5*)
- Within 3B/8 of the center of footing for footings on rock (*A10.6.3.2.5*)

Overall stability of slopes containing spread footing foundations is performed under AASHTO LRFD at the Service I Limit State (*A3.4.1, A10.6.2.2.4*). The resistance factors for overall slope stability analysis using LRFD were developed through direct calibration with ASD using the following:

$$\phi = \frac{\bar{\gamma}}{FS} \tag{Eq. 8-8}$$

where:

ϕ = Resistance factor (dim)
$\bar{\gamma}$ = Average load factor of driving forces on soil shear resistance (dim)
FS = Target factor of safety (dim)

Current common methods of analysis and particularly computer codes for evaluation of overall slope stability do not lend themselves readily to the LRFD concept for the Strength Limit State, which employs maximum and minimum load factors for permanent earth and dead loads. Consider, for example, a simple slope composed of dry soil and analyzed using a limiting equilibrium method of analysis circular arc failure surfaces. The loads and resisting forces on a typical slice of soil are shown in Figure 8-1. As indicated, the driving force and frictional component of resistance along the failure surface are a function of the soil weight, whereas the cohesive component of resistance along the failure surface is independent of the soil weight. For the typical condition discussed in Chapter 4, different load factors are applied to earth loads and surcharges for Strength Limit State evaluations depending on whether the loads have a stabilizing or destabilizing effect. Therefore, the load factors on the soil weight and any surcharge loads (and therefore the factored load effects) should vary depending on their location along the failure surface (i.e., $T = W_T \sin\alpha$ may either drive of resist movement). Because no current method of analysis readily accommodates variable soil weights, the LRFD Specification (AASHTO, 1997a) specifies evaluation of overall stability at the Service Limit State (i.e., $\gamma = 1.0$). Further, because no reliability-based calibration of typical slope stability methods has yet been performed, the LRFD Specification applies a single resistance factor to both the frictional and cohesive soil shear resistance, independent of the method of soil shear strength determination. The resistance factor specified in AASHTO (1997a) for a slope supporting or containing a foundation is 0.65 (Table 8-8), which results in an equivalent ASD safety factor of approximately 1.5 (from Eq. 8-8).

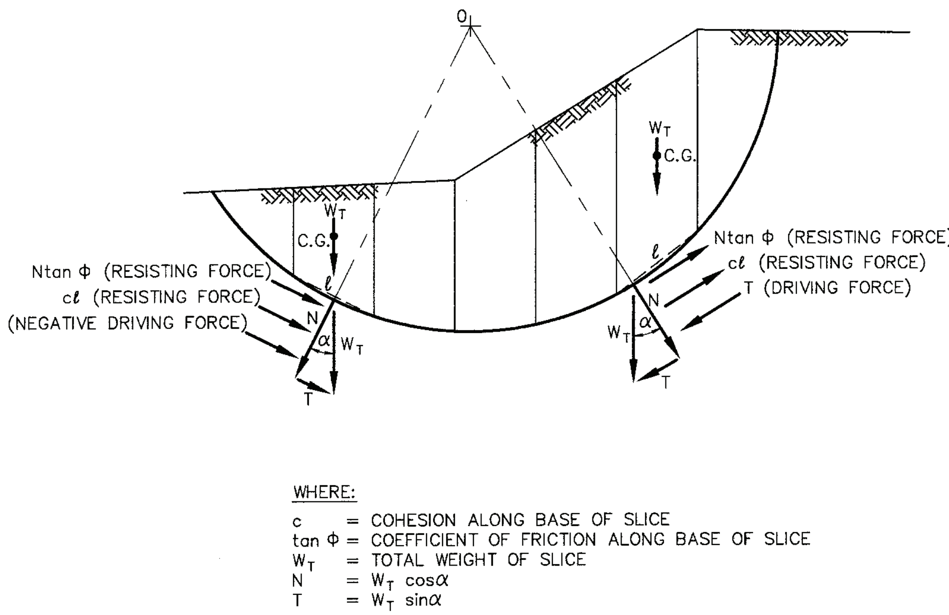


Figure 8-1
Circular Arc Slope Stability Failure Surface Showing Typical Soil Slice Forces

Resistance factors for the geotechnical design of spread footings are presented in Table 8-8. A majority of the resistance factors in the table are based on design procedures which are commonly used for ASD of spread footings. The resistance factors presented in the table were developed mostly by calibration with reliability theory where sufficient statistical information was available regarding particular design procedures or soil and soil-structure properties, tempered with engineering judgment for some cases. Where statistical information was insufficient, resistance factors were chosen by calibration with ASD (fitting) so that LRFD and current design would result in footings having similar dimensions.

8.2.2.3 Comparison of Spread Footing Design Using LRFD and ASD

To illustrate the relative differences between LRFD and ASD, the equivalent LRFD factor of safety (FS_{LRFD}) has been determined for each of the methods presented in Table 8-8 for estimating the geotechnical capacity of a spread footing with respect to bearing capacity, sliding and overall slope stability. In the table, FS_{LRFD} was determined as:

$$FS_{LRFD} = \bar{\gamma} / \phi \qquad \text{(Eq. 8-9)}$$

where:

$\bar{\gamma}$ = Average load factor (assumed = 1.45 for bearing capacity and sliding and taken = 1.00 for overall stability)

ϕ = Resistance factor from Table 8-8

The applicable ASD factor of safety (FS_{ASD}) from Tables 8-1, 8-2 and 8-5 for each category is also presented in Table 8-9.

Table 8-8 (A10.5.5-1)
Resistance Factors for Geotechnical Strength Limit State for Shallow Foundations
(Modified after AASHTO, 1997a)

METHOD/SOIL/CONDITION		RESISTANCE[(1)] FACTOR
Bearing Capacity	Sand	
	• Semi-empirical procedure using SPT data	0.45
	• Semi-empirical procedure using CPT data	0.55
	• Rational Method using ϕ_f from:	
	– SPT data	0.35
	– CPT data	0.45
	Clay	
	• Semi-empirical procedure using CPT data	0.50
	• Rational Method using S_u from:	
	– lab tests (UU Triaxial)	0.60
	– field Vane Shear tests	0.60
	– CPT data	0.50
	• Rock: Semi-empirical procedure	0.60
	• Plate Load Test	0.55
Sliding and Passive Pressure	• Precast concrete on sand using ϕ_f from:	
	– SPT data	0.90
	– CPT data	0.90
	• Concrete cast-in-place on sand using ϕ_f from:	
	– SPT data	0.80
	– CPT data	0.80
	• Precast or cast-in-place concrete on 150-mm thick sand subbase on clay where $S_u < 0.5\ \sigma_n$	
	• Using shear resistance (S_u) measured from:	
	– lab tests (UU Triaxial)	0.85
	– field Vane Shear tests	0.85
	– CPT data	0.80
	• Precast or cast-in-place concrete on 150-mm thick sand subbase on clay where $S_u > 0.5\ \sigma_n$	0.85
	• Soil on soil	1.00
	• Passive pressure component of sliding resistance	0.50
Overall Stability	• Geotechnical parameters are well defined and slope does not support/contain a structure	0.85[(2)]
	• Geotechnical parameters are based on limited data, or slope supports/contains a structure	0.65[(2)]

[(1)] Refer to Section 8.3 for description of design procedures for which ϕ's have been calibrated.
[(2)] From AASHTO LRFD A3.4.1

Table 8-9
Comparison of ASD Factor of Safety
with LRFD Equivalent Factor of Safety for Spread Footing Foundations

METHOD/SOIL/CONDITION		RESISTANCE FACTOR, ϕ	LRFD EQUIVALENT FACTOR OF SAFETY, $\bar{\gamma}/\phi^{(1)}$	ASD FACTOR OF SAFETY
Bearing Capacity	Sand			
	• Semi-empirical procedure using SPT data	0.45	3.2	3.0
	• Semi-empirical procedure using CPT data	0.55	2.6	$2.5^{(2)}$
	• Rational Method using ϕ_f from:			
	– SPT data	0.35	4.1	$2.5^{(2)}$
	– CPT data	0.45	3.2	$2.5^{(2)}$
	Clay			
	• Semi-empirical procedure using CPT data	0.50	2.9	$2.5^{(2)}$
	• Rational Method using S_u from:			
	– lab tests (UU Triaxial)	0.60	2.4	$2.5^{(2)}$
	– field Vane Shear tests	0.60	2.4	$2.5^{(2)}$
	– CPT data	0.50	2.9	$2.5^{(2)}$
	• Rock: Semi-empirical procedure	0.60	2.4	3.0
	• Plate Load Test	0.55	2.6	3.0
Sliding and Passive Resistance	• Precast concrete on sand using ϕ_f from:			
	– SPT data	0.90	1.6	1.5
	– CPT data	0.90	1.6	1.5
	• Concrete cast-in-place on sand using ϕ_f from:			
	– SPT data	0.80	1.8	1.5
	– CPT data	0.80	1.8	1.5
	• Precast or cast-in-place concrete on 150-mm thick sand subbase on clay where $S_u < 0.5\ \sigma_n$			
	• Using shear resistance (S_u) measured from:			
	– lab tests (UU Triaxial)	0.85	1.7	1.5
	– field Vane Shear tests	0.85	1.7	1.5
	– CPT data	0.80	1.8	1.5
	• Precast or cast-in-place concrete on 150-mm thick sand subbase on clay where $S_u > 0.5\ \sigma_n$	0.85	1.7	1.5
	• Soil on soil	1.00	1.45	1.5
	• Passive pressure component of sliding resistance	0.50	2.9	$1.5^{(3)}$
Overall Stability	Shallow foundations on/near slope	0.65	1.5	$1.5^{(4)}$

(1) An average load factor, $\bar{\gamma}$ = 1.45 is assumed for bearing capacity and sliding. The load factor for all stabilizing and destabilizing loads for overall stability is taken as 1.00.
(2) FHWA suggests a reduced FS = 2.5 when soil strength is estimated from laboratory or field strength tests.

[3] In ASD, passive resistance in front of a footing is often neglected in evaluating sliding resistance due to the relatively large displacement required to fully mobilize the passive resistance.
[4] A minimum factor of safety of 1.5 is specified for slopes supporting abutments or retaining walls by FHWA and AASHTO ASD where a detailed subsurface exploration and/or laboratory testing program has been performed to define subsurface conditions and soil properties.

The value of FS_{LRFD} in Table 8-9 was determined assuming $\bar{\gamma}$ = 1.45. Actually, γ can range from 1.25 for structure component loads to 1.75 for live loads so that FS_{LRFD} could vary from the value shown in the table depending on the relative proportion of live to dead load for a particular structure. However, assuming that $\bar{\gamma}$ = 1.45 (which corresponds to a dead to live load ratio of 3 to 2) represents a reasonable approximation, values of FS_{LRFD} are generally similar to FS_{ASD}, with FS_{LRFD} typically ranging from about 20% lower to 40% higher than FS_{ASD} for predictive methods generally considered more and less reliable, respectively. Thus, while general agreement exists between LRFD and ASD, the comparison depends on the load factor and method of capacity analysis used for design. Accordingly, whereas spread footing foundation designs performed using LRFD will be comparable to those using ASD, a precise or very close approximation between the two should not typically be expected.

8.2.2.4 Modification of Resistance Factors

As indicated in Section 8.2.2.2, the LRFD resistance factors for geotechnical design of spread footings in Table 8-8 were generally developed using a reliability-based calibration procedure, except for the resistance factor for overall stability, which is based on calibration with ASD. As described in Section 8.2.2.3, Table 8-9, application of these resistance factors in conjunction with $\bar{\gamma}$ = 1.45 results in an "equivalent" factor of safety ranging from 2.4 to 4.1 for bearing capacity and 1.6 to 2.9 for sliding, depending on the soil conditions, loading conditions and method of geotechnical capacity prediction.

In ASD, the designer or owner might decide to increase or decrease required factors of safety or allowable design stresses in consideration of a number of factors, such as:

- The potential consequences of a failure
- The extent or quality of information available from geotechnical exploration and testing
- Past experience with the soil conditions encountered and/or capacity prediction method used
- The level of construction control anticipated or specified
- The likelihood that the design loading conditions will be realized

When using LRFD, similar flexibility to vary the required level of safety should also be available and, in some cases, is inherent in the load and resistance factors used. Whereas the same factor of safety is generally used in ASD regardless of the source of loading, the equivalent factor of safety in LRFD (defined by Eq. 8-10) varies for a given resistance factor depending on the source of loading as a function of the available load factors. As stated previously, an average load factor of 1.45 was generally used to identify the equivalent safety factors in Table 8-9.

Although an average load factor of 1.45 is generally reasonable for a typical bridge abutment, pier or a retaining wall, the average load factor could theoretically range from a low of 1.25 (i.e., all dead load) to nearly 1.75 (i.e., extremely high live load). To modify the resistance factors for geotechnical design of footings to account for average load factors other than 1.45 and equivalent factors of safety other than those identified in Table 8-9, the following equation may be used:

$$\phi_m = \phi_T \text{ x } \left(\frac{FS_T}{FS_D} \right) \times \left(\frac{\bar{\gamma}_T}{\bar{\gamma}_D} \right) \qquad \text{(Eq. 8-10)}$$

where:

ϕ_m = Modified resistance factor (dim)
ϕ_T = Tabulated resistance factor (dim)
FS_T = Tabulated factor of safety (dim)
FS_D = Desired factor of safety (dim)
$\bar{\gamma}_T$ = Tabulated average load factor = 1.45 (dim)
$\bar{\gamma}_D$ = Actual average load factor including modification for operational importance (dim)

Modifying resistance factors may seem reasonable, but such modification may not be consistent with the goal of LRFD to achieve equal reliability against failure of structure components, unless the factor of safety accurately models the reliability of the footing capacity predictive method used.

The resistance factors may be more appropriately modified through application of the probabilistic procedures described in Chapter 3 to achieve the desired level of reliability if sufficient data are available.

8.2.3 Summarized Comparison of ASD and LRFD

As noted before, the process used to develop a spread footing foundation design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 8-2. Specific differences between the methods and other important issues are highlighted in the following section.

Other aspects of the spread footing design such as identifying special considerations (e.g., potential for loss of support through scour), developing a design foundation profile and determining requirements for construction control are inherent aspects of the design process required for both LRFD and ASD.

8.3 Performance Limits

Design of a spread footing foundation by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the footing size and details, and in establishing a suitable bearing level to meet the criteria for vertical, inclined and/or moment loading, the design of spread footing foundations requires consideration of many factors which can affect the spread footing foundation performance, including:

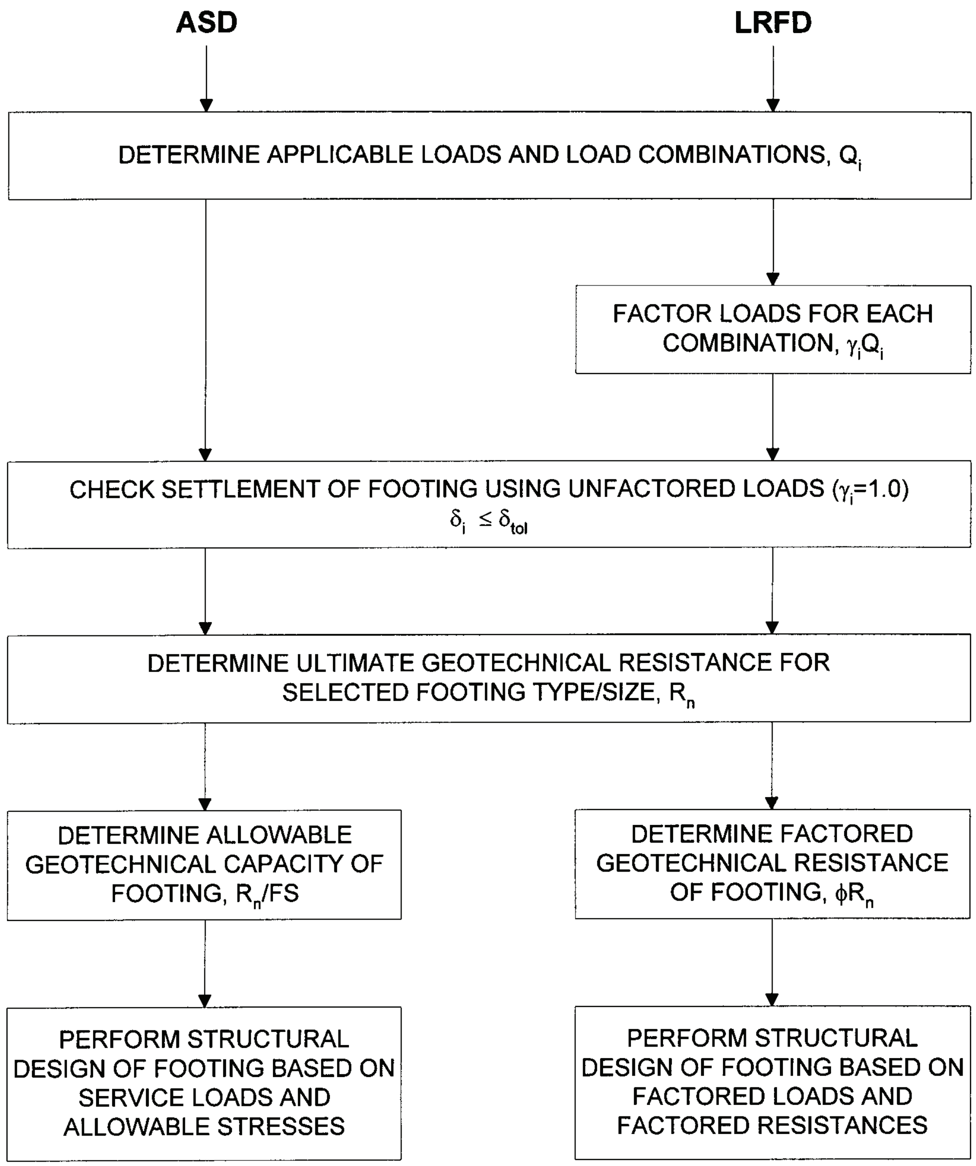


Figure 8-2
Generalized Flow Chart for Spread Footing Design by ASD and LRFD

- Bearing resistance to vertical and inclined loads and moments
- Sliding resistance to lateral loads
- Resistance to overturning forces and moments
- Resistance to uplift forces
- Resistance to effects of scour and frost

- Resistance to variable ground water levels, including the effect of seepage when footings support walls which do not provide adequate drainage
- Geometric constraints (e.g., nearby structures which could impose load on or be loaded by the footing)

For these performance limits, there is no difference between LRFD and ASD analysis procedures. The following sections highlight differences between LRFD and ASD in the performance criteria and application of design procedures.

8.3.1 Displacements and Tolerable Movement Criteria (*A10.6.2.2*)

The vertical and lateral displacements of spread footings must be evaluated for all applicable dead and live load combinations, and compared with tolerable movement criteria. Because evaluations of structure displacements by LRFD are made at the Service I Limit State where $\gamma = 1.0$ and $\phi = 1.0$, methods used to estimate settlement by LRFD are identical to those used for ASD. Consequently, footing settlements can be computed by conventional methods using semi-empirical correlations with in-situ test results or measurements by in-situ or laboratory test methods to estimate engineering soil properties. The potential for lateral displacement of footings should be estimated where the footing is:

- Subjected to inclined or lateral loads
- Placed on or near an embankment slope
- Subjected to loss of foundation support by scour
- Supported on a sloping bearing stratum (usually rock)

Vertical settlement can be a combination of the elastic, consolidation and secondary compression movements. In general, the settlement of footings on cohesionless soils, very stiff to hard cohesive soils, and rock with tight, unfilled joints will be elastic and will occur as load is applied. For footings on very soft to stiff cohesive soils, the potential for consolidation and secondary compression settlement components should be evaluated in addition to elastic settlement.

Methods for estimating settlement of footings are presented in the AASHTO LRFD Specification (1997a) (*A10.6.2.2.3*) and are described in Gifford, et al. (1986), Cheney and Chassie (1993) and Barker, et al. (1991a). The elastic settlements are estimated using elastic theory and a value of elastic modulus based on the results of in-situ or laboratory testing. The consolidation and secondary compression settlement on cohesive soils is estimated using consolidation theory and the results of laboratory consolidation tests.

The tolerable movement of spread footing foundations (*A10.6.2.2*) depends on structural criteria such as the type and size of the supported superstructure, as well as factors such as the cost and difficulty of implementing repairs in the future, rideability, aesthetics and safety. Based on such factors, limits on foundation movements have been set arbitrarily or based on empirical assumptions (e.g., 25 mm maximum vertical movement) without consideration of actual structural performance. However, due to the effects of creep, relaxation and redistribution of forces in bridge superstructures, bridges can accommodate substantially more settlement than traditionally allowed or anticipated in design. This conclusion is supported by the results of FHWA research (Moulton, et al., 1985) which involved a performance survey of more than 200 bridges supported on shallow foundations. The study showed that angular distortions (i.e., relative settlement of adjacent foundations divided by the span length) of 0.008 or less for simple-span structures and 0.004 or less for continuous-span structures are acceptable. Using these relationships, the maximum tolerable settlement between foundations can be much greater than normally assumed. The final selection of a tolerable movement should be made by the bridge designer in coordination with the geotechnical engineer.

8.3.2 Bearing Resistance (*A10.6.3.1 and A10.6.3.2*)

Spread footing foundations must be designed to resist vertical loads without bearing failure of the foundation soil or rock, and structural failure of the footing. Methods used for ASD to estimate the ultimate geotechnical bearing resistance of a footing can be used for LRFD. Therefore, the factored unit bearing resistance, q_R, can be determined from:

$$q_R = \phi\, q_n = \phi\, q_{ult} \qquad \text{(Eq. 8-11) } (\textit{A10.6.3.1.1-1})$$

where:

q_R = Factored unit bearing resistance (kPa)
q_n = Nominal unit bearing resistance (kPa)
ϕ = Resistance factor for bearing capacity from Table 8-7 (dim)
q_{ult} = Nominal (or ultimate) unit bearing resistance (kPa)

As the load on a footing is rarely concentric, the bearing pressure varies across the base of the footing. The variation in bearing pressure is generally assumed to be linear. For foundations on soil, compression of soil beneath the footing results in a redistribution of bearing stress to a more uniform value. Therefore, for foundations on soil, the factored bearing resistance is compared to the factored uniform unit bearing stress acting on the base of a concentrically-loaded footing area as follows:

$$q_R \geq \gamma\,\bar{q} = \frac{\sum \eta_i\,\gamma_i\,Q_i}{A'} \qquad \text{(Eq. 8-12)}$$

where:

$\gamma\,\bar{q}$ = Factored uniform unit bearing stress (kPa)
$\sum \eta_i \gamma_i Q_i$ = Factored vertical load effect (kN)
A' = Effective footing area (m^2)

and A' is determined (*A10.6.3.1.5*) as:

$$A' = (B - 2\,e_B)(L - 2\,e_L) \qquad \text{(Eq. 8-13)}$$

where:

B = Footing width (m)
L = Footing length (m)
e_B, e_L = Eccentricity of load resultant with respect to centroid of footing (m)

For foundations on rock, the factored bearing resistance is compared to the factored maximum unit bearing stress on the base of the effective footing area as follows (*A11.6.3.2*):

$$q_R \geq \gamma\, q_{max} \quad \text{(Eq. 8-14)}$$

where:

$\gamma\, q_{max}$ = Factored maximum unit bearing stress (kPa)

Various procedures are available for estimating the ultimate geotechnical bearing resistance of spread footings supported on soil or rock using rational or semi-empirical methods. ***When using the AASHTO LRFD Specification, only those methods referred to in Table 8-8 for which calibrated φ-factors have been developed can be used without developing other method-specific resistance factors.*** These methods include:

- Theoretical (rational) methods for footings on soil (*A10.6.3.1.2*) where the shear resistance of soil is developed from results of in-situ or laboratory strength tests (AASHTO, 1994; Barker, et al., 1991b)

- Semi-empirical methods for footings on soil (*A10.6.3.1.3*) where bearing resistance is developed directly from results of SPT for footings on cohesionless soil or from results of CPT for footings on cohesionless and cohesive soils (AASHTO, 1994)

- Semi-empirical methods for footings on rock (*A10.6.3.2.2*) where bearing resistance is developed using rock mass rating systems (Carter and Kulhawy, 1988)

- Methods for footings on soil or rock where shear resistance of foundation materials is developed from results of plate load tests (*A10.6.3.1.4 and A10.6.3.2.4*) (AASHTO, 1997a)

The resistance factors in Table 8-8 were developed for the specific design procedures referenced in the AASHTO LRFD Specification (1997a). Other procedures can be used for LRFD provided calibrated ϕ factors are developed using the methods described in Chapters 3 and 7. In lieu of more detailed calibration, Table 8-9 and Section 8.2.2.4 can be used as a guide in selecting and modifying resistance factors for various procedures to be consistent with ASD.

8.3.3 Sliding Resistance (*A10.6.3.3*)

Spread footing foundations must be designed to resist lateral and inclined loads without sliding failure of the foundation. Methods used for ASD to estimate the ultimate sliding resistance of a footing can be used for LRFD. Therefore, the factored sliding resistance of a footing subjected to inclined or lateral loading, Q_R, can be determined as:

$$Q_R = \phi Q_n = \left(\phi_\tau Q_\tau + \phi_{ep} Q_{ep} \right) \qquad \text{(Eq. 8-15) } (A.6.3.3\text{-}1)$$

where:

- Q_n = Nominal sliding resistance of footing (kN)
- ϕ_τ = Resistance factor for shear between footing and foundation material from Table 8-8 (dim)
- Q_τ = Nominal shear resistance between footing and foundation material (kN)
- ϕ_{ep} = Resistance factor for passive earth pressure component of sliding resistance from Table 8-8 (dim)
- Q_{ep} = Nominal passive resistance of foundation material available during the design life of the footing (kN)

The sliding resistance of a footing on cohesionless soil is based on the normal stress and the interface friction between the foundation and the soil. In the AASHTO LRFD Specification, it is assumed that at least 150 mm of cohesionless soil is placed beneath a footing bearing on a cohesive soil subgrade. For this case, the sliding resistance is assumed to be a combination of shear through the cohesive soil subgrade and the cohesionless subbase depending on the contact stress distribution along the base of the footing.

Often, the passive resistance component of sliding resistance, Q_{ep}, is ignored or reduced because it is difficult to assure that loss of ground (e.g., temporary excavation) or loss of contact by shrinkage will not occur in the future.

The resistance factors in Table 8-8 were developed for the specific design procedures referenced in the AASHTO LRFD Specification. Other procedures can be used for LRFD provided calibrated ϕ factors are developed using the methods described in Chapters 3 and 7. In lieu of more detailed calibration, Table 8-9 and Section 8.2.2.4 can be used as a guide in selecting and modifying resistance factors for various procedures to be consistent with ASD.

8.3.4 Load Eccentricity (Overturning) (*A10.6.3.1.5 and A10.6.3.2.5*)

Spread footing foundations must be designed to resist overturning which results from lateral and eccentric vertical loads. The criteria for evaluating overturning in ASD requires that the resultant force be maintained within B/6 and/or L/6 of the foundation centroid for footings on soil (*A10.6.3.1.5*), and within B/4 and/or L/4 for footings on rock (*A10.6.3.2.5*). For LRFD, the criteria were revised to reflect the factoring of loads in LRFD. As a result, the eccentricity of footings for factored loads must be less than B/4 and/or L/4 for footings on soil, and less than 3B/8 and/or 3L/8 for footings on rock. These limits were developed by direct calibration with ASD (Barker, et al., 1991a; Barker, et al., 1991b). The criteria for ASD and LRFD are illustrated below in Figure 8-3 for footings on soil and in Figure 8-4 for footings on rock.

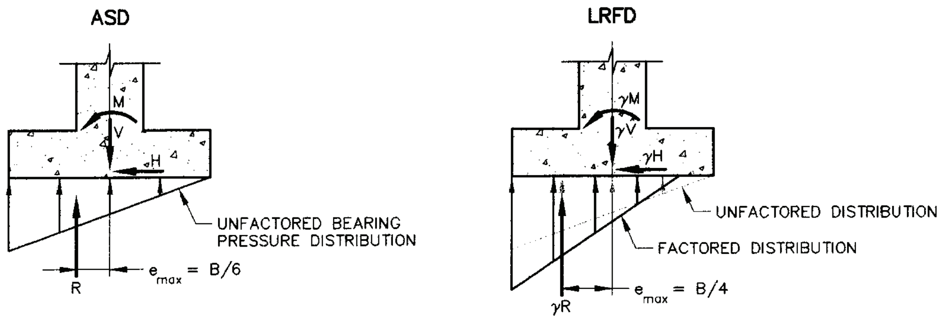


Figure 8-3
Comparison of Resultant Force Location
for Footings on Soil Using ASD and LRFD

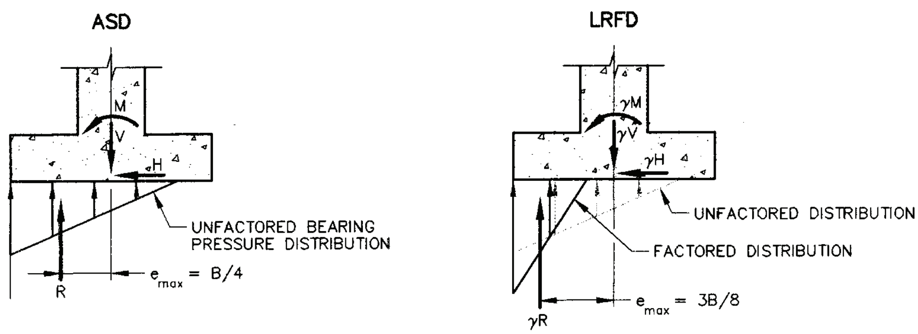


Figure 8-4
Comparison of Resultant Force Location
for Footings on Rock Using ASD and LRFD

As illustrated in the figures, the effect of factoring loads is to increase the eccentricity of the load resultant such that the permissible eccentricity is increased for LRFD.

8.3.5 Overall Stability (*A10.6.2.2.4*)
Slopes containing spread footing foundations must be designed to resist loads associated with soil, ground water, and footing and other surcharge pressures without a slope failure. Limiting equilibrium methods of analysis which employ the Modified Bishop, Simplified Janbu, Spencer or other generally accepted slope stability analysis methods can be used for LRFD as well as ASD. For LRFD and currently available methods of analysis, the analysis must first be performed at the Service I Limit State using unfactored loads and factored soil shear strengths. Thereafter, the suitability of the slope and foundation with respect to overall stability can be determined as follows:

$$\phi_R R_n \geq \gamma_i Q_i \qquad \text{(Eq. 8-16)}$$

where:

R_n = Net ultimate shearing resistance along failure surface (kN)
ΣQ_i = Net driving force along failure surface (kN)
ϕ_R = Resistance factor for overall stability from Table 8-8 (dim)
γ_i = Load factor for overall stability = 1.0 (dim)

The load factor of 1.00 and resistance factor of 0.65 in Table 8-8 for overall slope stability were developed to provide for an equivalent LRFD factor of safety of approximately 1.5 for slopes supporting or containing foundations. Tables 8-4, 8-5 and 8-8 and can be used as a guide in selecting and modifying resistance factors for overall slope stability to achieve other levels of safety consistent with ASD.

8.3.6 Structural Resistance (*A10.6.4*)

For structural design, the bearing pressure at the base of the footing is usually assumed to be uniform or to vary linearly across the bottom of the footing for concentric and eccentric loading, respectively (*A10.6.3.1.5*). The structural design is performed using the factored bearing pressure distribution and the factored resistance of the reinforced concrete footing.

8.3.7 Other Considerations

8.3.7.1 Footing Embedment (*A10.6.1.2*)

If undermining by scour is a potential problem, footings must be constructed below the depth of scour. Requirements for evaluation of scour are prescribed in the AASHTO LRFD Specification and procedures for evaluating scour at bridges are described in FHWA (1991). In regions of the U.S. where ground freezing is possible, foundations should be placed below the maximum depth of frost penetration to prevent damage from frost heave.

8.3.7.2 Buoyancy and Uplift (*A10.6.1.4 and A10.6.1.5*)

Where footings are constructed below the anticipated ground water table, the effects of buoyancy and uplift must be considered in evaluating the various performance limits. For sliding, the highest ground water table will be critical because buoyancy will reduce the sliding resistance. For bearing, the lowest water table will produce the maximum bearing pressure but the highest water table will produce the minimum bearing resistance. Where differential water levels exist on either side of a footing (e.g., for a retaining wall along a stream channel which is backfilled with poorly-draining soil), the effects of differential water loads on sliding and bearing resistance, and the potential for piping of soil fines below the footing should be evaluated.

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8.4 Student Exercise
Bearing Resistance of Spread Footings on Sand

The ultimate unit bearing resistance of a spread footing foundation in dense sand, q_{ult}, has been estimated at 1000 kPa. The load on the footing at the Strength I Limit State is composed of a dead load, Q_D, of 2000 kN (for which $\gamma_D = 1.25$) and a live load, Q_L, of 1000 kN (for which $\gamma_L = 1.75$).

Assuming a typical structure ($\eta_i = 1.0$), determine the minimum square footing size needed to satisfy bearing resistance requirements for the following:

1. LRFD (using Eq. 8-12), if the ultimate resistance was estimated by:

   a. Semi-empirical procedure using SPT data ($\phi = 0.45$)
   b. Semi-empirical procedure using CPT data ($\phi = 0.55$)
   c. Rational method using shear resistance (ϕ_f) estimated from SPT data ($\phi = 0.35$)

$$q_R \geq = \frac{\Sigma\, \eta_i\, \gamma_i\, Q_i}{A'} \qquad \text{(Eq. 8-12)}$$

2. ASD (using Eq. 8-2), if the minimum required factor of safety against a bearing capacity failure[(1)] is:

   a. 3.0 [Soil Strength (ϕ_f) based on SPT]
   b. 2.5 [Soil Strength (ϕ_f) based on Laboratory/Field Strength Tests]

$$Q \leq Q_{all} = R_n / FS \_\ \Sigma Q \leq (q_{ult}\, A)/FS \qquad \text{(Eq. 8-2)}$$

[(1)] Minimum factors of safety using FHWA criteria (Cheney and Chassie, 1993); AASHTO ASD minimum factor of safety is 3.0 for all cases.

Determination of Required Footing Size

Q_D (kN)	$\gamma_D\,Q_D$ (kN)	Q_L (kN)	$\gamma_L\,Q_L$ (kN)	ASD ΣQ (kN)	LRFD $\Sigma\gamma_i\,Q_i$ (kN)	q_{ult} (kPa)	LRFD $q_R=\phi q_{ult}$ (kPa)	ASD $q_{all}=q_{ult}/FS$ (kPa)	Minimum Required Footing Size, A_{req} (m x m)
LRFD: SEMI-EMPIRICAL PROCEDURE USING SPT DATA (ϕ=0.45)									
2000	2500	1000	1750	---		1000		---	
LRFD: SEMI-EMPIRICAL PROCEDURE USING CPT DATA (ϕ=0.55)									
2000	2500	1000	1750	---		1000		---	
LRFD: RATIONAL METHOD USING ϕ_f ESTIMATED FROM SPT (ϕ=0.35)									
2000	2500	1000	1750	---		1000		---	
ASD: MINIMUM FACTOR OF SAFETY = 3.0									
2000	---	1000	---		---	1000	---		
ASD: MINIMUM FACTOR OF SAFETY = 2.5									
2000	---	1000	---		---	1000	—		

8.5 Student Problem:
Footing Design on Soil Using LRFD and ASD

Problem: In the Student Problem in Chapter 4, Section 4.7, you developed unfactored and factored loads and moments for the design of a cantilever retaining wall supported on a spread footing. You will use that information for this problem to perform the geotechnical design of the wall foundation by LRFD and you will compare these results with a design already completed using ASD.

You recall from Chapter 4 that the cantilever retaining wall in Figure 8-5 is being considered for a grade separation between roadway lanes. The wall will be backfilled with a free draining granular fill such that the seasonal high water table will be below the bottom of the footing. The vehicular live load surcharge (LS) on the backfill is applied as shown in the figure.

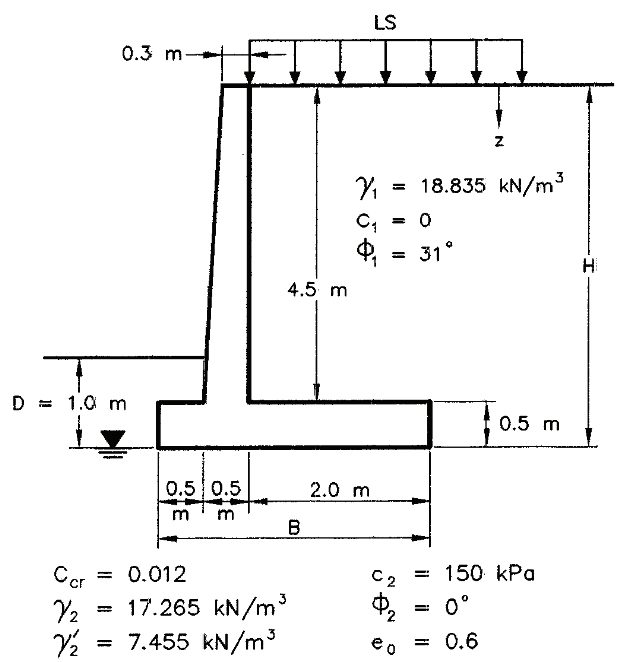


Figure 8-5
Schematic of Student Problem

During the subsurface exploration, you determined that the foundation soils are predominantly clay to a depth of 6 m below the proposed bottom of footing. Therefore, you have decided to place a 0.15-m thick blanket of compacted granular material below the footing to provide a uniform base for foundation construction and improve sliding resistance. In performing the wall design, you assume the following:

- Dense sand and gravel underlies the clayey foundation soils so that the elastic settlement of the dense sand and gravel will be negligible

- The proposed wall backfill will consist of a free draining granular fill

- The seasonal high water table will be at the bottom of the footing

Objective: Conduct a geotechnical design of a spread footing by LRFD and compare the results to those with a design prepared by ASD.

Approach: You will perform the evaluation using the following steps:

- Select footing length, L, and width, B, and determine unfactored and factored bearing pressure distribution

- Settlement: For ASD and LRFD, estimate footing settlement using unfactored loads and the applicable compression characteristics of the soil within the zone of influence and compare with tolerable movement criteria

- Bearing: For ASD, ensure that unfactored ultimate geotechnical bearing resistance, q_{ult}, of the footing divided by the factor of safety, FS, is greater than or equal to the design bearing stress due to unfactored load components, $\bar{q}$ or q_{max}, and for LRFD, ensure that the maximum bearing stress due to the factored load components, $\sum \eta_i \gamma_i Q_i$, is less than or equal to the factored geotechnical bearing resistance, $q_R = \phi\, q_{ult}$

- Sliding: For ASD, ensure that unfactored ultimate geotechnical lateral load capacity, Q_n, of the footing divided by the factor of safety, FS, is greater than or equal to the design load due to lateral load components, Q, and for LRFD, ensure that the sum of the factored lateral load components, $\sum \eta_i \gamma_i Q_i$, is less than or equal to the factored geotechnical lateral load resistance, $Q_R = \phi\, Q_n$

- Overturning: For ASD, ensure that the resultant unfactored vertical load component is located within L/6 and B/6 of the footing centroid, and for LRFD ensure that the factored resultant vertical load component is located within L/4 and B/4 of the footing centroid.

For convenience, the factored and unfactored loads and moments for critical load combinations from the example problem in Section 4.6 of Chapter 4 are presented in Tables 8-10, 8-11, 8-12 and 8-13.

Table 8-10 (4-17)
Unfactored and Factored Vertical Loads

Group/ Item Units	W_1 kN/m	W_2 kN/m	W_3 kN/m	P_{EV} kN/m	P_{LSV} kN/m	ΔP_{LSv} kN/m	P_{av} kN/m	V_{TOT} kN/m
V (Unf.)	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4
Strength I-a	28.6	9.5	31.8	169.5	59.8	22.4	52.8	374.4
Strength I-b	39.7	13.3	44.2	228.8	59.8	22.4	52.8	461.0
Strength IV	47.7	15.9	53.0	228.8	0	0	52.8	398.2
Service I	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4

Table 8-11 (4-18)
Unfactored and Factored Horizontal Loads

Group/Item Units	ΔP_{LSh} kN/m	P_{ah} kN/m	H_{TOT} kN/m
H (Unf.)	21.3	58.6	79.9
Strength I-a	37.3	87.9	125.2
Strength I-b	37.3	87.9	125.2
Strength IV	0	87.9	87.9
Service I	21.3	58.6	79.9

Table 8-12 (4-19)
Unfactored and Factored Moments from Vertical Forces (M_v)

Group/ Item Units	W_1 kN-m/m	W_2 kN-m/m	W_3 kN-m/m	P_{EV}kN-m/m	P_{LSV} kN-m/m	ΔP_{LSv} kN-m/m	P_{av} kN-m/m	M_{vTOT} kN-m/m
M_v (Unf.)	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0
Strength I-a	24.3	6.0	47.7	339.0	119.6	67.2	158.4	762.2
Strength I-b	33.8	8.4	66.2	457.7	119.6	67.2	158.4	911.3
Strength IV	40.5	10.1	79.5	457.7	0	0	158.4	746.2
Service I	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0

Table 8-13 (4-20)
Unfactored and Factored Moments from Horizontal Forces (M_h)

Group/Item Units	ΔP_{LSh} kN-m/m	P_{ah} kN-m/m	M_{hTOT} kN-m/m
M_h (Unf.)	53.3	97.9	151.2
Strength I-a	93.3	146.9	240.2
Strength I-b	93.3	146.9	240.2
Strength IV	0	146.9	146.9
Service I	53.3	97.9	151.2

Step 1: Calculate the Settlement of the Retaining Wall on Cohesive Soil Foundation

Assume embankment construction has been performed earlier and that consolidation settlement from the embankment loading has already occurred. The original ground surface is located 1 m above the footing level.

Divide the 6-m thickness of cohesive soil below the wall foundation into four layers as follows:

- $H_1 = 1.0$ m
- $H_2 = 1.0$ m
- $H_3 = 2.0$ m
- $H_4 = 2.0$ m

The depth of the footing below the existing ground surface is 1 m, so the depth to the center of each layer from the final ground surface in front of the wall is:

- $d_1 = 1.5$ m
- $d_2 = 2.5$ m
- $d_3 = 4.0$ m
- $d_4 = 6.0$ m

(A) Calculate the effective overburden pressure at the center of each layer before wall construction
The depth of footing below the existing ground surface is 1 m so that the footing is located at the annual high water level. The effective overburden stress at the center of each layer is:

$$\sigma'_{oi} = d_i \, \gamma'_2$$

$$\sigma'_{o1} = (1.0 \text{ m})(17.265 \text{ kN/m}^3) + (0.5 \text{ m})(7.455 \text{ kN/m}^3) = 21.0 \text{ kPa}$$
$$\sigma'_{o2} = (1.0 \text{ m})(17.265 \text{ kN/m}^3) + (1.5 \text{ m})(7.455 \text{ kN/m}^3) = 28.4 \text{ kPa}$$

σ'_{o3} = (1.0 m)(17.265 kN/m^3) +(3.0 m)(7.455 kN/m^3) = 39.6 kPa
σ'_{o4} = (1.0 m)(17.265 kN/m^3) +(5.0 m)(7.455 kN/m^3) = 54.5 kPa

(B) Calculate Increase in vertical pressure resulting from loading of the wall

Estimate the wall settlement at the Service I Limit State.

Using Service I loading from Table 8-10, the total vertical unfactored load (V_{TOT}) for calculation of settlement is 329.4 kN/m length of wall. Assuming the vertical load is uniformly distributed over the base width of the wall foundation (i.e., B = 3 m), the increase in pressure at the base of the footing per unit length of wall is:

$$q_o = (V_{TOT})/(B) = \frac{329.4\ \text{kN/m}}{3\ \text{m}} = 109.8\ \text{kPa}$$

(C) Estimate the consolidation settlement:

Because the cohesive foundation soil is assumed to have consolidated under embankment loading (i.e., $\sigma'_p = \sigma'_o$), the only consolidation settlement that will occur will be due to recompression under the foundation loading, after removal of excess embankment soil and wall construction. The recompression settlement can be computed from AASHTO LRFD A10.6.2.2.3c as:

$$S_c = \left(\frac{H_c}{1+e_o}\right)\left(C_{cr}\log\frac{\sigma'_f}{\sigma'_o}\right) = \frac{C_{cr}}{1+e_o}\Sigma\left[H_i\log\left(\frac{\sigma'_o+\Delta\sigma}{\sigma'_o}\right)\right] \qquad (A10.6.2.2.3c\text{-}1)$$

where:

C_{cr} = 0.012 and e_o = 0.6

Complete Table 8-14 and answer the conclusions below the table.

Table 8-14
Consolidation Settlement Calculation

Layer i	$(d_i - 1)/B$	$\Delta\sigma'/q_o$	$\Delta\sigma'$ (kPa)	σ'_{oi} (kPa)	H_i (m)	$\left[H_i\log\left(\frac{\sigma'_{oi}+\Delta\sigma}{\sigma'_{oi}}\right)\right]$
1	0.17	0.95	104.3	21.0	1.0	0.78 m
2	0.50	0.80	87.8	28.4	1.0	0.61 m
3	1.00	0.55	60.4	39.6	2.0	0.80 m
4	1.67	0.34	37.3	54.5	2.0	0.45 m
						Σ = 2.64 m

where:

d_i - 1 = Depth to the center of layer I below the footing from finished grade (m)
$\Delta\sigma'/q_o$ = Boussinesq stress contour at center of a continuous foundation (dim) (*A10.6.2.2.3a*)

From Table 8-14, the estimated total consolidation settlement is:

$$S_c - \frac{0.012}{1+0.6}(2.64\text{ m}) = 0.020\text{ m} = 20\text{ mm}$$

Assuming the maximum tolerable settlement, S_{max} = 25 mm, the total estimated settlement is acceptable.

Step 2: Eccentricity
The eccentricity of the retaining wall is checked in Table 8-15 as described in Section 8.3.4 by comparing the calculated eccentricity, e, for each loading group to the maximum allowed eccentricity (e_{max}) using the relationship:

$e_B = B/2 - X_o$

where:

B/2 = _____ m
X_o = Location of the resultant from the toe = $(M_{vTOT} - M_{hTOT})/V_{TOT}$
e_{max} = B/4 = _____ m/4 = _____ m

For each load group, the total vertical forces (V_{TOT}), horizontal forces (H_{TOT}), moments due to vertical forces (M_{vTOT}) and moments due to horizontal forces (M_{hTOT}) are obtained from Tables 8-10, 8-11, 8-12 and 8-13, respectively. **Complete Table 8-15 using the information from these tables. Note that the force and moment due to live load surcharge over the heel (P_{LSV}) are not included in the eccentricity (i.e., overturning) evaluation (i.e., $V_{TOT} = V_{TOT(Table\ 8\text{-}10)} - P_{LSV(Table\ 8\text{-}10)}$ and $M_{vTOT} = M_{vTOT(Table\ 8\text{-}12)} - P_{LSV(Table\ 8\text{-}12)}$.**

Table 8-15
Summary of Eccentricity Check

Group/Item Units	V_{TOT} (kN/m)	H_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o (m)	e_B (m)	e_{max} (m)
Strength I-a		125.2		240.2			
Strength I-b		125.2		240.2			
Strength IV		87.9		146.9			
Service I		79.9		151.2			

For all cases, e_B is $\leq$, = or $\geq$ e_{max} **(underline correct answer)**; therefore, the design is adequate/inadequate **(underline correct answer)** in regard to eccentricity.

Step 3: Bearing Resistance

(A) Estimate the Bearing Pressures

From Section 8.3.2, the adequacy for bearing capacity is developed based on a rectangular distribution of soil pressure ($\bar{q}$) over the reduced effective area as indicated in Figure 8-6. For a rectangular distribution:

$L' =$ 1 m (i.e., unit length of wall)

$B' = B - 2\,e_B$

$e_B = B/2 - X_o$

$X_o = (M_{vTOT} - M_{hTOT})/V_{TOT}$

$\gamma\bar{q} = V_{TOT}/L'\,B' = V_{TOT}/[B - 2(B/2 - X_o)] = V_{TOT}/2X_o$

Note that the force and moment due to live load surcharge over the heel (P_{LSV}) <u>are</u> included in the bearing resistance evaluation.

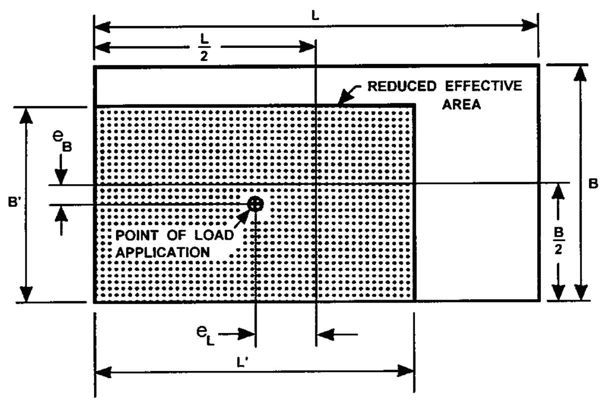


Figure 8-6 (*AC10.6.3.1.5-1*)
Reduced Footing Dimensions

Complete Table 8-16 using information from Tables 8-10, 8-12 and 8-13 for each applicable limit state.

Table 8-16
Summary of Factored Bearing Pressures

Group/Item Units	V_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o (m)	$\gamma\bar{q}$ (kPa)
Strength I-a			240.2		
Strength I-b			240.2		
Strength IV			146.9		
Service I			151.2		

(B) Evaluate Adequacy of Bearing Resistance
The factored bearing resistance, q_R, at the Strength Limit State is determined, based on LRFD (AASHTO, 1997a) (*A10.6.3.1.2b*) using:

$q_R = \phi\, q_{ult} = \phi\, (c\, N_{cm} + \gamma_2\, D_f\, N_{qm})$ (*A10.6.3.1.1-1 and A10.6.3.1.2b-1*)
ϕ = _____ From Table 8-8 for undrained strength based on lab UU tests

AASHTO LRFD (1997a) Article 10.6.3.1.2b provides guidance regarding the values of N_{cm} and N_{qm}.

q_R= _____ [(______ kPa)(3.08) + (______ kN/m^3)(_____ m)(0.45)]
q_R = (_____) (______ kPa) = ______ kPa > $\gamma\bar{q}$ from Table 8-16

Because the factored bearing resistance, q_R, is less than/exceeds **(underline correct answer)** the maximum factored uniform bearing stress, $\gamma\bar{q}$ = 158 kPa, the bearing resistance is adequate/inadequate **(underline correct answer)**.

Step 4: Sliding

Sliding of walls on clay is checked under AASHTO LRFD (1997a) using Figure 8-7.

From Section 8.3.3, the factored resistance, Q_R, against failure by sliding is:

$Q_R = \phi_\tau\, Q_\tau + \phi_{ep}\, Q_{ep}$ (Eq. 8-15)

where:

ϕ_τ = _____ from Table 8-8
Q_τ = Nominal shear resistance between footing and foundation material (kN)
Q_{ep} = Nominal passive resistance of foundation material available throughout the design life of the footing (kN)

From Figure 8-7, Q_τ is the lesser of:

- undrained shear strength or cohesion of the clay; or
- one-half the normal stress on the interface between the footing and the soil.

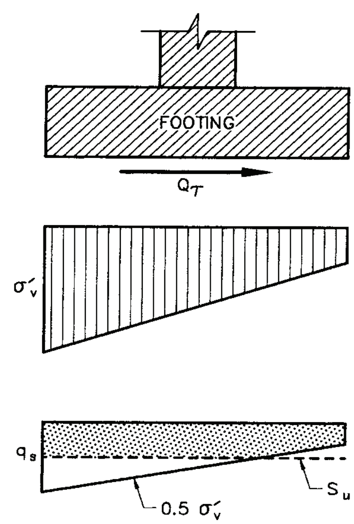
where the cohesion of the foundation soil (c_2) = 150.0 kPa and one-half the factored normal stress (γq_{max}) is given in Table 8-17. **As for the eccentricity check, the force and moment due to the live load surcharge over the heel (P_{LSV}) are <u>not</u> included in the sliding evaluation.**

The actual base pressure (i.e., normal stress at the foundation/soil contact) will have a trapezoidal shape except when the eccentricity is greater than B/6 (i.e., 0.5 m), at which point the base pressure distribution becomes triangular and acts over a reduced base width. From Table 8-15, $e_B < 0.5$ m for all limit states. The values of γq_{max} and γq_{min} in Table 8-17 are calculated; therefore, for a trapezoidal base pressure distribution as follows:

$$\gamma q_{max} = (V_{TOT}/B) + [(6)(V_{TOT})(e_B)/B^2]$$
$$\gamma q_{min} = (V_{TOT}/B) - [(6)(V_{TOT})(e_B)/B^2]$$

where V_{TOT} and e_B are obtained from Table 8-15.

Because $c_2 > \gamma q_{max}/2$ in all cases, the normal stress at the footing/soil interface is used in the calculation of Q_τ.



q_s = UNIT SHEAR RESISTANCE, EQUAL TO S_u OR $0.5\ \sigma'_v$, WHICHEVER IS LESS (kPa)
Q_T = SHADED AREA UNDER q_s DIAGRAM (kN)
σ'_v = VERTICAL EFFECTIVE STRESS kPa
S_u = UNDRAINED SHEAR STRENGTH kPa

Figure 8-7 (*A10.6.3.3-1*)
Procedure for Estimating Sliding Resistance of Footings on Clay
(AASHTO, 1997a)

For comparison with the total factored horizontal forces (H_{TOT}) from Table 8-11, Q_τ can be computed as follows:

$$Q_\tau = \frac{V_{TOT}}{2} = \frac{1}{2}\left(\frac{\gamma\, q_{max} + \gamma\, q_{min}}{2}\right)B$$

Using the relationships for γq_{max} and γq_{min}, the equation above for Q_τ, ϕ_τ from Table 8-8, and H_{TOT} from Table 8-11, complete Table 8-17 for each applicable limit state.

Table 8-17
Summary of Sliding Resistance

Group/Item Units	γq_{max} (kPa)	γq_{min} (kPa)	$\gamma q_{max}/2$ (kPa)	Q_τ (kN/m)	$\phi_\tau\, Q_\tau$ (kN/m)	H_{TOT} (kN/m)
Strength I-a						
Strength I-b						
Strength IV						
Service I						

Because the factored sliding resistance ($\phi_\tau\, Q_\tau$) calculated is greater/less **(underline correct answer)** than the factored horizontal loading for all Strength Limit States, the sliding resistance is satisfactory/unsatisfactory.

Summary

Table 8-18
Summary of Spread Footing Design by LRFD and ASD

Performance Limit	LRFD			ASD		
	Factored Resistance/ Eccentricity Limit	Factored Load/ Eccentricity	✔	Required FS/ Eccentricity Limit	Actual FS/ Eccentricity	✔
Eccentricity	0.25 B	0.07 B	✔	0.167 B	0.03 B	✔
Bearing Resistance	282 kPa	158 kPa	✔	3.0	4.2	✔
Sliding Resistance	134 kN/m	125 kN/m	✔	1.5	1.8	✔

As summarized in Table 8-18, a comparable design is achieved by LRFD and ASD. *Whereas the ASD factors of safety for bearing resistance and sliding are fixed, however, the LRFD resistance factors could possibly be increased with additional data accumulation and reliability-based calibration for similar soils and loading conditions.* ***As such, the LRFD provisions reflect the reliability of the soil strength estimates and capacity prediction models and provide a more rational basis for design than the ASD provisions. Therefore, with further data accumulation and calibration, more reliable and economical designs might be achieved using LRFD.***

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CHAPTER 9
DRIVEN PILE DESIGN

9.1 Introduction

For both ASD and LRFD, the design of driven pile foundations requires consideration of geotechnical and structural capacity and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between driven pile design by LRFD and ASD with respect to the comparison of loads and deformations to resistance and tolerable deformations

- Identifies the strength and serviceability performance limits which must be considered for pile design by LRFD

- Briefly summarizes methods commonly used for estimating the geotechnical and structural capacity and load-deflection behavior of piles and pile groups

- Presents examples of pile foundation designs by ASD and LRFD methods

9.2 Design Methods

With few exceptions, the design procedure for driven pile foundations using LRFD (A10.7) is identical to the procedure followed using ASD. Generally, the ultimate bearing capacity and settlement are checked for axially-loaded piles. Although design of laterally-loaded piles may be governed either by lateral capacity or deflection criteria, mobilization of the ultimate lateral capacity of the soil requires such large displacements that lateral capacity does not represent a realistic basis for laterally-loaded pile design. The following sections summarize the general design processes for driven pile foundation design using the ASD and LRFD approaches.

9.2.1 ASD Summary

Geotechnical Resistance

Existing practice for design of pile foundations follows the ASD approach. For ASD (AASHTO, 1997b), all uncertainty in the variation of applied loads transferred to the pile(s) and the ultimate geotechnical capacity of the soil and rock to support the load are incorporated in a factor of safety, FS. As a result, loads used for design, Q, consist of those actual forces estimated to be applied directly to the structure. In LRFD terminology, this is equivalent to applying a load factor of 1.0 to the estimated forces. Relative to axial loading of a pile, the ultimate axial geotechnical load capacity, $R_n = Q_{ult}$, is estimated using any number of available methods. Then the suitability of the design is evaluated by determining the allowable axial design load, Q_{all}, using:

$$Q \le Q_{all} = R_n / FS = Q_{ult} / FS \qquad \text{(Eq. 9-1)}$$

where:

Q = Design load (kN)
Q_{all} = Allowable design load (kN)
R_n = Q_{ult} = Ultimate geotechnical resistance of a pile (kN)
FS = Factor of safety (dim)

The required FS is generally specified by the governing agency, and may be constant or variable. The AASHTO ASD FS for the axial geotechnical capacity of a single pile is presented in Table 9-1.

Table 9-1
Factor of Safety on Ultimate Axial Geotechnical Capacity
Based on Level of Construction Control
(AASHTO, 1997b)

Basis for Design and Type of Construction Control	Increasing Design/Construction Control				
Subsurface Exploration	✔	✔	✔	✔	✔
Static Calculation	✔	✔	✔	✔	✔
Dynamic Formula	✔				
Wave Equation		✔	✔	✔	✔
CAPWAP Analysis			✔		✔
Static Load Test				✔	✔
Factor of Safety (FS)	3.50	2.75	2.25	2.00[1]	1.90

[1] For any combination of construction control that includes a static load test, FS = 2.0.

As Table 9-1 shows, AASHTO ASD for pile design permits the FS to vary depending on the level of control exercised during the design and construction phases. On this basis if R_n is developed using information from a site-specific subsurface exploration based on a static calculation and wave equation analysis, FS ≥ 2.75. However, if capacities are estimated using more reliable procedures and/or are confirmed through field testing procedures(e.g., by performing dynamic measurements during driving and wave equation analysis of pile driving operations or by conducting a static load test, a reduced factor of safety could be used to reflect the greater confidence level achieved by these improved methods for estimating and confirming the axial capacity of a pile.

Structural Resistance

The reduction of the structural capacity of a pile from an ultimate value (based on the yield or ultimate strength depending on material type) to an allowable value accounts for all uncertainty in the variation of applied loads and the ultimate structural capacity of the pile. Therefore, as with geotechnical design, the structural design of a pile or pile group is performed by ASD using actual estimated pile loads.

For structural design based on allowable stress, Eq. 9-1 can be written as:

$$\sum Q \leq \sigma_{all} \times A = P_{all} \qquad \text{(Eq. 9-2)}$$

where:

σ_{all} = Allowable axial stress in pile (kPa)
P_{all} = Allowable axial structural capacity (kN)
A = Cross sectional area of pile (m^2)

In ASD, the ultimate unit stress for each material type is reduced to an allowable value by application of a modifier to account for uncertainty in resistance and applied load. The modifiers are based on a comprehensive study by Davisson, et al. (1983) which evaluated the in-service stress level in piles based on structural analysis and case history data. The study used load and resistance factor concepts to evaluate permissible stress levels in piles, and incorporated the effects of driving stress levels, mislocation of piles, eccentricity of applied load, damage to piles due to driving and inherent variability in pile material properties of steel, concrete and timber piles. As an example, the maximum allowable stress, σ_{all}, for a steel pile is determined in ASD by the following:

$$\sigma_{all} = \left[\frac{(\phi)(ecc)(HDF)}{LF}\right] F_y \qquad \text{(Eq. 9-3)}$$

where:

σ_{all} = Maximum allowable stress (kPa)
ϕ = Reduction factor to account for manufacturing variability (dim)
ecc = Reduction factor for accidental load eccentricity (dim)
HDF = Reduction factor for hidden defects due to damage during pile driving (dim)
F_y = Yield strength of steel (kPa)
LF = Load factor (dim)

The maximum allowable stresses recommended for various pile types are presented in Table 9-2.

The allowable axial load, P_{all}, on a pile is:

$$P_{all} = \sum \sigma_{all} A \qquad \text{(Eq. 9-4)}$$

where:

P_{all} = Allowable axial load on pile (kN)
A = Cross-sectional area of the structural component for which σ_{all} is applicable (m^2)

Table 9-2
Allowable Stresses in Piles (GRL, 1996; AASHTO, 1997b)

Pile Type	Maximum Allowable Stress , σ_{all}, (kPa)
Steel • Driving Damage Likely • Driving Damage Unlikely	 $0.25\ F_y$ $0.33\ F_y$
Concrete-Filled Steel Pipe	$0.25\ F_y + 0.40\ f_c'^{(1)}$
Prestressed Concrete	$0.33\ f_c' - 0.27\ f_{pe}^{(2)}$
Round Timber • Douglas-Fir - Coast • Douglas Fir - Interior • Lodgepole Pine • Red Oak • Southern Pine • Western Hemlock	 8.3 7.6 5.5 7.6 8.3 6.9

(1) Applied over cross-sectional area of steel pipe and cross-sectional area of concrete; (2) Applied over gross cross-sectional area of concrete; F_v = Yield strength of steel (kPa); f_c' = Concrete compressive strength (kPa); and f_{pe} = Concrete compressive stress due to prestressing (kPa).

Deformation

In addition to axial geotechnical and structural capacity evaluation, the design of driven piles by ASD requires evaluations of pile deflections and comparisons with deformation criteria using the following.

$$\delta_i \leq \delta_n \qquad \text{(Eq. 9-5)}$$

where:

δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement established by designer (mm)

Tolerable movement criteria are usually a function of the type of structure, and for bridges depend primarily on whether the superstructure has fixed- or simply-supported spans. The design of laterally-loaded piles by ASD is usually governed by criteria for deflection under actual applied loads, except for pile groups containing batter piles for which the axial and/or bending capacity of the pile can control.

9.2.2 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. ***When properly developed and applied, the LRFD approach provides a consistent level of safety for the design of all structure components.*** As described in Section 4.3, the probability that a structure component will fail or perform unacceptably is no different than any other component. For design, the resistance and deformation of supporting soil and rock materials and structure components must satisfy the LRFD equations below. For the Strength Limit States:

$$\sum \eta_i \gamma_i Q_i \le \phi R_n = R_r \qquad \text{(Eq. 9-6) } (\textit{A1.3.2.1-1})$$

For the Service Limit States:

$$\sum \eta_i \gamma_i \delta_i \le \phi \delta_n \qquad \text{(Eq. 9-7)}$$

where:

η_i = Factors to account for effects of ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)
δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement (mm)

Relative to axial loading of a pile, the suitability of a pile with respect to the geotechnical and structural resistance can be obtained using Eq. 9-4, rewritten as:

$$\sum \eta_i \gamma_i Q_i \le \phi Q_{ult} = Q_r \qquad \text{(Eq. 9-8)}$$

and

$$\sum \eta_i \gamma_i Q_i \le \phi P_n = P_r \qquad \text{(Eq. 9-9)}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Factored load effect (kN)
ϕ = Resistance factor (dim)
Q_{ult} = Nominal (ultimate) geotechnical resistance of a pile (kN)
Q_R = Factored geotechnical resistance of a pile (kN)
P_n = P_{ult} = Nominal (ultimate) structural resistance of a pile (kN)
P_r = Factored structural resistance of a pile (kN)

The load factors and load factor combinations used for design were presented in Chapter 4. In general, values of $\gamma_i > 1.0$ are used to evaluate ultimate ground or structure capacity at the Strength Limit States, whereas the deformation performance of structures is evaluated at the Service I Limit State using $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads). In ASD (AASHTO, 1997b), values of $\gamma_i = 1.0$ (or γ_i

= 0.3 for wind loads) are used to evaluate structures for both strength (allowable stress) and serviceability (deflection). ***Therefore, analysis of pile deformations (e.g., settlement or lateral displacement) by LRFD and ASD are identical.***

When using Eq. 9-6 for driven pile foundation design at the Strength Limit States, the following values of η can normally be used:

- $\eta_D = \eta_R = 1.00$; and

- $\eta_I = 1.05$ for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures.

Determination of the operational importance of a structure (such as a bridge) is made by the facility owner as described in Chapter 4. The appropriate value of η_I is then applied throughout the superstructure design by the structural engineer. The value of η_I selected by the superstructure designer should then be applied in the foundation design. For the purpose of this chapter, the value of η_I is assumed equal to 1.0. When using Eq. 9-7 to evaluate a pile foundation at any Service Limit State, η_D, η_R, and $\eta_I = 1.0$.

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4 and in Section 9.2.2.1, and loads should be developed as described in Chapter 4. The ultimate resistance, R_n, should be determined for each type of resistance (e.g., axial, side, or group resistance) as described in Section 9.3.

Values of $\phi \leq 1.0$ are applied when evaluating pile foundation resistance for any strength limit state using Eq. 9-6. Currently, the value of $\phi = 1.0$ is applied when evaluating a pile foundation for any service limit state using Eq. 9-7. Selection and modification of resistance factors, ϕ, are described in Sections 9.2.2.2 through 9.2.2.4.

9.2.2.1 Limit States (*A10.5*)
In general, the design of driven pile foundations using LRFD requires evaluation of pile suitability at various Strength Limit States and the Service I Limit State. The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for piles designed at the Strength and Service I Limit States are summarized in Table 9-3. As conditions warrant, it may also be necessary to evaluate pile performance at other limit states (e.g., Extreme Event I for loading from earthquakes or Extreme Event II for vessel impact and ice loading).

Table 9-3
Strength and Service Limit States
for Design of Driven Pile Foundations

Performance Limit	Strength Limit State(s)	Service I Limit State
Bearing Resistance of Single Pile/Group	✔	
Pile/Group Punching	✔	
Settlement of Pile Group		✔
Tensile Resistance of Uplift-Loaded Piles	✔	
Pile/Group Lateral Displacement		✔
Structural Capacity of Axially/ Laterally-Loaded Piles	✔	

Methods of evaluating piles at these various limit states are described in Section 9.3.

9.2.2.2 Resistance Factors (*A10.5.5*)
Resistance factors for ***geotechnical design*** of friction piles in soil based on static and in-situ test methods of analysis were developed using the reliability-based calibration procedure described in Chapter 3. Resistance factors for geotechnical design of bearing piles in clay and rock and of friction and/or end bearing piles based on field testing of piles by Pile Driving Analyzer (PDA) and static load tests are based on direct calibration with ASD. The reliability-based calibration procedure involved:

- Estimating the level of reliability inherent in various methods for predicting pile capacity

- Observing the variation in reliability levels with different span lengths, dead- to live-load ratios, foundation geometry and methods used to estimate ultimate resistance and load combinations

- Selecting a target reliability, β_T, based on the margin of safety used for ASD

- Calculating resistance factors, ϕ, consistent with the target reliability index coupled with experience and judgment

This process was used to develop resistance factors for the design of driven pile foundations. Table 9-4 summarizes the results of analyses performed to develop ϕ values for estimating the axial resistance of piles. For these analyses, the variables included:

- Pile length (i.e., 10- and 30-m lengths)

- A dead to live load ratio of 3.69 which corresponds to a bridge with a span length of 75 m

- A target reliability index, β_T, of 2.0 to 2.5

- Method used to estimate R_n (Q_{ult}) (i.e., the α-, β-, and λ-methods based on soil shear strength, and methods based in CPT and SPT in-situ testing) as described in Section 9.3.2.1

Table 9-4
Resistance Factors for Driven Piles
for Estimating the Axial Geotechnical Pile Capacity
Using Reliability-Based Calibration
(modified after Barker, et al., 1991b)

Pile Length (m)	β_T	ϕ Values by Method of Axial Pile Capacity Estimation						
		α		β	λ		CPT	SPT
		Type I	Type II		Type I	Type II		
10	2.0	0.78	0.92	0.79	0.53	0.65	0.59	0.48
30	2.0	0.84	0.96	0.79	0.55	0.71	0.62	0.51
10	2.5	0.65	0.69	0.68	0.41	0.56	0.48	0.36
30	2.5	0.71	0.73	0.68	0.44	0.62	0.51	0.38
Average ϕ		0.78		0.74	0.56		0.55	0.43
Selected ϕ		0.70		0.50	0.55		0.55	0.45

Note: Type I refers to soils with S_u < 50 kPa; Type II refers to soils with S_u > 50 kPa

Table 9-4 shows that:

- Values of ϕ are not sensitive to pile length

- Except for axial pile capacity estimates using the β method, the selected values of φ are generally equal to the average value of ϕ

A ϕ factor for the β method much less than the average value was selected for use in the LRFD Specification based on the limited number of reported cases where the method had been used in conjunction with performance testing, and the engineering judgment of the code developers. The reliability-based procedure for determining ϕ using the Nordlund method for estimating the ultimate axial geotechnical capacity of a pile is presented in Chapter 7.

Resistance factors for the geotechnical design of driven pile foundations using the α, β and λ methods for axial capacity and for other design methods and considerations suggested in the AASHTO LRFD Specifications (AASHTO, 1996) are presented in Table 9-5.

Table 9-5 (*A10.5.4-2*)
Resistance Factors for
Geotechnical Strength Limit State for Axially Loaded Piles
(AASHTO, 1996)

METHOD/SOIL/CONDITION		RESISTANCE[(1)] FACTOR
Ultimate Bearing Resistance of Single Piles	Skin Friction: Clay	
	α-method	0.70
	β-method	0.50
	λ-method	0.55
	End Bearing: Clay and Rock	
	Clay	0.70
	Rock	0.50
	Skin Friction and End Bearing: Sand	
	SPT-method	0.45
	CPT-method	0.55
	Skin Friction and End Bearing: All Soils	
	Load Test	0.80
	Pile Driving Analyzer	0.70
Block Failure	Clay	0.65
Uplift Resistance of Single Piles	α-method	0.60
	β-method	0.40
	λ-method	0.45
	SPT-method	0.35
	CPT-method	0.45
	Load Test	0.80
Group Uplift Resistance	Sand	0.55
	Clay	0.55

(1) Refer to Section 9.3 for description of design procedures for which resistance factors have been calibrated.

The variation in ϕ-factors for determination of skin friction in clay provides a good indication of the concepts inherent in LRFD. The α-method, which is a total stress analysis method based on relating the undrained shear strength of the soil to the adhesion between the pile and soil, has been shown to give reasonable results for a wide range of soil conditions and therefore has a relatively high resistance factor. In contrast, the β-method, which is an effective stress method, has a lower resistance factor, due in part to the fact that this method lumps the effects of such factors as in-situ stress state, effective stress friction angle and pile-soil interface friction angle, all into a single parameter, β. Because the results of pile load tests or pile driving analyzer measurements during installation provide more reliable estimates of axial capacity of a pile compared to other methods (i.e., based on performance of an actual pile rather than estimates of soil properties), values of φ for these methods are the highest. Values of ϕ for uplift and group loading are proportionally lower than for compression loading to account for Poisson effects for uplift loading and interaction effects for group loading.

The resistance factors presented in Table 9-5 (AASHTO, 1996) assume that the indicated capacity prediction method is the only method used to estimate and/or field verify pile capacities. *In reality, static methods (e.g., α and SPT methods) are often used to predict a capacity which is subsequently field verified (i.e., by PDA and/or load tests).* ***The resistance factor used during the design stage should, therefore, reflect the reliability of the final method and extent of pile capacity verification.***

One relatively simple method to recalibrate static design method resistance factors for specified field verification methods based on experience with specific pile and soil conditions is to equate the factored static capacity prediction with the field-verified capacity prediction as follows:

$$Q_R = Q_{RSTATIC} = Q_{FIELD} \qquad \text{(Eq. 9-10)}$$

or:

$$\phi_{STATIC} Q_{ULT.STATIC} = \phi_{FIELD} Q_{ULT.FIELD} \qquad \text{(Eq. 9-11)}$$

where:

$Q_{RSTATIC}$ =	Factored resistance for static design method (kN)
Q_{RFIELD} =	Factored resistance for field capacity verification method (kN)
ϕ_{STATIC} =	Adjusted resistance factor for static design method (dim)
ϕ_{FIELD} =	Calibrated resistance factor for field capacity verification method (dim)
$Q_{ULT.STATIC}$ =	Nominal resistance predicted by static design method (kN)
$Q_{ULT.FIELD}$ =	Nominal resistance verified by field test procedure (kN)

Subsequent field capacity verification can then be performed by verifying a nominal (or ultimate) capacity identified as:

$$Q_{ULT} = Q_R / \phi_{FIELD} \qquad \text{(Eq. 9-12)}$$

In this manner, pile lengths estimated using static methods during the design stage would prove adequate based on field capacities verified during construction. Also, the β value for the final pile installation would be equal to the target reliability index, β_T, for the final (and most reliable) method of capacity verification. This procedure does, however, require accumulation and evaluation of pile capacity prediction and verification data for similar pile installations and subsurface conditions.

The 1997 Interims of the AASHTO LRFD Specifications (AASHTO, 1997a) include an initial

attempt to incorporate the specified method and extent of field construction control and capacity verification into the determination of resistance factors for axially loaded piles. However, the modified resistance factors are not based on any specific calibration, and in many cases may result in significantly more conservative designs than current ASD practice. Therefore, additional consideration and future revision of the 1997 AASHTO LRFD requirements are anticipated.

Resistance factors for ***structural design*** of piles using LRFD were developed through direct calibration with ASD using the following:

$$\phi = \bar{\gamma}\,\sigma_{all}/\sigma_n \qquad \text{(Eq. 9-13)}$$

where:

ϕ = Resistance factor (dim)
$\bar{\gamma}$ = Assumed average load factor = 1.45 (dim)
σ_{all} = Allowable stress in pile from Table 9-2 (kPa)
σ_r = Ultimate structural pile capacity (kPa)

For example, the ultimate capacity of a steel H-pile is defined as the yield strength, F_y, and the allowable stress in ASD for a steel H-pile for conditions where pile damage is likely during driving is 0.25 F_y. Therefore, the ϕ factor for steel H-piles for severe driving is:

$$\phi = \bar{\gamma}\,\sigma_{all}/\sigma_n$$
$$\phi = 1.45\ (0.25\ F_y/F_y) = 0.36$$

Values of ϕ for structural design of various pile types (rounded to the nearest 0.05) are listed in Table 9-6. The determination of the ultimate structural capacities of various pile types is discussed in Section 9.3.2.2.

The resistance factors for structural design of piles in the AASHTO LRFD Specifications (1997a) are applied to nominal structural resistance, P_n, which are reduced for inadvertent load eccentricity (*A6.9.4.1*) and reductions due to pile damage related to driving difficulty (*AC10.5.5*). The "effective" resistance factors in Table 9-6 include the base resistance factors for axial compression of steel piles (*0.90 in A6.5.4.2*), concrete piles (*0.75 in A5.5.4.2.1*) and timber piles (*0.90 in A8.5.2.2*) along with the reduction factor on nominal resistance, P_n, of 0.78 for H-piles and 0.87 for pipe piles to account for inadvertent load eccentricity (*A6.9.4.1*) and suggested reduction factors of 0.875 and 0.75 for moderately difficult and difficult driving, respectively (*AC10.5.5*). ***As indicated by Table 9-6, the effective resistance factors implied in the AASHTO LRFD Specification (1997a) for structural design of axially loaded piles are 35 to 40 percent to resistance factors derived through calibration to the AASHTO ASD Specification (1997b).*** The difference is due to the use of a load factor of 2.0 by Davisson (1983) in developing the ASD allowable stress levels for driven piles.

Table 9-6
Resistance Factors for Structural Design of Axially-Loaded Piles Calibrated to ASD

Pile Type	Effective Resistance Factor, ϕ		
	Calibrated to AASHTO ASD[1]	AASHTO[2] LRFD for Moderate Driving Difficulty	AASHTO[2] LRFD for Difficult Driving Difficulty
Steel H-Piles:		0.60	0.50
• Severe Driving Conditions	0.35		
• Good Driving Conditions	0.45		
Steel Pipe Piles:		0.65	0.55
• Severe Driving Conditions	0.35		
• Good Driving Conditions	0.45		
Prestressed Concrete	0.45	0.65	0.55
Concrete-Filled Pipe			
• Steel Pipe	0.35	-	-
• Concrete	0.55	-	-
Timber	0.55	0.75	0.65

[1] Resistance factors provide stresses approximately equivalent to allowable stresses in AASHTO (1997b) (Table 9-2) for $\bar{\gamma}$ = 1.45; [2] Resistance factors include AASHTO LRFD (1997a) reductions in nominal resistance to account for unintended eccentricity (*A6.9.4.1*) and driving difficulty (*AC10.5.5*), and assume $\bar{\gamma}$ = 1.45.

9.2.2.3 Comparison of Driven Pile Design Using LRFD and ASD

9.2.2.3.1 Geotechnical Design

To illustrate the relative differences between LRFD and ASD, the equivalent LRFD factor of safety (FS_{LRFD}) has been determined for each of the methods presented in Table 9-5 for estimating the axial geotechnical capacity of a single pile or pile group. As presented in Table 9-7, FS_{LRFD} was determined as:

$$FS_{LRFD} = \bar{\gamma}/\phi \qquad \text{(Eq. 9-12)}$$

where:

$\bar{\gamma}$ = Average load factor (assumed = 1.45)
ϕ = Resistance factor from Table 9-5.

The ASD factor of safety (FS_{ASD}) from Table 9-1 for each category is also presented in Table 9-7.

Table 9-7
Comparison of ASD Factor of Safety
with LRFD Equivalent Factor of Safety
Geotechnical Strength Limit State for Axially-Loaded Piles

Method/Soil Condition		Resistance Factor, ϕ	LRFD Equivalent Factor of Safety, $\bar{\gamma}/\phi$ (for $\bar{\gamma}$ =1.45)[1]	ASD Factor of Safety[2]
Ultimate Bearing Resistance of Single Piles	Skin Friction: Clay - α-method - β-method - λ-method	 0.70 0.50 0.55	 2.0 2.9 2.6	2.75
	End Bearing: Clay and Rock - Clay - Rock	 0.70 0.50	 2.0 2.9	2.75
	Skin Friction and End Bearing: Sand - SPT-method - CPT-method	 0.45 0.55	 3.2 2.6	2.75
	Skin Friction/End Bearing: All Soils - Load Test - Pile Driving Analyzer	 0.80 0.70	 1.8 2.0	 2.0 2.25
Block Failure	Clay	0.65	2.2	2.75
Uplift Resistance of Single Piles	α-method β-method λ-method SPT-method CPT-method Load Test	0.60 0.40 0.45 0.35 0.45 0.80	2.4 3.7 3.2 4.2 3.2 1.8	2.75
Group Uplift Resistance	Sand Clay	0.55 0.55	2.6 2.6	NA [3]

[1] An average load factor, $\bar{\gamma}$ = 1.45, is assumed for estimation of a typical FS_{LRFD}; [2] A range of factors of safety from 1.9 to 3.5 is recommended by AASHTO depending upon the degree of "construction control" implemented. A FS of 2.75 is applicable for those cases on which design is based only on subsurface exploration, static calculations and a wave equation analysis and is chosen for comparison purposes only herein.[3] The procedures used in LRFD are not equivalent to those in ASD. Therefore, the equivalent factors of safety for these methods cannot be directly compared.

The values of FS_{LRFD} in Table 9-7 were determined assuming $\bar{\gamma}$ = 1.45. Actually, γ_i can range from 1.25 for structure component loads to about 1.75 for live loads so that FS_{LRFD} could vary from the value shown in the table depending on the relative proportion of live to dead load for a particular structure. *Assuming that $\bar{\gamma}$ = 1.45 represents a reasonable approximation, $FS_{LRFD} \approx FS_{ASD}$ depending on the method of capacity estimation. While general agreement exists between LRFD and ASD, the comparison depends on the actual average load factor and method of capacity analysis. Therefore, while driven pile designs performed using LRFD will be comparable to those using ASD, a precise or very close approximation between the two should not typically be expected.*

9.2.2.3.2 Structural Design

In LRFD (AASHTO, 1997a), the resistance factors and capacity reduction factors account for the same capacity reductions cited in Section 9.2.1 for ASD, and uncertainty in loading is accounted for separately by a load factor. The factored axial resistance, P_r, is related to the allowable axial load, P_{all}, by the following:

$$P_{all} = \frac{\phi P_n}{\bar{\gamma}} = \frac{P_r}{\bar{\gamma}} \qquad \text{(Eq. 9-14)}$$

where:

ϕ = Resistance factor for structural design (dim)
P_n = Nominal (ultimate) structural resistance of pile (kN)
P_r = Factored structural resistance of pile (kN)
$\bar{\gamma}$ = Average load factor (dim)

The provisions for structural design of axially loaded concrete, steel and timber piles are contained in Sections 5, 6 and 8, respectively, of the AASHTO LRFD Specification (AASHTO, 1997a). The provisions in Section 6 (A6.9.4.1) specify reduced nominal capacities, P_n, for steel H-piles ($0.78F_yA_s$) to account for unintended eccentricities. Additional reduction factors are suggested in Section 10 (AC10.5.5) of the AASHTO LRFD Specification (AASHTO, 1997a) to reflect moderately difficult (0.875) and difficult (0.75) driving conditions. The "effective" AASHTO LRFD resistance factors for structural design presented in Table 9-6 include the reduction factors recommended and suggested in AASHTO. The resistance factors in Table 9-6 which are calibrated to ASD are calibrated to the maximum allowable stresses in Table 9-2. All resistance factors in Table 9-2 assume $\bar{\gamma}$ = 1.45.

9.2.2.4 Modification of Resistance Factors

9.2.2.4.1 Geotechnical Design

As stated in Section 9.2.2.2, the LRFD resistance factors for geotechnical design of piles in Table 9-5 were developed using a reliability-based calibration procedure. Application of these resistance factors in conjunction with $\bar{\gamma}$ = 1.45 results in an "equivalent" factor of safety ranging from 1.8 to 4.2, depending on the soil conditions, loading conditions and method of geotechnical capacity prediction, as described in Section 9.2.2.3.1.

In ASD, the designer or owner might decide to increase or decrease required factors of safety or allowable design stresses in consideration of a number of factors, such as:

- The potential consequences of a failure
- The extent or quality of information available from geotechnical exploration and testing
- Past experience with the soil conditions encountered and/or capacity prediction method used
- The level of construction control anticipated or specified
- The likelihood that the design loading conditions will be realized

When using LRFD, similar flexibility to vary the required level of safety should also be available. Additionally, whereas the same factor of safety is generally used in ASD regardless of the source of loading, the equivalent factor of safety in LRFD (defined by Eq. 9-13) varies for a given resistance factor depending on the source of loading. As stated previously, an average load factor of 1.45 was used to identify the equivalent safety factors in Table 9-7 and to develop the resistance factors in Table 9-5. Although $\bar{\gamma}$ = 1.45 is generally reasonable for a typical bridge abutment or pier, or a retaining wall, the average load factor could theoretically range from a low of 1.25 (i.e., all dead load) to nearly 1.75 (i.e., extremely high live load).

To modify the resistance factors for geotechnical design of piles to account for average load factors other than 1.45 and equivalent factors of safety other than those identified in Table 9-7, the following equation may be used:

$$\phi_m = \phi_T \times \left(\frac{FS_T}{FS_D} \right) \times \left(\frac{\bar{\gamma}_T}{\bar{\gamma}_D} \right) \qquad \text{(Eq. 9-15)}$$

where:

ϕ_m = Modified resistance factor (dim)
ϕ_T = Tabulated resistance factor from Table 9-5 (dim)
FS_T = Tabulated factor of safety from Table 9-7 (dim)
FS_D = Desired factor of safety (dim)
$\bar{\phi}_T$ = Average load factor = 1.45 (dim)
$\bar{\phi}_D$ = Actual average load factor including modification for operational importance (dim)

9.2.2.4.2 Structural Design

Unlike resistance factors for geotechnical design, the resistance factors provided in Table 9-5 for structural design of piles were developed using direct calibration with ASD, generally based on the allowable stresses in Table 9-2.

To modify the resistance factors for structural design of piles to account for $\bar{\gamma}$ other than 1.45 and allowable stresses other than those presented in Table 9-2, the following equation may be used:

$$\phi_m = \phi_T \times \left(\frac{\sigma_{allA}}{\sigma_{allT}} \right) \times \left(\frac{\bar{\gamma}_T}{\bar{\gamma}_D} \right) \qquad \text{(Eq. 9-16)}$$

where:

ϕ_T = Tabulated resistance factor from Table 9-6 (dim)
σ_{allA} = Actual allowable stress (kPa)
σ_{allT} = Tabulated allowable stress from Table 9-2 (kPa)

Modifying resistance factors may seem reasonable, but such modification may not be consistent with the goal of LRFD to achieve equal reliability against failure of structure components, unless the factor of safety accurately models the reliability of the pile capacity predictive method used. The resistance factors may be more appropriately modified through application of the probabilistic procedures described in Chapter 3 to achieve the desired level of reliability if a sufficient amount of data is available.

9.2.3 Summarized Comparison of LRFD and ASD

As noted before, the process used to develop a pile foundation design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 9-1. Specific differences between the methods and other important issues are highlighted in the following section. Other aspects of pile design such as identifying special considerations (e.g., potential for loss of lateral support or negative loading), developing design foundation profile and determining requirements for construction control (e.g., dynamic monitoring or load testing) are inherent aspects of the design process required for both LRFD and ASD.

9.3 Performance Limits

Design of a driven pile foundation by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the pile section and details to meet these criteria for axial, lateral and/or moment loading, the design of driven pile foundations requires consideration of many factors which can affect pile performance, including:

- Axial and lateral resistance of pile and pile group
- Axial and lateral deformation of pile and pile group
- Scour and loss of axial/lateral resistance
- Negative or downdrag loading
- Effect of variable ground water levels and buoyancy
- Degradation of the pile section
- Uplift loading on pile and pile group
- Pile spacing and the effects of group action including effects of pile battering
- Punching failure or excessive settlement of piles overlying weak or compressible soil

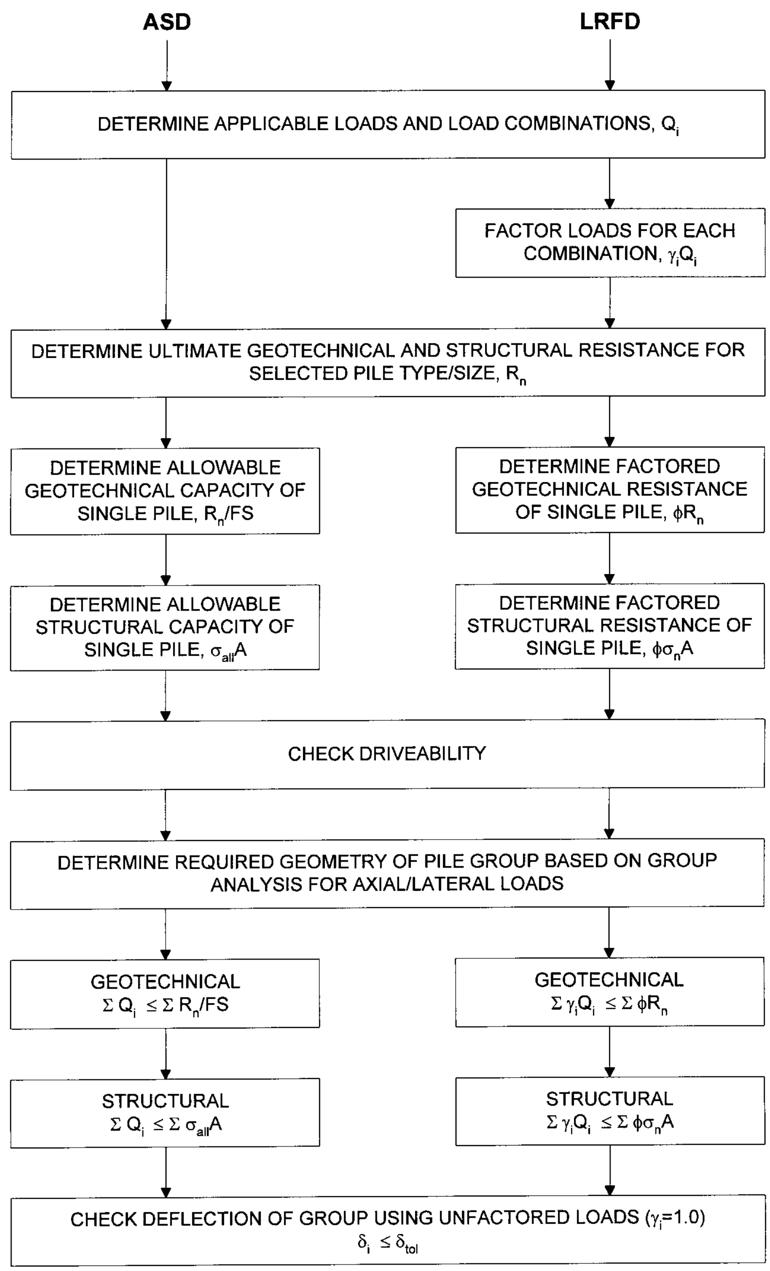


Figure 9-1
Generalized Flow Chart for Driven Pile Design by LRFD and ASD

For these design factors, there is no difference between LRFD and ASD analysis procedures. The following sections highlight differences between LRFD and ASD in the performance criteria and application of design procedures.

9.3.1 Displacements and Tolerable Movement Criteria (*A10.7.2*)

The vertical and lateral displacement of driven piles must be evaluated for all applicable dead and live load combinations, and compared with tolerable movement criteria. ***Because evaluations of structure displacements by LRFD are made at the Service I Limit State where $\gamma = 1.0$ and $\varphi = 1.0$, methods used to estimate settlement and lateral displacement by LRFD are identical to those used for ASD.*** Consequently, axial compression (or tension) of the pile section can be computed by elastic methods and axial and lateral ground displacements can be determined by conventional methods using empirical correlations with in-situ test results or measurements by in-situ or laboratory test methods to estimate engineering soil properties. Lateral displacements need to be evaluated if:

- Piles are subjected to inclined or lateral load
- Piles are placed on or near an embankment slope
- Loss of lateral foundation support by scour is possible

For pile groups in cohesionless soils, settlements will occur immediately as the pile group is loaded. Pile groups in cohesive soils, however may experience long-term consolidation settlement as well as immediate elastic settlement. Elastic settlement generally predominates in overconsolidated clays, whereas consolidation settlement generally predominates in normally consolidated clays. The consolidation settlement of pile groups in cohesive soil is typically estimated using the equivalent footing procedure (GRL, 1996) (*A10.7.2.1*). During the past decade, computer programs have become available for axial and lateral load-deflection analysis of deep foundations based on the t-z and p-y curve methods of analysis.

The tolerable axial and lateral movement of driven piles should be based on criteria developed by the structural engineer for the superstructure, or by consideration of the effects of foundation movements on adjacent structures. The AASHTO LRFD Specification (AASHTO, 1997a) restricts lateral movements to 38 mm (*A10.7.2.2*). In some cases, the tolerable lateral movement is fixed at a small displacement (e.g., ≤ 12 mm based on observed acceptable performance and engineering judgment) to ensure acceptable structure performance.

9.3.2 Axial Resistance

9.3.2.1 Geotechnical Resistance (*A10.7.3*)

Pile foundations must be designed to resist axial loads without structural failure of the pile, and without excessive deflection. Methods used for ASD to estimate the ultimate axial geotechnical resistance of a single pile can be used for LRFD. Therefore, the ultimate axial geotechnical resistance of piles subjected to axial loading, R_n, can be determined as:

$$R_n = Q_{ult} = Q_p + Q_s \qquad \text{(Eq. 9-17)}$$

and the factored axial geotechnical resistance, Q_R, can be determined as:

$$R_r = Q_R = \phi Q_{ult} = \phi_{qp} Q_p + \phi_{qs} Q_s$$ (Eq. 9-18) (*A10.7.3.2-2*)

for which:

$$Q_p = q_p A_p$$ (Eq. 9-19) (*A10.7.3.2-3*)

and

$$Q_s = q_s A_s$$ (Eq. 9-20) (*A10.7.3.2-4*)

where:

ϕ_{qp}, ϕ_{qs} = Resistance factors from Table 9-5 (dim)
Q_p, Q_s = Ultimate pile tip and side resistance (kN)
q_p, q_s = Unit tip and side resistance (kPa)
A_p, A_s = Area of pile tip and side surface (m^2)

Various procedures are available for estimating the ultimate axial geotechnical capacity of driven piles in soil using semi-empirical methods and in-situ testing, and for piles bearing on or in rock using semi-empirical methods. ***When using the AASHTO LRFD Specification, only those methods referred to in Table 9-5 for which calibrated φ-factors have been developed can be used without developing other method-specific resistance factors. These methods for a single pile include:***

- Static Methods of Analysis

  - α-method - A semi-empirical, total stress static method of analysis for estimating the ultimate unit side resistance, q_s, and the ultimate unit tip resistance, q_p, as a function of the undrained shear strength, S_u, of cohesive soil (GRL, 1996) (*A10.7.3.3.2a*).

  - β-method - A semi-empirical, effective stress method of analysis for estimating q_s and q_p in soil as a function of the effective overburden pressure (GRL, 1996) (*A10.7.3.3.2b*).

  - λ-method - A semi-empirical, effective stress method of analysis for estimating q_s in soil as a function of the passive effective lateral earth pressure (AASHTO, 1997a) (*A10.7.3.3.2c*).

  - End-Bearing Pile on Rock - Semi-empirical method (Canadian Geotechnical Society, 1992) for estimating q_p as a function of the uniaxial compressive strength of the rock, σ_c, and the spacing of discontinuities (AASHTO, 1997a) (*A10.7.3.5*).

- In-Situ Methods of Analysis

- SPT-method - Semi-empirical developed by Meyerhof (1976) which correlates q_s and q_p with the SPT blow count for cohesionless soils (GRL, 1996) (*A10.7.3.4.2*).

- CPT-method - Semi-empirical method developed by Nottingham and Schmertmann (1975) which correlates q_s and q_p with CPT results for cohesionless soils (GRL, 1996) (*A10.7.3.4.3*).

- Methods Based on Field Testing of Pile

  - PDA Method - A method for estimating total load capacity based on monitored performance of driven piles and wave equation analyses (GRL, 1996) (*A10.7.3.6*).

  - Static Load Test - A method for estimating total load capacity based on tests (ASTM, 1996) representative of pile, load and subsurface conditions expected for the prototype piles (AASHTO, 1997a) (*A10.7.3.6*).

With exception of the λ-method, these methods are used by FHWA for evaluating the geotechnical axial capacity of driven piles (GRL, 1996). In addition, FHWA recommends (GRL, 1996) use of a semi-empirical procedure by Nordlund (1963) for estimating q_s and q_p for piles in cohesionless soils, and recognizes (GRL, 1996) LPC-method for estimating pile capacity based on CPT results (Bustamante and Gianeselli, 1983). FHWA also recognizes use of wave equation methods of analysis (without PDA) for estimating static pile capacity in soil (GRL, 1996). These and other methods can be used for LRFD provided calibrated ϕ-factors are developed using the methods described in Chapters 3 and 7.

9.3.2.2 Structural Resistance (*A10.7.4*)

In addition to meeting the requirements for adequate geotechnical resistance of piles subjected to axial and lateral loading, the axial and flexural resistance of the pile section must also be adequate for all potential loading cases. The ultimate unit structural capacity, σ_n, of a pile is based on the yield strength, F_y, for steel, the unconfined compressive strength, f_c', for concrete, F_y and f_c' for reinforced concrete and composite sections, and the crushing strength parallel to the grain, F_{co}, for timber. To evaluate an axially-loaded pile for structural resistance, the maximum factored stress, σ_{max}, is compared to the factored unit resistance, σ_r, which is determined as follows:

$$\sigma_r = \Sigma\,\phi\,\sigma_n \qquad \text{(Eq. 9-21)}$$

where:

ϕ = Resistance factor from Table 9-6 (dim)
σ_n = Nominal (ultimate) unit structural resistance (F_y, f_c' or F_{co}) for pile material (kPa)

Relative to axial loading of a pile, Eq. 9-20 can be multiplied by the cross-sectional area of the pile and written as:

$$P_r = \sum \phi \, P_n \tag{Eq. 9-22}$$

where:

P_n = Nominal (ultimate) structural resistance of pile (kN)
P_r = Factored structural resistance of pile (kN)

The ϕ factor used in Eq. 9-21 and presented in the left column of Table 9-6 was developed by direct calibration with ASD to include the effects of manufacturing variability, accidental load eccentricity and hidden defects due to damage during pile driving as described in Section 9.2.2.2.

9.3.3 Lateral Resistance (*A10.7.3.8, A10.7.3.11*)

Pile foundations must be designed to resist lateral loads without structural failure of the pile, and without excessive deflection (i.e., without exceeding the tolerable lateral movement). Although passive failure of the soil into which the pile is embedded is a potential failure mode, such failure occurs only at relatively large deflections which generally exceed tolerable movements. Therefore, design of piles subjected to lateral loads is commonly based on structural capacity and load-deflection behavior considerations. For piles subjected to both axial and lateral loading, the structural resistance of the pile as a beam-column must be checked using appropriate interaction equations calibrated with consideration of the load factors being utilized.

Methods used for ASD to estimate the lateral resistance of a single pile or pile group can also be used for LRFD. For homogeneous foundation materials and a fixed- or free-head condition at the pile cap, simplified methods of analysis (e.g., GRL, 1996; and Barker, et al., 1991a) can be used. For more complex foundation and loading conditions and intermediate levels of fixity at the pile cap, the analysis of laterally-loaded piles and pile groups is usually accomplished using computer programs based on the p-y curve method of lateral load-deflection analysis. These programs include COM624P (Wang and Reese. 1993) for individual vertical piles in level or sloping ground and GROUP (Reese, et al., 1994) for a group of vertical and battered piles in level ground. These methods of analysis incorporate p-y curves to model the nonlinear lateral load-deflection behavior for each soil type along the length of the pile. When using these methods at the LRFD Strength Limit State, the model pile or pile group is subjected to the factored lateral and axial loads and factored moment, and the resulting factored axial and bending stresses are compared with the factored axial and bending capacities of the pile.

In general, the design of laterally-loaded piles involves:

1. Determine the maximum lateral ground line deflection at the Service Limit State and the maximum moment at the Strength Limit State for an individual pile considering installation method (e.g., jetting or preaugering) for the selected pile section.

2. Repeat Step 1 if the lateral ground line deflection exceeds the tolerable deformation or results in axial or combined stresses which exceed the maximum factored axial resistance of the pile or do not meet the appropriate interaction criteria.

3. The pile section is acceptable for the design loads if neither the lateral groundline deflection nor the stress criteria from Step 2 is exceeded.

In practice, the piles in a group are tied together with a pile cap which is typically embedded below the ground surface. In the analyses of the lateral load resistance of this type of system, the contribution of passive resistance of the embedded portion of the pile cap is typically neglected. Neglecting passive resistance of the embedded pile cap can provide a significant degree of additional conservatism at both service and ultimate load levels.

FHWA promotes the use of vertical piles to resist lateral loads. However, if the lateral loads acting on a foundation are large, batter piles can be used. Batter piles transmit lateral loads predominantly as an axial force into the soil, and should be avoided when:

- Installations where negative loading or downdrag forces can cause bending of the pile section

- Installations in areas of high seismicity where high lateral forces from earthquake loading can result in local overstressing of piles near the cap/batter pile contact, and punching of piles through the cap

Methods such as GROUP (Reese, et al., 1994) used for ASD to estimate the lateral resistance of a pile group containing batter piles can also be used for LRFD.

9.3.4 Other Considerations

9.3.4.1 Group Effects (*A10.7.3.10*)

Adjustments used in ASD to account for the effects of group action can also be used for LRFD. For axial loading, the effect of closely-spaced piles can be considered in analyses by modifying the nominal resistance of each pile using an efficiency factor, η, to account for the:

- Spacing between piles
- Type and consistency of soil into which the piles are driven
- Contact between the soil and the pile cap

In cohesive soils, the ultimate axial resistance of a pile group may be less than the cumulative resistance of the individual piles due to overlapping of zones of shear deformation in the soil between the piles (*A10.7.3.10.2*). In stiff cohesive soils, there is no loss in resistance due to group effects. However, in soft cohesive soils where the pile cap is not in contact with the underlying soil, the resistance of a pile group will be less than the cumulative resistance of an equal number of individual piles if the center to center (CTC) spacing between piles is less than 6 D. For closer pile spacings, the resistance of each pile can be estimated by multiplying the nominal capacity by η, where:

- η = 0.65 for CTC spacing of 2.5 D
- η = 1.0 for CTC spacing of 6.0 D
- η determined by linear interpolation for intermediate pile spacings

The ultimate axial capacity of a pile group in cohesive soil is then the lesser of:

- The sum of the individual resistances of each pile in the group, modified by η

- The resistance of an equivalent pier consisting of the piles and the block of soil bounded within the piles

For a pile group in cohesionless soil (*A10.7.3.10.3*), η = 1.0 regardless of the spacing of piles or contact between the pile cap and the ground. No reduction in capacity is used for a pile group in sand due to the increase in soil density that usually occurs when piles are driven into sand.

If a pile group is embedded in a stiff soil deposit overlying a weaker deposit (*A10.7.3.10.4*), the potential exists for a punching failure of the pile group through the stiff soil into the soft soil. In addition, the potential for settlements in the soft layer must be evaluated.

9.3.4.2 Negative Loading (*A10.7.1.4*)
Methods in ASD to estimate the magnitude and location of the maximum downdrag load can also by used in LRFD. General comments regarding downdrag load estimation are presented in Chapter 4. Briaud and Tucker (1994a) indicate that the potential for downdrag loading should be considered in design when the indicators in Table 9-8 are present. In terms of performance limits, downdrag generally poses a foundation settlement concern for friction piles and for end-bearing piles bearing on a very stiff layer such as very dense sand or rock.

Table 9-8
Conditions When Downdrag Should be Considered in Design
(after Briaud and Tucker, 1994a)

1	Total settlement of the ground surface > 10 mm
2	Settlement of ground surface after pile driving > 1 mm
3	Height of embankment filling on ground surface > 2 m
4	Thickness of soft compressible layer >10 m
5	Water table drawn down >4 m
6	Piles length > 25 m

WARNING: Downdrag can occur even if the above conditions are not met.

9.3.4.3 Uplift Loading (*A10.7.1.9*, *A10.7.3.7*)
Uplift loading on piles can be caused by lateral loads from supported structures, buoyancy effects or expansive soils. Evaluation of the ultimate uplift resistance of a single pile or pile group by ASD

and LRFD are conducted using the same methods used to estimate ultimate axial side resistance, Q_S, of the pile or group to axial compression loading, but with a 20 percent reduction in Q_S. In the AASHTO LRFD Specification (AASHTO, 1997a), the 20 percent reduction in Q_S for uplift is incorporated in the resistance factors for uplift in Table 9-5. In addition, the structural resistance of the pile and the pile/pile cap connection must also be checked.

9.3.4.4 Driving Stresses and Driveability (*A10.7.1.16*)

The effects of the pile driving process must be evaluated during design to ensure that the pile can be installed without damage that could affect the capacity of the pile to perform within acceptable limits. A driveability evaluation is needed because the highest pile stresses are usually developed during driving to facilitate penetration of the pile to the depth needed to mobilize the required resistance. However, the high strain rate and temporary nature of loading during pile driving allow a substantially higher stress level to be used during installation than for service. The driveability of candidate pile-hammer-system combinations can be evaluated using wave equation analyses (GRL, 1996).

The LRFD Specification (AASHTO, 1997a) provides performance limits for pile stresses during driving consistent with the current AASHTO ASD Specification (AASHTO, 1997b). These driving stress criteria are summarized in Table 9-9, and include limitations of (unfactored) driving stresses in piles based on:

Table 9-9
Permissible Stresses During Pile Driving
(AASHTO, 1997a)

Pile Type	Stress Level (MPa)	Resistance Factor	AASHTO LRFD Provision
Steel			
• Compression:	$0.90\ \phi\ F_y$	1.00	(*A6.5.4.2*)
• Tension:	$0.90\ \phi\ F_y$	1.00	(*A6.5.4.2*)
Concrete:			
• Compression	$0.85\ \phi\ f'_c$	1.00	(*A5.5.4.2.1*)
• Tension	$0.70\ \phi\ F_y$ of Steel Reinforcement	1.00	(*A5.5.4.2.1*)
Prestressed Concrete:			
• Normal environments			
- Compression	$\phi\ (0.85\ f'_c - f_{pe})$ [1]	1.00	(*A5.5.4.2.1*)
- Tension	$\phi\,(0.25\sqrt{f_c\_} + f_{pe})$ [1]	1.00	(*A5.5.4.2.1*)
• Severe corrosive environments			
- Tension	$\phi\ f_{pe}$	1.00	(*A5.5.4.2.1*)
Timber:			
• Compression	$\phi\ F_{co}$	1.15	(*A8.5.2.2*)
• Tension	$\phi\ F_{co}$	1.15	(*A8.5.2.2*)

(1) f_c' and f_{pe} in MPa.

- Yield strength of steel in steel piles

- Ultimate compressive strength of the gross concrete section $\pm$ the effective prestress after losses for prestressed concrete piles loaded in tension or compression, respectively

- Base resistance for the selected species of timber piles loaded in compression parallel to the grain

9.3.4.5 Fixity of Pile-Cap Connection (*A10.7.3.8*)
The load-deflection response and the magnitude and location of maximum moment of a laterally-loaded pile or pile group depends on the fixity of the pile butt into the cap. Unless a detailed structural analysis is performed, the fixity of the pile-cap connection is usually assumed to vary between fully fixed and 50 percent partially fixed. The key factors that must be considered in determining or estimating fixity at the pile-cap connection include:

- Depth of pile embedment into the cap
- Magnitude of bending moment at pile-cap connection
- Pile type and geometry
- Pile-to-pile cap connection detail

Based on the results of full-scale load tests (e.g., Castilla, et al., 1984; Shahawy and Issa, 1992), a depth of embedment of 2D to 3D into the pile cap provides full fixity for most service load conditions. Because the degree of pile-cap fixity is usually established empirically, there is no difference in the approach taken for pile foundations designed by either ASD or LRFD.

9.4 Student Problem:
Comparison of Pile Designs Using ASD and LRFD

Problem: You are to design an axially loaded pile group to support the bridge pier illustrated in the following problem. This problem presents SPT, CPT and instrumented pile load test data for closed-end steel pipe piles driven to a depth of 12 m into a deep sand deposit. In situ and load test data presented in the example provide a basis for comparing ASD and LRFD concepts for a driven pile foundation. (Note: T his problem has been simplified for classroom purposes from the typical design case which would include both horizontal and axial loads. A more typical problem is presented in Sections 9.5 and 9.6).

Objectives: To demonstrate the procedures for driven pile design by LRFD, and to compare the results with those obtained using ASD.

Approach: To perform the designs, you should take following steps:

- Establish unfactored design loads due to structure components, wearing surfaces and utilities, and vehicular live load
- Determine appropriate load factors and load combinations and calculate the total factored load effects
- Estimate the unfactored (ASD) and factored (LRFD) axial resistance of a single pile based on correlation with the results of SPTs
- Estimate the unfactored and factored axial resistance of a single pile based on correlation with the results of CPTs
- Estimate the unfactored and factored axial resistance of a single pile based on the results of full-scale load tests
- Establish the allowable (ASD) and factored (LRFD) structural capacity of a single pile
- Determine the required number of piles in the group based on geotechnical and structural criteria

Step 1: Establish Unfactored Loads

The unfactored vertical loads on the pile group shown in Figure 9-2 are:

DC = Dead load of structural components and non-structural attachments = 4600 kN
DW = Dead load of wearing surfaces and utilities = 3900 kN
LL = Vehicular live load = 3450 kN

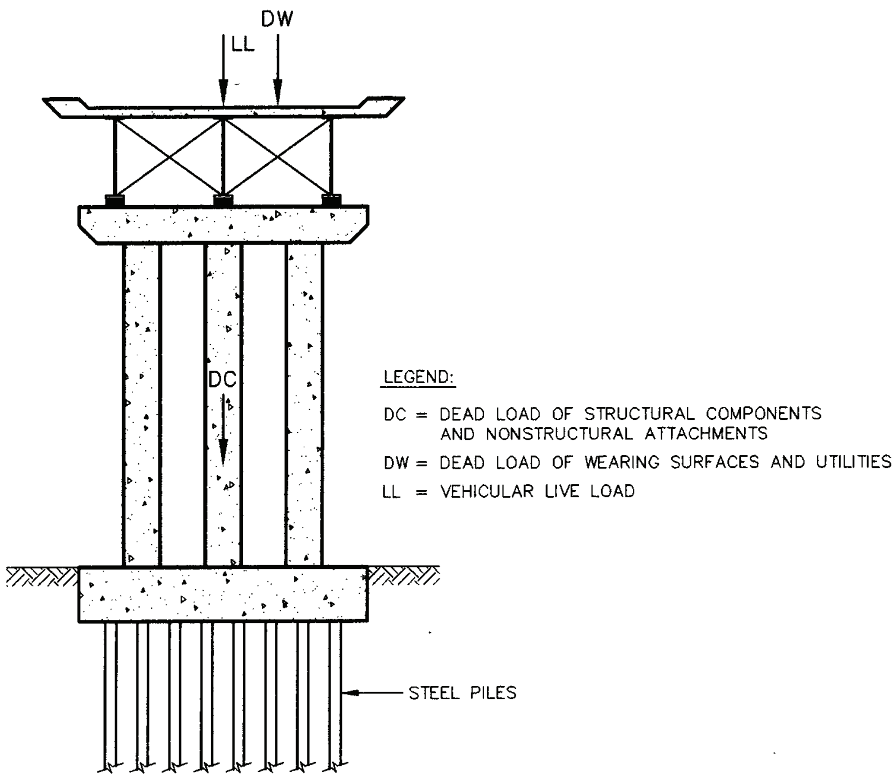


Figure 9-2
Pile Group Loading

From which the total unfactored load, Q, is:

Q = DC + DW + LL
Q = ______ kN + ______ kN + ______ kN
Q = ______ kN

Step 2: Determine Load Factors and Factored Loads
For this design example, consider only Strength I and Service I Limit States.

For the Strength I and Service I Limit States, Eq. 9-6 is used to compute the factored load effect:

$$\sum \eta_i \gamma_i Q_i$$

Assume a typical structure such that $\eta_i = 1.0$.

Complete Table 9-10 by selecting load factors for the Strength I and Service I Limit States from Table 4-10 in Chapter 4:

Table 9-10
Load Factors

Limit State	γ_{DC}	γ_{DW}	γ_{LL}
Strength I			
Service I			

The total factored load effects are then calculated as follows. For the Strength I Limit State:

$$\sum \eta_i \gamma_i Q_i = \eta_i \, [\gamma_{DC}\, DC + \gamma_{DW}\, DW + \gamma_{LL}\, LL]$$
$$\sum \eta_i \gamma_i Q_i \;= \_\_\_\_\_[(\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN}) + (\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN}) + (\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN})]$$
$$\sum \eta_i \gamma_i Q_i \;= \_\_\_\_\_\_ \text{ kN}$$

For the Service I Limit State:

$$\sum \eta_i \gamma_i Q_i \;= \eta_i \, [\gamma_{DC}\, DC) + \gamma_{DW}\, DW + \gamma_{LL}\, LL]$$
$$\sum \eta_i \gamma_i Q_i \;= \_\_\_\_\_[(\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN}) + (\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN}) + (\_\_\_\_\_)(\_\_\_\_\_\_ \text{ kN})]$$
$$\sum \eta_i \gamma_i Q_i \;= \_\_\_\_\_\_ \text{ kN}$$

Step 3: Estimate Axial Capacity of Single Pile from SPTs
Figure 9-3 shows the generalized soil profile, pile geometry and idealized SPT blow count profile for use in design based on several borings performed near the pier location. Using Meyerhof (1976) (*A10.7.3.4.2*), the estimated ultimate axial capacity of the pile driven into the soil profile in Figure 9-3 is:

$$Q_p = q_p\, A_p = (8800 \text{ kPa})\, \pi\, (0.46 \text{ m}/2)^2 = 1460 \text{ kN} \qquad \text{(Eq. 9-19)}$$

and

$$Q_s = q_s\, A_s = (28 \text{ kPa})\, 2\pi\, (0.23 \text{ m})\, (11.00 \text{ m}) = 445 \text{ kN} \qquad \text{(Eq. 9-20)}$$

where:

Q_{ult}, Q_p, Q_s = Ultimate total, tip and side resistance (kN)
q_p, q_s = Unit tip and side resistance (kPa)
A_p, A_s = Area of pile tip and side (m^2)

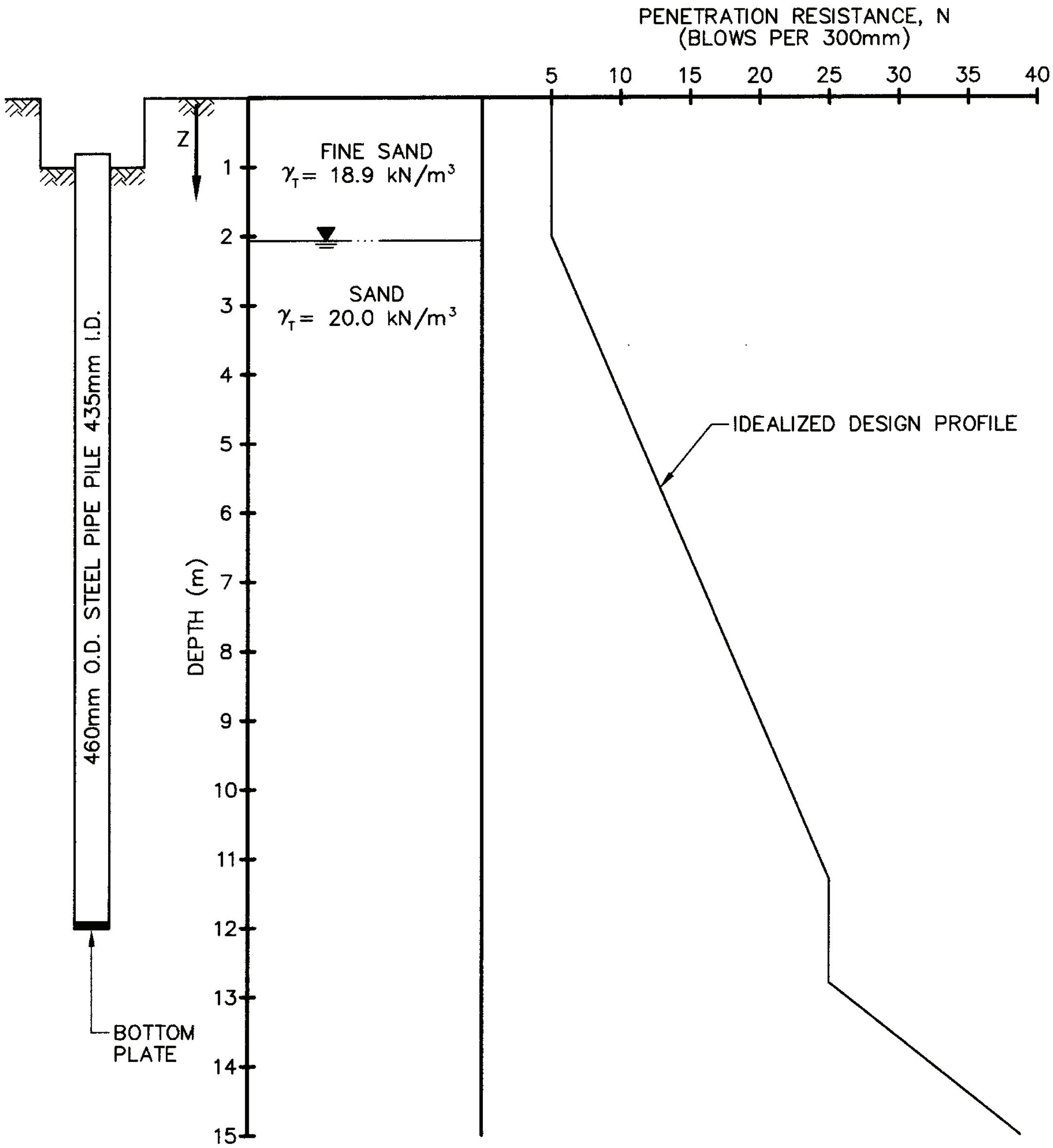


Figure 9-3
Generalized Problem Geometry and SPT Design Envelope

Using Eq. 9-17, the ultimate axial resistance of a single pile is:

$Q_{ult} = Q_p + Q_s =$ ______ kN + ______ kN (Eq. 9-17)
$Q_{ult} =$ ______ kN

From Eq. 9-18, the factored axial resistance of a single pile is:

$$Q_R = \phi\, Q_{ult} = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s \qquad \text{(Eq. 9-18)}$$

From Table 9-5, the resistance factors for the SPT method are:

- ϕ_{qp} =
- ϕ_{qs} =

The factored bearing resistance is then:

$$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s = \_\_\_\_ (\_\_\_\_\_ \text{ kN}) + \_\_\_\_ (\_\_\_\_\_ \text{ kN}) = \_\_\_\_\_ \text{ kN}$$

Step 4: Estimate Axial Capacity of Single Pile from CPTs
Figures 9-4 and 9-5 show a representative pile and idealized tip and sleeve friction profiles used for design from a typical mechanical cone penetration sounding performed near the pier location referred to in Step 3.

Using Nottingham and Schmertmann (1975) (A10.7.3.4.3), estimate ultimate axial capacity of the pile driven into the soil profile in Figures 9-4 and 9-5.

$$Q_p = q_p\, A_p = (11\ 300 \text{ kPa})\ \pi\ (0.46 \text{ m}/2)^2 = 1880 \text{ kN} \qquad \text{(Eq. 9-19)}$$

The ultimate unit side resistance of the pile is estimated using the CPT sleeve resistance values from Figure 9-4 and the following relationship:

$$Q_s = K_s \left[\sum_{i=1}^{N_1} \left(\frac{L_i}{8 D_i} \right) f_{si}\, a_{si}\, h_i + \sum_{i=N_i}^{N_2} f_{si}\, a_{si}\, h_i \right] \qquad (\textit{A10.7.3.4.3c-1})$$

For a steel pile with a length to diameter (z/D) ratio of 11.00 m/0.46 m = 23.9, the correction factor, K_s for a mechanical cone (from Figure A10.7.3.4.3c-1) is 0.39. The incremental values of Q_{si} along the pile are presented in Table 9-11.

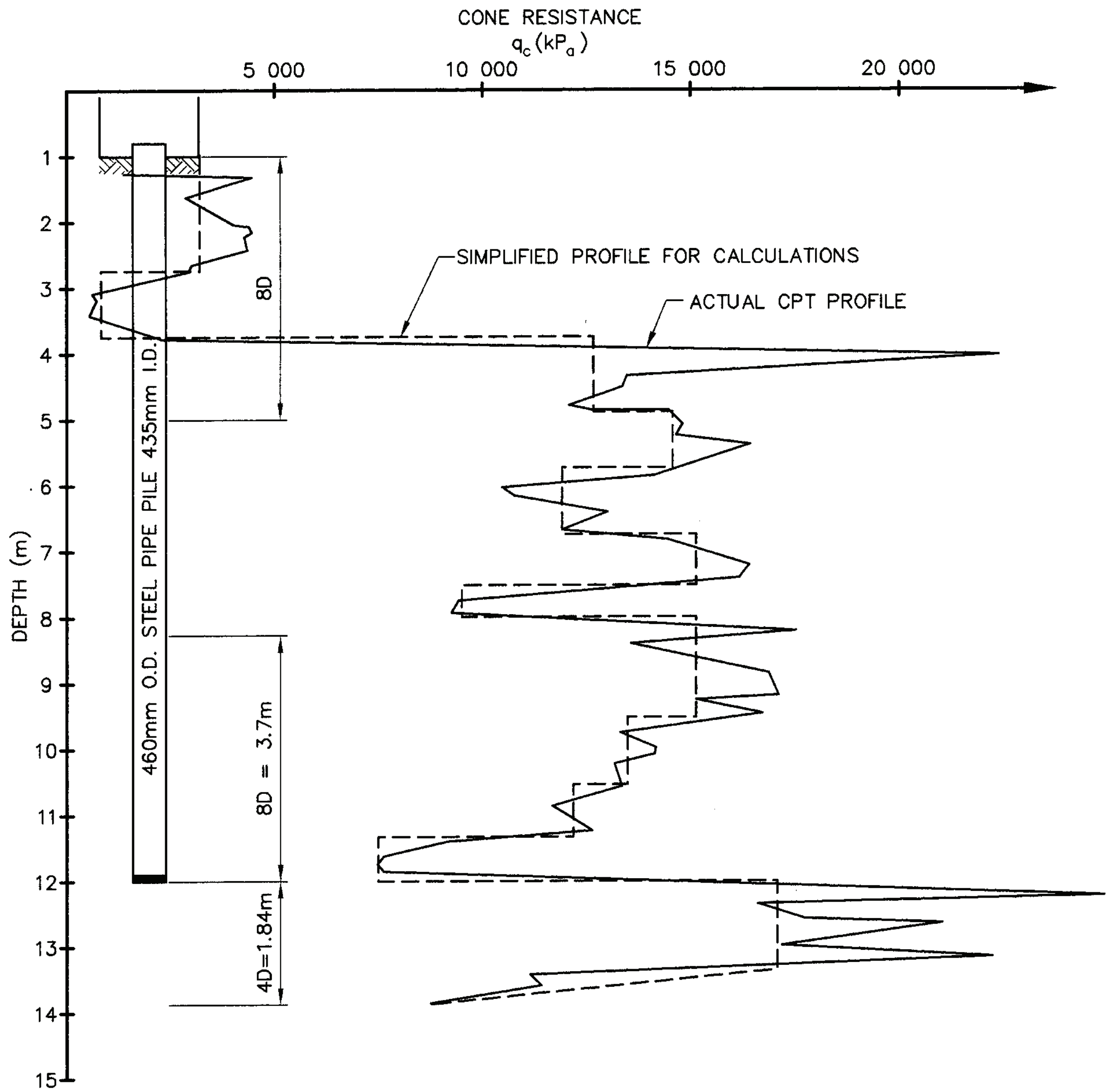


Figure 9-4
Representative Pile and CPT Tip Resistance Profile Used in Example

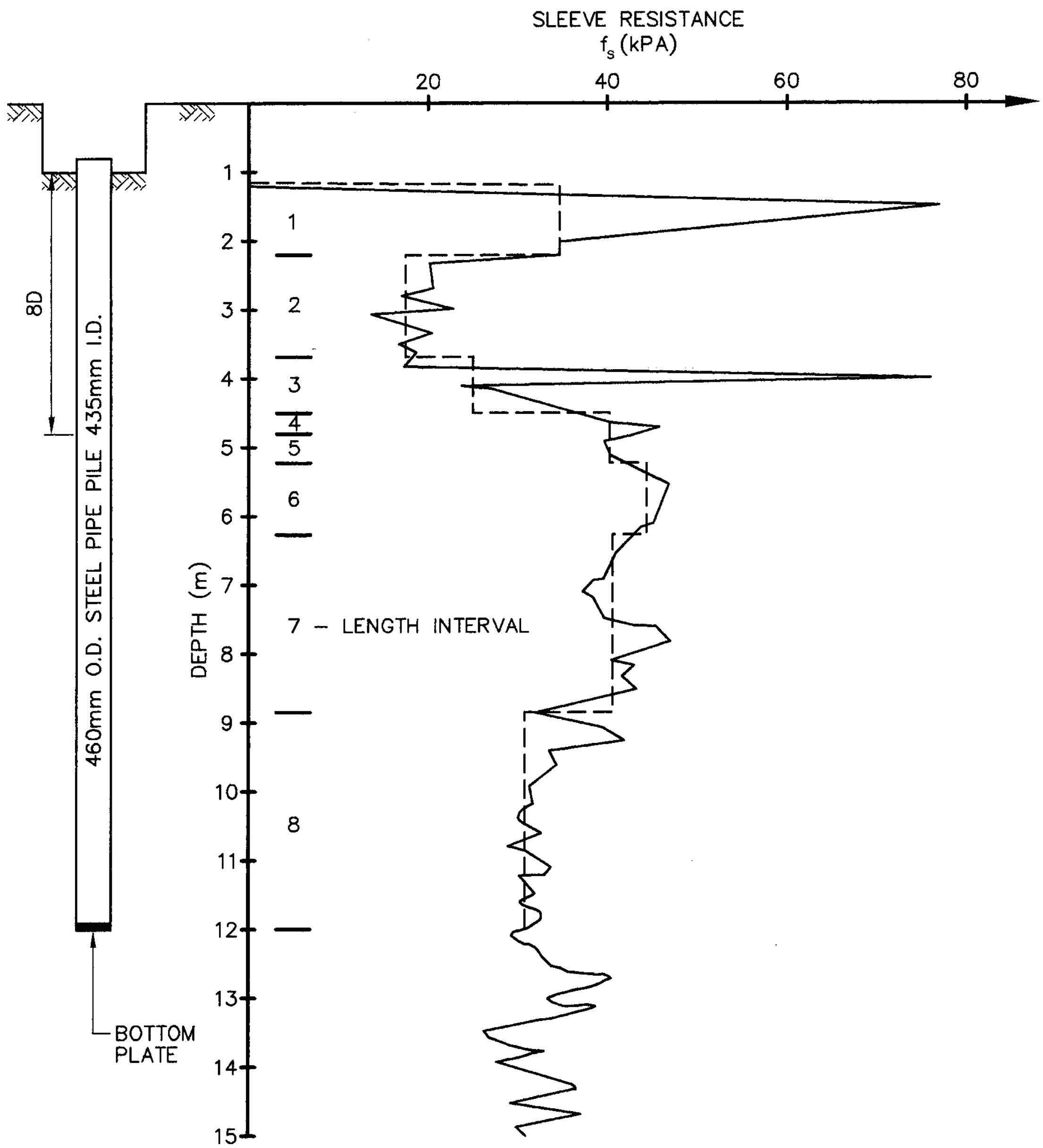


Figure 9-5
Representative Pile and CPT Side Resistance Profile Used in Example

Table 9-11
Tabulation of Side Resistance Along Pile Length

Pile Segment, I	L_i (m)	D_i (m)	f_{si} (kPa)	a_{si} (m^2/m)	h_i (m)	Q_{si} (kN)
1	0.643	0.46	345	1445	1.105	4
2	1.885	0.46	173	1445	1.380	7
3	2.970	0.46	249	1445	0.790	9
4	3.523	0.46	403	1445	0.315	7
5	NA	NA	403	1445	0.480	11
6	NA	NA	441	1445	1.000	25
7	NA	NA	403	1445	2.650	60
8	NA	NA	307	1445	3.190	55
					TOTAL Q_s:	178

where the variables are the same as those in Step 3.

From Eq. 9-17, the ultimate axial resistance of a single pile is:

$$Q_{ult} = Q_p + Q_s = \_\_\_\_\_\_ \text{ kN} + \_\_\_\_\_\_ \text{ kN} = \_\_\_\_\_\_ \text{ kN} \qquad \text{(Eq. 9-17)}$$

The factored axial resistance of a single pile is determined from Eq. 9-18 as:

$$Q_R = \phi\, Q_{ult} = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s \qquad \text{(Eq. 9-18)}$$

From Table 9-5, the resistance factors for the CPT method are:

- $\phi_{qp} =$
- $\phi_{qs} =$

The factored bearing resistance is then:

$$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s = \_\_\_\_\_ (\_\_\_\_\_\_ \text{ kN}) + \_\_\_\_\_ (\_\_\_\_\_\_ \text{ kN}) = \_\_\_\_\_\_ \text{ kN}$$

Step 5: Estimate Axial Capacity of Single Pile from Full-Scale Load Tests
From a full-scale axial load test on a pile at the site (Vesic, 1970), the ultimate tip and side resistance of a pile driven to a depth of 12 m at the site are:

- Q_s = 1180 kN
- Q_p = 1900 kN

from which the ultimate axial resistance based on Eq. 9-17 is:

$$Q_{ult} = Q_p + Q_s = \_\_\_\_\_\_ \text{ kN} + \_\_\_\_\_\_ \text{ kN} = \_\_\_\_\_\_ \text{ kN} \quad \text{(Eq. 9-17)}$$

The factored axial resistance of a single pile from Eq. 9-18 is:

$$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s \quad \text{(Eq. 9-18)}$$

From Table 9-5, the resistance factors for bearing resistance of a single pile based on a load test are as follows:

- ϕ_{qp} =
- ϕ_{qs} =

Therefore, the factored bearing resistance is:

$$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s = \_\_\_\_\_ (\_\_\_\_\_\_ \text{ kN}) + \_\_\_\_\_ (\_\_\_\_\_\_ \text{ kN}) = \_\_\_\_\_\_ \text{ kN}$$

Step 6: Check the Axial Structural Capacity of a Single Pile
From Table 9-2, the maximum allowable stress, σ_{all} (AASHTO, 1997b), for axial compression loading of an unfilled steel pipe pile is:

- For severe driving:
  $\sigma_{all} = 0.25\ F_y$

- For good driving:
  $\sigma_{all} = 0.33\ F_y$

For ASTM A709M, Grade 250, F_y = 250 000 kPa, the allowable axial stress is:

- For severe driving:
  $\sigma_{all} = \_\_\_\_\_\ F_y = \_\_\_\_\_\ (\_\_\_\_\_\_\_\_ \text{ kPa})$
  $\sigma_{all} = \_\_\_\_\_\_\_\_ \text{ kPa}$

- For good driving:
  $\sigma_{all} = \_\_\_\_\_\ F_y = \_\_\_\_\_\ (\_\_\_\_\_\_\_\_ \text{ kPa})$
  $\sigma_{all} = \_\_\_\_\_\_\_\_ \text{ kPa}$

The maximum allowable axial load, P_{all}, is:

- For severe driving:
  $P_{all} = \sigma_{all}\ A_s$
  $P_{all} = (\_\_\_\_\_\_ \text{ kPa})(0.0176 \text{ m}^2) = \_\_\_\_\_\_ \text{ kN}$

- For good driving:
  $P_{all} = \sigma_{all}\ A_s$

$P_{all} = (\_\_\_\_\_\_ \text{ kPa})(0.0176 \text{ m}^2) = \_\_\_\_\_\_ \text{ kN}$

For LRFD, the factored axial resistance, P_r, of an unfilled steel pipe pile from Eq. 9-21 and Eq. 9-22 is:

$$P_r = \phi\ P_n = \phi\ \sigma_n\ A_s = \phi\ F_y\ A_s$$

From Table 9-6, the resistance factors for axial compression loading of an unfilled steel pipe pile are:

- For severe driving:
  $\phi =$
- For good driving:
  $\phi =$

For ASTM A709M, Grade 250 steel:

- For severe driving:
  $P_r = \phi\ F_y\ A_s$
  $P_r =$ (_____)(______ kPa)(0.0176 m^2) = _____ kN
- For good driving:
  $P_r = \phi\ F_y\ A_s$
  $P_r =$ (_____)(______ kPa)(0.0176 m^2) = _____ kN

Step 7: Determine the Required Pile Group Size
For ASD, the number of piles, x, required to support the total (unfactored) design load, Q, can be computed based on Eqs. 9-1 and 9-2 from the following:

$$\frac{Q}{(Q_{ult}/FS)} \quad ; \quad x \geq \frac{Q}{P_{all}}$$

where:

$Q_{ult}/FS =$ Allowable geotechnical resistance (kN)
$P_{all} =$ Allowable structural resistance (kN)

From Table 9-1, the required factors of safety (FS) for axial load resistance based on various predictive methods are:

- Static Calculation (SPT or CPT): FS = 3.5
- Static Load Test: FS = 2.0

Based on these safety factors and the results of Steps 1 through 6, complete Table 9-12 to determine the number of piles need to resist the design load based on geotechnical and structural resistance by

ASD.

Table 9-12
Determination of Required Pile Group Size Based on ASD

Resistance	FS or σ_{all}	Design Group Load Q (kN)	Single Pile Ultimate Resistance Q_{ult} (kN)	Single Pile Allowable Resistance Q_{ult}/FS or P_{all} (kN)	Required Number of Piles, x
Geotechnical:					
• SPT Method	3.5	11 950			
• CPT Method	3.5	11 950			
• Static Load Test	2.0	11 950			
Structural:					
• Severe Driving	0.25 F_y	11 950	---		
• Good Driving	0.33 F_y	11 950	—		

For LRFD, the number of piles, x, required to support the factored design load, Q, can be computed based on Eqs. 9-7 and 9-8 from the following:

$$\frac{\Sigma\, \eta_i\, \gamma_i\, Q_i}{Q_R} \quad ; \quad x \geq \frac{\Sigma\, \eta_i\, \gamma_i\, Q_i}{P_r}$$

where:

$\Sigma\, \eta_i\, \gamma_i\, Q_i$ = Sum of the factored axial loads (kN)
Q_R = Factored geotechnical resistance (kN)
P_r = Factored structural resistance (kN)

Based on the results of Steps 1 through 6, complete Table 9-13 to determine the number of piles need to resist the design load based on geotechnical and structural resistance by LRFD.

Table 9-13
Determination of Required Pile Group Size Based on LRFD

Resistance	Resistance Factor ϕ	Factored[1] Design Group Load $\Sigma\ \eta_i\ \gamma_i\ Q_i$ (kN)	Single Pile Resistance		Required Number of Piles, x
			Ultimate Q_{ult} (kN)	Factored Q_R or P_r (kN)	
Geotechnical: • SPT Method	0.45	17 638			
• CPT Method	0.55	17 638			
• Static Load Test	0.80	17 638			
Structural: • Severe Driving	0.35	17 638	-		
• Good Driving	0.45	17 638	-		

[1] For the Strength I Limit State

Summary

This example illustrates the design of axially loaded pile groups by LRFD and compares the results to design by ASD. As shown on Tables 9-12 and 9-13:

- *The geotechnical resistance controls the design for static methods of axial capacity prediction and the structural resistance controls the design based on static load testing for both the ASD and LRFD procedures*

- *Comparison with Table 9-12 shows that the LRFD procedures based on the reliability-based calibration (i.e., SPT and CPT) are somewhat less conservative than the ASD procedure for the conditions analyzed*

- *LRFD design procedures based on direct calibration with ASD (i.e., static load test and structural resistance) provide essentially identical results with ASD (minor differences occur due to differences in the actual average load factor and the load factor assumed in the resistance factor calibration*)

As indicated in Section 9.2.2.2, the AASHTO LRFD Specifications do not yet satisfactorily incorporate the impact of field capacity verification (e.g., PDA or load tests) on the reliability of pile installations for which design capacities are initially predicted using static design methods. Therefore, this example assumes that the piles are designed on the basis of static methods without field capacity verification.

9.5 Student Exercise:
Pile Capacity Evaluation by Nordlund Method

The development of a resistance factor for evaluation of the axial geotechnical resistance of a pile by Nordlund's method was presented in Chapter 7, Sections 7.3 and 7.4. This exercise compares LRFD and ASD for the Nordlund pile capacity prediction method for the Classroom Example in Section 9.4, and compares the results with the methods used in the Classroom Example. Nordlund's method for estimation of pile side and tip resistance in cohesionless soil (GRL, 1996) utilizes the following equation:

$$Q_{ult} = \sum_{d=0}^{d=D} K_\delta C_F p_d \frac{\sin(\delta + \omega)}{\cos \omega} C_d \Delta d + \alpha_t N'_q A_t p_t$$

where:

- $d =$ Depth (m)
- $D =$ Embedded pile length (m)
- $K_\delta =$ Coefficient of lateral earth pressure at depth d (dim)
- $C_F =$ Correction factor for K_δ when $\delta \neq \phi_f$ (dim)
- $p_d =$ Effective overburden pressure at the center of depth increment d (kPa)
- $\delta =$ Friction angle between pile and soil (deg)
- $\omega =$ Angle of taper from vertical (deg)
- $\phi_f =$ Soil friction angle estimated from SPT (deg)
- $C_d =$ Pile perimeter at depth d (m)
- $\Delta d =$ Length of pile segment (m)
- $\alpha_t =$ Dimensionless factor (dependent on pile depth-width relationship) (dim)
- $N'_q =$ Bearing capacity factor (dim)
- $A_t =$ Pile toe area (m^2)
- $p_t =$ Effective overburden pressure at the pile toe (kPa)

To focus solely on the differences between LRFD and ASD, the details of Nordlund's method as applied to this problem are not presented herein. The results of Nordlund's method are tabulated below. For the steel pipe piles and soil conditions in the Example in Section 9.4:

- $\omega =$ 0
- $\alpha_t =$ 0.65
- $N'_q =$ 55
- $A_t =$ 0.166 m^2
- $P_t =$ 139.7 kPa

and values of other parameters in the Nordlund equation are tabulated below:

Depth Interval (m)	K_δ (dim)	p_d (kPa)	ϕ'_f (deg)	C_F (dim)	δ (deg)	C_d (m)	Δd (m)
1 - 2	0.92	28.4	29	0.92	23	1.45	1
2 - 7	0.91	63.3	31	0.91	24	1.45	5
7 – 11.25	0.90	110.4	33	0.90	26	1.45	4.25
11.25 - 12	0.90	139.7	34	0.90	27	1.45	0.75

From which the ultimate pile capacity is:

Q_{ult} = 1385 kN

For the pile capacity prediction shown above, complete the following tables (reproduced from the Classroom Example in Section 9.4) by filling in the blank cells associated with Nordlund's method. **Recall from Table 9-1 that for ASD a factor of safety of 3.5 is applicable (where pile capacity is estimated using a static analysis) and that for LRFD an average resistance factor of about 0.60 was developed for Nordlund's method in Chapter 7. Recall also that the required number of piles, x, can be obtained as follows:**

- **For ASD:**

$$x \geq \frac{Q}{(Q_{ult}/FS)}$$

- **For LRFD:**

$$x \geq \frac{\sum \eta_i \gamma_i Q_i}{Q_R}$$

Determination of Required Pile Group Size Based on ASD

Resistance	FS or σ_{all}	Design Group Load Q (kN)	Single Pile Ultimate Resistance Q_{ult} (kN)	Single Pile Allowable Resistance Q_{ult}/FS or P_{all} (kN)	Required Number of Piles, x
Geotechnical:					
• SPT Method	3.5	11 950	1900	542.9	22
• CPT Method	3.5	11 950	2057	587.7	21
• Static Load Test	2.0	11 950	3080	1540	8
• Nordlund Method	3.5	11 950	1385	________	_______
Structural:					
• Severe Driving	0.25 F_y	11 950	---	1098	11
• Good Driving	0.33 F_y	11 950	---	1450	9

Determination of Required Pile Group Size Based on LRFD

Resistance	Resistance Factor ϕ	Factored[1] Design Load $\Sigma\, \eta_i\, \gamma_i\, Q_i$ (kN)	Single Pile Resistance		Required Number of Piles, x
			Ultimate Q_{ult} (kN)	Factored Q_R or P_r (kN)	
Geotechnical:					
• SPT Method	0.45	17 638	1900	855	21
• CPT Method	0.55	17 638	2057	1131	16
• Static Load Test	0.80	17 638	3080	2464	8
• Nordlund Method	0.60	17 638	1385	_______	_____
Structural:	0.35	17 638	-	1538	12
• Severe Driving	0.45	17 638	-	1977	9
• Good Driving					

[1] For the Strength I Limit State

9.6 Design Example 1: Design of Pile Support for Bridge Pier

Problem: The design of a bridge pier supported by pile group bearing on competent sandstone is illustrated in the following example. In this example, the piles will be designed to support the loads developed in the Design Example of Chapter 4 (Section 4.9). The soil profile consists of 11.7 m of NC medium stiff clay and the pier group will be comprised of HP 310 × 110 piles. This problem differs from the problem presented in 9.4 in the following respects:

- The piles are end bearing
- The pile group is subjected to biaxial bending and bi-directional lateral forces in addition to a vertical load

Objective: To illustrate the procedure for driven pile design using LRFD for an end-bearing pile group subjected to axial, lateral and moment loading.

Approach: To perform the evaluation, the following steps are taken:

- Establish factored load combinations and select critical load combination(s)
- Determine the factored geotechnical and structural axial resistance for a single pile
- Determine the factored structural moment resistances about the strong and weak axes for a single pile
- Estimate the number of piles required to support the factored loads and determine the geometric arrangement of piles in the group
- Determine the factored axial loads and bending moments, about both axes, for each pile in the pile group
- For the most heavily loaded piles, compare the factored axial and bending resistances with the factored axial, bending and lateral loads
- Determine whether the axial and lateral displacements of the pile group are acceptable
- Evaluate other critical factored loading combinations
- Determine whether any load case could result in uplift forces on any of the piles
- If for any factored load combination the pile group is incapable of adequately supporting the factored loading, either due to inadequate factored resistance or excessive deformation, increase the number of piles and repeat the

analysis. If the pile group is capable of supporting the factored loads, evaluate whether the design is over conservative, i.e., whether the selected number of piles is excessive. If so, optimize the piles design.

Step 1: Establish Factored Load Combinations

The potentially critical factored loads were developed in Chapter 4 (Table 4-29) and are reproduced herein as Table 9-14.

Table 9-14
Summary of Factored Loads for Critical Foundation Design Load Combinations

LOAD CASE		MAXIMUM LOADS					MINIMUM LOADS				
		Moments		Axial Load (kN)	Horiz. Load		Moment		Axial Load (kN)	Horiz. Load	
		Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)	Long. (kN-m)	Trans. (kN-m)		Long. (kN)	Trans. (kN)
A	STR I	4772	5632	12 644	317	77	---	---	---	---	---
A	STR III	1046	3723	9617	74	270	952	3723	6585	74	270
A	SER I	2912	3803	8962	220	115	---	---	---	---	---
B	STR I	4772	5632	12 644	317	77	---	---	---	---	--
B	SER I	3269	3454	8960	261	85	---	---	---	---	---
C	STR I	5427	4432	11 967	317	77	5262	4432	8935	317	77
C	STR III	1046	3723	9617	74	270	952	3723	6585	74	270
D	STR I	5427	4432	11 967	317	77	5262	4432	8935	317	77
D	STR V	---	---	---	---	---	4861	3899	8497	324	112
D	SER I	3614	2801	8594	261	85	---	---	---	---	---
E	STR I	---	---	---	---	---	2952	6203	8575	201	49
E	SER I	2060	4114	8398	157	100	---	---	---	---	---
F	STR I	---	---	---	---	---	2952	6203	8575	201	49

Shaded areas duplicate another Load/Limit State Case

Load Cases A/B, Strength I appear to be critical as they have the highest transverse moment, vertical load and longitudinal horizontal force. This loading will be used in the initial analysis and design. Load Cases C/D, Strength I could be critical, as they have equally high horizontal loads and a higher longitudinal moment, although the transverse moment and vertical load are lower. Cases A/C, Strength III have the highest transverse horizontal load, but it is unlikely that these two cases could be critical as the other four load components are considerably smaller. The remaining cases have smaller loads and can be ignored. Therefore, this problem will be analyzed initially using the following factored loading from Load Case A, Strength I:

- Longitudinal Moment, M_L = 4772 kN-m
- Transverse Moment, M_T = 5632 kN-m

- Vertical Load, P_V = 12 644 kN
- Horizontal Load, P_H = 317 kN
- Transverse Load, P_T = 77 kN

Step 2: Establish the Factored Geotechnical Axial Resistance
The piles will be driven through a medium stiff clay and will be end bearing on a competent sandstone. For properly driven piles bearing on hard rock, a very high geotechnical resistance will be available. Therefore, assume the geotechnical resistance is adequate and that the resistance will depend on the structural capacity of the pile section.

Step 3: Establish the Required Resistance Considerations Which Must Be Satisfied
At shallow depth, the piles will carry axial loading as well as biaxial bending moments. Each pile must be capable of resisting the interactive compressive stresses due to axial loading and bending. In this zone, interaction criterion such as the following must be satisfied:

$$\frac{P_u}{P_{ri}} + \frac{8.0}{9.0}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \le 1.0 \qquad (\mathit{A6.9.2.2\text{-}2})$$

where:

P_u = Factored axial load (kN)
P_{ri} = Factored interactive compressive resistance (*A6.9.2.1*) (kN)
M_{ux}, M_{uy} = Factored moment for strong and weak axes, respectively (kN-m)
M_{rx}, M_{ry} = Factored flexural resistance for strong and weak axes, respectively (A6.10.6 and A6.12) (kN-m)

and

$P_{ri} = \phi_{ci}\, P_n = 0.60\, F_y\, A_s$
$M_{rx} = \phi_{fi}\, M_{nx} = 0.85\, F_y\, S_x$
$M_{ry} = \phi_{fi}\, M_{nx} = 0.85\, F_y\, S_y$
ϕ_{ci} = Axial resistance factor for combined axial load and flexure (dim)
ϕ_{fi} = Flexural resistance factor for combined axial load and flexure (dim)

[The interactive resistance factors, ϕ_{ci} and ϕ_{fi}, used in this problem were selected based on strength reductions for pile material properties and unanticipated load eccentricity as recommended by Davisson, et al. (1983), and differ somewhat from values prescribed in AASHTO (1997a) and AISC (1986) as follows:

AISC: $\phi_{ci} = 0.85$; $\phi_{fi} = 0.90$
AASHTO LRFD: $\phi_{ci} = 0.70$; $\phi_{fi} = 0.78$ (*A6.5.4.2* and *A6.9.4.1*)
This problem: $\phi_{ci} = 0.60$; $\phi_{fi} = 0.85$]

At some depth, the moments will decrease to a negligible level, and only axial forces will be present. Each pile must be capable of resisting the compressive stresses due to the axial loading. In this zone, the following criteria must be satisfied:

$P_u/P_{ra} \leq 1.0$

where:

$P_{ra} = \phi_c P_n$
$\phi_c =$ 0.35 for hard driving (Table 9-6)
$\phi_c =$ 0.45 for easy driving (Table 9-6)

Step 4: Establish the Factored Structural Axial Resistance for a Single Pile
For an HP 310 × 110 section, $A_s = 0.0141\ m^2$, and $F_y = 250\ 000$ kPa (Grade 250). Because the piles will be driven a relatively short distance through a medium stiff clay, little driving damage should occur. Therefore, assume $\phi_a = 0.45$.

$$P_{ra} = \phi_c P_n = \phi_c F_y A_s = 0.45\ (250\ 000\ kN/m^2)\ (0.0141\ m^2) = 1586\ kN$$

Step 5: Establish Factored Structural Bending Resistance For Both Axes for a Single Pile
For an HP 310 × 110 section, $S_x = 0.00154\ m^3$, and $S_y = 0.00050\ m^3$.

$$M_{rx} = \phi_{fi} F_y S_x = (0.85)\ (250\ 000\ kN/m^2)\ (0.00154\ m^3) = 327\ kN\text{-}m$$

$$M_{ry} = \phi_{fi} F_y S_y\ (0.85)\ (250\ 000\ kN/m^2)\ (.00050\ m^3) = 106\ kN\text{-}m$$

Step 6: Estimate the Required Number of Piles and Arrange Piles Geometrically
From Step 1, the maximum factored axial load, $P_V = 12\ 644$ kN. Therefore, number of piles, n_{piles}, required to carry P_V is:

$$n_{piles} = P_V/P_{ra} = 12\ 644\ kN/1586\ kN\ \text{per pile} = 8\ \text{piles}$$

As a first approximation, assume that twice as many piles (i.e., 16) will be required to resist all the applied loads. The validity of this assumption will be checked later in the design.

In developing a preliminary pile cap and pile layout geometry, consider the following:

- The pile cap plan dimensions should be at least as large as the pier, which measures 3.5 m × 1.5 m.

- As the bending moments and lateral forces for the subject pier can act in either the positive or negative directions, the pile cap should be made concentric to the pier. (Note: For a structure, such as an abutment, for which the direction of overturning moments and horizontal loads cannot be reversed, piles are usually offset to resist the imposed load eccentricity).

- The pile bending stresses will accrue primarily due to the lateral forces. The maximum lateral longitudinal and transverse forces, 317 and 270 kN, respectively, are similar. Therefore, orient one-half, i.e. 8, of the piles with their strong axes parallel to the longitudinal direction, and the other 8 parallel to the transverse direction.

The pile layout geometry shown in Figure 9-6 satisfies the above-discussed considerations.

Step 7: Determine Factored Axial Loads and Bending Moments for Each Pile
The factored loads, presented in Step 1, the pile properties, presented in Steps 4 and 5, and the pile layout geometry, presented in Step 6, were input into the computer program GROUP. The results of this analysis are presented below. The pile axial loads (in kN) are shown in Figure 9-7.

The average pile axial load due to the vertical load is:

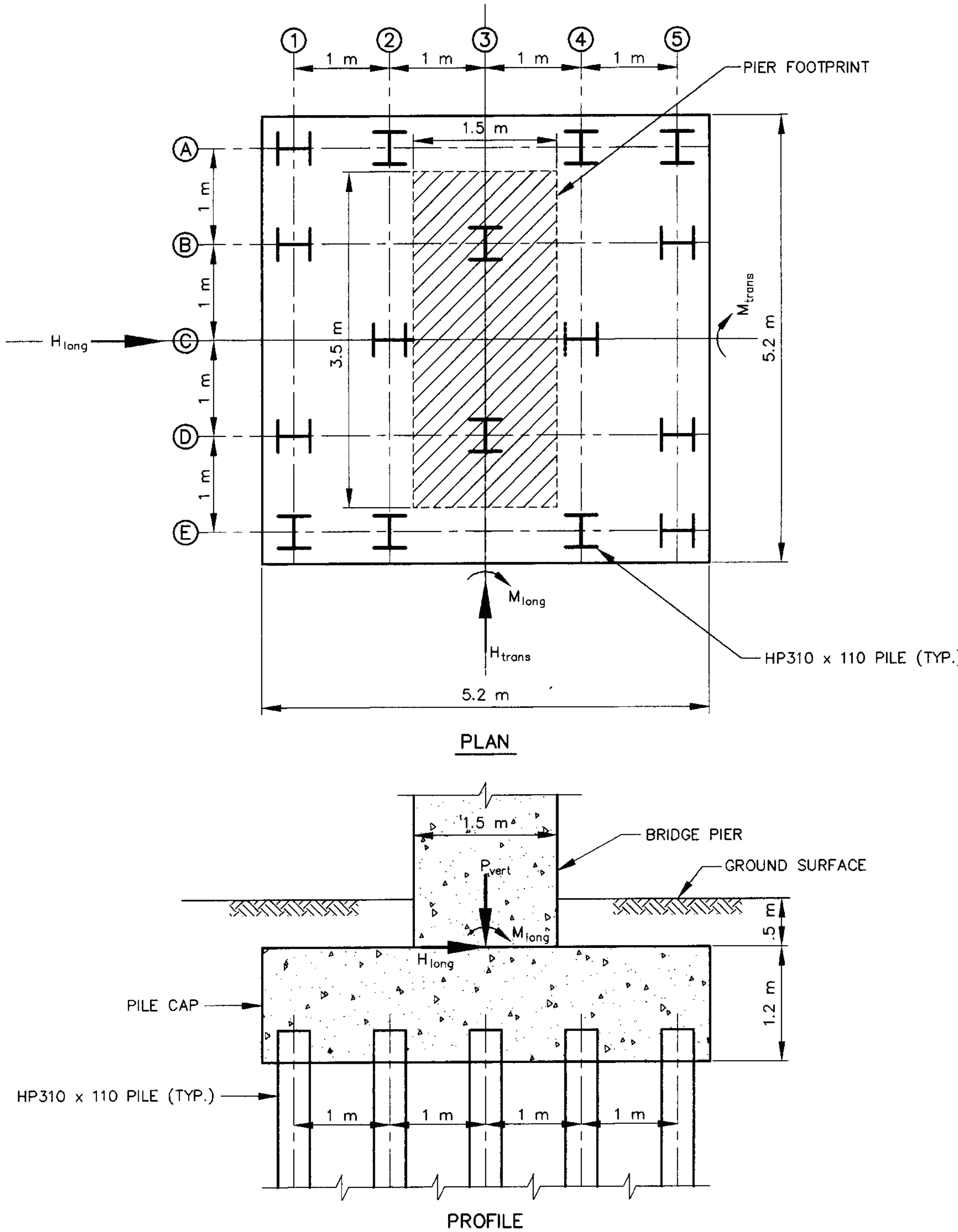
= 12 644 kN/16 piles
= 790 kN/pile

Half of the piles (i.e. 8) shown in Figure 9-7 carry less than the average load, and 8 piles carry more. The axial load imbalance is caused primarily by the biaxial overturning moments.

The maximum pile bending moments about both the strong and weak axes (in kN-m) are shown in Figures 9-8 and 9-9, respectively.

The pile bending moments are primarily caused by the lateral forces. The analysis indicates that piles oriented with their strong axes parallel to the direction of applied lateral load carry a high proportion of the resulting bending moment.

In performing the GROUP analysis, the degree of pile-cap fixity must be specified (i.e., fixed, pinned or some intermediate level). The degree of fixity is related to the bending moment at the pile to cap interface, with the degree of fixity decreasing with increasing moment. The results presented in Figures 9-7, 9-8 and 9-9 were based on an assumed fixed condition. The results of the analysis indicate that the pile bending moments are a relatively small fraction of the bending capacity of the piles. Therefore, the assumed fixed condition is reasonable.



Pile Cap and Pile Layout

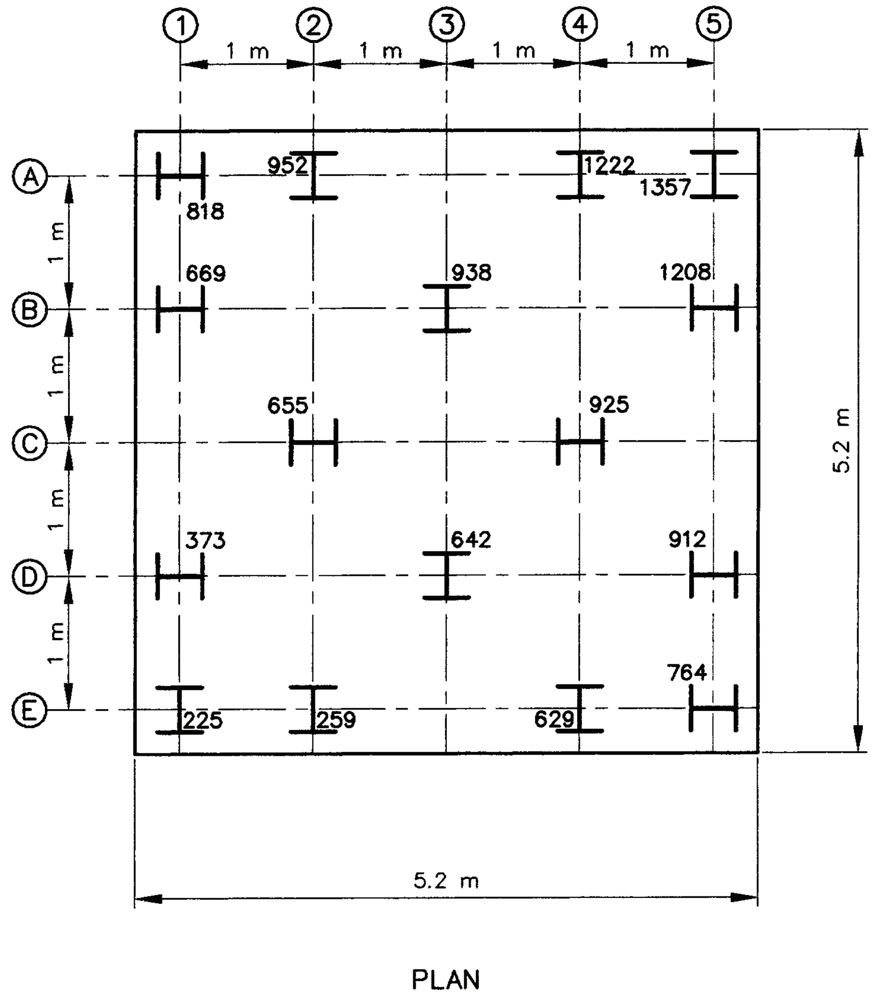


Figure 9-7
Axial Load, P_u, Summary

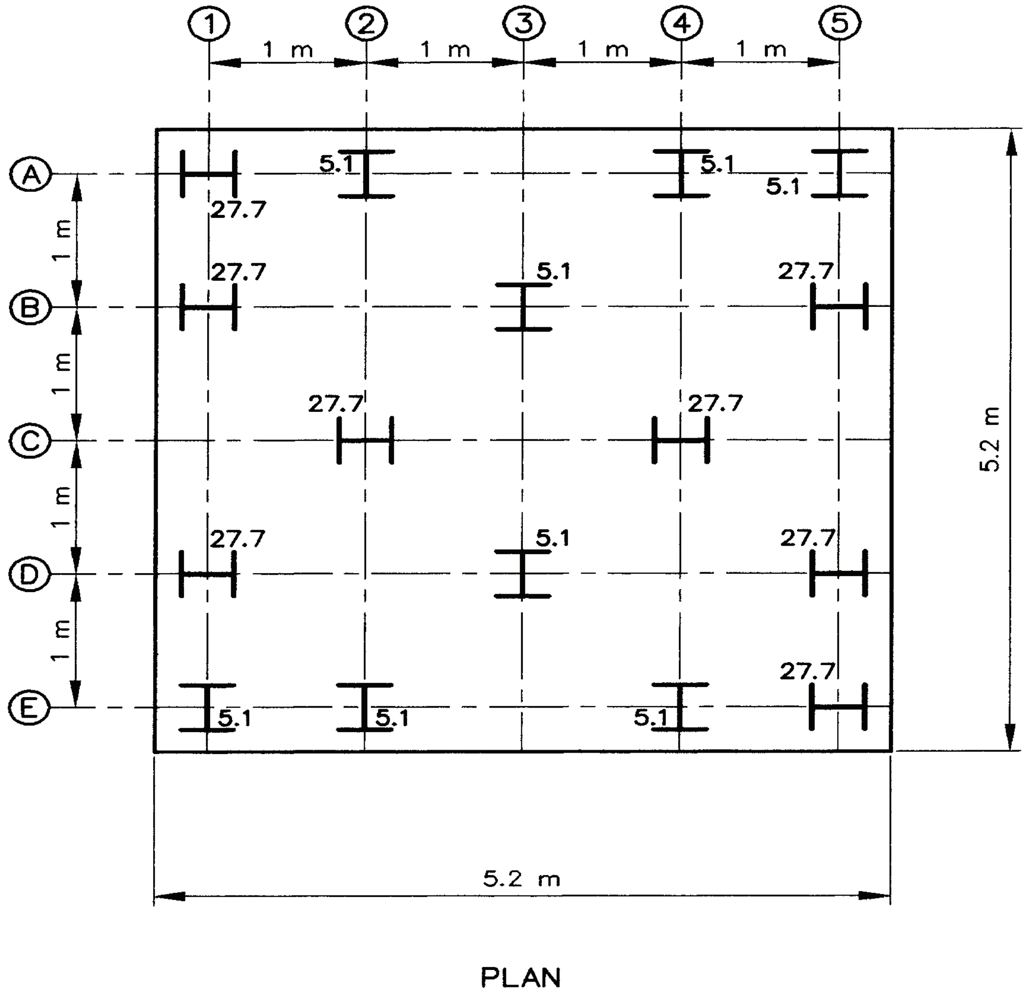


Figure 9-8
Strong Axis Maximum Bending Moment, M_{ux}, Summary

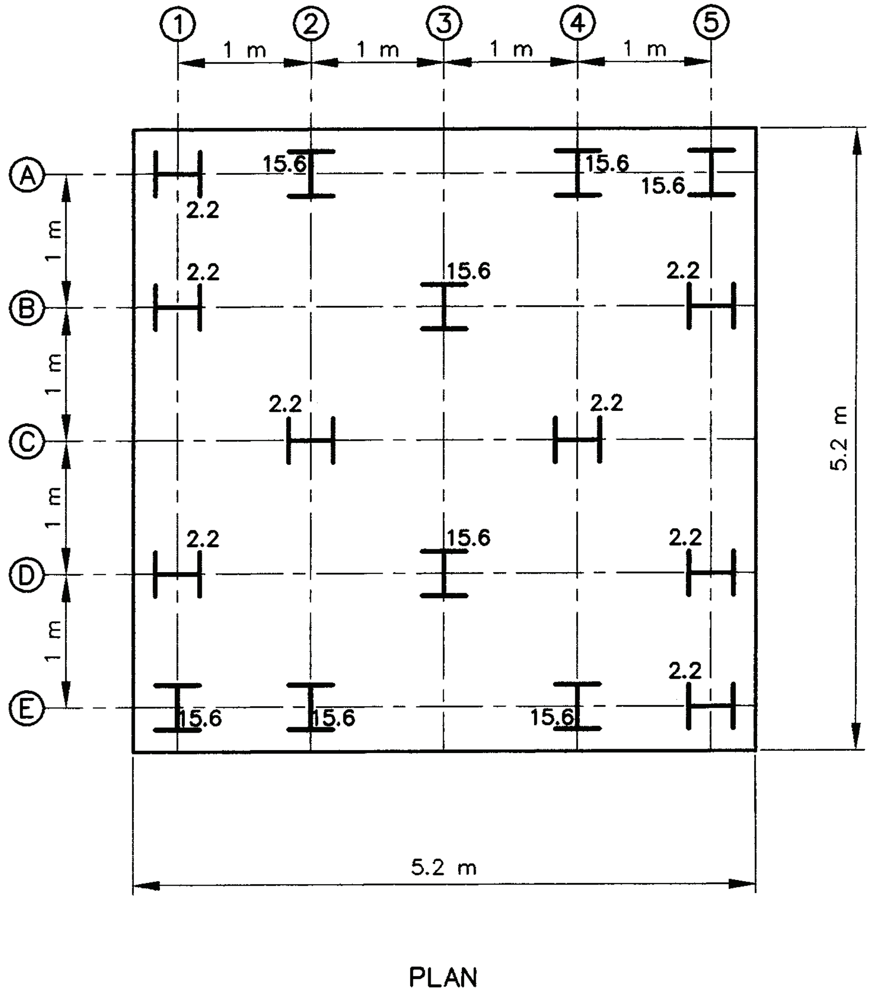


Figure 9-9
Weak Axis Maximum Bending Moment, M_{uy}, Summary

Step 8: Determine Structural Resistance Adequacy
Piles A4, A5 and B5 are the most heavily loaded. Their ability to resist the loads and moments presented in Figures 9-7, 9-8 and 9-9 will be evaluated by both the interactive axial and bending stress equation,

$$\frac{P_u}{P_{ri}} + \frac{8.0}{9.0}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1.0 \qquad (A6.9.2.2\text{-}2)$$

and the axial stress only equation,

$$\frac{P_u}{P_{ra}} \leq 1.0$$

$P_{ri} = \phi_{fi}\ F_y\ A_s = 0.60\ (250\ 000\ kN)(0.0141\ m^2) = 2115\ kN$ (ϕ_{ci} from Step 3)
$M_{rx} = 325$ kN-m (from Step 5)
$M_{ry} = 105$ kN-m (from Step 5)
$P_{ra} = 1574$ kN (from Step 4)

Pile Interaction Equation

$$A4: \frac{1222}{2115} + \frac{8.0}{9.0}\left(\frac{5.1}{327} + \frac{15.6}{106}\right) = 0.578 + \frac{8.0}{9.0}(0.016 + 0.147) = 0.72 < 1.0\ O.K.$$

$$A5: \frac{1357}{2115} + \frac{8.0}{9.0}\left(\frac{5.1}{327} + \frac{15.6}{106}\right) = 0.642 + \frac{8.0}{9.0}(0.016 + 0.147) = 0.79 < 1.0\ O.K$$

$$B5: \frac{1208}{2115} + \frac{8.0}{9.0}\left(\frac{27.7}{327} + \frac{2.2}{106}\right) = 0.571 + \frac{8.0}{9.0}(0.085 + 0.021) = 0.67 < 1.0\ O.K$$

For the axial load only equation, only pile A5 needs to be evaluated since it carries the highest axial load.

$$\frac{1357}{1586} = 0.86 < 1.0\ \ O.K$$

The above-shown analysis indicates that the selected pile group is capable of resisting the Cases A/B, Strength I factored loads.

Step 9: Determine Whether Deformations Are Acceptable
The Service I Case loads are to be used to determine whether deformations are acceptable. However, a separate deformation analysis need not be conducted if the deformations resulting from the critical load application are small, as the Service I case deformations would be smaller.

The GROUP analysis for the critical load case, presented in Step 7, yielded deformation data. The pile cap settlements at each of the pile locations (in mm) is shown in Figure 9-10.

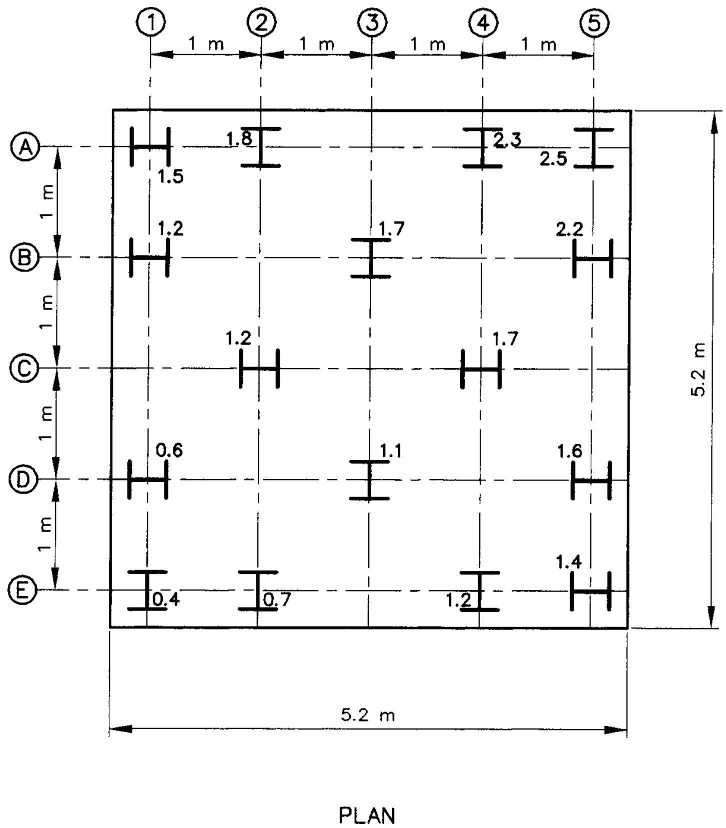


Figure 9-10
Pile Settlements

The lateral displacements in the longitudinal and transverse directions were determined by GROUP to be 1.8 and 0.5 mm, respectively.

These displacements, even though they are based on the critical loading, are small. Therefore, there is no need to evaluate deformations due to the smaller Service I loads.

Step 10: Examine Whether Any Other Load Conditions Are More Critical
In Step 1, Load Cases A/B, Strength 1 were identified as probably being most critical. The succeeding analyses were conducted using these loads. Step 1, however, also pointed out that possibly Load Cases C/D, Strength 1 could be more critical. The latter loading was subsequently analyzed via GROUP and was found to be non-critical.

Step 11: Check for Pile Uplift
The load data need to be evaluated to determine whether there are any load cases which result in uplift on any of the piles. A high tensile pile force could overcome the piles' uplift resistance, and cause load to be transferred to other piles, possibly causing overstressing. Pile uplift would occur if the uplift effect of the sum of the longitudinal and transverse moments could overcome the axial

compressive load.

All of the load cases were checked to determine whether any tensile pile forces could exist. This evaluation showed that, for all the load cases, all the piles were in compression.

Step 12: Examine Whether Design Is Overconservative
In Step 8, it was determined that Pile A5 was the most heavily stressed. The axial load only criterion was the more critical. The analysis showed that Pile A5 was loaded to 86.2 percent of its capacity. Therefore, some excess capacity exists. A reduced pile quantity can be estimated as follows:

Revised pile quantity = 86.2 percent (16 piles) = 14 piles

The cost savings which would accrue from reducing the number of piles from 16 to 14 would be fairly minor. If it were considered worthwhile, Steps 6 through 11 could be repeated. For this problem, a small degree of over conservatism will be accepted and the number of piles will remain at 16.

9.7 Design Example 2:
Design of Pile Support for Bridge Pier Subjected to Downdrag Loading

Problem: Design of a pile group subjected to negative skin friction forces will be examined in the following example. Downdrag forces will be applied to the pile group designed in Design Example 1 in Section 9.7. It will be assumed that the ground water within the pile-penetrated soil will be artificially lowered a sufficient amount to cause soil compression and a resulting imposition of negative skin friction forces on the pile group. The soil has an average shear strength of 37.5 kPa and an average buoyant unit weight of 8.5 kN/m^3.

Objective: To illustrate the impact of soil downdrag force on the design of a pile group.

Approach: To perform the evaluation, the following steps are taken:

- Determine unfactored soil downdrag force
- Determine the factored downdrag force
- Evaluate ability of the pile group designed in Design Example 1 to resist the additional downdrag force
- If pile group redesign is required, explore potential redesign options

Step 1: Determine Unfactored Downdrag Force
Negative skin friction affects that portion of a pile in which the soil moves downward relative to the pile. As the pile group from Design Example 1 rests on rock and the pile tip movement will therefore be negligible, it will be assumed that the entire pile length will be subjected to the downdrag effect.

Negative skin friction is similar to a pile's resistance to settlement, except that it acts on the pile in the opposite direction. It can therefore be estimated using the α or β methods.

α method (***A10.7.3.3.2a***)

$$q_s = \alpha\ S_u$$

where:

S_u = 37.5 kPa
α = adhesion ≈ 1 for clays with S_u = 37.5 kPa (*Fig. A10.7.3.3.2a-1*)

Therefore:

$$q_s = 1.0 \text{ x } 37.5 \text{ kPa} = 37.5 \text{ kPa}$$

β method (***A10.7.3.3.2b***)

$$q_s = \beta\ \sigma_v'$$

$$\sigma_{v\_} = \left(\frac{1.7\ \text{m} + 11.7\ \text{m}}{2} \right) \text{x}\ 8.5\ \text{kN/}\ \text{m}^3 = 57.0\ \text{kN/}\ \text{m}^2 = 57.0\ \text{kPa}$$

where:

β = 0.3 for NC clays (*Fig. A10.7.3.3.2b-1*)

Therefore:

q_s = (0.3)(57.0 kPa) = 17.1 kPa

$$\bar{q}_s = \frac{37.5\ \text{kPa} + 17.1\ \text{kPa}}{2} = 27.3\ \text{kPa}$$

The average for the two methods is:

The pile area over which the negative skin friction acts is:

A = Length $\times$ Perimeter = (10 m)(2{0.310 m + 0.306 m}) = 12.32 m

The downdrag force per pile is:

DD = q_s A = (27.3 kPa)(12.32 m^2) = 336 kN

Step 2: Determine Factored Downdrag Force
The load factor for downdrag from Chapter 4 (Table 4-11) is 1.8. Therefore, the factored downdrag force is:

ϕ DD = 1.8 (336 kN) = 605 kN

Step 3: Determine Resistance to Downdrag Force
The 16-member pile group from Design Example 1 had some excess load carrying capacity remaining. Recall that the "axial pile force only" case governed the pile design and that Pile A5 was the most critically loaded member. In Step 8, it was shown that the factored load was 1357 kN while the factored resistance was 1586 kN. Therefore, an excess resistance capacity of 229 kN was available.

Because the factored downdrag force per pile is 605 kN, the factored loads exceed the factored resistance by 376 kN. Therefore, the designed pile group has insufficient factored resistance to carry the factored loads and must be redesigned.

Step 4: Evaluate Potential Redesign Options
As the pile group designed in 9.7 has insufficient factored resistance to withstand the additional downdrag force, the pile group must be redesigned. The following options will be evaluated:

- Use a heavier H pile section
- Increase the pile spacing
- Use more piles
- Coat the piles to reduce the factored downdrag load

Prior to proceeding with redesign, the sources of the axial load acting on Pile A5 must be quantified.

The total factored axial force is equal to the force determined in Design Example 1, 1357 kN, plus the 605 kN factored downdrag force, or 1962 kN.

This total factored axial load accrues due to the following loads:

- The pier vertical load
- The pier overturning moments
- The pier lateral forces
- The soil downdrag force

Vertical Load Component

$$\frac{\text{Total factored vertical force}}{\text{No. of piles}} = \frac{12\,644 \text{ kN}}{16 \text{ piles}} = 790 \text{ kN/pile}$$

Overturning Moment Component
To evaluate this load component, the "section modulus" of the pile group, Σd^2, where d represents the distance from each pile to the center of gravity of the pile group, must be determined about both axes as follows:

$$\Sigma d^2_x = 8\ (2\text{ m})^2 + 6\ (1\text{ m})^2 = 38\text{ m}^2$$

As the pile group is symmetrical with respect to the calculation of the $\Sigma\ d^2_s$:

$$\Sigma d^2_y = \Sigma d^2_x = 38\text{ m}^2$$

The vertical force caused by an overturning moment is:

$$= \frac{(\text{moment})\,(\text{pile moment arm})}{\Sigma d^2}$$

For the longitudinal moment,

$$\text{vertical force} = \frac{(4772\ \text{kN-m})\,(2\ \text{m})}{38\ \text{m}^2} = 251\ \text{kN}$$

For the transverse moment,

$$\text{vertical force} = \frac{(5632\ \text{kN-m})\,(2\ \text{m})}{38\ \text{m}^2} = 296\ \text{kN}$$

The sum effect of the two overturning moments is a vertical force of 547 kN for pile A5.

Lateral Force Component

The vertical force caused by the lateral forces can be determined by subtracting the vertical force, 790 kN, and the moment force, 547 kN, effects from the total vertical force of 1357 kN as determined by Step 7 in Design Example 1. The resulting calculation shows the lateral forces to cause a relatively minor vertical load of 20 kN on pile A5.

In summary, the total factored vertical load acting on Pile A5 accrues from the following components:

Vertical Load	= 790 kN
Overturning Moments	= 547 kN
Lateral Force	= 20 kN
Soil Downdrag	= 605 kN
TOTAL	=1962 kN

Step 5 - Option A: Use Heavier H-Pile Member

The required member can be determined using the following relationship:

$$\frac{P_u}{P_{ra}} \leq 1.0$$

from Step 3, Section 9.6

$$\frac{1962\ \text{kN}}{\phi\, F_y\, A_s} \leq 1.0$$

where:

ϕ_c = 0.45; and
F_y = 250 000 kPa

Therefore,

$$\frac{P_u}{P_{ra}} = \frac{1962\ \text{kN}}{(112\,500\ \text{kPa})(A_s)} \leq 1.0$$

and

$$A_s \geq 0.0175\ \text{m}^2$$

A HP 380 × 152 section, having a cross-sectional area of 0.0194 m^2, would be adequate. (Note: As this member has a 19 percent larger perimeter than the 310 x 110 pile for which downdrag was calculated, the pile downdrag would be increased by a similar proportion, i.e. 0.19 × 605 kN, or 115 kN. This increase in factored load would require a member with a cross-sectional area of 0.0185 m^2, which the HP 380 × 152 member satisfies.)

Step 6 - Option B: Increase the Pile Spacing

Increasing the pile spacing decreases the vertical force component due to overturning moment, shown to be equal to 547 kN in Step 4, by increasing $\sum d^2$. In Step 3, it was shown that Pile A5 would be overstressed by 376 kN upon the application of the downdrag force. If the vertical load due to the overturning moment were to be reduced by 376 kN down to 171 kN, 16 HP 310 × 110 piles would have adequate factored resistance, although a greater pile spacing, and therefore larger pile cap, would be required.

As shown in Figure 9-6, the 16 piles are arranged on a 1 m × 1 m grid. Increasing the grid size decreases the Pile A5 factored vertical load due to overturning moments as shown in Table 9-15 below:

Table 9-15
Pile Spacing Effect

Grid Size (m × m)	$\sum d^2$ (m^2)	Factored Vertical Load (kN)
1.0 × 1.0	38.0	547
1.5 × 1.5	85.5	365
2.0 × 2.0	168	248
2.5 × 2.5	238	219
3.0 × 3.0	342	182

Table 9-15 shows that increasing pile spacing alone is not a feasible option. Even at the 3 m × 3 m grid size, which would require a very large 13.2 m × 13.2 m pile cap, the factored vertical load due to overturning moments is insufficiently reduced.

Step 6 - Option C: Increase the Number of Piles
Increasing the number of piles will reduce the factored load on Pile A5 for the following reasons:

- The vertical pier load will be spread out over more members

- The vertical load due to the overturning moments will be decreased due to an increase in $\Sigma\, d_x^2$ and $\Sigma\, d_y^2$

As shown in Figure 9-6, the original design consisted of H piles occupying 16 of the available 25 locations on the 5 x 5 grid. Now, add an additional 9 piles, so that a pile occupies each grid intersection.

The factored vertical load imposed on Pile A5 due to the vertical pier load will now be:

$$\frac{12\,644 \text{ kN}}{25 \text{ piles}} = 506 \frac{\text{kN}}{\text{pile}}$$

The factored vertical load due to overturning moments is calculated as follows:

$$\Sigma\, d_x^2 = 10(2\text{ m})^2 + 10(1\text{ m})^2 = 50\text{ m}^2$$

$$\Sigma\, d_y^2 = \Sigma\, d_y^2 = 50\text{ m}^2$$

$$\text{vertical load} = \frac{(4772 + 5632)\text{ kN-m x }(2\text{ m})}{50\text{ m}^2} = 416\text{ kN}$$

The factored soil downdrag force will remain unchanged at 605 kN. Assume that the small vertical load due to the horizontal forces remains unchanged at 20 kN. In summary, the following vertical forces act on Pile A5.

Vertical load effect	= 506 kN
Overturning moment effect	= 416 kN
Lateral force effect	= 20 kN
Soil downdrag effect	= 605 kN
TOTAL	= 1547 kN

Therefore,

$$\frac{P_u}{P_{ra}} \leq 1.0$$

$$\frac{1586\text{ kN}}{1574\text{ kN}} \leq 1.0$$

$$0.98 < 1.0 \quad \text{O.K.}$$

Therefore, 25 HP 310 × 110 piles have sufficient factored resistance.

Step 7 - Option D: Reduce Downdrag Load By Coating Piles
By coating piles with bitumen, the adhesion between the pile and soil can be reduced, leading to a proportionate reduction in the downdrag force. According to Vesic, et al. (1977), the unit downdrag force acting on a bitumen-coated pile can be estimated as follows:

$$q_s = \overline{N}_o \, \overline{p}_o$$

where:

$\overline{N}_0$ = Factor ranging from 0.01 to 0.05
$\overline{p}_o$ = Mean horizontal effective stress

Assuming k_o = 0.5 and assigning a conservative value of 0.05 to $\overline{N}_0$,

$$q_s = 0.05 \text{ x } 0.5\left(\frac{1.7 \text{ m x } 11.7 \text{ m}}{2}\right) \text{x } 8.5 \text{ kN/ m}^3 = 1.4 \text{ kPa}$$

Note that this is a significant reduction from the uncoated pile q_s value of 27.3 kPa calculated in Step 1. Applying this stress to the 12.32 m^2 pile contact area results in an unfactored downdrag load of 17 kN. Applying a load factor of 1.8 results in a factored downdrag load equal to 31 kN.

In Step 3, it was shown that the 16-member pile group designed in Design Example 1, utilizing HP 310 × 110 members, had some excess load resistance capacity prior to considering soil downdrag. Pile A5 was most heavily loaded at 1357 kN compared to its factored resistance of 1586 kN. Adding the factored 31 kN downdrag force acting on the coated pile increases the factored load transmitted to Pile A5 to 1388 kN, still somewhat less than its factored resistance.

Step 8 - Summarize Redesign Options
In summary, four options were analyzed. Three were found to be feasible as follows:

Option A - Use 16 HP 380 × 152 piles
Option C - Use 25 HP 310 × 110 piles
Option D - Use 16 bitumen-coated HP 310 × 110 piles

Selection of the preferred option should be based on cost considerations.

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CHAPTER 10
DRILLED SHAFT DESIGN

10.1 Introduction

For both ASD and LRFD, the design of drilled shaft foundations requires consideration of geotechnical and structural capacity and deformation limits. As with pile design, the design processes require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between drilled shaft design by LRFD and ASD
- Identifies the strength and serviceability performance limits which must be considered for shaft design by LRFD
- Briefly summarizes methods commonly used for estimating the geotechnical and structural capacity and load-deflection behavior of a shaft and shaft group
- Presents an example of a drilled shaft group design by ASD and LRFD methods

10.2 Design Methods

With few exceptions, the procedure for design of drilled shaft foundations using LRFD (A10.8) is identical to the procedure followed using ASD. Generally, the ultimate bearing capacity and settlement are checked for axially-loaded shafts. As with driven piles, design of laterally-loaded shafts may be governed either by lateral capacity or deflection criteria. Again, however, mobilization of the ultimate lateral capacity of the soil requires such large displacements that lateral capacity does not represent a realistic basis for laterally-loaded shaft design. The following sections summarize the general design processes for drilled shaft foundation design using the ASD and LRFD approaches.

10.2.1 ASD Summary

Existing practice for design of drilled shaft foundations follows the ASD approach. For ASD (AASHTO, 1997b), all uncertainty in the variation of applied loads transferred to the shaft(s) and the ultimate geotechnical capacity of the soil and rock to support the load are incorporated in a factor of safety, FS. As a result, loads used for design, Q, consist of those actual forces estimated to be applied directly to the structure. In LRFD terminology, this is equivalent to applying a load factor of 1.0 to the estimated forces. Relative to axial loading of a shaft, the ultimate axial geotechnical load capacity, Q_{ult}, is estimated using any number of available methods. Then the suitability of the design is evaluated by determining the allowable axial design load, Q_{all}, using:

$$Q = Q_{all} = \frac{R_n}{FS} = \frac{Q_{ult}}{FS} \qquad \text{(Eq. 10-1)}$$

where:

Q = Design load (kN)
Q_{all} = Allowable design load (kN)
R_n = Q_{ult} = Ultimate geotechnical capacity of a shaft (kN)
FS = Factor of safety (dim)

The required FS is generally specified by the governing agency, and may be constant or variable. The FS used for AASHTO ASD for the axial geotechnical capacity of a single shaft is presented in Table 10-1. If Q_{ult} is developed using information from a site-specific subsurface exploration based on a semi-empirical calculation, FS. 2.50. However, if capacities are estimated using a static field load test, FS = 2.0 could be used to reflect the greater confidence level achieved by the improved method for estimating and confirming the axial capacity of a shaft. As the table shows, ASD for shaft design assumes that a reasonable level of quality control will be exercised during the design and construction phases. If normal levels of field construction quality control cannot be assured however, AASHTO ASD requires the use of higher (but unspecified) safety factors.

Table 10-1
Factor of Safety on Ultimate Axial Geotechnical Capacity
(AASHTO, 1997b)

Basis for Design	Required Minimum Factor of Safety (FS)(1)
Semi-Empirical Calculation	2.5
Load Test	2.0

(1) Assuming normal level of field quality control during shaft construction

FHWA (Reese and O'Neill, 1988 and Cheney and Chassie, 1993) suggests a typical range for safety factors of 2.0 to 3.0 (where good-quality geotechnical data are available), and provide guidance for a variable FS considering the level of construction control and structure design life, as shown in Table 10-2.

Table 10-2
Factors of Safety on Ultimate Geotechnical Capacity
Based on Design Life and Level of Construction Control
(Reese and O'Neill, 1988)

Design Life (Type of Structure)	Required Minimum Factor of Safety (FS)[1]		
	Poor Quality Control	Normal Quality Control	Good Quality Control
200 to 500 years (large bridges & monumental structures)	3.5	2.3	1.7
75 to 100 years (typical rail & road bridges & large buildings)	2.8	1.9	1.5
25 to 50 years (industrial buildings)	2.3	1.7	1.4

[1] Assumes good-quality geotechnical information and reliable model.

The structural design of drilled shafts is described in detail in Reese and Allen (1977) and is covered briefly by Reese and O'Neill (1988). Although geotechnical design of drilled shafts may follow ASD procedures, structural design is currently performed in accordance with the American Concrete Institute (ACI) code which uses the LRFD method.

In addition to axial geotechnical and structural capacity evaluation, the design of drilled shafts by ASD requires evaluations of shaft deflections and comparisons with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 10-2)}$$

where:

δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement established by designer (mm)

Tolerable movement criteria are usually a function of the type of structure, and for bridges depend primarily on the span length and whether the superstructure has fixed- or simply-supported spans. The design of laterally-loaded shafts by ASD is often governed by criteria for deflection under actual applied loads, although design of large diameter shafts and shaft groups containing batter shafts may be controlled by the axial and/or bending capacity of the shaft(s).

10.2.2 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical capacity in factors of safety, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. ***When properly developed and applied, the LRFD approach provides a generally consistent level of safety for the design of all structure components.*** Thus, the probability that any one structure component will fail or perform unacceptably is no different than any other component. As described in Section 4.3, the resistance and deformation of supporting soil and rock materials and structure components must satisfy the LRFD equations below. For the Strength Limit States:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 10-3) } (\mathit{A1.3.2.1\text{-}1})$$

For the Service Limit States:

$$\sum \eta_i \gamma_i \delta_i \leq \phi \delta_n \qquad \text{(Eq. 10-4)}$$

where:

η_i = Factors to account for effects of ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)
δ_i = Estimated displacement (m)
δ_n = Tolerable displacement (m)

Relative to axial loading of a shaft, the suitability of a shaft with respect to the geotechnical and structural resistance can be obtained using Eq. 10-3, rewritten as:

$$\sum \eta_i \gamma_i Q_i \leq \phi Q_{ult} = Q_R \qquad \text{(Eq. 10-5)}$$

and

$$\sum \eta_i \gamma_i Q_i \leq \phi P_n = P_r \qquad \text{(Eq. 10-6)}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Factored load effect (kN)
ϕ = Resistance factor (dim)
Q_{ult} = Ultimate geotechnical resistance of a shaft (kN)
Q_R = Factored geotechnical resistance of a shaft (kN)
P_n = P_{ult} = Nominal (ultimate) structural resistance of a shaft (kN)
P_r = Factored structural resistance of a shaft (kN)

The load factors and load factor combinations used for design were presented in Chapter 4. In general, values of $\gamma_i > 1.0$ are used to evaluate ultimate ground or structure capacity at the Strength Limit States, whereas the deformation performance of structures is evaluated at the Service I Limit State using $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads). In ASD (AASHTO, 1997b), values of $\gamma_i = 1.0$ (or

γ_i = 0.3 for wind loads) are used to evaluate structures for both strength (allowable stress) and serviceability (deflection). ***Therefore, analysis of shaft deformations (e.g., settlement or lateral displacement) by LRFD and ASD are identical.***

When using Eq. 10-3 for drilled shaft foundation design at the Strength Limit States, the following values of η can normally be used:

- $\eta_D = \eta_R = 1.00$
- $\eta_I =$ 1.05 for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures.

Determination of the operational importance of a structure (such as a bridge) is made by the facility Owner as described in Chapter 4. The appropriate value of η_I is then applied throughout the superstructure design by the structural engineer. The value of η_I selected by the superstructure designer should then be applied in the foundation design. For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

When using Eq. 10-4 to evaluate a drilled shaft foundation at any Service Limit State, η_D, η_R, and η_I = 1.0.

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4 and in Section 10.2.2.1, and loads should be developed as described in Chapter 4. The ultimate resistance, R_n, should be determined for each type of resistance (e.g., axial, side, or group resistance) as described in Section 10.3. Values of $\phi \leq 1.0$ are applied when evaluating drilled shaft foundation resistance for any strength limit state using Eq. 10-3. Currently, the value of $\phi = 1.0$ is applied when evaluating a drilled shaft foundation for any service limit state using Eq. 10-4. Selection and modification of resistance factors, ϕ, are described in Sections 10.2.2.2 through 10.2.2.4.

10.2.2.1 Limit States (*A10.5*)

In general, the design of drilled shaft foundations using LRFD parallels that of pile foundations, and requires evaluation of shaft suitability at various Strength Limit States and the Service I Limit State. The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for shafts designed at the Strength and Service I Limit States are summarized in Table 10-3. As conditions warrant, it may also be necessary to evaluate shaft performance at other limit states (e.g., Extreme Event I for loading from earthquakes or Extreme Event II for vessel impact and ice loading). Methods of evaluating shafts at the various limit states are described in Section 10.3.

Table 10-3
Limit States for Design of Drilled Shaft Foundations

Performance Limit	Strength Limit State(s)	Service I Limit State
Bearing Resistance of Single Shaft/Group	✔	
Shaft/Group Punching	✔	
Settlement of Shaft Group		✔
Tensile Resistance of Uplift-Loaded Shafts	✔	
Lateral Displacement of Shaft/Group		✔
Structural Capacity of Axially/ Laterally-Loaded Shafts	✔	

10.2.2.2 Resistance Factors (*A10.5.5*)
Resistance factors for geotechnical design of shafts socketed into rock using the Horvath and Kenney method (1979) and Carter and Kulhawy method (1987) and for shafts in clay using the Reese and O'Neill method (1988) are based on static and in-situ test methods of analysis and were developed using the reliability-based calibration procedure described in Chapter 3. Resistance factors for geotechnical design of shafts based on other semi-empirical methods and field testing of shafts are based on direct calibration with ASD. The reliability-based calibration procedure involved:

- Estimating the level of reliability inherent in various methods for predicting shaft capacity
- Observing the variation in reliability levels with different span lengths, dead- to live-load ratios, foundation geometry and methods used to estimate ultimate resistance and load combinations
- Selecting a target reliability, β_T, based on the margin of safety used for ASD
- Calculating resistance factors, ϕ, consistent with the target reliability index coupled with experience and judgment

This process was used to develop resistance factors for the design of drilled shaft foundations. As an example of this process, Table 10-4 summarizes the results of analyses performed to develop ϕ values for estimating the axial resistance of shafts. For these analyses, the variables included:

- Shaft length (i.e., 3-, 10- and 30-m lengths)
- A dead to live load ratio of 3.69 which corresponds to a bridge with a span length of 75 m

- Target reliability, β_T (i.e., 2.5 and 3.0)

- The Horvath and Kenney (1979), Carter and Kulhawy (1987) and Reese and O'Neill (1988) methods for estimating ultimate shaft resistance, Q_s, as described in Section 10.3.2.1

Table 10-4 shows that:

- Values of ϕ are not highly sensitive to shaft length

- Selected values of ϕ for the Reese & O'Neill and Horvath & Kenney methods are less than the average value of ϕ, whereas the selected value of ϕ for the Carter & Kulhawy method is greater than the average ϕ value

Table 10-4
Resistance Factors for Drilled Shafts for Estimating the Ultimate Axial Shaft Capacity Using Reliability-Based Calibration
(modified Barker, et al., 1991b)

Shaft Length (m)	β_T	ϕ Value by Method of Axial Shaft Capacity Estimation		
		Reese & O'Neill (1988) Shafts in Clay	Horvath & Kenney (1979) Shafts in Rock	Carter & Kulhawy (1987) Shafts in Rock
3	2.5	---	0.70	0.49
10	2.5	0.72	0.73	0.56
30	2.5	0.80	---	---
3	3.0	---	0.56	0.37
10	3.0	0.72	0.59	0.43
30	3.0	0.71	---	---
	Average ϕ	0.74	0.65	0.46
	Selected ϕ	0.65	0.65	0.55

The relative ϕ factor values for shafts in rock were selected for use in the LRFD Specification based on the limited number of reported cases where the methods had been used in conjunction with performance testing, and the engineering judgment of the code developers. Values of ϕ for the geotechnical design of drilled shaft foundations using these methods for axial capacity and for other design considerations are presented in Table 10-5.

As indicated in Table 10-5, resistance factors have not been developed for drilled shafts in cohesionless soils due to a lack of field load test data. Shaft side resistance in cohesionless soils is highly dependent upon installation practice and quality control. Because large displacements are generally required to mobilize shaft base resistance in cohesionless soils, the base resistance is generally limited on the basis of service (settlement) criteria. The LRFD Specification (AASHTO, 1997a) recommends estimation of shaft and base resistance in cohesionless soils using all applicable methods, and selection of a factored capacity based on judgment and experience with similar conditions (*A10.8.3.4.3*).

The value of ϕ for structural design of axially-loaded, reinforced concrete drilled shafts (*A5.5.4.2.1*) is shown in Table 10-6. The calculation of the ultimate structural capacities of drilled shafts is described in Section 10.3.2.2.

Table 10-5 (*A10.5.5-3*)
Resistance Factors for Geotechnical Strength Limit States
for Axially Loaded Drilled Shafts
(AASHTO, 1997a)

<table>
<tr><th colspan="3">Method/Soil/Condition</th><th>Resistance [1] Factor</th></tr>
<tr><td rowspan="7">Ultimate Bearing Resistance of Single Drilled Shaft</td><td>Side Resistance - Clay</td><td>α-method (Reese & O'Neill, 1988)</td><td>0.65</td></tr>
<tr><td>Base Resistance - Clay</td><td>Total Stress (Reese & O'Neill, 1988)</td><td>0.55</td></tr>
<tr><td>Side Resistance - Sand</td><td>Various</td><td>---[2]</td></tr>
<tr><td>Base Resistance - Sand</td><td>Various</td><td>---[2]</td></tr>
<tr><td>Side Resistance - Rock</td><td>• Carter & Kulhawy (1988)
• Horvath & Kenney (1979)</td><td>0.55
0.65</td></tr>
<tr><td>Base Resistance - Rock</td><td>• CGS (1992)
• Pressuremeter Method (CGS, 1992)</td><td>0.50
0.50</td></tr>
<tr><td>Side & Base Resistance</td><td>Load Test</td><td>0.80</td></tr>
<tr><td>Block Failure</td><td>Clay</td><td></td><td>0.65</td></tr>
<tr><td rowspan="4">Uplift Resistance of Single Drilled Shaft</td><td>Clay</td><td>• α-method
• Belled Shafts (Reese & O'Neill, 1988)</td><td>0.55
0.50</td></tr>
<tr><td>Sand</td><td>Various</td><td>---[2]</td></tr>
<tr><td rowspan="2">Rock</td><td>• Carter & Kulhawy (1988)
• Horvath & Kenney (1979)</td><td>0.45
0.55</td></tr>
<tr><td>Load Test</td><td>0.80</td></tr>
<tr><td>Group Uplift Resistance</td><td>Sand
Clay</td><td></td><td>0.55
0.55</td></tr>
</table>

[1] Refer to Section 10.3 for description of design procedures for which ϕ factors have been calibrated.
[2] ϕ factors not been developed for drilled shafts in cohesionless soils due to a lack of adequate field data.

Table 10-6
Resistance Factor for Structural Design of Axially-Loaded Drilled Shafts
(AASHTO, 1997a)

Shaft Type	Resistance Factor
Reinforced Concrete with Spiral Reinforcement	0.75
Reinforced Concrete with Tie Reinforcement	0.75

10.2.2.3 Comparison of Drilled Shaft Design Using LRFD and ASD

To illustrate the relative differences between LRFD and ASD, the equivalent LRFD factor of safety (FS_{LRFD}) has been determined for each of the methods presented in Table 10-5 for estimating the axial geotechnical capacity of a shaft or group. As presented in Table 10-7, FS_{LRFD} was determined as:

$$FS_{LRFD} = \bar{\gamma}/\phi \qquad \text{(Eq. 10-7)}$$

where:

$\bar{\gamma}$ = Average load factor (assumed = 1.45)

ϕ = Resistance factor from Table 10-5

The ASD factor of safety (FS_{ASD}) from Table 10-1 for each category is also presented in Table 10-7.

The values of FS_{LRFD} in Table 10-7 were determined assuming $\bar{\gamma}$ = 1.45. Actually, γ_i can range from 1.25 for structure component loads to about 1.75 for live loads so that FS_{LRFD} could vary from the value shown in the table depending on the relative proportion of live to dead load for a particular structure. *Assuming that $\bar{\gamma}$ = 1.45 represents a reasonable approximation, $FS_{LRFD} \approx FS_{ASD}$ depending on the method of capacity estimation. Thus, while general agreement exists between LRFD and ASD, the comparison depends on the load factor and method of capacity analysis. Therefore, while drilled shaft designs performed using LRFD will be comparable to those using ASD, a precise or very close approximation between the two should not necessarily be expected.*

10.2.2.4 Modification of Resistance Factors

As stated in Section 10.2.2.2, each of the LRFD resistance factors for geotechnical design of shafts in Table 10-5 was developed using either a reliability-based calibration procedure or direct calibration with ASD, tempered with engineering judgment and experience. Application of these resistance factors in conjunction with $\bar{\gamma}$ = 1.45 results in an "equivalent" factor of safety ranging from 1.8 to 3.2, depending on the soil conditions, loading conditions and method of geotechnical capacity prediction, as described in Section 10.2.2.3.

Table 10-7
Comparison of ASD Factor of Safety
with LRFD Equivalent Factor of Safety
Geotechnical Strength Limit State for Axially-Loaded Shafts

Method/Soil/Condition			Resistance Factor, ϕ	LRFD Equivalent Factor of Safety $\bar{\gamma}/\phi$, for $\bar{\gamma}$ = 1.45[1]	ASD Factor of Safety[2]
Ultimate Bearing Resistance of Single Drilled Shaft	Side Resist. - Clay	α-method	0.65	2.2	2.5
	Base Resist. - Clay	Total Stress	0.55	2.6	2.5
	Side Resist. - Sand	Various	---[3]	---[3]	---[3]
	Base Resist. - Sand	Various	---[3]	---[3]	---[3]
	Side Resist. - Rock	• Carter & Kulhawy • Horvath & Kenney	0.55 0.65	2.6 2.2	2.5 2.5
	Base Resist. - Rock	• CGS • Pressuremeter	0.50 0.50	2.9 2.9	2.5 2.5
	Side/Base Resist.	Load Test	0.80	1.8	2.0
Block Failure	Clay		0.65	2.2	2.5
Uplift Resistance of Single Drilled Shafts	Clay	• α-method • Belled Shafts	0.55 0.50	2.6 2.9	2.5 2.5
	Sand	Various	---[3]	---[3]	---[3]
	Rock	• Carter & Kulhawy • Horvath & Kenney	0.45 0.55	3.2 2.6	2.5 2.5
		Load Test	0.80	1.8	2.0
Group Uplift Resistance	Sand or Clay		0.55	2.6	NA[4]

[1] An average load factor, $\bar{\gamma}$ = 1.45, is assumed for a typical FS_{LRFD}. [2] From AASHTO ASD, a range of factors of safety from 1.4 to 3.5 is suggested by FHWA, depending on structure type, design life and the degree of "construction control" exercised. A FS of 2.5 is consistent with FHWA guidelines for a typical road bridge having a design life of 50 to 100 years and constructed using "poor" quality control, and is chosen for comparison purposes only herein. [3] See Table 10-5. [4] The procedures used in LRFD are not equivalent to those in ASD. Therefore, the equivalent factors of safety for these methods cannot be directly compared.

In ASD, the designer or owner might decide to increase or decrease required factors of safety in consideration of a number of factors, such as:

- The potential consequences of a failure
- The extent or quality of information available from geotechnical exploration and testing
- Past experience with the soil conditions encountered and/or capacity

prediction method used

- The level of construction control anticipated or specified
- The likelihood that the design loading conditions will be realized (often dictated by the structure design life).

When using LRFD, similar flexibility to vary the required level of safety should also be available. Additionally, whereas the same factor of safety is generally used in ASD regardless of the source of loading, the equivalent factor of safety in LRFD (defined by Eq. 10-7) varies for a given resistance factor depending on the source of loading. As stated previously, an average load factor of 1.45 was used to identify the equivalent safety factors in Table 10-7. Although an average load factor of 1.45 is reasonable for a typical bridge abutment or pier , or a retaining wall, the average load factor could theoretically range from a low of 1.25 (i.e., all dead load) to nearly 1.75 (i.e., extremely high live load).

To modify the resistance factors for geotechnical design of shafts to account for average load factors other than 1.45 and equivalent factors of safety other than those identified in Table 10-7, the following equation may be used:

$$\phi_m = \phi_T \times \left(\frac{FS_T}{FS_D} \right) \times \left(\frac{\bar{\gamma}_T}{\bar{\gamma}_D} \right) \qquad \text{(Eq. 10-8)}$$

where:

ϕ_m = Modified resistance factor (dim)
ϕ_T = Tabulated resistance factor from Table 10-5 (dim)
FS_T = Tabulated factor of safety from Table 10-7 (dim)
FS_D = Desired factor of safety (dim)
$\bar{\gamma}_T$ = Average load factor = 1.45 (dim)
$\bar{\gamma}_D$ = Actual average load factor including modification for operational importance (dim).

Modifying resistance factors may seem reasonable, but such modification may not be consistent with the goal of LRFD to achieve near equal reliability against failure of structure components, unless the factor of safety accurately models the reliability of the shaft capacity predictive method used. The resistance factors may be more appropriately modified through application of the probabilistic procedures described in Chapters 3 and 5 to achieve the desired level of reliability if sufficient data are available.

10.2.3 Summarized Comparison of LRFD and ASD

As noted before, the process used to develop a drilled shaft foundation design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 10-1. Specific differences between the methods and other important issues are highlighted in the following section.

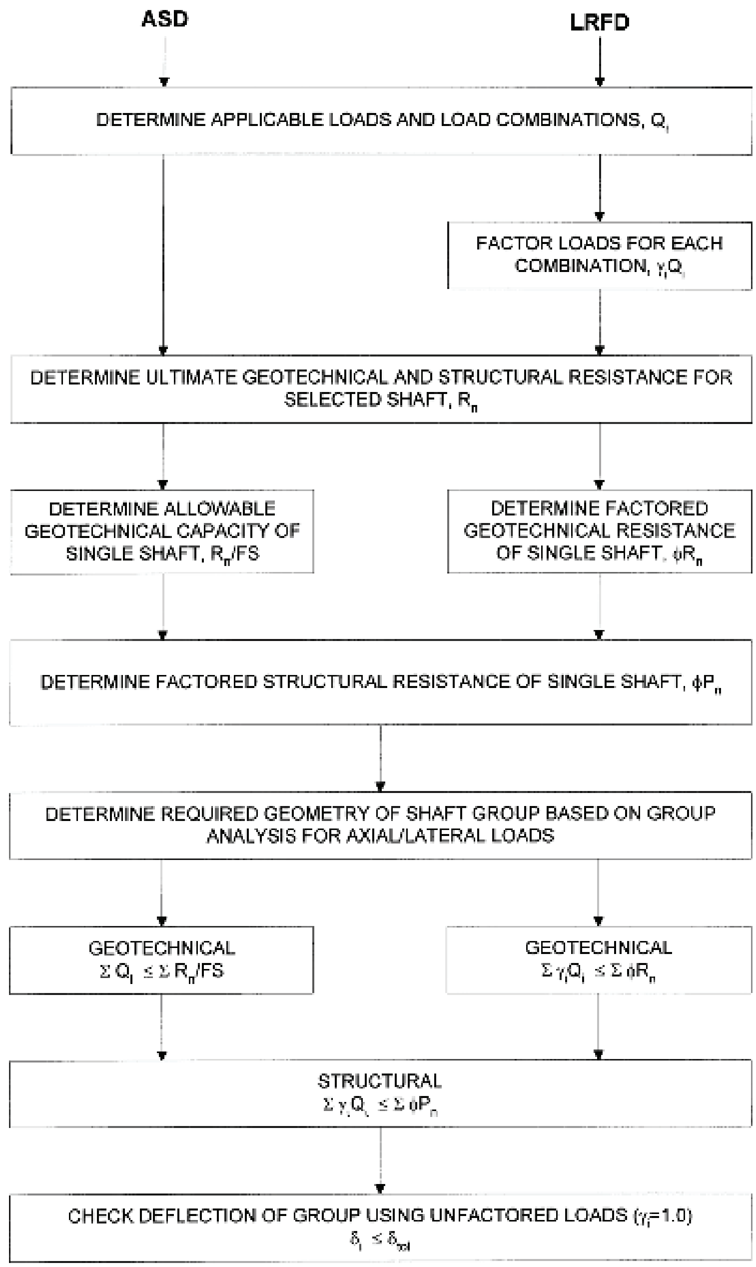


Figure 10-1
Generalized Flow Chart for Drilled Shaft Design
by LRFD and ASD

Other aspects of the drilled shaft design such as identifying special considerations (e.g., potential for loss of lateral support or negative loading), developing the design foundation profile and determining requirements for construction control (e.g., the load testing) are inherent aspects of the design process required for both LRFD and ASD.

10.3 Performance Limits

Design of a drilled shaft foundation by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the shaft section and details to meet these criteria for axial, lateral and/or moment loading, the design of drilled shaft foundations requires consideration of many factors which can affect shaft performance, including:

- Axial and lateral resistance of shaft and shaft group
- Axial and lateral deformation of shaft and shaft group
- Scour and loss of axial/lateral resistance
- Negative or downdrag loading
- Effect of variable ground water levels and buoyancy
- Effect of placement procedures and construction quality control
- Uplift loading on shaft and shaft group
- Shaft spacing and the effects of group action including effects of shaft battering
- Punching failure or excessive settlement of shafts overlying weak or compressible soil

For these design factors, there is no difference between LRFD and ASD analysis procedures. The following sections highlight differences between LRFD and ASD in the performance criteria and application of design procedures.

10.3.1 Displacements and Tolerable Movement Criteria (*A10.8.2*)

The vertical and lateral displacement of drilled shafts must be evaluated for all applicable dead and live load combinations, and compared with tolerable movement criteria. Because evaluations of structure displacements by LRFD are made at the Service I Limit State where $\gamma = 1.0$ and $\phi = 1.0$, methods used to estimate settlement and lateral displacement by LRFD are identical to those used for ASD. Consequently, axial compression (or tension) of the shaft section can be computed by elastic methods and axial and lateral ground displacements can be determined by conventional methods using empirical correlations with in-situ test results or measurements by in-situ or laboratory test methods to estimate engineering soil properties. Lateral displacements need to be evaluated if:

- Shafts are subjected to inclined or lateral loads
- Shafts are placed on or near an embankment slope
- Loss of lateral foundation support by scour is possible

For drilled shaft groups in cohesionless soils or rock, settlements will generally occur immediately as the group is loaded. Drilled shaft groups in cohesive soils, however may experience long-term consolidation settlement as well as immediate elastic settlement. Elastic settlement generally predominates in overconsolidated clays, whereas consolidation settlement generally predominates in normally consolidated clays. The consolidation settlement of shaft groups in cohesive soil is typically estimated using the equivalent footing procedure described in Chapter 9 for pile groups.

During the past decade, computer programs have become available for axial and lateral load-deflection analysis of deep foundations based on the axial load-axial displacement (i.e., t-z) and lateral load-lateral displacement (i.e., p-y) curve methods of analysis. The tolerable axial and lateral movement of drilled shafts should be based on criteria developed by the structural engineer for the superstructure, or by consideration of the effects of foundation movements on adjacent structures. In some cases, the tolerable lateral movement is fixed at a small displacement (e.g., $\leq$ 12 mm based on observed acceptable performance and engineering judgment) to ensure acceptable structure performance.

10.3.2 Axial Resistance

10.3.2.1 Geotechnical Resistance (*A10.8.3*)

Drilled shaft foundations must be designed to resist axial loads without structural failure of the shaft, and without excessive deflection. Methods used for ASD to estimate the ultimate axial geotechnical resistance of a single shaft can be used for LRFD. Therefore, the ultimate axial geotechnical resistance of shafts subjected to axial loading, R_n, can be determined as:

$$R_n = Q_{ult} = Q_p + Q_s \qquad \text{(Eq. 10-9)}$$

and the factored axial geotechnical resistance, Q_R, can be determined as:

$$R_r = Q_R = \phi Q_{ult} = \phi_{qp} Q_p + \phi_{qs} Q_s \qquad \text{(Eq. 10-10)}$$

for which:

$$Q_p = q_p A_p \qquad \text{(Eq. 10-11)}$$

and

$$Q_s = q_s A_s \qquad \text{(Eq. 10-12)}$$

where:

ϕ_{qp}, ϕ_{qs} = Resistance factors from Table 10-5 (dim)
Q_p, Q_s = Ultimate shaft tip and side resistance (kN)
q_p, q_s = Unit tip and side resistance (kPa)
A_p, A_s = Area of shaft tip and side surface (m^2)

Various procedures are available for estimating the ultimate axial geotechnical capacity of drilled shafts in soil using semi-empirical methods and in-situ testing, and for shafts bearing on or in rock using semi-empirical methods. ***When using the LRFD Specification, only those methods referred to in Table 10-5 for which calibrated ϕ-factors have been developed can be used without developing other method-specific resistance factors.*** Calibrated design methods for shafts in cohesive soil (clay) and rock, and uncalibrated design methods for shafts in cohesionless soil referenced in the LRFD Specification for a single shaft include:

- Shafts in Cohesive Soil (*A10.8.3.3*)

  – Side Resistance - The semi-empirical α-method for estimating the ultimate unit side resistance as a function of the undrained shear strength, S_u, determined by UU triaxial testing (Reese and O'Neill, 1988) (*A10.8.3.3.1*).

  – Tip Resistance - The total stress method of analysis developed by Skempton for estimating the ultimate unit tip resistance as a function of undrained shear strength, S_u, and shaft geometry (Reese and O'Neill, 1988) (*A10.8.3.3.2*).

- Shafts in Cohesionless Soil (*A10.8.3.4*)

  – Side Resistance - Two effective stress methods for estimating ultimate unit side resistance as a function of vertical effective stress along the shaft (Touma and Reese, 1974 and Reese and O'Neill, 1988), and three methods for estimating ultimate unit side resistance based on direct correlation with Standard Penetration Resistance, N of soil along the shaft (Meyerhof, 1976; Quiros and Reese, 1977; and Reese and Wright, 1977) (*A10.8.3.4.2*).

  – Tip Resistance - Four methods for estimating ultimate unit tip resistance based on direct correlation with the relative density or Standard Penetration Resistance, N, of soil below the shaft tip (Touma and Reese, 1974; Meyerhof, 1976; Reese and Wright, 1977; and Reese and O'Neill, 1988) (*A10.8.3.4.3*).

- Shafts in Rock (*A10.8.3.5*)

  – Side Resistance - Two semi-empirical methods for estimating ultimate unit side resistance based on the uniaxial compressive strength, q_u, of the rock into which the shaft is socketed (Horvath and Kenney, 1979 and Carter and Kulhawy, 1987) (*AC10.8.3.5*).

  – Tip Resistance - Semi-empirical methods for estimating ultimate unit tip resistance based on the uniaxial compressive strength of rock below the shaft tip and on direct correlation with the results of in-situ pressuremeter tests performed in rock below the shaft tip (Canadian Geotechnical Society, 1992) (*AC10.8.3.5*).

- Method Based on Field Testing of Shaft (*A10.8.3.6*)

– Static Load Test - A method for estimating total load capacity based on tests (ASTM, 1997) representative of shaft, load and subsurface conditions expected for the prototype shafts (AASHTO, 1997a).

With exception of the methods for estimating the side resistance of shafts in cohesionless soils based on Standard Penetration Resistance and for the Touma and Reese, Meyerhof and Reese and Wright methods for estimating the tip resistance of shafts in cohesionless soil, the methods described above are used by FHWA (Reese and O'Neill, 1988) for evaluating the axial geotechnical capacity of drilled shafts. Other methods can be used for LRFD provided calibrated ϕ-factors are developed using the methods described in Chapters 3 and 7.

10.3.2.2 Structural Resistance (*A10.8.4*)

In addition to meeting the requirements for adequate geotechnical resistance of shafts subjected to axial and lateral loading, the axial and flexural resistance of the shaft section must also be adequate for all potential loading cases. The ultimate structural capacity of a reinforced concrete shaft is based on the yield strength, F_y, of the reinforcing steel and the unconfined compressive strength, f_c', of the concrete. To evaluate an axially-loaded shaft for structural resistance, the factored axial load, P_u, is compared to the factored axial resistance, P_r, which is determined as follows:

$$P_r = \sum \phi \, P_n \qquad \text{(Eq. 10-13)}$$

where:

ϕ = Resistance factor from Table 10-6 (dim)
P_n = Nominal (ultimate) structural resistance of shaft (kN)
P_r = Factored structural resistance of pile (kN)

The nominal structural resistance of a reinforced concrete drilled shaft subjected to an axial load can be computed as follows (AASHTO, 1997a):

- For members with spiral reinforcement:

$$P_n = 0.85\left\lfloor 0.85 f_c'\left(A_g - A_{st}\right) + A_{st}F_y \right\rfloor \qquad \text{(Eq. 10-14) (\textit{A5.7.4.4-2})}$$

- For members with tie reinforcement:

$$P_n = 0.8\left\lfloor 0.85\, f_c'\left(A_g - A_{st}\right) + A_{st}F_y \right\rfloor \qquad \text{(Eq. 10-15) (\textit{A5.7.4.4-3})}$$

P_n = Nominal (ultimate) capacity of the shaft section (kN)
f_c' = Specified compressive strength of the concrete (kPa)
A_g = Gross area of the concrete section (m^2)
A_{st} = Area of the reinforcing steel (m^2)
F_y = Yield strength of the reinforcing steel (kPa)

The ϕ-factors used in Eq. 10-13 in combination with Eq. 10-14 and Eq. 10-15 and presented in Table 10-6 are generally consistent with the existing ACI Code and include the effects of manufacturing variability and small accidental load eccentricity.

The current ACI code specifies load factors for dead and live loads of 1.4 and 1.7, respectively, whereas the LRFD Specification specifies maximum dead load factors ranging from 1.25 to 1.50 and maximum live load factors ranging from 1.35 to 1.75. Further, the ACI code specifies resistance factors, ϕ, of 0.75 and 0.70 for spirally reinforced and tied shafts, respectively, whereas the LRFD Specification specifies a ϕ value of 0.75 for both types of reinforcement. ***Based on a review of the load factors in Chapter 4 and ϕ values described above, and as summarized in Table 10-8, the level of safety provided for structural design of drilled shafts in the AASHTO LRFD Specifications (1997a), as measured by an "equivalent safety factor," is typically about 10 to 15 percent less than that provided under the ACI code, except for high live to dead load ratios.***

Table 10-8
Comparison of ACI and AASHTO LRFD
Load and Resistance Factors for Drilled Shafts

Shaft Reinforcement Type	ACI LRFD			AASHTO LRFD		
	Resistance Factor ϕ	Load Factor γ	Equivalent Safety Factor γ/ϕ	Resistance Factor ϕ	Load Factor γ	Equivalent Safety Factor γ/ϕ
Spiral	0.75	1.4	1.87	0.75	1.25	1.67
		1.7	2.27		1.75	2.33
Tied	0.70	1.4	2.0	0.75	1.25	1.67
		1.7	2.43		1.75	2.33

10.3.3 Lateral Resistance (*A10.8.3.8*)

Drilled shaft foundations must be designed to resist lateral loads without structural failure of the shaft, and without excessive deflection (i.e., without exceeding the tolerable lateral movement). Although passive failure of the soil into which the shaft is embedded is a potential failure mode, such failure occurs only at relatively large deflections which generally exceed tolerable movements. Therefore, design of shafts subjected to lateral loads is commonly based on structural capacity and load-deflection behavior considerations. For shafts subjected to both axial and lateral loading, the structural resistance of the shafts as beam-columns must be checked using appropriate interaction equations calibrated with consideration of the load factors being utilized.

Methods used for ASD to estimate the lateral resistance of a single shaft or group can also be used for LRFD. For homogeneous foundation materials and a fixed- or free-head condition at the cap, simplified methods of analysis such as the Broms Method (Reese, 1984) can be used. For more complex foundation and loading conditions and intermediate levels of fixity at the cap, the analysis of laterally-loaded shafts and drilled shaft groups is usually accomplished using computer programs based on the p-y curve method of lateral load-deflection analysis. These programs include COM624P (Wang and Reese. 1993) for individual vertical shafts in level or sloping ground and GROUP (Reese, et al., 1994) for a group of vertical and battered shafts in level ground. These methods of analysis incorporate p-y curves to model the nonlinear lateral load-deflection behavior for each soil type along the length of the shaft. When using these methods at the LRFD Strength Limit State, the model shaft or drilled shaft group is subjected to the factored lateral and axial loads and factored moment, and the resulting factored axial and bending stresses are compared with the factored axial and bending resistance of the shaft.

In general, the design of laterally-loaded shafts involves:

1. Determine the maximum lateral ground line deflection at the Service Limit State and the maximum moment at the Strength Limit State for an individual shaft considering installation method for the selected shaft section.

2. Repeat Step 1 if the lateral ground line deflection exceeds the tolerable deformation or results in axial or combined stresses which exceed the maximum factored axial resistance of the shaft or do not meet the appropriate interaction criteria.

3. The shaft section is acceptable for the design loads if neither the lateral ground line deflection nor the stress criteria from Step 2 is exceeded.

In practice, the shafts in a group are tied together with a cap which is typically embedded below the ground surface. In the analyses of the lateral load resistance of this type of system, the contribution of passive resistance of the embedded portion of the cap is typically neglected. Neglecting passive resistance of the embedded cap can provide a significant degree of additional conservatism at both service and ultimate load levels.

FHWA promotes the use of vertical shafts to resist lateral loads. However, if the lateral loads acting on a foundation are large, batter shafts can be used. Batter shafts transmit lateral loads predominantly as an axial force into the soil, and should be avoided when:

- Installations where negative loading or downdrag forces can cause bending of the shaft section

- Installations in areas of high seismicity where high lateral forces from earthquake loading can result in local overstressing of shafts near the cap/batter shaft contact, and punching of shafts through the cap

Methods such as GROUP (Reese, et al., 1994) used for ASD to estimate the lateral resistance of a drilled shaft group containing batter shafts can also be used for LRFD.

10.3.4 Other Considerations

10.3.4.1 Group Effects (*A10.8.3.9*)

Adjustments used in ASD to account for the effects of group action can also be used for LRFD. The effect of closely-spaced shafts on axial load resistance can be considered in analyses by modifying the p-y relationships using an efficiency factor, η, to account for the:

- Spacing between shafts
- Type and consistency of soil into which the shafts are constructed
- Contact between the soil and the cap

In cohesive soils, the ultimate axial resistance of a drilled shaft group may be less than the cumulative resistance of the individual shafts due to overlapping of zones of shear deformation in the soil between the shafts (*A10.8.3.9.2*). In stiff cohesive soils, there is no loss in resistance due to group effects. However, in soft cohesive soils where the cap is not in contact with the underlying soil and in cohesionless soils (*A10.8.3.9.3*), the resistance of a group will be less than the cumulative resistance of an equal number of individual shafts if the center-to-center (CTC) spacing between shafts is less than 6D, and the group efficiency factors, η, in Section 9.3.4.1 apply.

If a drilled shaft group is embedded in a stiff soil deposit overlying a weaker deposit (*A10.8.3.9.3*), the potential exists for a punching failure of the group through the stiff soil into the soft soil. In addition, the potential for settlements in the soft layer must be evaluated.

10.3.4.2 Negative Loading (*A10.8.1.5*)

Methods in ASD to estimate the magnitude and location of the maximum downdrag load can also by used in LRFD. General comments regarding downdrag load estimation are presented in Chapter 4. Briaud and Tucker (1994a) indicate that the potential for downdrag loading should be considered in design when the indicators in Table 10-9 are present. In terms of performance limits, downdrag generally poses a foundation settlement concern for friction shafts and for end-bearing shafts bearing on a very stiff layer such as very dense sand or rock.

10.3.4.3 Uplift Loading (*A10.8.3.7*)

Uplift loading on shafts can be caused by lateral loads from supported structures, buoyancy effects or expansive soils. Evaluation of the ultimate uplift resistance of a single shaft or shaft group by ASD and LRFD are conducted using the same methods used to estimate ultimate axial side resistance, Q_S, of the shaft or group to axial compression loading, but with a 20 percent reduction in Q_S. The resistance factors for uplift in Table 10-5 incorporate a 20 percent reduction in Q_S for uplift loading. In addition, the structural resistance of the shaft and the shaft/cap connection must also be checked.

Table 10-9
Conditions When Downdrag Should be Considered in Design
(after Briaud and Tucker, 1994a)

1	Total settlement of the ground surface > 10 mm
2	Settlement of ground surface after shaft construction > 1 mm
3	Height of embankment filling on ground surface > 2 m
4	Thickness of soft compressible layer >10 m
5	Water table drawn down >4 m
6.	Shaft length > 25 m

WARNING: Downdrag can occur even if the above conditions are not met.

10.3.4.4 Fixity of Cap Connection (*A10.8.3.8*)

The load-deflection response and the magnitude and location of maximum moment of a laterally-loaded shaft or group depends on the fixity of the butt into the cap. Unless a detailed structural analysis is performed, the fixity of the cap connection is usually assumed to vary between fully fixed and 50 percent partially fixed. The key factors that must be considered in determining or estimating fixity at the cap connection include:

- Depth of shaft embedment into the cap
- Magnitude of bending moment at shaft-cap connection
- Shaft type and geometry
- Shaft-to-cap connection detail

Based on the results of full-scale load tests (e.g., Castilla, et al., 1984; Shahawy and Issa, 1992), a depth of embedment of 2D to 3D provides full fixity for most service load conditions. Because the degree of shaft-cap fixity is usually established empirically, there is no difference in the approach taken for drilled shaft foundations designed by either ASD or LRFD.

10.4 Design Example:
Comparison of Drilled Shaft Designs Using ASD and LRFD

Problem: The design of a drilled shaft group supporting a bridge pier is illustrated in the following example. In this example, the drilled shafts will be designed to support the loads developed in the Design Example in Chapter 4, Section 4.9. The soil profile and soil material properties are shown on Figure 10-2. This example considers design of a group of drilled shafts socketed into rock.

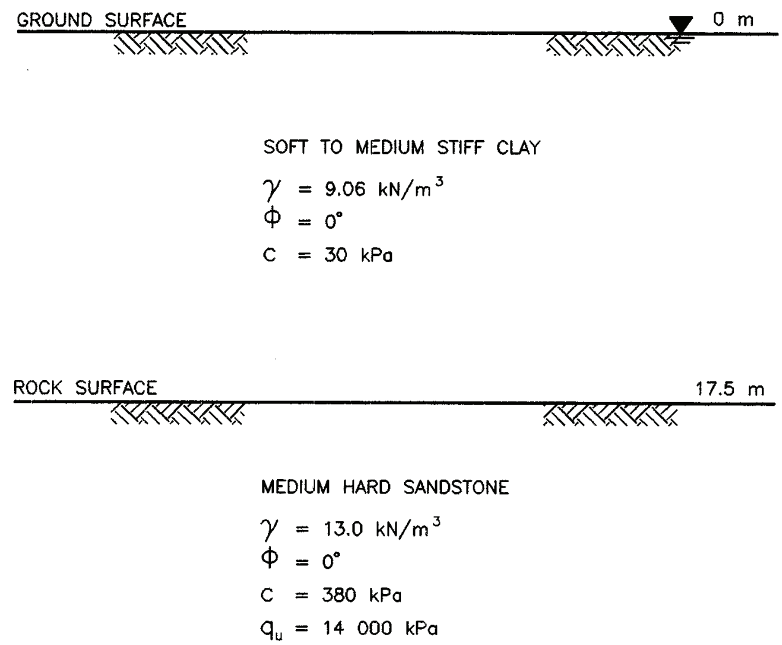


Figure 10-2
Soil Profile for Example

Objective: To demonstrate the relationship between ASD and LRFD using an example drilled shaft design by both methods.

Approach: To perform the evaluation, the following steps are taken:

- Establish unfactored design loads due to structure components, wearing surfaces and utilities, and vehicular live load
- Determine appropriate load factors and load combinations and calculate the total factored load effects
- Estimate the unfactored (ASD) and factored (LRFD) axial geotechnical resistance of a single shaft

- Determine the required number and length of shafts in the group based on geotechnical and structural criteria

- Check the allowable (ASD) and factored (LRFD) axial structural resistance of a single shaft

- Using software program STIFF1 (Ensoft, Inc., Austin, Texas, 1987) determine the flexural rigidity and factored bending resistance for each shaft in the group

- Using software program GROUP, Version 3.0 (Ensoft, Inc., Austin, Texas, 1994) determine the maximum factored bending moment in an individual shaft and the maximum deformations for the shaft group

- Determine whether the combined shaft stresses and shaft group deformations are acceptable

Step 1: Establish Unfactored Loads

The unfactored bridge pier loads and moments summarized in Table 4-21 of Chapter 4 are used in this example. As described in Step 3(I) of the Design Example in Section 4.8, Load Cases A and D, Strength I and Load Cases A, B and E, Service I are the critical factored and unfactored load combinations as they represent the critical combinations of axial load and moment in the transverse and longitudinal directions. The loading from Load Case A, Strength I will be used in the initial analysis and design. The total unfactored loads for Load Case A, from Table 4-21, are:

- Longitudinal moment = DC + DW + BR_2 + H_u + M_u = ______ kN-m
- Transverse moment = BR_2 +H_u + M_u + LL_1 = ______ kN-m
- Vertical (axial) load = DC + DW + LL_1 = ______ kN
- Longitudinal horizontal load = BR_2 + H_u = ______ kN
- Transverse horizontal load = BR_2 + H_u = ______ kN

Step 2: Establish Factored Loads

The corresponding factored loads for Load Case A, Strength I with maximum load factors were summarized in Table 4-29 as follows:

- Longitudinal moment = ______ kN-m
- Transverse moment = ______ kN-m
- Vertical (axial) load = ______ kN
- Longitudinal horizontal load = ______ kN
- Transverse horizontal load = ______ kN

Step 3: Determine Ultimate Axial and Factored Geotechnical (Bearing) Resistance of a Single Shaft

In this example, shaft tip resistance and side resistance in soil overburden and tip resistance in rock were neglected. Side resistance in the underlying sandstone layer is assumed to provide the entire axial load resistance. For shafts embedded into rock with uniaxial compressive strength greater than 1900 kPa, the Horvath and Kenney (1979) method is recommended for determining geotechnical side resistance.

From the AASHTO LRFD Specification (AC10.8.3.5), the ultimate unit side resistance of a drilled shaft in rock is according to Horvath and Kenney (1979):

$$q_s\,(\text{MPa}) = 0.21\sqrt{q_u \text{ or } f_{c'}\,(\text{MPa}),\ \text{whichever is lower}}$$

where:

q_s = Unit side resistance (MPa)
q_u = Uniaxial compressive strength of rock = 14 MPa
f_c' = Ultimate compressive strength of shaft concrete = 27.6 MPa

Therefore:

$$q_s = 0.21\sqrt{14\ \text{MPa}} = 0.786\ \text{MPa} = 786\ \text{kPa}$$

The ultimate side resistance of a single shaft is, then:

$$Q_{ult} = Q_s = q_s\,A_s \qquad \text{(Eq. 10-12)}$$

where:

Q_{ult} = Ultimate axial resistance (kN)
Q_s = Ultimate side resistance (kN)
A_s = Side wall area (m^2)

For a shaft diameter = 0.9144 m,

$$A_s = \pi\ 0.9144\ H_s$$

and the ultimate shaft resistance, Q_{ult}, in kN is:

$$Q_{ult} = Q_s = q_s\,A_s = \_\_\_\_\_\_\ H_s$$

where:

H_s = Embedment length into the sandstone layer (m)

The factored bearing resistance, from Eq. 10-5, is:

$$Q_R = \phi_{qs}\,Q_s = \_\_\_\_\_\_ \text{ x } \_\_\_\_\_\_\ H_s = \_\_\_\_\_\_\_\_\_\_\ H_s$$

where:

ϕ_{qs} = 0.65 from Table 10-5 for side resistance based on Horvath and Kenney (1979)

Step 4: Determine Required Shaft Length for ASD:
Based on consideration of the pier geometry, the shaft group layout in Figure 10-3 was selected.

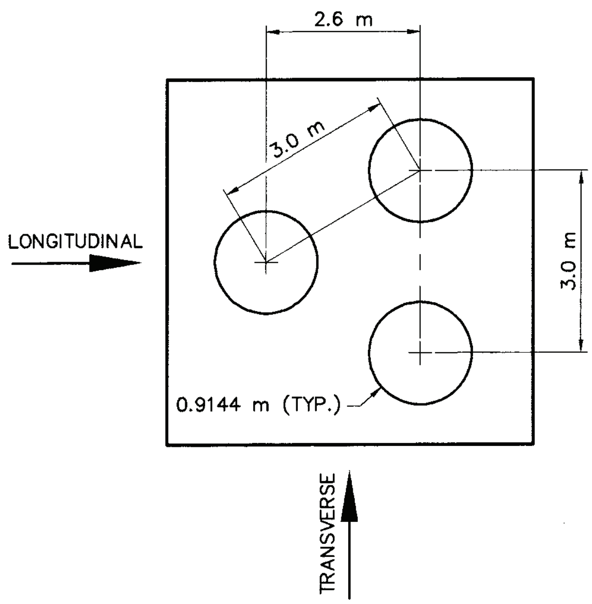


Figure 10-3
Shaft Group Layout for Example

Based on Eq. 10-1, the required rock socket embedment depth can be computed using the following relationship:

$$\frac{3\,Q_{ult}}{FS} = \frac{3\,[(\_\_\_\_\_\_\_\ \text{kN/m})\ H_s]}{FS} \geq Q = \_\_\_\_\_\_\_\ \text{kN}$$

Using FS = 2.5,

$$H_s \geq \frac{7991\ \text{kN x } 2.5}{2258\ \text{kN/m x } 3} = 2.95\ \text{m}$$

The total shaft depth is 17.5 m + ______ m = ______ m.

Step 5: Determine Required Shaft Length for LRFD:
Based on Eq. 10-5, the required rock socket embedment depth for each shaft can be computed using the following relationship:

$3\ Q_R = 3\ (\phi_{qs}\ Q_s) = 3\ (\_\_\_\_\_\_\ \text{kN/m})\ H_s \geq \sum \eta_i \gamma_i Q_i\ = \_\_\_\_\_\_\ \text{kN}$

$H_s \geq \_\_\_\_\_\_\ \text{m}$

The total shaft depth is 17.5 m + ______ m = ______ m.

Step 6: Select Longitudinal Steel Reinforcing:
Select a reasonable preliminary longitudinal steel ratio, ρ = 2.5%,

The gross shaft area, A_g, is:

$$A_g = \frac{\pi\, d^2}{4} = \pi \frac{(0.9144\ \text{m})^2}{4} = 0.657\ \text{m}^2$$

The required steel area is then:

$$A_s = (0.657\ \text{m}^2)(0.025) = 0.0164\ \text{m}^2$$

Select 12 #14 bars (0.043 m diameter),

$$\text{actual}\ \rho = 0.00145\ \text{m}^2 \times 12 \big/ 0.657\ \text{m}^2 = 0.0265 = 2.65\%$$

Provide a minimum clear cover = 0.0762 m. Therefore, the center-to-center bar spacing around the perimeter of the shaft is:

$$\text{Spacing} = \pi\,(0.9144\ \text{m} - 0.0762\ \text{m} \times 2 - 0.043\ \text{m} \big/ 12 = 0.188\ \text{m}$$

and the clear spacing between bars = 0.188 m - 0.043 m = 0.145 m.

The preliminary reinforcing is shown on Figure 10-4.

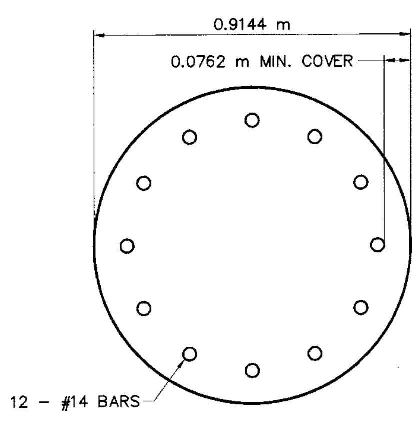


Figure 10-4
Preliminary Longitudinal Shaft Reinforcement for Example

Step 7: Check Axial Structural Resistance:
The ACI 318-95 Building code uses LRFD methodology, but utilizes load factors which vary somewhat from those employed in AASHTO. From Eq. 10-13 and Eq. 10-14, the factored axial structural resistance for a single shaft with spiral reinforcement under the AASHTO and ACI codes is:

$$P_r = \phi\, P_n = \phi\, 0.85\, [0.85\, f_c'\, (A_g - A_s) + A_s\, F_y]$$

According to Table 10-6, $\phi = 0.75$. Therefore, for $f_c' = 27\,580$ kPa and $F_y = 413\,700$ kPa, the factored axial structural resistance of one shaft, P_R, is:

$$P_r = (\_\_\_)(0.85)[(0.85)(\_\_\_\_\_\_\_\ \text{kPa})(0.657\ \text{m}^2 - 0.0174\ \text{m}^2) + (\_\_\_\_\_\_\_\ \text{KPa})(0.0174\ \text{m}^2)$$

$$P_r = \_\_\_\_\_\_\_\_\_\_\ \text{kN}$$

The factored resistance of a single shaft in the three shaft group is then checked using the following relationship.

$$P_r \geq \sum \eta_i \gamma_i Q_i$$

$$\_\_\_\_\_\_\_\ \text{kN} > \frac{\_\_\_\_\_\_\_\ \text{kN}}{3}\ \ \text{OK or NG}$$

Step 8: Check Shaft Bending Resistance:
Using the software program STIFF1, the ultimate (unfactored) moment resistance, M_n, of a single shaft was found to be:

$$M_n = 2570\ \text{kN-m}$$

From AASHTO (1997a), the factored moment resistance, M_r, is then:

$$M_r = \phi\, M_n = (0.9)\, 2570\ \text{kN-m} = 2313\ \text{kN-m} \qquad (\textit{A5.7.3.2.1-1})$$

where:

ϕ = 0.90 = AASHTO LRFD resistance factor for flexural resistance of reinforced concrete (A5.5.4.2.1)

Using the software program GROUP, Version 3.0 (Reese, et al., 1994) and factored loads, the maximum factored moments on any shaft in the transverse and longitudinal directions were determined to be:

M_{UT} = 20 kN-m
M_{UL} = 207 kN-m

where:

M_{UT} = Maximum factored transverse moment (kN-m)
M_{UL} = Maximum factored longitudinal moment (kN-m)

Because, $M_r >> M_{UT}$ and M_{UL} and the shafts are satisfactory with respect to bending resistance.

Note that the structural resistance of the shafts must also be checked for combined axial compression and biaxial flexure interaction and for shear.

Step 9: Displacements and Tolerable Movement Criteria
Because evaluations of structure displacements by LRFD are made at the Service I Limit state where $\gamma = 1.0$ and $\phi = 1.0$, methods used to estimate settlement and lateral displacement by LRFD are identical to those used for ASD, and are expressed by Eq. 10-2 as follows:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 10-2)}$$

Using GROUP program (Reese, et al., 1994) for lateral loaded pile/shaft group analysis, the shaft cap deflections, δ_i, were estimated as follows:

Vertical Settlement	9.7 mm
Longitudinal Horizontal Deflection	0.68 mm
Longitudinal Rotation	-0.000187 rad
Transverse Horizontal Deflection	0.089 mm
Transverse Rotation	-0.000159 rad

According to Moulton (1986), tolerable differential settlement, δ_n, for 2 to 4 span bridges is 25 mm or 0.004 to 0.005 times the span length, and the tolerable deflection, δ_n, at the connection of the superstructure with the pier is 38 mm. Therefore, the above estimated displacements are less than the tolerable deflections for the Service Limit State.

Summary
This example presents the design of a group of drilled shafts socketed into rock to support a typical bridge pier. The geotechnical design was performed by both ASD and LRFD. *Although the AASHTO LRFD resistance factors for geotechnical design of drilled shafts socketed into rock were obtained through a reliability-based calibration, the ASD and LRFD approaches result in nearly identical designs.* The structural design of the shafts, as is typical, was performed only by LRFD. Deflection evaluations by LRFD, which consider only unfactored (service) loads, are identical to deflection evaluations made by ASD.

10.5 Student Exercise:
Development of Resistance Factor for Design of Drilled Shafts in Sand

Problem: The AASHTO LRFD Specification does not include specific Strength Limit State resistance factors for the various methods available to estimate drilled shaft capacity in sand. Determine the ASD calibrated resistance factor for these methods for dead to live load ratios, Q_D/Q_L, of 1.0, 2.0 and 3.0 (which correspond to approximate span lengths of 17 m, 34 m and 51 m), dead load factor, γ_D, of 1.25 and a live load factor, γ_L, of 1.75 (i.e., the Strength I Limit State). Recall that the equation for this calibration (Eq. 7-7, Chapter 7) is:

$$\phi = \frac{\gamma_D \, Q_D / Q_L + \gamma_L}{FS\,(1 + Q_D / Q_L)} \qquad \text{(Eq. 7-7)}$$

From Section 10.2.1, the AASHTO ASD factor of safety for these type of predictive methods is 2.5.

Q_D/Q_L	Dead Load Factor, γ_D	Live Load Factor, γ_L	Resistance Factor, ϕ	Average Load Factor, $\bar{\gamma}$
1.0	1.25	1.75		
2.0	1.25	1.75		
3.0	1.25	1.75		

CHAPTER 11
CONVENTIONAL RETAINING WALL AND ABUTMENT DESIGN

11.1 Introduction

For both Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD), the design of conventional retaining walls and abutments requires consideration of geotechnical capacity, overall stability, structural capacity and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between gravity wall and abutment design by LRFD and ASD

- Identifies the strength and serviceability performance limits which must be considered for gravity wall and abutment design by LRFD

11.2 Design Methods

With the exception of differences in foundation design noted in Chapters 8, 9 and 10, the procedure for design of conventional gravity and cantilever retaining walls and abutments using LRFD (A11.6) is identical to that followed using ASD. Generally, the ultimate bearing capacity, resistance to sliding, overall stability and settlement of the wall foundation are checked, and the wall is checked for lateral deflection. The following sections summarize the general design processes for conventional retaining wall and abutment design using the ASD and LRFD approaches.

11.2.1 ASD Summary

Existing practice for geotechnical design of conventional retaining walls and abutments typically follows the ASD approach for design of spread footing foundations presented in Chapter 8. Where a retaining wall or abutment is supported with piles or drilled shafts, the design of the wall foundation follows the ASD approach outlined in Chapter 9 or 10, as appropriate.

The design of conventional retaining walls and abutments may often be governed by criteria for lateral deflection or tilt. Therefore, the design of a retaining wall or abutment by ASD requires an estimation of foundation settlements and lateral displacements (geotechnical) and deflections (structural) associated with mobilization of design earth pressures, and comparison of estimated deflections with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 11-1)}$$

where:

δ_i = Estimated displacement or differential displacement (mm)
δ_n = Tolerable displacement or differential displacement established by the designer (mm)

After all geotechnical deformation and capacity criteria are met and a suitable foundation type, size and geometry is selected, the structural design of the wall or abutment is performed using service loads and allowable stresses (AASHTO, 1997b), or by LFD or LRFD (AASHTO, 1997b or ACI), consistent with the superstructure design.

11.2.2 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. The geotechnical design of conventional retaining walls and abutments typically follows the LRFD approach for design of spread footing foundations presented in Chapter 8. Where a retaining wall or abutment is supported with piles or drilled shafts, the design of the wall foundation follows the ASD approach outlined in Chapter 9 or 10, as appropriate. Structural design of the wall is then performed by LRFD procedures using factored loads and resistances.

Using the guidelines described in Chapters 3 and 4, load magnitudes, values of load factor and load factor combinations for each applicable limit state must be developed. ***The key issues in the design of retaining walls and abutments by LRFD is the application of maximum and minimum load factors for dead, earth and surcharge loads as described in Chapter 4 and illustrated in the example problem in Section 4.6.***

11.2.2.1 Limit States (*A11.5*)

The design of conventional walls and abutments using LRFD requires evaluation of foundation suitability at various Limit States (i.e., applicable Strength Limit States and the Service I Limit State). The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for a typical retaining wall or abutment supported on a spread footing designed at the Strength and Service I Limit States are summarized in Table 11-1. As conditions warrant, it may also be necessary to evaluate foundation performance at other limit states (e.g., Extreme Event I for loading from earthquakes).

More detailed discussion of procedures used to evaluate the various Performance Limits is provided in Section 11.3, and in Chapters 8, 9 and 10.

Table 11-1
Limit States for Design of Gravity Retaining Walls and Abutments

Performance Limit	Strength Limit State(s)	Service Limit State
Location of Base Resultant Force	✔	
Bearing Resistance	✔	
Sliding Resistance	✔	
Overall Stability	✔	
Settlement and Horizontal Movement		✔
Structural Capacity of Wall Elements	✔	

11.2.2.2 Resistance Factors (*A11.5.6*)

Resistance factors for geotechnical design of spread footings with respect to bearing capacity and sliding resistance were discussed in Chapter 8. Resistance factors for geotechnical and structural design of driven pile and drilled shaft foundations were discussed in Chapters 9 and 10, respectively.

As indicated in Section 8.2.2.2, the eccentricity criteria for design of spread footings under AASHTO LRFD were calibrated directly with existing AASHTO ASD criteria through analysis of cantilever and gravity retaining walls and abutments. The calibration study (Barker, et al., 1991b) considered a wide range of parameters.

In general, the calibration was performed by developing ASD wall designs resulting in critical base pressure eccentricities (e_B = 0.167 B or B/6 for footings on soil and e_B = 0.25 B or B/4 for footings on rock), factoring the loads and recomputing the eccentricity for LRFD. The computation process considered the following parameter variations:

- Varying wall heights from 1.5 m to 7.5 m and abutment heights from 4.5 m to 7.6 m while maintaining constant base widths and other dimensions
- Varying wall and abutment base widths while maintaining constant heights and other dimensions
- Varying stem location while maintaining constant height and base width
- Varying the friction angle, ϕ_f, of the retained earth and foundation soil
- Varying the magnitude of dead and live load applied to abutments

- Varying the ratio of dead to live load applied to abutments from 1.0 to 3.0
- Considering and neglecting wind loads

A summary of the results from the calibration process are presented in Table 11-2. The tabulation indicates the parameter which was varied in a given analysis, and the LRFD eccentricity resulting for the parameter value which produced the critical ASD eccentricity. ***As indicated in Table 11-2, the selected LRFD eccentricity limitation for retaining walls is approximately equal to the average value from the calibration, whereas the selected LRFD eccentricity limitation for abutments is somewhat greater (or less conservative) than the average value from the calibration.***

Table 11-2
Summary of Parametric Study Results
from LRFD Eccentricity Calibration
(after Barker, et al., 1991b)

<table>
<tr><th rowspan="3">Structure Type</th><th rowspan="3">Varied Parameter</th><th colspan="4">e_B/B</th></tr>
<tr><th colspan="2">Footing on Soil</th><th colspan="2">Footing on Rock</th></tr>
<tr><th>ASD</th><th>LRFD</th><th>ASD</th><th>LRFD</th></tr>
<tr><td rowspan="6">Abutments</td><td>Stem Location</td><td>0.167</td><td>0.22</td><td>0.250</td><td>0.31</td></tr>
<tr><td>Soil Friction Angle</td><td>0.167</td><td>0.22</td><td>0.250</td><td>0.32</td></tr>
<tr><td>Wall Height</td><td>0.167</td><td>0.23</td><td>0.250</td><td>0.33</td></tr>
<tr><td>Base Width with:
• Q_D = 35 kN
• Q_L = 35 kN</td><td>0.167</td><td>0.21</td><td>0.250</td><td>0.30</td></tr>
<tr><td>• Q_D = 54 kN
• Q_L = 18 Kn</td><td>0.167</td><td>0.22</td><td>0.250</td><td>0.33</td></tr>
<tr><td>• Q_D = 93 kN
• Q_L = 31 kN</td><td>0.167</td><td>0.21</td><td>0.250</td><td>0.31</td></tr>
<tr><td rowspan="3">Retaining Walls</td><td>Stem Location</td><td>0.167</td><td>0.25</td><td>0.250</td><td>0.42</td></tr>
<tr><td>Wall Height</td><td>0.167</td><td>0.25</td><td>0.250</td><td>0.36</td></tr>
<tr><td>Base Width (w/ Q_D = Q_L = 0)</td><td>0.167</td><td>0.24</td><td>0.250</td><td>0.35</td></tr>
<tr><td colspan="2">Average e_B/B
• Abutments
• Walls</td><td>
0.167
0.167</td><td>
0.218
0.247</td><td>
0.250
0.250</td><td>
0.316
0.376</td></tr>
<tr><td colspan="2">Selected e_B/B</td><td>0.167</td><td>0.250</td><td>0.250</td><td>0.375</td></tr>
</table>

The calibration was performed in 1991 using the AASHTO LFD (AASHTO, 1997b) load factors before the LRFD load factors listed in Chapter 4 had been finalized. Subsequent analyses by the original calibrators for a limited number of retaining wall and abutment design cases indicated that the differences in load factors does not have a significant effect on the calibration.

11.2.2.3 Comparison of Wall Design Using LRFD and ASD

The differences between the geotechnical design of a conventional retaining wall or abutment by ASD and LRFD relate to differences in the foundation design as discussed in Chapters 8, 9 and 10.

11.2.2.4 Modification of Resistance Factors

Resistance factors for retaining wall and abutment foundation design can be modified as described in Chapters 8, 9 and 10. Due to the variation in load factors for various load effects applicable for a wall or abutment design, and the range of load combinations which may apply depending on the problem geometry, modification of resistance factors or eccentricity criteria for these designs should be based on a parametric study covering expected potential variations in loading conditions and problem geometry. As a minimum, such a parametric study should include the anticipated range of wall heights and dead to live load combinations to define the applicable range of the average load factor for net destabilizing forces.

11.2.3 Summarized Comparison of ASD and LRFD

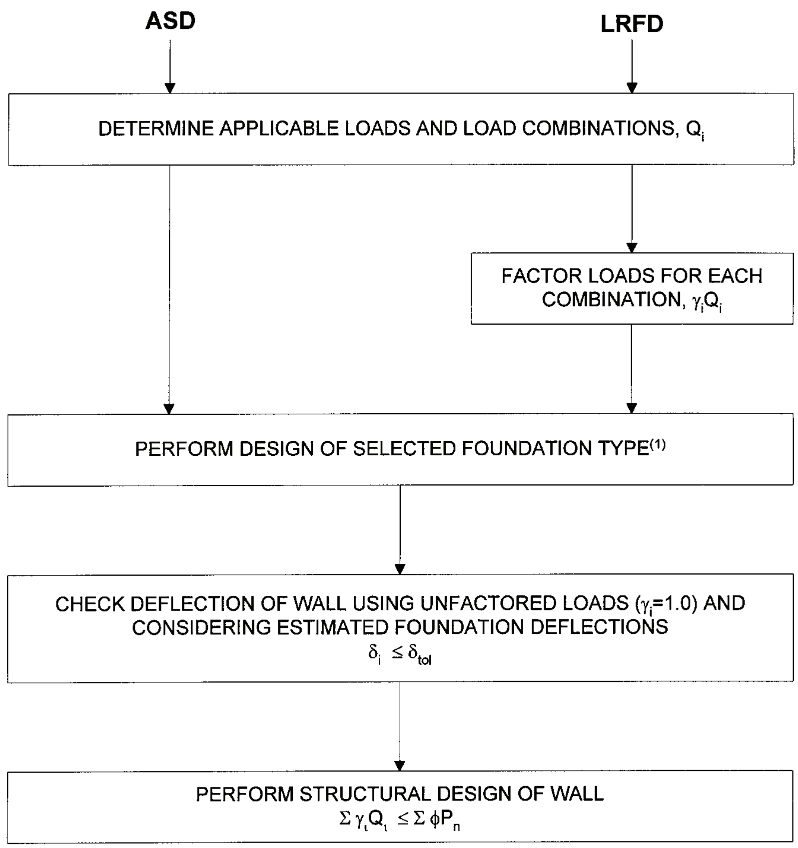
As noted before, the process used to develop a conventional retaining wall or abutment design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 11-1. Specific differences between the methods and other important issues are highlighted in the following section.

Other aspects of retaining wall or abutment design such as identifying special considerations (e.g., potential for loss of support through scour), developing a design foundation and retained soil profile and determining requirements for construction control are inherent aspects of the design process required for both LRFD and ASD.

11.3 Performance Limits

Design of a conventional retaining wall or abutment by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the wall geometry and details, and in establishing a suitable bearing level to meet the criteria for vertical, inclined and/or moment loading, the design of these structures requires consideration of many factors which can affect wall or abutment performance, including:

- Bearing resistance to vertical and inclined loads and moments
- Sliding resistance to lateral loads
- Resistance to overturning forces and moments
- Resistance to uplift forces
- Resistance to effects of scour and frost
- Resistance to variable ground water levels, including the effect of seepage when footings support walls which do not provide adequate drainage
- Geometric constraints (e.g., nearby structures which could impose load on or be loaded by the wall)



(1) Refer to Chapters 8, 9 and 10.

Figure 11-1
Generalized Flow Chart for Conventional Retaining Wall and Abutment Design by ASD and LRFD

11.3.1 Displacements and Tolerable Movement Criteria (*A11.6.2*)

The vertical and lateral displacements of conventional retaining walls and abutments must be evaluated for all applicable dead and live load combinations and compared with tolerable movement criteria. Vertical and lateral movements of wall foundations can be estimated and evaluated using the procedures and criteria described in Chapters 8, 9 and 10. At a minimum, conventional retaining walls and abutments must deflect (either by foundation movement or structural deformation) a sufficient magnitude to permit mobilization of the shear strength of the backfill soil and development of the design (usually active) earth pressure on the wall stem. In general, lateral movements of walls on shallow foundations can be estimated assuming the wall rotates or translates as a rigid body due to the effects of earth loads and differential settlements along the base of the wall.

The tolerable movement criteria for abutments should generally follow the guidance provided in Section 8.3.1 for spread footing foundations. For other retaining walls, tolerable movement criteria should be developed with consideration of the function and type of wall, anticipated service life, and consequences of unacceptable movements (e.g., affect of wall movements on adjacent facilities).

11.3.2 Geotechnical Resistance *(A11.6.3)*

The geotechnical resistance of retaining walls and abutments can be evaluated for the selected foundation type as described in Chapters 8, 9 and 10. The overall stability of an abutment or retaining wall, including the retained ground and foundation, should be evaluated for all walls using limit equilibrium methods of slope stability analysis at the Service Limit State as described in Chapter 8.

11.3.3 Structural Resistance *(A11.6.4)*

The design of conventional walls and abutments must meet the requirements for the structural design of concrete structures. The structural design of footings, driven piles and drilled shafts is discussed briefly in Chapters 8, 9 and 10, respectively. Structural design of the wall stem and base slab is performed using the factored loads developed for the geotechnical design of the wall foundation and the factored structural resistance of the reinforced and unreinforced concrete wall or abutment.

11.3.4 Other Considerations

11.3.4.1 Loss of Passive Resistance *(A11.6.3.6)*

The sliding resistance of a retaining wall is controlled by the (1) shear resistance between the wall foundation and the foundation subgrade, and by the (2) passive resistance in front of the wall due to embedment. The component of sliding resistance provided by embedment is often ignored during design due to the possibility that permanent or temporary excavations in front of the wall could occur during the service life of the structure and lead to partial or complete loss of passive resistance. Additional shear resistance can be developed by keying when the foundation materials are rock or very stiff soils, or by increasing the width of the foundation.

11.3.4.2 Drainage *(A11.6.6)*

The lateral load on walls is affected by (1) the shear strength and unit weight of the soil backfill, and by (2) the presence of water in the backfill. For these reasons, free-draining, granular soils are usually specified for backfill to minimize the lateral earth pressure and to ensure that a permanent water level cannot be maintained above the level of weep holes in the wall. The use of free-draining, granular backfills is also beneficial in limiting the development of uplift pressures along the base of the wall due to seepage from the backfill soil.

11.4 Design Example:
Cantilever Retaining Wall on Spread Footing by LRFD and ASD

The geotechnical design of a cantilever retaining wall supported on a spread footing foundation generally involves two steps:

1. Determining the loads acting on the retaining wall

2. Designing a wall foundation which meets applicable settlement, resultant eccentricity, bearing resistance and sliding resistance criteria

Unfactored and factored loads and moments for design of a cantilever retaining wall were developed in the Classroom Example in Chapter 4. The geotechnical design of a spread footing to support the wall is presented in the Classroom Example in Chapter 8.

CHAPTER 12
PREFABRICATED MODULAR WALL DESIGN

12.1 Introduction

Similar to the requirements described in Chapter 11 for conventional retaining walls and abutments, both Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD) of prefabricated modular retaining walls requires consideration of geotechnical capacity, overall stability and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between prefabricated modular wall design by LRFD and ASD

- Identifies the strength and serviceability performance limits which must be considered for prefabricated modular wall design by LRFD

- Presents an example of a prefabricated modular wall design by LRFD

12.2 Design Methods

With the exception of differences in foundation design noted in Chapter 8, the procedure for design of prefabricated modular retaining walls using LRFD (A11.10) is identical to that followed using ASD. Generally, the ultimate bearing capacity, resistance to sliding, base pressure resultant eccentricity, overall stability and settlement of the wall foundation are checked, and the wall is checked for lateral deflection. (Note: Sliding, eccentricity and overall stability should be checked at the base of each module.) The following sections summarize the general design processes for modular retaining wall design using the ASD and LRFD approaches.

12.2.1 ASD Summary

With the exception of differences in the contribution of soil weight in the evaluation of overturning stability and eccentricity, the procedures for design of modular retaining walls by ASD are identical to those described in Chapter 11 for conventional retaining walls and abutments. In this regard, ASD design of modular retaining walls differs from the design of conventional retaining walls as follows (AASHTO, 1997b):

- Foundation bearing capacity is evaluated assuming that dead loads are resisted by point supports per unit length at the rear and front of the modules, and assuming that 80 percent of the contained backfill weight is transferred to the front and rear supports. (If a footing or pad is placed under the entire module area, 100 percent of the contained backfill weight and the full foundation area are considered for bearing capacity.)

- Evaluation of base pressure resultant eccentricity (overturning) considers that only 80 percent of the contained backfill weight is effective in resisting overturning moments.

- Sliding stability is evaluated assuming 100 percent of the contained backfill is effective in resisting sliding.

The design of prefabricated modular walls may be governed by criteria for lateral deflection or tilt. Therefore, the design of a modular retaining wall or abutment by ASD requires an estimation of foundation settlements and lateral displacements associated with mobilization of design earth pressures, and comparison of estimated deflections with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 12-1)}$$

where:

δ_i = Estimated displacement or differential displacement (mm)

δ_n = Tolerable displacement or differential displacement established by the designer (mm)

After all geotechnical deformation and capacity criteria are met, the structural design of the wall is performed using service loads and allowable stresses or by LFD (AASHTO, 1997b), or by LRFD (AASHTO, 1997a or ACI).

12.2.2 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. The geotechnical design of prefabricated modular retaining walls typically follows the LRFD approach for design of spread footing foundations presented in Chapter 8. Structural design of the wall is then performed by LRFD procedures using factored loads and resistances.

Using the guidelines described in Chapters 3 and 4, load magnitudes, values of load factor and load factor combinations for each applicable limit state must be developed. ***The key issue in the design of modular walls by LRFD is the application of maximum and minimum load factors for dead, earth and surcharge loads as described in Chapter 4 and illustrated in the example problem in Section 12.4.***

12.2.2.1 Limit States (*A11.5*)

The design of prefabricated modular walls using LRFD requires evaluation of foundation suitability at various Performance Limit States (i.e., applicable Strength Limit States and the Service I Limit

State). The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for a modular wall designed at the Strength and Service I Limit States are summarized in Table 12-1. As conditions warrant, it may also be necessary to evaluate foundation performance at other limit states (e.g., Extreme Event I for loading from earthquakes). More detailed discussion of procedures used to evaluate the various Strength and Service Limit States is provided in Section 12.3 and in Chapter 8.

Table 12-1
Strength and Service Limit States
for Design of Prefabricated Modular Walls

Design Consideration	Strength Limit State(s)	Service I Limit State
Location of Base Resultant Force	✔	
Bearing Resistance	✔	
Sliding Resistance	✔	
Overall Stability	✔	
Settlement and Horizontal Movement		✔
Structural Capacity of Wall Modules	✔	

12.2.2.2 Resistance Factors (*A11.5.6*)

Resistance factors for geotechnical design of spread footings with respect to bearing capacity and sliding resistance were discussed in Chapter 8. As indicated in Section 8.2.2.2 in Chapter 8, the eccentricity criteria for design of spread footings under AASHTO LRFD were calibrated directly with existing AASHTO ASD. As discussed in Chapter 11, the calibration for eccentricity resulted in limiting the resultant factored load to within B/4 for footings on soil 3B/8 for footings on rock).

12.2.2.3 Comparison of Wall Design Using LRFD and ASD

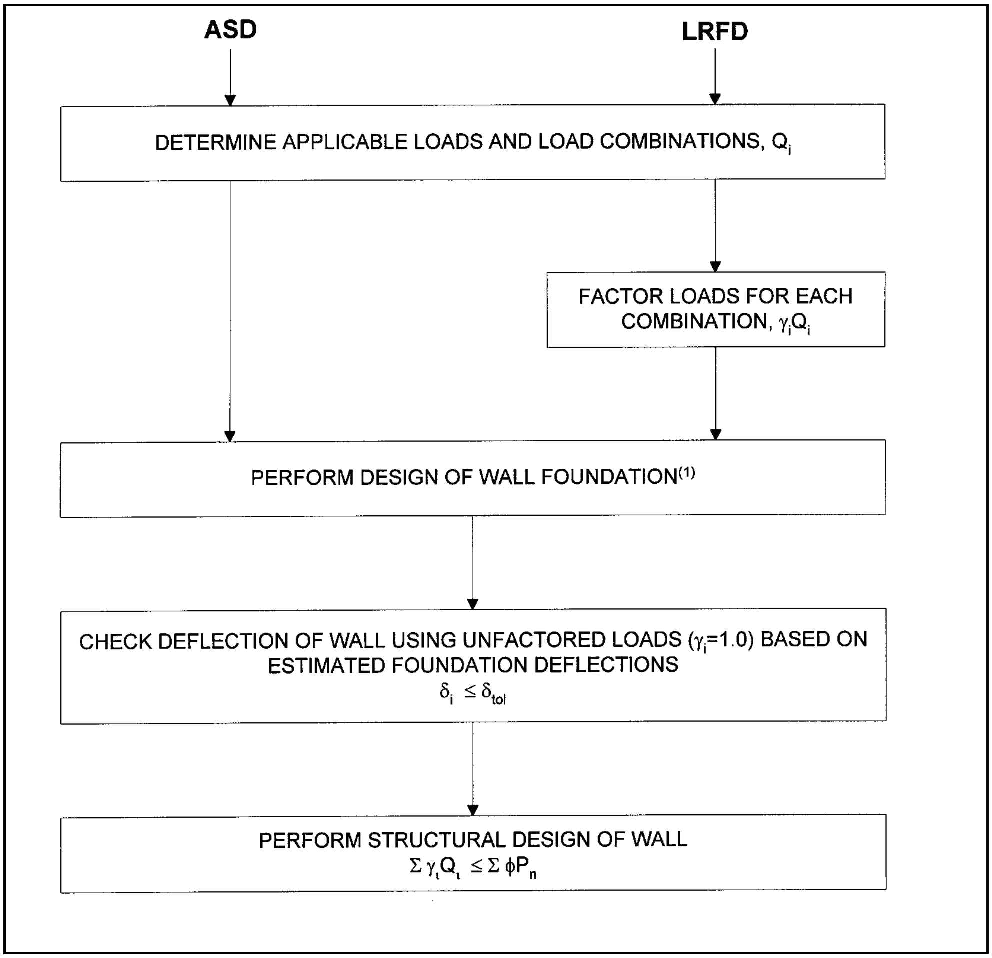
The differences between the geotechnical design of a prefabricated modular retaining wall or abutment by ASD and LRFD relate to differences in the foundation design and application of factored earth pressures as discussed in Chapters 8 and 10, respectively.

12.2.2.4 Modification of Resistance Factors

Resistance factors for retaining wall and abutment foundation design can be modified as described in Chapters 8. Due to the variation in load factors for various load effects applicable for a wall or abutment design, and the range of load combinations which may apply depending on the problem geometry, modification of resistance factors or eccentricity criteria for these designs should be based on a parametric study covering expected potential variations in loading conditions and problem geometry. As a minimum, such a parametric study should include the anticipated range of wall heights and dead to live load combinations to define the applicable range of the average load factor for net destabilizing forces.

12.2.3 Summarized Comparison of ASD and LRFD

As noted before, the process used to develop a prefabricated modular retaining wall design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 12-1. Specific differences between the methods and other important issues are highlighted in Section 12.3.



(1) Refer to Chapter 8.

Figure 12-1
Generalized Flow Chart for Modular Retaining Wall Design

12.3 Performance Limits

Design of a prefabricated modular retaining wall by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the wall geometry and details, and in establishing a suitable bearing level to meet the criteria for vertical, inclined and/or moment loading, the design of these structures requires consideration of many factors which can affect wall or abutment performance, including:

- Bearing resistance to vertical and inclined loads and moments
- Sliding resistance to lateral loads
- Resistance to overturning forces and moments
- Resistance to effects of scour and frost
- Resistance to variable ground water levels, including the effect of seepage when footings support walls which do not provide adequate drainage
- Geometric constraints (e.g., nearby structures which could impose load on or be loaded by the footing)

12.3.1 Displacements and Tolerable Movement Criteria (*A11.10.2*)

The vertical and lateral displacements of prefabricated modular retaining walls must be evaluated for all applicable dead and live load combinations and compared with tolerable movement criteria. Vertical and lateral movements of wall foundations can be estimated and evaluated as described in Chapter 8. Similar to conventional retaining walls, prefabricated modular retaining walls must deflect (either by foundation movement or structural deformation) a sufficient magnitude to permit mobilization of the shear strength of the backfill soil and development of the design (usually active) earth pressure on the wall stem. In general, lateral movements of walls on shallow foundations can be estimated assuming the wall rotates or translates as a rigid body due to the effects of earth loads and differential settlements along the base of the wall.

Tolerable movement criteria for prefabricated modular walls should be developed with consideration of the function and type of wall, anticipated service life, and consequences of unacceptable movements (e.g., affect of wall movements on adjacent facilities).

12.3.2 Geotechnical Resistance (*A11.10.3*)

The geotechnical resistance of prefabricated modular retaining walls can be evaluated as described in Chapter 8. Because the interior of modular walls are backfilled with soil to complete their construction, and because the interior of the backfill soil can move with respect to the retaining module (for open bottom modules) if the module is uplifted or overturned, the full weight of the backfilled structure is not effective in resisting eccentric (overturning) loads. Therefore, the soil load resisting overturning is limited to a maximum of 80 percent of the weight of the soil in the modules. In addition, bearing resistance is evaluated by assuming that a minimum of 80 percent of the weight of soil in the modules is transferred to Point (or line) supports at the front and rear of the module. The overall stability of an abutment or retaining wall, including the retained ground and foundation, should be evaluated for all walls using limit equilibrium methods of slope stability analysis as described in Chapter 8.

12.3.3 Structural Resistance (*A11.10.4*)

The design of conventional walls and abutments must meet the requirements for the structural design of concrete structures. The structural design of footings is discussed briefly in Chapter 8. Structural design of the module members, and base slab if any, is performed using the factored loads developed for the geotechnical design of the wall foundation and the factored structural resistance of reinforced concrete for concrete systems, and the factored structural resistance of steel for steel systems.

12.3.4 Other Considerations

12.3.4.1 Loss of Passive Resistance (*A11.10.3.7*)

The sliding resistance of a retaining wall is controlled by (1) the shear resistance between the wall foundation and the foundation subgrade, and (2) the passive resistance in front of the wall due to embedment. The component of sliding resistance provided by embedment is often ignored during design due to the possibility that permanent or temporary excavations in front of the wall could occur during the service life of the structure and lead to partial or complete loss of passive resistance. Additional shear resistance can be developed for modular walls by inclining the base of the wall or by increasing the base width of the wall.

12.3.4.2 Drainage (*A11.10.6*)

The lateral load on and within the elements of modular walls is affected by (1) the shear strength and unit weight of the soil backfill, and (2) the presence of water in the backfill. For these reasons, free-draining, granular soils should specified for backfill to minimize the lateral earth pressure and to ensure that a permanent water level cannot be maintained behind the wall. The use of free-draining, granular backfills is also beneficial in limiting the development uplift pressures along the base of the wall due to seepage from the backfill soil.

12.4 Design Example:
Geotechnical Design of Prefabricated Modular Retaining Wall

Problem: This example illustrates the development of unfactored and factored loads needed for the geotechnical design of a modular retaining wall and the geotechnical design of the wall using the AASHTO LRFD Specifications.

The modular retaining wall in Figure 12-2 is being considered for a grade separation between roadway lanes. The wall will be backfilled with a free draining granular fill similar to the retained embankment and the seasonal high water table is 3 m below the bottom of the footing. The vehicular live load surcharge (LS) on the backfill will be applied as shown in the figure.

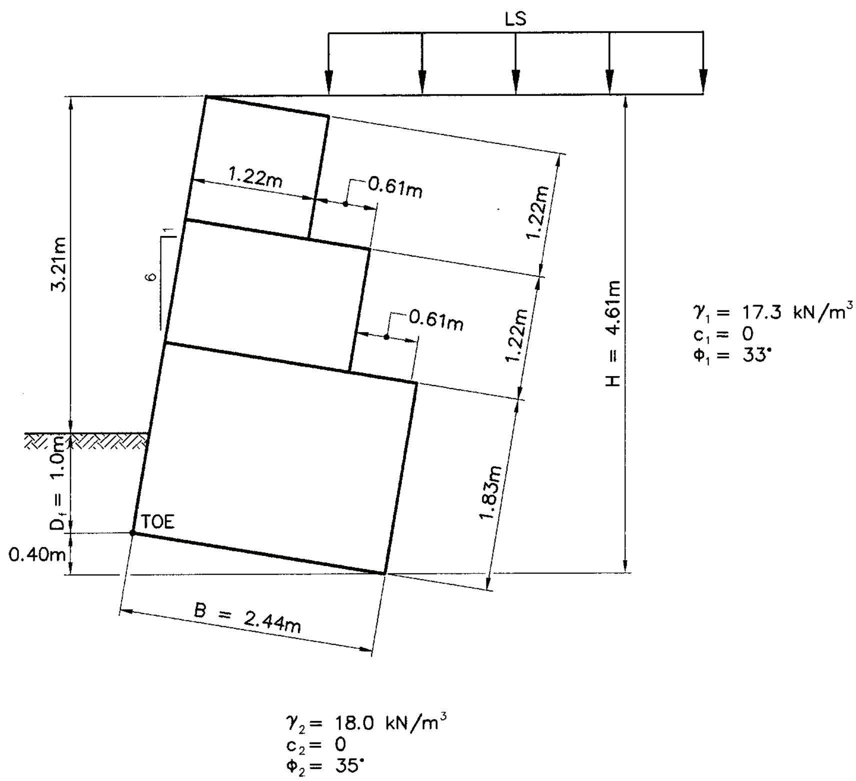


Figure 12-2
Schematic of Example Problem

During the subsurface exploration, it was determined, based on CPT results, that the foundation soils are predominantly a dense sand to a depth of more than 6 m below the proposed bottom of footing.

In performing the wall design, the following are assumed:

- Dense sand and gravel underlies the foundation so that the elastic settlement of the dense sand and gravel will be negligible

- Wall and wall modules will be backfilled with free-draining granular fill

- The seasonal high water table is 3 m below the base of the wall

- The base of the wall will be supported at the rear and front of the modules by concrete bearing pads

Objective: Demonstrate the procedure for geotechnical design of a prefabricated modular retaining wall by LRFD and compare the results with those obtained using ASD.

Approach: To perform the evaluation, the following steps are taken:

- The loads and resulting moments due to wall components, earth pressures and live load surcharge are calculated

- The appropriate load factors and combinations are determined and applied to the unfactored loads and moments to determine the factored loading conditions

- Based on the factored vertical loads and resisting and overturning moments, adequacy against overturning will be determined by checking the eccentricity

- The bearing capacity is checked by first determining the maximum bearing pressure for each strength limit loading case and comparing it to the ultimate bearing resistance multiplied by the resistance factor

- The sum of the factored horizontal forces for each strength limit loading case are compared to the factored horizontal resistance to determine the adequacy against sliding

Step 1: Unfactored Loads

(A) Dead Load of Structural Components (DC) and Module Fill (EV)
Referring to Figure 12-3 and the unit properties of the modular wall in Table 12-2, calculate the unfactored dead loads.

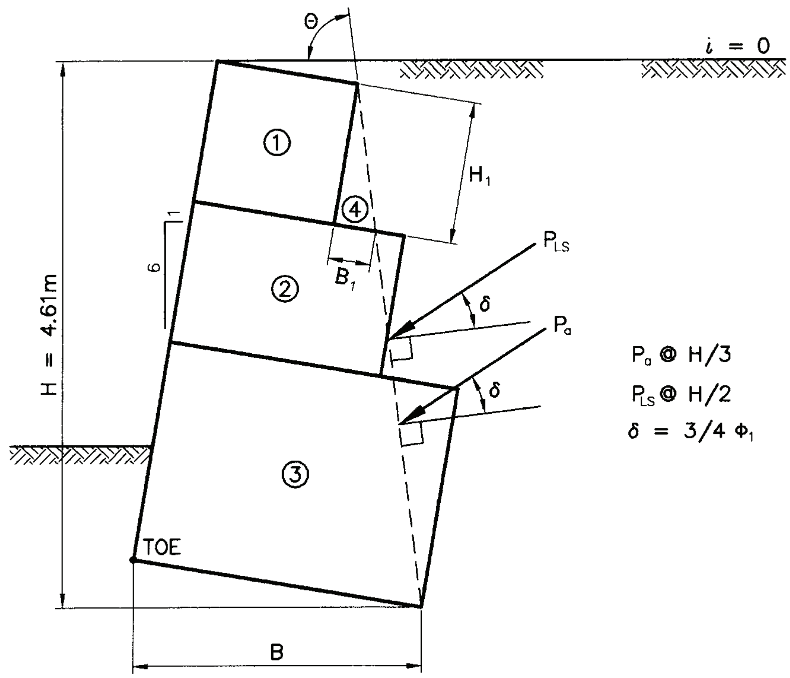


Figure 12-3
Retaining Wall Area Designation for Weight of Concrete and Soil

The weights of the concrete modules and retained fill are summarized in Table 12-2.

Table 12-2
Unfactored Unit Properties of 2.44-m Long Wall Modules

Unit No.	Height/Width (m)	Weight of Concrete, WC (kN)	WC/m (kN/m)	Volume of Fill, FV (m^3/m)	Weight of Fill/m, WF[(1)] (kN/m)
1	1.22/1.22	32.2	13.2	0.89	15.4
2	1.22/1.83	38.1	15.6	1.53	26.5
3	1.83/2.44	65.8	27.0	3.27	56.6
	Total	136.1	55.8	5.69	98.5

(1) Density of fill = 17.3 kN/m^3

(B) Vertical Earth Pressure on Back of Modules (EV)
Unit Weight of Soil γ_1 = 17.3 kN/m^3. Therefore, the weight of soil on top of module 2 is:

$$P_{EV1} = W_4 = 0.5\, B_1\, H_1\, \gamma_1 = 0.5\ (0.35\ m)(1.22m)(17.3\ kN/m^3) = 3.7\ kN/m$$

(C) Live Load Surcharge (LS)
A live load surcharge is applied when vehicle loads will be supported on the backfill within a distance equal to H behind the wall. The live load surcharge is applied as an equivalent height of soil for the design vehicle loading (h_{eq}) using Table 4-2 from Chapter 4, and a wall height of 4.61 m. By interpolation, h_{eq} = 0.964 m.

Using the unit weight of the soil backfill (i.e., γ_1 = 17.3 kN/m^3), the unit vertical component of LS is:

$$p_{LSV} = \gamma'\, h_{eq} = 17.3\ kN/m^3\ (0.964\ m) = 16.7\ kPa$$

For a wall friction angle $\delta = 3/4\phi = 24.8^o$ (*Table AC11.10.1-1*), backslope $\theta = 83.5°$ from the horizontal, horizontal backfill surface ($i = 0^o$) and soil friction angle $\phi_1 = 33^o$, the active earth pressure coefficient, k_a, is obtained from the equation in AASHTO Figure A11.10.3.2-2:

$$k_a = \frac{\sin^2(\theta+\phi)}{\sin^2\theta\,\sin(\theta-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\,\sin(\phi-i)}{\sin(\theta-\delta)\,\sin(\theta+i)}}\right]^2}$$

from which:

$$k_a = \frac{\sin^2(83.5°+33°)}{\sin^2 83.5°\,\sin(83.5°-24.8°)\left[1+\sqrt{\dfrac{\sin(33°+24.8°)\,\sin(33°-0°)}{\sin(83.5°-24.8°)\,\sin(83.5°+0°)}}\right]^2} = 0.315$$

Then from Eq. 4-7 in Chapter 4:

$$\Delta p = k_a\, \gamma_s'\, h_{eq} = 0.315\ (17.3\ kN/m^3)\ (0.964\ m) = 5.3\ kPa \qquad \text{(Eq. 4-7)}$$

Using a rectangular distribution inclined as shown in Figure 12-3, the horizontal and vertical components of live load surcharge and vertical earth pressure acting on the wall are:

$$P_{LSH} = \Delta p\ H \cos(90^o - \theta + \delta) = (5.3\ kPa)\ (4.6\ m) \cos(6.5^o + 24.8^o) = 20.7\ kN/m$$
$$P_{LSV} = \Delta p\ H \sin(90^o - \theta + \delta) = (5.3\ kPa)\ (4.6\ m) \sin(6.5^o + 24.8^o) = 12.6\ kN/m$$

(D) Horizontal and Vertical Components of Active Earth Pressure (EH)
The active earth pressure is assumed to vary linearly with the depth of soil backfill as given by:

$$p = k_a \gamma_s' z \qquad (A3.11.5.1\text{-}1)$$

At the base of the footing (i.e., @ z = H), p = 0.315 (17.3 kN/m^3) (4.61 m) = 25.1 kPa

The resultant of the active earth pressure is inclined at the wall friction angle, δ with respect to plane surface along the back of the modular wall. The horizontal and vertical components of the earth pressure (triangular distribution) acting on the wall are:

$P_{aH} = ½\ pH \cos(\delta + 90^o - \theta) = 0.5\ (25.1\ kPa)(4.61\ m) \cos(24.8^o + 6.5^o) = 49.5\ kN/m$
$P_{aV} = ½\ pH \sin(\delta + 90^o - \theta) = 0.5\ (25.1\ kPa)(4.61\ m) \sin(24.8^o + 6.5^o) = 30.0\ kN/m$

(E) Summary of Unfactored Loads

Table 12-3
Unfactored Vertical Loads and Resisting Moments

Item	V (kN/m)	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
WC_1	13.2	1.20	15.9
WC_2	15.6	1.30	20.4
WC_3	27.0	1.35	36.5
WF_1	15.4	1.20	18.5
WF_2	26.5	1.30	34.6
WF_3	56.6	1.35	76.6
P_{EV1}	3.7	1.97	7.3
P_{aV}	30.0	2.24	67.3
P_{LSV}	12.6	2.15	27.0
V_{TOT}	200.6		

Table 12-4
Unfactored Horizontal Loads and Overturning Moments

Item	H (kN/m)	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
P_{LSH}	20.7	1.91	39.6
P_{aH}	49.5	1.14	56.4
TOTAL	70.2		96.0

Step 2: Load Factors

From Tables 4-10 and 4-11 in Chapter 4, the applicable load factors are summarized in Table 12-5. Strength I-a and I-b represent the Strength I Limit State using maximum and minimum load factors, respectively, from Table 4-11 in Chapter 4.

Table 12-5
Load Factors

Group	γ_{DC}	γ_{EV}	γ_{LSV}	γ_{LSH}	γ_{EH} (active)	Probable Use
Strength I-a	0.90	1.00	1.75	1.75	1.50	BC/EC/SL
Strength I-b	1.25	1.35	1.75	1.75	1.50	BC (max. value)
Strength IV	1.50	1.35	-	-	1.50	BC (max. value)
Service I	1.00	1.00	1.00	1.00	1.00	Settlement

Notes: BC - Bearing Capacity; EC - Eccentricity; SL - Sliding; γ_{EH} applies to both P_{aV} and P_{aH}

By inspection:

- Strength I-a (minimum vertical and maximum horizontal loads) will govern for the case of sliding and overturning (eccentricity)

- For the case of bearing capacity (maximum), both Strength I-b and Strength IV must be evaluated

Step 3: Factored Loads and Factored Moments

Table 12-6
Factored Vertical Loads

Item	V (unfactored) (kN/m)	Strength I-a[1] (kN/m)	Strength I-b[1] (kN/m)	Strength IV[1] (kN/m)	Service I[1] (kN/m)
WC_1	13.2	11.9	16.5	19.8	13.2
WC_2	15.6	14.0	19.5	23.4	15.6
WC_3	27.0	24.3	33.7	40.5	27.0
WF_1[1]	15.4	12.3	16.6	16.6	12.3
WF_2[1]	26.5	21.2	28.6	28.6	21.2
WF_3[1]	56.6	45.3	61.1	61.1	45.3
Σ WF	98.5	78.8	106.3	106.3	78.8
P_{EV1}	3.7	3.7	5.0	5.0	3.7
P_{LSV}	12.6	22.0	22.0	0	12.6
P_{aV}	30.0	45.0	45.0	45.0	30.0
V_{TOT}	200.6	199.7	248.0	240.0	180.9

(1) 80% of the soil weight inside the modules is considered for the factored vertical loads.

Table 12-7
Factored Horizontal Loads

Item	H (unfactored) (kN/m)	Strength I-a (kN/m)	Strength I-b (kN/m)	Strength IV (kN/m)	Service I (kN/m)
P_{LSH}	20.7	36.2	36.2	0	20.7
P_{aH}	49.5	74.3	74.3	74.3	49.5
H_{TOT}	70.2	110.5	110.5	74.3	70.2

Table 12-8
Factored Moments from Vertical Forces (M_v)

Item	M (unfactored) (kN-m/m)	Strength I-a[1] (kN-m/m)	Strength I-b[1] (kN-m/m)	Strength IV[1] (kN-m/m)	Service I[1] (kN-m/m)
WC_1	15.9	14.3	19.9	23.8	15.9
WC_2	20.3	18.3	25.4	30.5	20.3
WC_3	36.5	32.9	45.6	54.8	36.5
WF_1	18.5	14.8	20.0	20.0	14.8
WF_2	34.6	27.7	37.4	37.4	27.7
WF_3	76.6	61.7	82.7	82.7	61.3
P_{EV1}	7.3	7.3	9.8	9.8	7.3
P_{LSV}	27.0	47.3	47.3	0	27.0
P_{aV}	67.3	100.9	100.9	100.9	67.3
M_{vTOT}	304.0	324.7	389.0	359.9	278.1

(1) 80% of the soil weight inside the modules is considered for the factored vertical loads.

Table 12-9
Factored Moments from Horizontal Forces (M_h)

Item	M (unfactored) (kN-m/m)	Strength I-a (kN-m/m)	Strength I-b (kN-m/m)	Strength IV (kN-m/m)	Service I (kN-m/m)
P_{LSH}	39.6	69.2	69.2	0	39.6
P_{aH}	56.4	84.7	84.7	84.7	56.4
M_{hTOT}	96.0	153.9	153.9	84.7	96.0

Step 4: Eccentricity

The eccentricity of the retaining wall is checked in Table 12-10 by comparing the calculated eccentricity, e, for each loading group to the maximum allowed eccentricity (e_{max}) of B/4 specified in AASHTO LRFD (1997a) (*A10.6.3.1.5*):

$$e = B/2 - X_o$$

where:

X_o = Location of the resultant from the toe=$(M_{vTOT} - M_{hTOT})/V_{TOT}$
B = 2.44 m (cos 9.462°) = 2.41 m

e_{max} = B/4 = 2.41 m/4 = 0.60 m

For each load group, the total vertical forces (V_{TOT}), horizontal forces (H_{TOT}), moments due to vertical forces (M_v) and moments due to horizontal forces (M_h) are obtained from Tables 12-6, 12-7, 12-8 and 12-9, respectively.

Table 12-10
Summary of Eccentricity Check

Group	V_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o (m)	e (m)	e_{max} (m)
Strength I-a	199.7	324.7	153.9	0.86	0.34	0.60
Strength I-b	248.0	389.0	153.9	0.95	0.26	0.60
Strength IV	240.0	359.9	84.7	1.15	0.06	0.60
Service I	180.9	278.1	96.0	1.01	0.20	0.60

For all cases, e is less than or equal to e_{max}; therefore, the design is adequate with regard to eccentricity.

Note that for ASD, e_{max} = B/6, or 0.40 m for a 2.41 m wide footing. Because ASD is equivalent to the Service I Limit State in Table 12-10, the design also meets the ASD eccentricity requirements.

Step 5: Bearing Resistance

(A) Estimate the Bearing Pressures

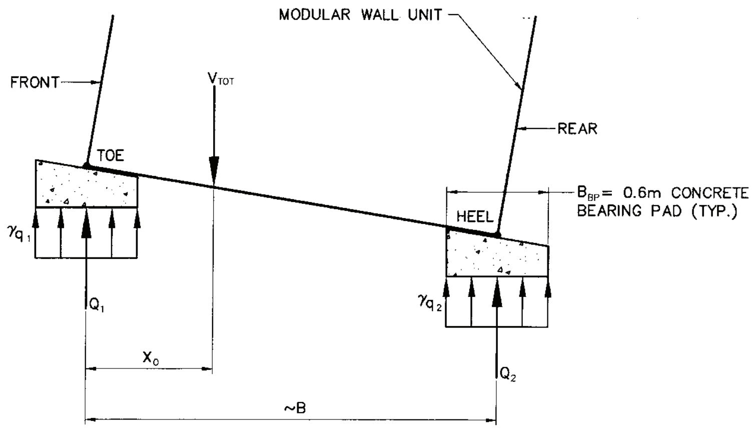
The adequacy for bearing resistance is developed based on a rectangular distribution of soil pressure over point supports (i.e., bearing pads) at the rear and front of the modules as indicated in Figure 12-4. In accordance with AASHTO (1997a) (*A11.10.3.3*) 80 percent of the soil weight within the modules is considered to be transferred to the front and rear support points or bearing pads. Therefore, the critical bearing case will be for the individual bearing pads rather than for the full base width of the wall.

For a rectangular distribution with the resultant bearing pressure located near the toe and heal respectively:

$$Q_1 = V_{TOT}\left(\frac{B - X_o}{B}\right)$$

$$Q_2 = V_{TOT} - Q_1 = V_{TOT}\, X_o/B$$

The uniform pressure will be distributed over the bearing pad width, B_{BP} = 0.6 m:



$q_{max} = q_1 = Q_1/B_{BP}$
$q_{min} = q_2 = Q_2/B_{BP}$

Figure 12-4
Bearing Pads at Rear and Front of Modules

Table 12-11
Summary of Maximum Bearing Pressures

Group	V_{TOT} (kN/m)	X_o (m)	Q_1 (kN/m)	Q_2 (kN/m)	γq_{max} (kPa)	γq_{min} (kPa)
Strength I-a	199.7	0.86	128	71	213	118
Strength I-b	248.0	0.95	150	98	250	163
Strength IV	240.0	1.15	125	115	208	192
Service I	180.9	1.01	105	76	175	127

(B) Evaluate Adequacy of Bearing Resistance
The factored bearing resistance, q_R, at the Strength Limit State is determined, based on LRFD (AASHTO, 1997a) using:

$q_R = \phi\ q_n = \phi\ q_{ult}$ (*A10.6.3.1.1-1*)
$q_{ult} = 0.5\ \gamma_s\ B\ C_{W1}\ N_{\gamma m} + \gamma_s\ C_{W2}\ D_f\ N_{qm}$ (*A10.6.3.1.2c-1*)

Values of C_{W1}, C_{W2}, $N_{\gamma m}$ and N_{qm} can be obtained using the equations and tables in the AASHTO LRFD Specification (*A10.6.3.1.2*). The resistance factor, ϕ, from Table 8-8 in Chapter 8 for bearing capacity of a spread footing on sand determined by the rational method using ϕ_f estimated from CPT results is 0.45. Therefore:

$$q_R = \phi\ q_{ult} = 0.45\ (0.5\ \gamma_s\ B\ C_{W1}\ N_{\gamma m} + \gamma_s\ C_{W2}\ D_f\ N_{qm})$$

$$q_R = 0.45\ [(0.5)(18.0\ \frac{kN}{m^3})(0.60\ m)(1.0)(25.0)+(18.0\frac{kN}{m^3})(1.0)(1.0\ m)(25.1)]$$

$$q_R = 0.45\ [135\ kPa + 451.8\ kPa] = 0.45\ [586.8\ kPa] = 264\ kPa$$

Because the factored bearing resistance, q_R, exceeds the maximum factored bearing stress, q_{max} = 250 kPa, the bearing resistance is adequate.

Relative to ASD, as represented by the Service I Limit State, the factor of safety against bearing capacity failure is q_{ult}/q_{max} = 586.8 kPa/175 kPa = 3.3. Therefore, the design is also acceptable with respect to bearing capacity by ASD.

Step 6: Sliding

For sliding at the bearing pad level, sliding of the pads and shear of the soil between the pads is checked. As per Step 5, 80 percent of the soil fill weight is assumed to be transferred to the bearing pads (*A11.10.3.3*). The remaining soil fill weight (20 percent) is assumed to be transferred directly to the foundation soil. Using this assumption, the sliding resistance is calculated using Eq. 8-15 as follows:

$$Q_R = \phi_\tau\ Q_\tau + \phi_{ep}\ Q_{ep}$$

where:

ϕ_τ = 0.80 for cast-in-place concrete on sand (Table 8-8, Chapter 8)
ϕ_τ = 1.00 for soil on soil (Table 8-8, Chapter 8)
ϕ_{ep} = 0.5 (Table 8-8, Chapter 8)
Q_τ = Nominal shear resistance between footing and foundation material (kN)
Q_{ep} = Nominal passive resistance of foundation material available throughout the design life of the footing (kN)(assumed equal to 0)

Q_τ will have units of force per unit length of wall when using the vertical forces from Table 12-6.

For individual bearing pads, the sliding resistance is computed as follows:

$$Q_\tau = V \tan \delta \qquad (A10.6.3.3\text{-}2)$$

For sliding of footings on soil:

$$Q_{\tau 1} = V_{TOT} \tan \delta$$

For sliding of soil on soil:

$$Q_{\tau 2} = \gamma_{EV} \, (0.2 \, \Sigma \, WF) \tan \phi_f$$

where:

$\delta =$ ϕ_f for cast-in-place footings
$V_{TOT} =$ Total factored vertical load including 80% of module backfill weight (kN/m)
$\phi_f =$ Angle of internal friction of foundation soil (deg) = 35°
$\gamma_{EV} =$ Load factor for earth load (dim)
$\Sigma WF =$ Total unfactored weight of module backfill (kN/m) = 98.5 kN/m (Table 12-6)

Therefore, the total sliding resistance, Q_τ, is:

$$Q_\tau = Q_{\tau 1} + Q_{\tau 2}$$

and the factored sliding resistance, Q_R, is:

$$Q_R = \phi_{\tau 1} \, Q_{\tau 1} + \phi_{\tau 2} \, Q_{\tau 2}$$
$$Q_R = \phi_{\tau 1} \, (V_{TOT} \tan \phi_f) + \phi_{\tau 2} \, [\gamma_{EV} \, (0.2 \, \Sigma \, WF) \tan \phi_f]$$

For the Strength Limit States:

$$Q_R = 0.8 \, (V_{TOT} \tan 35°) + 1.0 \, [\gamma_{EV} \, (0.2 \, \Sigma \, WF) \tan 35°]$$
$$Q_R = [(0.8 \, V_{TOT}) + \gamma_{EV} \, (0.2 \, \Sigma \, WF)] \tan 35°$$

For the Service Limit State ($\phi_\tau = 1.0$):

$$Q_R = (V_{TOT} + 0.2 \, \Sigma \, WF) \tan 35°$$

Table 12-12
Summary of Sliding Resistance

Group	γ_{EV}	V_{TOT} (kN/m)	γ_{EV} (0.2 Σ WF) (kN/m)	$\phi_\tau \, Q_\tau$ (kN/m)	H_{TOT} (kN/m)
Strength I-a	1.00	199.7	19.7	125.6	110.5
Strength I-b	1.35	248.0	26.6	157.6	110.5
Strength IV	1.35	240.0	26.6	153.1	74.3
Service I	1.00	180.9	19.7	140.4	70.2

Because the factored sliding resistance (ϕ_τ Q_τ) calculated is greater than the factored horizontal loading for all strength limit states, the design is acceptable by LRFD. By ASD (as represented by the Service I Limit State), the factor of safety against sliding = Q_τ/H_{TOT} = 140.4 kN/m / 70.2 kN/m = 2.0, which is acceptable ($\geq$ 1.5 as indicated in Table 8-1 of Chapter 8).

Summary

Table 12-13
Summary of Spread Footing Design by LRFD and ASD

Performance Limit	LRFD			ASD		
	Factored Resistance/ Eccentricity Limit	Factored Load/ Eccentricity	✔	Required FS/ Eccentricity Limit	Actual FS/ Eccentricity	✔
Eccentricity	0.25 B	0.14 B	✔	0.167 B	0.08 B	✔
Bearing Resistance	264 kPa	250 kPa	✔	3.0	3.3	✔
Sliding Resistance	158 kN/m	110 kN/m	✔	1.5	2.0	✔

The design of a prefabricated modular wall, as illustrated in this example problem, essentially involves the design of a geometry (i.e., base width) and bearing pads which satisfy the eccentricity, bearing resistance and sliding resistance criteria described in Chapter 8 for spread footings. As discussed in chapter 8, the resistance factors for bearing and sliding were developed from a reliability-based calibration, whereas the eccentricity were calibrated directly to ASD.

As summarized in Table 12-13, an acceptable design is achieved in this example by both LRFD and ASD. *Whereas the ASD factors of safety, FS, for bearing and sliding resistance are fixed, however, the LRFD resistance factors could possibly be increased with additional data accumulation and reliability-based calibration for similar soils and loading conditions.* ***It is possible therefore, that with further calibration, more reliable and more economical designs may be achieved using LRFD.***

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CHAPTER 13
ANCHORED WALL DESIGN

13.1 Introduction

For both Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD), the design of anchored retaining walls requires consideration of geotechnical capacity of anchorages and embedment of vertical wall elements, structural capacity of anchorages, vertical wall elements and facings, overall stability and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between anchored wall design by LRFD and ASD

- Identifies the strength and serviceability performance limits which must be considered for anchored wall design by LRFD

- Presents an example of an anchored retaining wall design by LRFD

13.2 Design Methods

With the exception of differences in foundation design noted in Chapters 8, 9 and 10, the procedure for design of anchored retaining walls using LRFD (A11.8) is identical to that followed using ASD. The geotechnical aspects of anchored wall design include evaluation of anchor pullout resistance, the bearing and passive resistance of the vertical wall elements, overall stability and foundation and wall movements. In addition, the structural capacity of the anchors, vertical wall elements and facing (temporary and permanent) must also be evaluated. The following sections present general design considerations and summarize the general design processes for anchored wall design using the ASD and LRFD approaches.

13.2.1 General Design Considerations

As shown in Figure 13-1, anchored walls are comprised of anchors constructed using prestressing strand or steel bars, discrete vertical wall elements (i.e., usually rolled steel sections) spaced approximately 2 to 3 m along the length of the wall, and facing (i.e., timber or precast concrete lagging or shotcrete, and cast-in-place concrete for some permanent walls). In some instances, typically in urban environments, the vertical wall element consists of continuous reinforced concrete panels constructed using diaphragm wall methods which rely on a bentonite or polymer slurry to provide temporary support during excavation of the panel. As the excavation is made, anchors are installed into drilled holes and extend through the wall face to a bearing plate supported on the face of the vertical wall element or a wale between adjacent vertical wall elements.

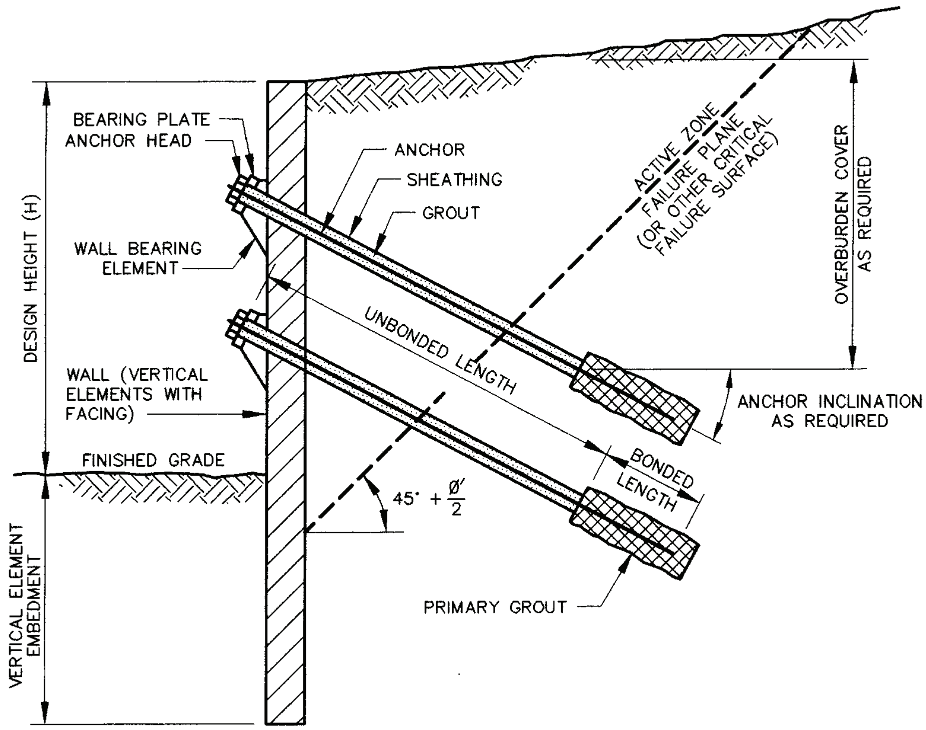


Figure 13-1 (*A11.8.1-1*)
Anchored Wall Nomenclature

Because most anchored walls are constructed from the top down, wall construction proceeds by:

- Installing discrete vertical wall elements by driving or concreting into a predrilled hole
- Excavating to a level of between 2 m to 4 m to install the upper level of anchors, and installing facing support, as required
- Drilling the anchor hole at a typical angle of about 10^{o} to 30^{o} to the horizontal, then installing and grouting the anchor in the bond length
- Stressing and testing each anchor, locking off the anchor at the design load, and if the anchor has grease and sheathing corrosion protection, grouting the unbonded length
- Repeating the excavation, anchor hole drilling, installation, grouting and testing, and facing installation for each anchor level to the base of the wall

- Constructing a wall facing in front of the wall for permanent construction

The geometry of wall construction shown in Figure 13-1 is typical for most anchored walls. The length of anchor is designed to provide an adequate bond length to resist loads on the wall and to extend a reasonable distance beyond the critical failure surface. Although the inclination of anchors is usually limited to a maximum of about 30^o to the horizontal to minimize axial loads on the vertical wall elements, steeper inclinations can be used to avoid interference with buried structures or to reach more suitable materials which provide a higher bond capacity. The vertical spacing of anchors can be optimized to provide a balanced design between the number of anchors and bending of the vertical wall element.

Additional detail relative to the design, installation and field testing of anchors and anchored walls is provided in Cheney (1988).

13.2.2 ASD Summary

Existing practice for geotechnical design of anchored walls follows the ASD approach (AASHTO, 1997b), wherein all uncertainty in the variation of applied loads transferred to the wall elements and the ultimate geotechnical capacity of the soil and rock to support the loads are incorporated in a factor of safety, FS. Structural design of anchors, vertical wall elements and facing is based on allowable stresses, σ_{all}, which account for uncertainty in the variation in applied loads and structural capacity. As a result, loads used for design, Q, consist of those actual forces estimated to be applied directly to the structure. In LRFD terminology, this process is equivalent to applying a load factor of 1.0 to the estimated forces. In ASD, six primary performance and failure conditions are evaluated in the design of an anchored wall:

- Settlement and lateral movement
- Pullout capacity of anchors (i.e., soil/grout and grout/tendon bonds)
- Bearing capacity of vertical wall elements
- Passive resistance of vertical wall elements
- Overall stability
- Structural capacity of anchors, vertical wall elements and wall facing

The design of anchored walls by ASD can be controlled by deformation or settlement considerations. Thus, the design of anchored walls by ASD requires estimation of foundation settlement and wall movements under the applied loads, and comparison of estimated settlement and wall movement with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 13-1)}$$

where:

δ_i = Estimated displacement or differential displacement (mm)
δ_n = Tolerable displacement or differential displacement established by the designer (mm)

The actual wall movements will be controlled primarily by the stiffness of wall and anchorages, the engineering characteristics of the supported ground, and the quality control exercised during construction. Because highway structures are not typically supported by anchored walls, tolerable movement criteria are usually a function of the proximity of the wall to deformation-sensitive structures. However for walls supporting an abutment, designs may be governed by criteria for lateral deflection or tilt as described in Chapter 11. The basis for estimation of ground movements behind anchored walls and criteria for evaluating their performance are discussed in Section 13.3.1.

Following the preliminary selection of the spacing of anchors to support the apparent earth pressure distribution assumed for design, the ultimate pullout capacity of anchors, R_n, can be estimated as described in Section 13.3, and the bearing and lateral capacity of vertical wall elements, R_n, can be estimated by available theoretical or semi-empirical procedures as described in Chapters 8 and 9. The suitability of the design with respect to anchor pullout and to bearing and lateral capacity are then evaluated by determining the allowable load components, Q_{all}, using:

$$Q \leq Q_{all} = R_n / FS = Q_{ult} / FS \qquad \text{(Eq. 13-2)}$$

where:

Q = Design load (kN)
Q_{all} = Allowable design load (kN)
R_n = Q_{ult} = Ultimate geotechnical capacity of the foundation element (kN)
FS = Factor of safety (dim)

The required FS with respect to bearing capacity and sliding are generally specified by the governing agency, and may be constant or variable. The AASHTO ASD (AASHTO, 1997b) and FHWA (Cheney, 1988) FS criteria for anchor pullout, bearing capacity and passive lateral capacity of vertical wall elements, and overall stability are shown in Table 13-1. The AASHTO and FHWA FS values assume that designs are based on soil and rock properties determined through appropriate field and laboratory testing and not presumptive properties.

Table 13-1
Factors of Safety for Anchored Walls
(AASHTO, 1997b; Cheney, 1988)

Failure Condition	Required Minimum Factor of Safety (FS)	
	AASHTO	FHWA
Anchor Pullout • Soil • Rock	 2.5 3.0	 2.5 3.0
Bearing Capacity	Refer to Chapters 8, 9 and 10	
Lateral (Passive) Capacity	1.5	1.5
Overall Stability	Refer to Chapter 8	

The overall stability of the retained earth and foundation must also be evaluated using procedures described in Section 8.2.1.

The structural capacity of the anchors, vertical wall elements and facing is checked as follows:

$$Q \leq \sigma_{all} \text{ x } A = P_{all} \qquad \text{(Eq. 13-3)}$$

where:

σ_{all} = Allowable tensile or flexural stress (kPa)

P_{all} = Allowable tensile or flexural structural capacity (kN)

A = Cross-sectional area of structural element (m^2)

AASHTO (1997b) and FHWA (Cheney, 1988) indicate maximum allowable anchor loads, P_{all}, as indicated in Table 13-2.

Table 13-2
Maximum Allowable Tensile Loads for Ground Anchors
(AASHTO, 1997b; Cheney, 1988)

Anchor Type	Allowable Load (1)
Temporary(2)	$0.80\ P_n$ (2)(3)
Permanent	$0.60\ P_n$ (2)

(1) P_n = Guaranteed Ultimate Tensile resistance (GUTS) of anchor (kN)
(2) Implied by Division II anchor stressing and load testing requirements (i.e., to 1.33Q) and assuming that temporary stressing to 80% GUTS is permissible.
(3) For acceptance testing only.

Alternatively, the structural design of the vertical wall elements and facing may be performed by LFD (AASHTO, 1997b) or LRFD (AASHTO, 1997a) or ACI.

13.2.3 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. ***When properly developed and applied, this approach can provide a consistent level of safety for the design of all structure components.*** Thus, the probability that a structure component will fail or perform unacceptably is no different than any other component. As described on Section 4.3, the resistance and deformation of supporting soil and rock materials and structure components must satisfy the LRFD equations below. For the Strength Limit States:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 13-4) } \textit{(A1.3.2.1-1)}$$

For the Service Limit States:

$$\sum \eta_i \gamma_i \delta_i \leq \phi \delta_n \qquad \text{(Eq. 13-5)}$$

where:

η_i = Factors to account for effects of ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)
δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement (mm)

Relative to the external stability of the wall and the structural adequacy of system components, the suitability of an anchored wall with respect to the geotechnical and structural resistance can be obtained using Eq. 13-4, rewritten as:

$$\sum \eta_i \gamma_i Q_i \leq \phi Q_{ult} = Q_R \qquad \text{(Eq. 13-6)}$$

and

$$\sum \eta_i \gamma_i Q_i \leq \phi Q_i = Q_i \qquad \text{(Eq. 13-7)}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Factored load effect (kN)
ϕ = Resistance factor (dim)
Q_{ult} = Ultimate geotechnical resistance (kN)
Q_R = Factored geotechnical resistance (kN)
P_n = P_{ult} = Nominal (ultimate) structural resistance of vertical wall elements, anchors, facing and connections (kN)
P_r = Factored structural resistance of vertical wall elements, anchors, facing and connections(kN)

The load factors and load factor combinations used for design were presented in Chapter 4. In general, values of $\gamma_i > 1.0$ are used to evaluate ultimate ground or structure capacity at the Strength Limit States, whereas the deformation performance of structures is evaluated at the Service I Limit State using $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads). In ASD (AASHTO, 1997b), values of $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads) are used to evaluate structures for both strength (allowable stress) and serviceability (deflection). ***Because the AASHTO LRFD Specification for anchored walls was developed based on direct calibration with AASHTO ASD, the results of analyses by LRFD and ASD to evaluate external wall stability and the structural adequacy of wall components are similar, and for foundation and wall deformations are identical.***

When using Eq. 13-4 for anchored wall design at the Strength Limit States, the following values of η can normally be used:

- $\eta_D = \eta_R = 1.00$
- $\eta_I =$ 1.05 for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures.

For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

When using Eq. 13-5 to evaluate an anchored retaining wall at any Service Limit State, η_D, η_R, and $\eta_I = 1.0$.

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4, and loads should be developed as described in Chapter 4. The ultimate resistance, R_n, should be determined for each type of resistance (e.g., bearing criteria, sliding, overall stability, and anchor pullout and structural adequacy) as described in Section 13.3.

Values of $\phi \leq 1.0$ are applied when evaluating geotechnical or structural resistance for any strength limit state using Eq. 13.4. Currently the value of $\phi = 1.0$ is applied when evaluating an anchored retaining wall for any service limit state using Eq. 13-5. Selection and modification of resistance factors, ϕ, are described in Sections 13.2.3.2 and 13.2.3.4.

13.2.3.1 Limit States (*A11.5*)

In general, the design of anchored retaining walls using LRFD requires evaluation of foundation suitability at various Performance Limit States (i.e., applicable Strength Limit States and the Service I Limit State). The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for a typical anchored retaining wall designed at the Strength and Service I Limit States are summarized in Table 13-3. As conditions warrant, it may also be necessary to evaluate wall performance at other limit states (e.g., Extreme Event I for loading from earthquakes).

Table 13-3
Strength and Service Limit States for Design of Anchored Walls

Design Consideration	Strength Limit State(s)	Service I Limit State
Anchor Pullout	✔	
Structural Resistance of Anchor	✔	
Structural Resistance of Vertical Wall Elements	✔	
Passive and Bearing Resistance of Vertical Wall Elements	✔	
Structural Resistance of Facing	✔	
Overall Stability	✔	
Foundation and Wall Movement		✔

Additional guidance for the design of anchored walls is provided in Cheney (1988).

13.2.3.2 Resistance Factors (*A11.5.6*)

Resistance factors for the design of anchored walls are presented in Table 13-4. The resistance factors for passive resistance of vertical wall elements, anchor pullout and structural design of anchors were developed based on engineering judgment and calibration with current allowable stress design procedures for anchored walls. The provisions of Sections 5, 6 and/or 8 of the AASHTO LRFD Specification (1997a) should be followed in evaluating the structural resistance of wall components.

The variation of ϕ-factors for geotechnical resistance represents the judged relative reliability of the different methods of estimating resistance.

13.2.3.3 Comparison of Anchored Wall Design Using LRFD and ASD

13.2.3.3.1 Geotechnical Design

To illustrate the relative differences between LRFD and ASD, the equivalent LRFD factor of safety (FS_{LRFD}) has been determined for each of the methods presented in Table 13-4 for estimating the geotechnical capacity of passive resistance of embedded vertical wall elements and the pullout capacity of anchors. As presented in Table 13-5, FS_{LRFD} was determined as:

$$FS_{LRFD} = \bar{\gamma} / \phi \qquad \text{(Eq. 13-8)}$$

where:

$\bar{\gamma}$ = Average load factor; and
ϕ = Resistance factor from Table 13-4.

The ASD factor of safety (FS_{ASD}) from Table 13-1 for each category is also presented in Table 13-5.

Table 13-4 (*A11.5.6-1*)
Resistance Factors for Anchored Walls
(AASHTO, 1997a)

Wall Type and Condition		Resistance Factor [1]
Bearing Resistance of Vertical Wall Elements		Refer to Chapter 8, 9 or 10
Passive Resistance of Vertical Wall Elements		
	• Soil	0.60
	• Rock	0.60
Pullout Resistance of Anchors	• Soil	
	– Correlations w/ SPT	0.65
	– Pullout Load Tests	0.70
	• Clay (Correlations w/ S_u)	
	– From lab tests	0.65
	– From field vane shear tests	0.65
	– From pullout load tests	0.70
	• Rock	
	– Correlations w/ rock type only	0.55
	– C_o from lab tests – soft rock only	0.60
	– Laboratory rock-grout bond tests	0.75
	– Pullout load tests	0.80
Tensile Resistance of Anchors	• Temporary	0.90
	• Permanent	1.00

(1) Refer to Section 13.3 for description of design procedures for which resistance factors have been calibrated.

Values of FS_{LRFD} in Table 13-5 were determined assuming $\gamma = 1.35$ and 1.50, which represent at-rest and active earth pressure, respectively. Actually, γ_i will typically range from 1.50 for active earth pressure to about 1.75 for live load surcharge so that FS_{LRFD} will normally be slightly higher than the values shown in the table depending on the relative proportion of live to earth load for a particular structure. In general, the larger the load factor (i.e., the greater the proportion of live load surcharge), the higher the equivalent factor of safety.

The LRFD resistance factors for anchor pullout, although calibrated to ASD, have been adjusted to consider reliability of the basis for the anchor resistance estimate. Higher resistance factors are assigned where more data on soil/rock strength and/or anchor pullout resistance are available. Therefore, as indicated by the values of FS_{LRFD} and FS_{ASD} in Table 13-5, the LRFD design would be somewhat less conservative than ASD with respect to anchor pullout.

Relative to passive resistance of embedded vertical wall elements, the LRFD resistance factors were calibrated to a higher FS (2.5) than currently used in ASD (1.5) and, therefore, provide a more conservative design than ASD.

Table 13-5
Comparison of FS_{ASD} and FS_{LRFD} for Anchored Walls

Method/Soil/Condition		Resistance Factor ϕ	Load Factor[1] γ	FS_{LRFD}[1]	FS_{ASD}[2]
Passive Resistance of Vertical Wall Elements	Soil	0.60	1.35 1.50	2.2 2.5	1.5 1.5
	Rock	0.60	1.35 1.50	2.2 2.5	1.5 1.5
Pullout Resistance of Anchors	Sand: • Correlation w/ SPT	0.65	1.35 1.50	2.1 2.3	2.5 2.5
	• Pullout load tests	0.70	1.35 1.50	1.9 2.1	2.5 2.5
	Clay: •Correlation w/ S_u from lab tests	0.65	1.35 1.50	2.1 2.3	2.5 2.5
	•Correlation w/ S_u from vane tests	0.65	1.35 1.50	2.1 2.3	2.5 2.5
	•Pullout load tests	0.70	1.35 1.50	1.9 2.1	2.5 2.5
Pullout Resistance of Anchors	Rock: •Correlation with rock type only	0.55	1.35 1.50	2.5 2.7	3.0 3.0
	•C_o from lab tests - soft rock only	0.60	1.35 1.50	2.3 2.5	3.0 3.0
	•Laboratory rock-grout bond tests	0.75	1.35 1.50	1.8 2.0	3.0 3.0
	• Pullout load tests	0.80	1.35 1.50	1.7 1.9	3.0 3.0

(1) γ = 1.35 for at-rest earth pressure and 1.50 for active earth pressure; (2) From Table 13-1

13.2.3.3.2 Structural Design

In ASD, the ultimate unit stress or ultimate load capacity for each material type is reduced to an allowable value by application of a modifier to account for uncertainty in resistance and applied load. The allowable tensile loads for ground anchors recommended by AASHTO (1996b) and FHWA (Cheney, 1988) are listed in Table 13-2.

To illustrate the relative differences between LRFD and ASD with respect to anchor tensile capacity, the equivalent LRFD allowable load has been determined for each of the loading conditions identified in Table 13-4. As presented in Table 13-6, $P_{all(LRFD)}$ was determined as:

$$P_{all(LRFD)} = \frac{\phi}{\gamma} P_n \quad \text{(Eq. 13-9)}$$

where:

$P_{all(LRFD)}$ = Equivalent allowable anchor tensile load (kN)
ϕ = Resistance factor from Table 13-4 (dim)
$\bar{\gamma}$ = Average load factor (dim)
P_n = Ultimate anchor tensile resistance or Guaranteed Ultimate Tensile Strength (GUTS) (kN)

The FHWA allowable anchor tensile loads for each loading condition from Table 13-1 are also presented in Table 13-6, as is the ASD allowable stress for flexural resistance of steel beams (AASHTO, 1997b).

Table 13-6
Comparison of ASD Allowable Load/Stress with the LRFD Equivalent Load/Stress for Structural Design of Ground Anchors

Loading Condition		Resistance Factor ϕ	Load Factor [(1)] γ	LRFD Equivalent Allowable Load/Stress $P_{all(LRFD)}/\sigma_{all(LRFD)}$	ASD Allowable Load/Stress P_{all}/σ_{all}	
					AASHTO	FHWA
Tensile Resistance of Anchors	• Temporary	1.0	1.35	0.74 P_n	0.80 P_n	0.80 P_n
			1.50	0.67 P_n	0.80 P_n	0.80 P_n
	• Permanent	0.9	1.35	0.67 P_n	0.60 P_n	0.60 P_n
			1.50	0.60 P_n	0.60 P_n	0.60 P_n

(1) γ = 1.35 for at-rest earth pressure and 1.50 for active earth pressure.

As described in Section 13.2.3.3.1 with respect to geotechnical design, the actual average load factor will typically range between 1.50 (for active earth pressure) to somewhat less than 1.75 (for a large live load surcharge), such that the actual LRFD equivalent allowable loads and stresses will be somewhat lower (i.e., more conservative) than the values listed in Table 13-6 for active earth pressures.

Other aspects of anchored wall design such as identifying special considerations (e.g., settlement of soil behind the wall, sequence of construction, developing a design foundation and retained soil profile and determining requirements for construction control) are inherent aspects of the design process required for both LRFD and ASD.

13.2.3.4 Modification of Resistance Factors

Resistance factors for anchored wall design using LRFD were developed based on direct calibration with ASD. Therefore, resistance factors for various aspects of anchored wall design can be modified using the procedures summarized in Chapter 7.

13.2.3.4.1 Geotechnical Design

As stated in Section 13.2.3.2, the LRFD resistance factors for geotechnical design of anchored walls in Table 13-4 were developed based on direct calibration with ASD. As shown in Table 13-5, application of the resistance factors results in "equivalent" safety factors of 1.9 to 2.7 for anchor pullout 2.5 and for passive resistance of embedded vertical wall elements for walls supporting active soil pressure (i.e., $\gamma = 1.50$). For walls subjected to live load surcharge, the "equivalent" factor of safety would be somewhat higher.

In ASD, the designer or owner might decide to increase or decrease required factors of safety or allowable design stresses in consideration of a number of factors, such as:

- The potential consequences of a failure
- The extent or quality of information available from geotechnical exploration and testing
- Past experience with the soil conditions encountered and/or capacity prediction method used
- The level of construction control anticipated or specified
- The likelihood that the design loading conditions will be realized

When using LRFD, similar flexibility to vary the required level of safety should also be available. Additionally, whereas the same factor of safety is generally used in ASD regardless of the source of loading, the equivalent factor of safety in LRFD (defined by Eq. 13-8) varies for a given resistance factor depending on the source of loading, such as described previously with respect to live load surcharge.

To modify the resistance factors for geotechnical design of piles to account for average load factors and equivalent factors of safety other than those identified in Table 13-5, the following equation may be used:

$$\phi_m = \phi_T \text{ x } \left(\frac{FS_T}{FS_D}\right) \text{ x } \left(\frac{\gamma_T}{\overline{\gamma}_D}\right) \qquad \text{(Eq. 13-10)}$$

where:

ϕ_m = Modified resistance factor (dim)
ϕ_T = Tabulated resistance factor from Table 13-4 or 13-5 (dim)
FS_T = Tabulated factor of safety from Table 13-5 (dim)
FS_D = Desired factor of safety (dim)
γ_T = Load factor from Table 13-5 (dim)
$\overline{\gamma}_D$ = Actual average load factor including modification for operational importance (dim)

13.2.3.4.2 Structural Design

As for geotechnical design, the resistance factors provided in Table 13-4 for structural design of anchors and vertical wall elements were developed using direct calibration with ASD.

To modify the resistance factors for structural design of anchors to account for average load factors, $\bar{\gamma}$, other than 1.35 or 1.50 and "equivalent" allowable loads or stresses other than those presented in Table 13-6, the following equations may be used:

$$\phi_m = \phi_T \text{ x} \left(\frac{\sigma_{allA}}{\sigma_{allT}}\right) \text{x} \left(\frac{\bar{\bar{\gamma}}_T}{\gamma_D}\right) \qquad \text{(Eq. 13-11)}$$

or

$$\phi_m = \phi_T \text{ x} \left(\frac{P_{allA}}{P_{allT}}\right) \text{x} \left(\frac{\bar{\bar{\gamma}}_T}{\gamma_D}\right) \qquad \text{(Eq. 13-12)}$$

where:

ϕ_T = Resistance factor from Table 13-6 (dim)
σ_{allA} = Actual allowable stress (kPa)
σ_{allT} = Allowable stress from Table 13-6 (kPa)
P_{allA} = Actual allowable load (kN)
P_{allT} = Allowable load (kN)

Modifying resistance factors may seem reasonable, but such modification may not be consistent with the goal of LRFD to achieve equal reliability against failure of structure components, unless the factor of safety accurately models the reliability of the predictive method used.

The resistance factors may be more appropriately modified through application of the probabilistic procedures described in Chapter 3 to achieve the desired level of reliability if a sufficient amount of data is available.

13.2.4 Summarized Comparison of ASD and LRFD

As noted before, the process used to develop an anchored wall design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 13-2. Specific differences between the methods and other important issues are highlighted in Section 13.3.

13.3 Performance Limits

13.3.1 Displacements and Tolerable Movement Criteria (*A11.8.3*, *A3.11.5.6*)

The displacement of anchored walls must be evaluated at the Service I Limit State for all applic-

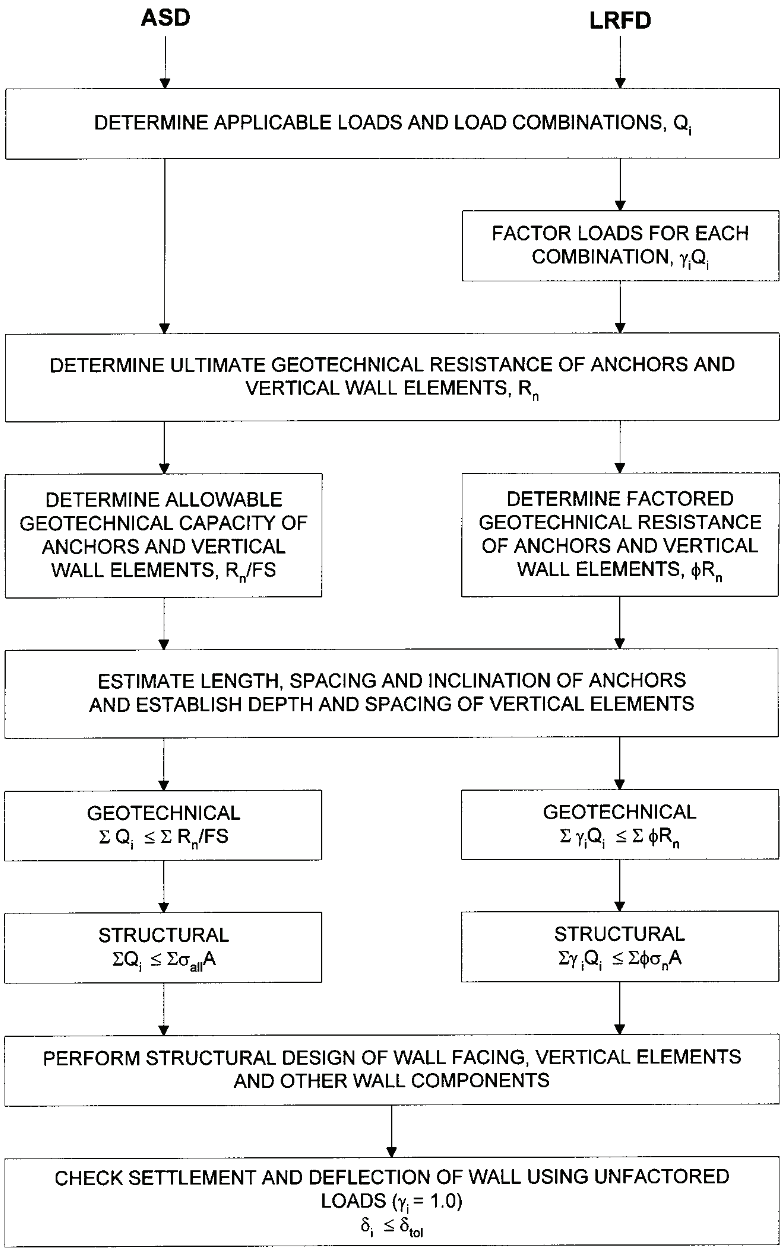
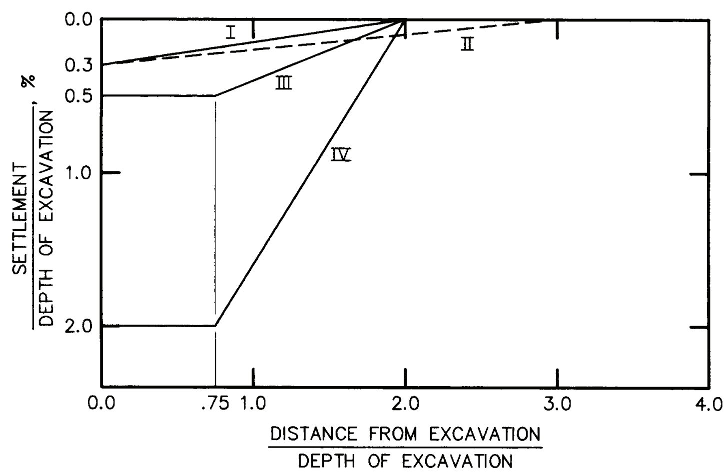


Figure 13-2
Generalized Flow Chart for Anchored Wall Design by ASD and LRFD

able load combinations. ***Because evaluations of structure displacements by LRFD are made at the Service I Limit State where $\gamma = 1.0$ and $\phi = 1.0$, methods used to estimate settlement and lateral displacement by LRFD are identical to those used for ASD.*** The vertical and lateral displacement of anchored walls is a complex soil-structure interaction problem, and deformation analyses can be performed using modified forms of beam on elastic foundation theory or finite element analyses. For many projects, however, such analyses are not warranted unless deformation-sensitive structures are in close proximity to the wall. Alternatively, for anchored walls built using proven wall construction technology, Figure 13-3 can be used as a guide to estimate ground surface settlement behind the walls. The corresponding maximum lateral movement for walls supporting sand and stiff to hard clay soils usually does not exceed about 0.3 percent of the depth of excavation. Figure 13-3 does not consider the effects of other construction activities (e.g., dewatering, ground heave at the wall base, or poor construction quality).



CURVE I = Sand

CURVE II = Stiff to very hard clay

CURVE III = Soft to medium clay, factor of Safety against basal heave $\left(= \frac{5.1\ S_u}{\gamma H + q}\right)$ Equal to 2.0

CURVE IV = Soft to medium clay, factor of Safety against basal heave $\left(= \frac{5.1\ S_u}{\gamma H + q}\right)$ Equal to 1.2

Figure 13-3 (*AC3.11.5.6-1*)
Settlement Profiles Behind Anchored Walls
(Modified after Clough and O'Rourke 1990)

Because anchored walls are not typically used to support highway superstructures, the tolerable movement of the wall should be developed based on consideration of the effects of possible wall movements on other structures and facilities near the wall.

13.3.2 Anchor Pullout (*A11.8.4.2*)

Prestressed anchors used for anchored wall construction must be designed to resist pullout of the bonded length of anchor in soil or rock. The minimum bonded length, L, is estimated as:

$$L \geq \sum \gamma_i Q_i / \phi Q_a \qquad \text{(Eq. 13-13)}$$

where:

L =	Bond Length (m)
$\sum \gamma_i Q_i$ =	Summation of factored anchor loads from contributory area (kN)
ϕ =	Resistance factor in Table 13-4 (dim)
Q_a =	Ultimate unit resistance between grout/soil/rock in anchor bond zone (kN/m)

The factored anchor load is developed using an apparent earth pressure distribution, which depends on the soils to be supported as described in Chapter 4, and various assumptions regarding the distribution of load from the tributary area of wall face. In the proportional method, the top anchor row is assumed to support the tributary area of pressure between the top of the wall and the midpoint between the upper two anchor levels; and the bottom anchor row is assumed to support the pressure between the base of the wall and the midpoint between the two lowest anchor rows. Alternately, the embedded portion of the vertical wall element can be assumed to support the pressure between the base of the exposed wall and the midpoint between the base and the lowest anchor row. Intermediate anchor rows are assumed to support the pressure from the midway points between vertically adjacent anchor rows and pairs of vertical wall elements.

As described in FHWA research reports (Cheney, 1988; Weatherby, 1982), various procedures are available for estimating the ultimate unit geotechnical resistance of ground anchors in soil and rock using semi-empirical correlations or in-situ testing. The LRFD Specification, for which resistance factors in Table 13-4 were developed, uses the following methods:

- Cohesionless Soils - Based on soil type and compactness with reference to SPT (Cheney, 1988) or based on field pullout load tests.

- Cohesive Soils - Based on soil type and stiffness with reference to undrained shear strength (Cheney, 1988) or based on field pullout load tests.

- Rock - Based on correlation with rock type (Cheney, 1988) or rock shear strength testing (Weatherby, 1982), or based on laboratory rock-grout bond tests or field pullout load tests.

Tables 13-7 and 13-8 provide conservative values of Q_a for anchors in soil and rock, respectively. These values are intended for preliminary design or evaluation of the feasibility of straight shaft anchors installed in small-diameter holes which are grouted using low pressures. Pressure-grouted anchors usually achieve higher capacities. Because anchor capacity in soil and rock can be strongly influenced by the method of anchor hole advancement, hole diameter, anchor type, type of grout and grouting pressure, selection of the anchor type and determination of Q_a for final design should be made by the specialty geotechnical contractor selected for wall construction.

Table 13-7 (*A11.8.4.2-1*)
Ultimate Unit Resistance of Anchors in Soil
(AASHTO, 1997a; AASHTO, 1997b; Cheney, 1988)

Soil Type	Compactness or SPT Resistance (Blows per 0.30 m)		Ultimate Unit Anchor Resistance, Q_a (kN/m)
Sand and Gravel	Loose	4-10	145
	Medium	10-30	220
	Dense	30-50	290
Sand	Loose	4-10	100
	Medium	10-30	145
	Dense	30-50	190
Sand and Silt	Loose	4-10	75
	Medium	10-30	100
	Dense	30-50	130
Soil Type	**Unconfined Compressive Strength, 2 S_u (kPa)**		**Ultimate Unit Anchor Resistance Q_a (kN/m)**
Silt-Clay Mixture	Stiff	100 - 240	30
	Hard	240 – 380	60

Table 13-8 (*A11.8.4.2-2*)
Ultimate Unit Resistance of Anchors in Rock
(AASHTO, 1997a; AASHTO, 1997b; Cheney, 1988)

Rock Type	Ultimate Unit Anchor Resistance, Q_a (kN/m)
Granite or Basalt	730
Dolomitic Limestone	585
Soft Limestone/Sandstone	440
Slate and Hard Shale	365
Soft Shales	145

13.3.3 Bearing Resistance (Vertical Wall Element)

The bearing resistance of continuous vertical wall elements (e.g., concrete diaphragm walls) can be evaluated as discussed in Chapter 8 for spread footing foundations. The bearing resistance of discrete vertical wall elements (e.g., soldier piles) can be evaluated as discussed in Chapter 9 or 10 for driven pile or drilled shaft foundations. In either case, the back face of the exposed wall is assumed to be frictionless so that the base of the vertical element is designed to support the vertical component of the factored anchor force and the factored weight of the wall face.

13.3.4 Passive Resistance (Vertical Wall Element) (*A11.8.4.4*)

The passive resistance of continuous vertical wall elements should be evaluated as discussed in Chapter 8 for sliding resistance of deeply-embedded footings. The passive resistance of discrete vertical wall elements should be evaluated using classical earth pressure theory. For the resistance of laterally loaded pile foundations (Goldberg, et al., 1975), it is assumed that the passive resistance acts over a width equal to three times the width of the embedded portion of the vertical wall element.

13.3.5 Overall Stability (*A11.8.4.3*)

The overall stability of anchored wall should be evaluated as described in Section 8.5.3. In performance of the slope stability analyses, consideration should be given to using a method of analysis (e.g., STABL5; Carpenter, 1986) that includes the effects of anchor forces in assessing the resistance of a slope to instability.

13.3.6 Structural Resistance (*A11.8.5*)

13.3.6.1 Vertical Wall Elements (*A11.8.5.2*)

Discrete vertical wall elements must be designed to resist all applicable earth and water pressure, surcharge, anchor and seismic loadings, and the vertical component of the anchor loads and other vertical loads within the tributary area between adjacent vertical wall elements. In designing these elements, fixed horizontal support can be assumed at each anchor level and at the bottom of the wall if the elements are sufficiently embedded below the base of the wall.

Unless beam on elastic foundation, finite element or other methods of soil-structure interaction analysis are used, the maximum bending moment, M_{max}, in the vertical wall element may be determined using the proportional method to compute anchor forces and wall element stresses. For the wall section above the top anchor row, the vertical wall element is designed as a cantilever and M_{max} is determined as:

$$M_{max} = 0.5\,pLx^2 \qquad \text{(Eq. 13-14) } (\textit{AC11.8.5.2-1})$$

For wall sections between rows, the vertical wall element is designed as a simply-supported beam and M_{max} is determined as:

$$M_{max} = 0.125\,pLx^2 \qquad \text{(Eq. 13-15) } (\textit{AC11.8.5.2-2})$$

If the wall is not embedded or the lateral resistance of the embedded length is neglected, the vertical wall element below the lowest anchor level can be designed as a cantilever and M_{max} can be determined using Eq. 13-14. If the embedded portion of the vertical wall element is assumed to provide support and embedment is not required to resist unbalanced forces due to base instability, M_{max} can be determined for the section of wall below the lowest anchor row of anchors using:

$$M_{max} = 0.333\,pLx^2 \qquad \text{(Eq. 13-16) } (\textit{AC11.8.5.2-3})$$

For sections of vertical wall elements sections of vertical wall elements spanning three or more equally-spaced anchor levels, the vertical wall element can be designed as continuous beams and

M_{max} can be determined as:

$$M_{max} = 0.100\ pLx^2$$ (Eq. 13-17) (*AC11.8.5.2-4*)

where:

M_{max} = Factored maximum flexural moment (kN-m)
p = Average factored lateral pressure carried by vertical wall facing (kPa)
L = Horizontal spacing between vertical wall elements (m)
x = Height of vertical element between anchors or supports (m)

M_{max} is considered to be the positive design moment between anchors and the negative design moment at the anchors. If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

13.3.6.2 Permanent Facing (*A11.8.5.3*)

The maximum spacing between vertical wall elements should be determined based on the relative stiffness of the vertical elements and the type and condition of soil to be supported. The horizontal spacing between vertical elements typically varies between 2 to 3 meters. If timber facing is specified, the timber should be stress-grade pressure treated.

Facing can be designed assuming simple support between elements, with or without soil arching, or assuming the facing behaves as a continuous support across several elements. Based on these assumptions, the value of the maximum factored flexural moment, M_{max}, on a unit (1 meter) width or height of facing may be determined as:

- For simple spans without soil arching:

$$M_{max} = 0.125\ pL^2$$ (Eq. 13-18) (*AC11.8.5.3-1*)

- Simple span (soil arching):

$$M_{max} = 0.083\ pL^2$$ (Eq. 13-19) (*AC11.8.5.3-2*)

- Continuous:

$$M_{max} = 0.100\ pL^2$$ (Eq. 13-20) (*AC11.8.5.3-3*)

where:

M_{max} = Factored maximum flexural moment on facing (kN-m/m)
p = Average factored lateral pressure acting on wall facing (kPa)
L = Horizontal spacing between vertical wall elements (m)

If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

Eq. 13-18 is applicable for simply supported facing behind which the soil will not arch between vertical supports (e.g., in soft cohesive soils or for rigid concrete facing placed tightly against the in-place soil). Eq. 13-19 is applicable for simply supported facing behind which the soil will arch between vertical supports (e.g., in granular or stiff cohesive soils with flexible facing or rigid facing behind which there is sufficient space to permit the in-place soil to arch). Eq. 13-20 is applicable for facing which is continuous over several vertical supports (e.g., reinforced shotcrete).

Temporary timber lagging thickness is generally based on empirical relationships with excavation depth (Goldberg, et al., 1975) rather than on a rigorous structural design.

13.2.7 Other Considerations

13.3.7.1 Corrosion (*A11.8.7*)

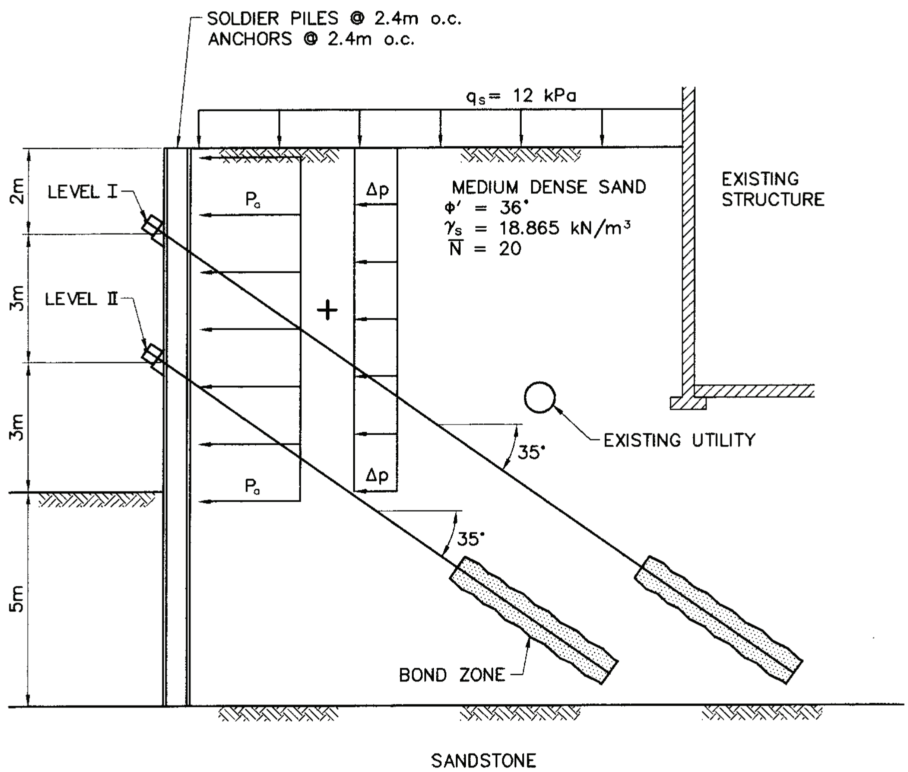
Except for temporary walls, the designer must provide adequate protection for anchor and anchor hardware against aggressive ground or the presence of stray currents which can corrode and lead to loss of service life for the primary resistance element of anchored walls. Typically, corrosion resistance for anchors is provided by encapsulation by electrostatically-applied, resin-bonded epoxy coatings, grease and grout. For anchor hardware, protection is provided by electrostatically-applied, resin-bonded epoxy coatings or by use of a sacrificial thickness.

13.3.7.2 Drainage (*A11.8.9*)

As with other wall types, the lateral load on an anchored wall is affected by the shear strength and unit weight of the retained ground, and by the presence of water in the retained ground. Because anchored walls are typically constructed from the top down and support in-situ soil and rock materials however, the designer should consider installing drainage features between the ground and the facing to minimize permanent water loads on the wall and to preclude uncontrolled seepage through the permanent wall face. For anchored soldier pile and lagging walls, geocomposite drains can be installed between the ground the facing to intercept and convey seepage to drainage control structures. For anchored diaphragm walls, seepage can be controlled by water stops between panels and continuous pours of high-quality, flowable concrete.

13.4 Student Problem:
Anchored Soldier Pile Wall Design by LRFD

Problem: You are to design an anchored retaining wall by LRFD. Figure 13-4 shows the problem geometry, in which a cut-and-cover excavation is required for construction of a roadway tunnel. The final excavation depth will be 8 m and the excavation will be supported by a soldier pile and timber lagging wall incorporating two levels of anchors. The high water table is below the bottom of the excavation, and the vehicular live load surcharge (LS = q_S) on the backfill is applied as shown in the figure.



γ = Unit weight of retained soil (kN/m^3)
q_s = Vehicular live load surcharge (kPa)
p_a = Lateral earth pressure (kPa)
Δp = Lateral earth pressure due to vehicular live load (kPa)

Figure 13-4
Schematic of Example Problem

During the subsurface exploration, it was determined that the foundation soils consist of medium dense sand to a depth of 13 m below the ground surface, underlain by hard, sandstone bedrock. In performing the wall design, you can assume the following:

- The anchor derive their capacity wholly within the medium dense sand

- Only the end of excavation stage needs to be checked for this problem (Note: For a complete design, each stage of the excavation should be checked)

- The high water table will be below the bedrock surface

Objective: To demonstrate the procedure for anchored retaining wall design by LRFD

Approach: To perform the anchor wall design, you should take the following steps:

- Compute and tabulate the unfactored loads and moments required for design at the applicable limits states
- Determine and tabulate the factored loads and moments required for design at the applicable limits states
- Estimate wall movements at the Service I Limit State
- Evaluate the global stability of the excavation
- Evaluate the earth pressures on the excavation support system
- Estimate the required anchor bond zone length
- Determine the required soldier pile section using the applicable factored loads and simplified structural analysis procedures
- Determine the required section for the timber lagging
- Evaluate the vertical stability of the wall

Solution:

Step 1: Calculate Loads

The first step in determining the loading conditions is to assess the earth pressures on the wall. The apparent earth pressure diagram shown in Figure 13-4 is selected in accordance with recommendations made by Peck, et. al (1974) and AASHTO (1997a, 1997b) for wall systems with multiple levels of support in cohesionless soils (*A3.11.5.6*). A live load surcharge of 12 kPa (equivalent to a uniform soil surcharge of about 0.63 m) is used for design. The calculations for development of the lateral earth pressure and lateral pressure due to vehicular live load surcharge are shown below.

<u>Unfactored Apparent Earth Pressure (EH):</u>

$P_a = 0.65\ k_a\ \gamma_s\ H = 0.65\ \tan^2(45° - \phi_f/2)\ \gamma_s\ H$ (*A3.11.5.6-1*)
$P_a = 0.65\ \tan^2(45° -$ ____ $°/2)$ (______ kN/m^3) (____ m)
$P_a =$ _______kPa

<u>Unfactored Surcharge Pressure (LS):</u>

$\Delta p = k_a\ q_s = \tan^2(45° -$ ____ $°/2)$ (_____) kPa = _______kPa (*A3.11.6.1-1*)

The apparent earth pressure and surcharge pressure diagrams are then distributed to the various members of the system assuming tributary areas extending equidistant between the support levels as shown below.

The unfactored horizontal force to be supported at Level I is the sum of the apparent earth and vehicular surcharge lateral pressures from the ground surface down to a depth of 3.5 m (z_I = 3.5 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:
$P_1 = P_a z_I$ = (_____ kPa)(____ m) = ______ kN/m

Unfactored horizontal load from vehicular surcharge lateral pressure:
$P_2 = \Delta p\ z_I$ = (____ kPa)(____ m) = ______ kN/m

Total unfactored horizontal load:
$P_I = P_1 + P_2$ = ______ kN/m + ______ kN/m = ______ kN/m

The load factors from Chapter 4 (Tables 4-10 and 4-11) for horizontal earth pressure, EH, and live load surcharge, LS, are:

γ_{EH} = ______
γ_{LS} = ______

Therefore, the total factored horizontal load is:

$\gamma P_I = \gamma_{EH} P_1 + \gamma_{LS} P_2$ = (_____ kN/m) + (______kN/m) = _____ kN/m

The unfactored horizontal force to be supported at Level II is the sum of the apparent earth and vehicular surcharge lateral pressures from a depth of 3.5 m to a depth of 6.5 m (z_{II} = 3.0 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:
$P_1 = P_a z_{II}$ = (_____ kPa)(____ m) = _____ kN/m
Unfactored horizontal load from vehicular surcharge lateral pressure:
$P_2 = \Delta p\ z_{II}$ = (_____ kPa)(____ m) = _____ kN/m

Total unfactored horizontal load:
$P_{II} = P_1 + P_2$ = _____ kN/m + _____ kN/m = _____ kN/m

Applying the load factors for EH and LS, the total factored horizontal load is:

$\gamma P_{II} = \gamma_{EH} P_1 + \gamma_{LS} P_2$ =____ (_____ kN/m) + ____ (_____ kN/m) =_____ kN/m

The unfactored horizontal force to be supported at the base of the wall for design of the soldier pile embedment is the sum of the apparent earth and vehicular surcharge lateral pressures from a depth of 6.5 m to the base of the wall (z_{III} = 1.5 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:
$P_1 = P_a z_{III}$ = (_____ kPa)(____ m) = _____ kN/m

Unfactored horizontal load from vehicular surcharge lateral pressure:
$P_2 = \Delta p\ z_{III}$ = (_____ kPa)(____ m) = _____ kN/m

Total unfactored horizontal load:
$P_{III} = P_1 + P_2$ = _____ kN/m + _____ kN/m = _____ kN/m

Applying the load factors for EH and LS, the total factored horizontal load at the wall base is:

$\gamma P_{III} = \gamma_{EH} P_1 + \gamma_{LS} P_2$ =____ (_____ kN/m) + ____ (_____ kN/m) =_____ kN/m

These calculations are summarized in the table below.

Load	Unfactored EH (kN/m)	Unfactored LS (kN/m)	Unfactored Horizontal Load (kN/m)	Factored Horizontal Load (kN/m)
Level I Anchor				
Level II Anchor				
Excavation Base				

In addition to anchor design and soldier pile embedment, it is necessary to consider load distribution and load factoring for structural design of the soldier piles and timber lagging. The distribution and factoring of the apparent earth pressure and vehicular surcharge lateral pressures to the anchors provides the necessary loading information for analysis of the vertical stability of each pile.

Step 2: Estimate Lateral Wall Deflection and Settlement Profile Behind Wall

Figure 13-3 shows the typical settlement profiles behind a wall of this type assuming good construction practices are followed. For this problem, it is evident from Curve I in Figure 13-3 that the maximum settlement at the wall will be about 0.3 percent of the excavation depth or 0.024 m. The lateral wall deformations should be similar in magnitude to this settlement. It is assumed that this level of deformation is acceptable for this problem.

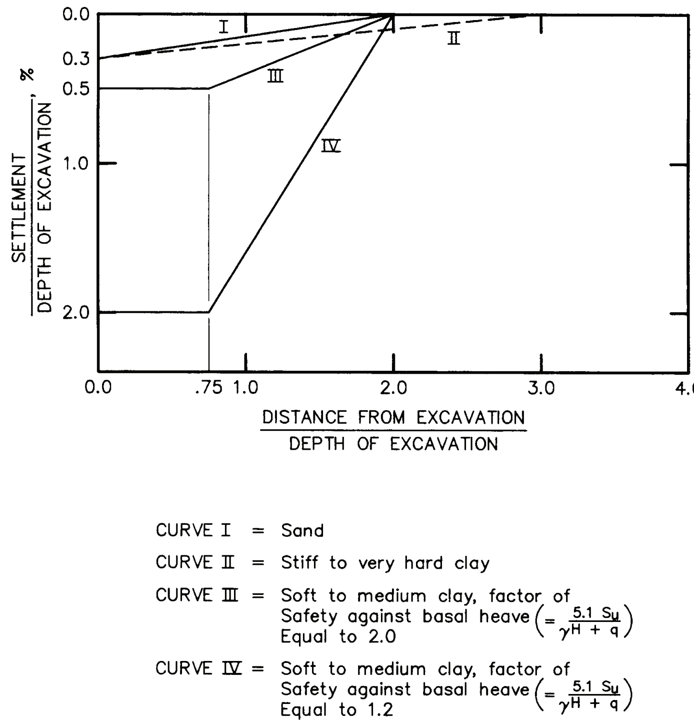


Figure 13-3 (*AC3.11.5.6-1*)
Settlement Profiles Behind Anchored Walls
(Modified after Clough and O'Rourke 1990)

Step 3: Global Stability

An analysis of the global stability of the excavation is not presented herein, but would be checked using a limit equilibrium analysis (*A11.8.4.3*) and is assumed to be adequate for this example.

Step 4: Preliminary Geotechnical Design of Anchors

As with ASD, design of an anchored wall includes preliminary estimation of anchor capacity to permit selection of a reasonable anchor spacing and design of facing and other structural components of the wall. For this problem, you can assume that the anchors will develop their capacity within the medium dense sand deposit. In accordance with AASHTO (1997a) and FHWA (Cheney, 1988) recommendations presumptive ultimate anchor capacity, Q_a, as provided in Table 13-7 for estimating the length of the bond zone in sand for each anchor. The bond zone lengths are estimated as shown in the following calculations. The adequacy of anchor capacity is confirmed during construction in the same manner as for ASD, by means of proof and performance tests. For the purposes of this example, the structural capacity of the anchor strands is assumed to be adequate.

Table 13-7 (*A11.8.4.2-1 Excerpt*)
Ultimate Unit Resistance of Anchors in Soil

(AASHTO, 1997a; AASHTO, 1997b; Cheney, 1988)

Soil Type	Compactness or SPT Resistance (Blows per 0.30 m)		Ultimate Unit Anchor Resistance, Q_a (kN/m)
Sand	Loose	4-10	100
	Medium	**10-30**	**145**
	Dense	30-50	190

Anchor Level I
Using the anchor inclination, i, of 35° from the horizontal, unfactored and factored horizontal loads for Anchor Level I from Step 1, the total unfactored and factored anchor loads for a spacing, s, of 2.4 meters along the length of the wall between anchors are:

$Q = P_{aI}$ s/(cos i) = (_____ kN/m)(____ m)/(cos ___°) = ______ kN
γ Q = γ P_I s/(cos i) = (_____ kN/m)(____ m)/(cos ___°) = ______ kN

From Table 13-4, the resistance factor for anchor pullout in sand based on correlation with Standard Penetration Test is:

ϕ = _______

For preliminary design, the ultimate anchor capacity, Q_a, for estimation of anchor bond length is:

Q_a = _____ kN/m (from Table 13-7 for medium dense sand)

Using Eq. 13-13, the estimated anchor bond length, L, is:

$L \geq \gamma Q/\phi Q_a$ (*Eq. 13-13*)
L = _____ kN/(____)(_____ kN/m) = _____ m

Anchor Level II
Using the anchor inclination, i, of 35° from the horizontal, the unfactored and factored horizontal loads for Anchor Level II from Step 1, the total unfactored and factored anchor loads for a spacing, s, of 2.4 meters along the length of the wall between anchors are:

$Q = P_{aII}$ s/(cos i) = (_____ kN/m)(____ m)/(cos ___°) = ______ kN
γ Q = γ P_{II} s/(cos i) = (_____ kN/m)(____ m)/(cos ___°) = ______ kN

Then using Eq. 13-13, the estimated anchor bond length, L, is:

$L \geq \gamma Q/\phi Q_a$ (*Eq. 13-13*)

L = _____ kN/(____)(_____ kN/m) = _____ m

The table below summarizes the bond zone lengths for Anchors at Levels I and II.

Anchor Level	LRFD Bond Zone Length (m)
Level I	
Level II	

Step 5: Soldier Pile Embedment

The passive resistance of the embedded section of soldier pile must resist the lateral earth load at the excavation base. From Step 1, the unfactored and factored horizontal loads at the excavation base are 42.9 kN/m and 65.5 kN/m. For a pile spacing, s, of 2.4 m, the total unfactored and factored loads on each embedded pile section are:

$$Q = P_{aIII}\ s = (42.9 \text{ kN/m})(2.4 \text{ m}) = 103.0 \text{ kN}$$
$$\gamma\ Q = \gamma\ P_{III}\ s = (65.5 \text{ kN/m})(2.4 \text{ m}) = 157.2 \text{ kN}$$

As depicted in Figure 13-5, the net passive resistance of each embedded pile is obtained assuming that passive pressure on the front of the pile acts over a width equal to three times the pile flange width and that the active pressure on the back of the pile acts over the actual pile flange width.

From Step 1, the active earth pressure coefficient is:

$$k_a = \tan^2 (45° - \phi/2) = \tan^2 (45° - 36°/2) = 0.26$$

(Note that $k_a = 0.26$ is also obtained using AASHTO Eq. A3.11.5.3-1 and A3.11.5.3-2 for a vertical wall, level backfill and no wall friction.)

The passive earth pressure coefficient, k_p, for a vertical wall, horizontal ground surface and no wall friction ($\delta = 0°$) and $\phi_f = 36°$ is obtained from AASHTO (1997a) Figure A3.11.5.4-1 as:

$$k_p = (0.36)(11.6) = 4.18$$

From Table 13-4, the resistance factor, ϕ, for passive resistance of embedded vertical wall elements in soil is 0.60.

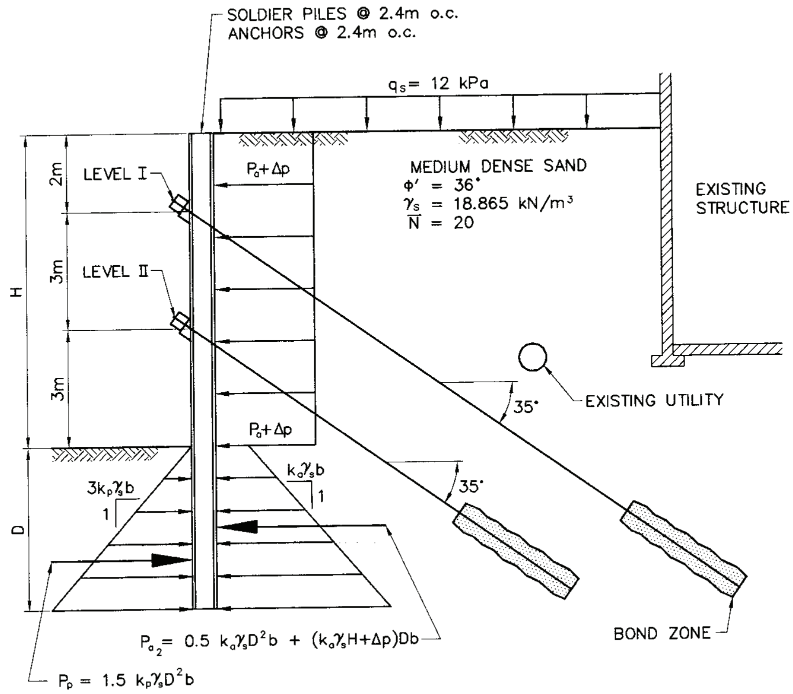


Figure 13-5
Lateral Earth Pressure Diagram
for Soldier Pile Embedment into Cohesionless Soil

From Step 1, the load factor for active earth pressure, γ_{EH}, is 1.50 and for live load surcharge, γ_{LS}, is 1.75. For a 310 mm HP soldier pile (b = 0.31 m), the factored active pressure from Figure 13-5 is:

$$\gamma\ P_{a2} = \gamma_{EH}\ 0.5\ k_a\ \gamma_s\ D^2\ b + (\gamma_{EH}\ k_a\ \gamma_s\ H + \gamma_{LS}\ \Delta p)\ D\ b$$
$$\gamma\ P_{a2} = 1.5(0.5)(0.26)(18.865\ kN/m^3)D^2(0.31\ m)$$
$$+ [1.5(0.26)(18.865\ kN/m^3)(8\ m) + 1.75(3.1\ kPa)]D(0.31\ m)$$
$$\gamma\ P_{a2} = 1.14\ D^2 + 19.9\ D$$

and the factored passive pressure from Figure 13-5 is:

$$\phi\ P_p = \phi\ (1.5\ k_p\ \gamma_s\ D^2\ b)$$
$$\phi\ P_p = 0.60\ (1.5)(4.18)(18.865\ kN/m^3)(D^2)(0.31\ m)$$
$$\phi\ P_p = 22.0\ D^2$$

Each soldier pile must resist the factored lateral load, γ Q, computed as the unit factored horizontal load at the excavation base from Step 1 times the pile spacing, s, of 2.4 m so that:

$$\gamma\ Q = (65.5\ kN/m)(2.4\ m) = 157.2\ kN$$

Equating the net factored passive resistance of the embedded vertical element to the load at the excavation base:

$$\gamma\, Q = \phi\, P_p - \gamma\, P_{a2}$$
$$157.2 \text{ kN} = 22.0\, D^2 - 1.14\, D^2 - 19.9\, D$$

or

$$0 = D^2 - 0.95\, D - 7.54$$

from which the required depth of embedment is:

$$D = 3.3 \text{ m}$$

Step 6: Vertical Stability of Wall

The soldier piles must support the vertical component of anchor load by side and base resistance of the soldier pile within the depth of embedment.

Tip Resistance

The nominal unit tip resistance, q_p, of a pile in cohesionless soil is:

$$q_p = \frac{38\, N_{corr}\, D_b}{D} < q_\ell \qquad (A10.7.3.4.2a\text{-}1)$$

for which:

$$N_{corr} = 0.77 \log_{10}\left(\frac{1920}{\sigma'_v}\right) N \qquad (A10.7.3.4.2b\text{-}1)$$

and:

N_{corr} = Representative SPT blow count near the pile tip corrected for σ'_v (blows/0.3 m)
N = Measured SPT blow count (blows/0.3 m)
D = Pile width or diameter (m)
D_b = Depth of penetration in bearing stratum (m)
q_ℓ = Limiting tip resistance taken as 400 N_{corr} for sands and 300 N_{corr} for silt (kPa)
σ'_v = Effective overburden pressure at pile tip (kPa)

For a pile tip at 3.3 m below the excavation base (or 3.3 m + 8.0 = 11.3 m) below the original ground surface:

$$N_{corr} = 0.77 \log_{10}\left(\frac{1920}{(18.865 \text{ kN/m}^2)(11.3 \text{ m})}\right) 20 = 14.7$$

and

$$q_p = \frac{(38)(14.7)(3.3\,\mathrm{m})}{0.31\,\mathrm{m}} = 5946\,\mathrm{kPa} < q_\ell = (400)(14.7) = 5880\,\mathrm{kPa}$$

Use q_p = 5880 kPa

For a typical pile tip plug area of 0.30 m $\times$ 0.31 m = 0.093 m^2 for an HP 310 pile section, the nominal bearing resistance for a 3.3 m embedment is:

$Q_p = q_p A_p$ (Eq. 9-19)
Q_p = (_____ kPa)(_____ m^2) = _____ kN

Side Resistance
The nominal unit side resistance, q_s, of an H-pile in cohesionless soil is:

$$q_s = 0.96\ \overline{N}$$

where:

q_s = Unit skin friction for driven piles (kPa)
$\overline{N}$ = Average (uncorrected) SPT blow count along pile shaft (blows/0.3 m)

from which:

$$q_s = 0.96\ (20) = 19.2\ \mathrm{kPa}$$

For a typical pile perimeter area of 2 (0.30 m) + 2 (0.31 m) = 1.22 m^2/m for an HP 310 pile section, the nominal side resistance for a 3.3 m embedment is:

$Q_s = q_s A_s$ (Eq. 9-20)
Q_s = (_____ kPa)(_____ m^2/m)(_____ m) = _____ kN

Factored Bearing Resistance
The total factored bearing resistance of each soldier pile is:

$$Q_R = \phi\ Q_n = \phi_{qp}\ Q_p + \phi_{qs}\ Q_s \qquad \text{(Eq. 9-18)}$$

The resistance factor for a single pile for capacity estimation based directly on SPT results is obtained from Table 9-5 in Chapter 9 as:

$$\phi_{qp} = \phi_{qs} = \phi =$$

from which:

Q_R = ____ (_____ kN) + ____ (____ kN)
Q_R = _____ kN

Factored Load
The total factored vertical load on an individual soldier pile is calculated as follows:

Factored vertical load:

Level I factored vertical load, $\gamma\ V_I$ = (____ kN/m) (tan ___°) (___ m) = _____ kN
Level II factored vertical load, $\gamma\ V_{II}$ = (____ kN/m) (tan ___°) (___ m) = _____ kN
Total factored vertical load, $\gamma\ V = \gamma\ V_I + \gamma\ V_{II}$= ____ kN + ____ kN = ____ kN

For the final LRFD check, the factored resistance effect, Q_R, must be greater than or equal to the factored load effect, γ Q using Eq. 13-7, as shown below.

$\gamma\ Q \leq Q_R$
_____ kN $\leq$ or > _____ kN   OK or NG (Select One)

Therefore, a soldier pile embedment of 3.3 m is insufficient to adequately resist the vertical components of the anchor forces. By inspection, the additional skin friction available through the remaining 1.7 m depth of soil would not increase the resistance sufficiently to resist the anchor forces. Therefore, drive soldier piles to the top of rock at a depth of 5 m below the excavation base. For the purpose of this example, it is presumed that the pile bearing resistance on the sandstone bedrock is adequate.

Step 7: Soldier Pile Structural Design

The soldier piles considered in this problem are subjected to flexure due to the imposed earth pressures and to axial load from the two levels of anchors. Therefore, the structural analysis of these soldier piles must consider the interaction effects of combined axial load and flexure. As specified in the LRFD Specification (AASHTO, 1997a) Article A6.9.2.2, Eq. A6.9.2.2-2 applies for $P_u/P_r \geq 0.2$:

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1 \qquad (\textit{A6.9.2.2-2})$$

where:

P_u = Factored axial load (kN)
P_r = Factored compressive resistance (kN)
M_{ux} = Factored flexural moment about the x axis (kN-m)
M_{rx} = Factored flexural resistance about the x axis (kN-m)
M_{uy} = Factored flexural moment about the y axis (kN-m)
M_{ry} = Factored flexural resistance about the y axis (kN-m)

The AASHTO LRFD Specification (1997a) provides simplified design equations for flexural analysis of the vertical supporting elements dependent upon the average factored lateral pressure, distance between vertical elements and height of the section of vertical element being considered. Because the load and resistance factors in LRFD are calibrated for this approach, this example problem uses these simplified design equations. Using these simplified equations, the maximum moment, M_{max}, will occur below the bottommost row of anchors as:

$$M_{max} = 0.333 \text{ p L x}^2 \qquad (\textit{AC11.8.5.2-3})$$

where:

p = Average lateral pressure (kPa)
L = Spacing between vertical elements (m)
x = Height of the section for the vertical element being considered (m)

In the AASHTO LRFD Specification (1997a), the average lateral pressure is taken as the average factored lateral pressure and the maximum flexural moment is the maximum factored flexural moment. The factored maximum moment below the Level II anchors is:

$$M_{max} = 0.333 \text{ p L x}^2 \qquad (\textit{AC11.8.5.2-3})$$

where:

$$p = (1.5)(25.5 \text{ kPa}) + (1.75)(3.1 \text{ kPa}) = 43.7 \text{ kPa}$$
$$M_{max} = 0.333 \text{ (43.7 kPa)(2.4 m)(3 m)}^2 = 314.3 \text{ kN-m}$$

From Section 6 of the AASHTO LRFD Specification (Steel Structures), structural design of the soldier piles must also include consideration of the interaction effects from combined axial load and flexure. From the calculations in Step 6, the factored axial load is 477.0 kN.

From the AASHTO LRFD Specification (1997a), Article 6.5.4.2, the resistance factors for flexure and compression in LRFD are:

$\phi_f = 1.00$ (flexure)
$\phi_c = 0.90$ (compression)

First trying an HP 310 $\times$ 94 pile, the nominal moment capacity, M_n, of the HP soldier piles is determined from the AASHTO LRFD Specification (1997a) Articles 6.12.2.2.1, 6.10.6.2 and 6.10.5.2.1-1 and Metric Properties of Structural Shapes (AISC, 1992) as:

$$M_n = M_p$$
$$M_n = Z_x F_y$$
$$M_n = (14.5 \times 10^{-4} \text{ m}^3)(250\,000 \text{ kPa})$$
$$M_n = 362.5 \text{ kN-m}$$

where:

M_p = Plastic Moment resistance kN-m
Z_x = Plastic section modulus = 14.5×10^{-4} m^3
F_y = Steel yield stress = 250 000 kPa

Therefore:

$$M_r = \varphi_f M_n = 1.00 \text{ (362.5 kN-m)} = 362.5 \text{ kN-m}$$

From AASHTO LRFD (1997a) Article 6.9.4.1, the nominal axial capacity, P_n, of the soldier pile is:

$$P_n = 0.66^{\lambda} F_y A_s$$

$$\lambda = \left(\frac{K \ell}{r_s \pi}\right)^2 \frac{F_y}{E} = 0.045$$

which is applicable for $\lambda \leq 2.25$, where λ is the normalized column slenderness factor defined as:
for:

A_s = Gross cross-sectional area = 11.9×10^{-3} m^2
F_y = Yield strength = 250 000 kPa
E = Modulus of elasticity = 200×10^6 kPa
K = Effective length factor = 0.8 (from Table *AC4.6.2.5-1*)
ℓ = Unbraced length = 3 m
r_s = Radius of gyration about the plane of buckling = 0.128 m

Through substitution back into the equation for the nominal axial capacity we find that:

$$P_n = 0.66^{0.045} F_y A_s = 0.98 F_y A_s = 0.98 \text{ (250 000 kPa)}(11.9 \times 10^{-3} \text{ m}^2) = 2916 \text{ kN}$$

Then the factored axial capacity, P_r is:

$$P_r = \phi_c P_n = 0.90 P_n = 0.90 \text{ (2916 kN)} = 2624 \text{ kN}$$

At this point, the axial resistance component of the interaction equation can be evaluated using:

$$\frac{P_u}{P_r} = \frac{477.0 \text{ kN}}{2624 \text{ kN}} = 0.18 < 0.2$$

For this problem, it is reasonable to expect that there will be no significant bending in the plane of the wall and therefore we can assume that M_{uy} is zero. The factored flexural resistance about the x axis is:

$$M_{rx} = \phi M_n = (1.00) \text{ 362.5 kN-m} = 362.5 \text{ kN-m}$$

Therefore, the moment resistance component of the interaction equation is:

$$\frac{M_{ux}}{M_{rx}} = \frac{314.3\ \text{kN-m}}{362.5\ \text{kN-m}} = 0.87$$

Substituting into the interaction equation, we find the following:

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = 0.18 + \frac{8}{9}(0.87) = 0.95 \le 1 \qquad (A6.9.2.2\text{-}2)$$

Therefore, this soldier pile provides a section which is adequate for the design loading conditions for LRFD. In addition to this combined stress check, the slenderness of the soldier beam should be checked to assure that buckling will not control design. For this example, slenderness is not a controlling factor.

Step 8: Timber Lagging Design

As for the structural design of the soldier piles, in this problem you can use the simplified design equations to select a timber lagging section. The maximum factored moment is:

$$M_{max} = 0.083\ p\ L^2 \qquad (AC11.8.5.3\text{-}2)$$

where:

p = Average factored lateral pressure = (1.50)(25.5 kPa) + (1.75)(3.1 kPa) = 43.7 kPa
L = Lagging span =2.4 m - 0.306 m = 2.094 m (Note: Pile flange width = 306 mm)

Substituting into Eq. AC11.8.5.3-2, the maximum factored moment is:

$$M_{max} = 0.083\ (43.7\ \text{kPa})(2.094\ \text{m})^2 = (15.9\ \text{kN-m/m})$$

The factored flexural capacity, M_r, of the lagging is determined by multiplying the resistance factor of 0.85 for flexure of wood from Article 8.5.2.2 of the AASHTO LRFD Specification (1997a) by the nominal flexural resistance, M_n, as follows:

$$M_r = \phi\ M_n = 0.85\ M_n \qquad (A8.6.1\text{-}1)$$

where the nominal flexural resistance of wood, M_n, from Article 8.6.2 of the AASHTO LRFD Specification (1997a) is:

$$M_n = F_b\ S\ C_s \qquad (A8.6.2\text{-}1)$$

and:

F_b = Specified nominal resistance in flexure = 29 000 kPa (*Table AC8.4.1.1.4-1*)
S = Section modulus (m^3)
C_s = Size effect factor (dim)

Evaluation of the size effect factor and the specified resistance in flexure both require calculations which are not presented herein, although the results shown are consistent with the material and geometry specified. For this problem, use Select Structural Southern Pine 102 mm thick and 254 mm wide for the timber lagging. For this material, F_b = 29 000 kPa, $S = 4.4 \times 10^{-4}$ m^3, C_s = 1.0 and the nominal flexural resistance of the lagging is:.

$$M_n = (29\ 000\ \text{kPa})(4.4 \times 10^{-4}\ \text{m}^3)(1.0) = 12.8\ \text{kN-m}$$

The factored flexural resistance, M_r, is then:

$$M_r = 0.85\ (12.8\ \text{kN-m}) = 10.8\ \text{kN-m}$$

The maximum factored moment on a 254 mm wide lagging section is:

$$M_{max} = (15.9\ \text{kN-m/m})\ (0.254\ \text{m}) = 4.0\ \text{kN-m}$$

Therefore, the lagging is acceptable.

As notted in Section 13.3.6.2, the thickness of timber lagging for temporary support is generally selected based on empirical relationships with excavation depth (Goldberg, et al., 1975). For an excavation depth of 8 m, a lagging thickness of 76 mm to 102 mm is prescribed for soldier piles spaced at 2.5 m in medium dense sand. It is apparent from the above calculation that thinner lagging would be acceptable.

Summary

- ***This example problem illustrates the design of an anchored soldier pile and lagging retaining wall by LRFD***

- ***The wall consists of driven steel H-piles supported by two rows of anchors and spanned using timber lagging***

- ***This example is intended to illustrate the LRFD process and procedures and may not represent the conditions the optimal design for the conditions identified* (*an actual design would be optimized to identify the most economical combination of pile size and spacing; anchor location, spacing and capacity; and lagging thickness*)**

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CHAPTER 14
MECHANICALLY-STABILIZED EARTH (MSE) WALL DESIGN

14.1 Introduction

For both Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD), the design of mechanically-stabilized earth (MSE) walls requires consideration of geotechnical capacity, overall stability, structural capacity and deformation limits. The design processes therefore require both establishment of criteria for acceptable stress and deformation levels, and comparison of these criteria with stress and deformation levels estimated from the design. This chapter:

- Describes primary differences between MSE wall design by LRFD and ASD

- Identifies the strength and serviceability performance limits which must be considered for MSE wall design by LRFD

- Briefly summarizes methods commonly used for evaluating the external and internal stability of MSE walls

- Present examples of MSE wall designs by LRFD

14.2 Design Methods

With few exceptions, the procedure for design of MSE walls using LRFD (A11.9) is identical to that followed using ASD. For MSE walls, the external stability of the wall, internal stability of wall components and foundation and wall movements are evaluated for the earth and external loads the wall must support. External stability considerations include bearing capacity, sliding, overturning and overall stability. Internal stability considerations include pullout and rupture of reinforcements, capacity of reinforcement connections to the wall face, and structural capacity of the wall facing. Settlement and lateral wall deformations must also be checked. The following sections summarize the general design processes for MSE wall design using the ASD and LRFD approaches.

14.2.1 General Design Considerations

Mechanically stabilized earth (MSE) walls are comprised of a reinforced soil mass, and a discrete modular precast concrete facing which is vertical or near vertical. The reinforced soil mass consists of select granular backfill. The tensile reinforcements and their connections may be proprietary, and may employ either metallic (i.e., strip- or grid-type) or polymeric (i.e., sheet-, strip-, or grid-type) reinforcement.

MSE walls may be used where conventional gravity, cantilever, or counterfort concrete retaining walls are considered, and they are particularly well suited where substantial total and differential settlements are anticipated.

The size of the reinforced soil mass is based on consideration of:

- Requirements for stability and geotechnical resistance as described for gravity walls

- Requirements for structural resistance within the reinforced soil mass and the panel units, and for the development of reinforcement extending beyond assumed failure zones

- Traditional requirements for a reinforcement length of not less than 70% of the wall height, or 2.4 m, whichever is greater

14.2.2 ASD Summary

Existing practice for geotechnical design of MSE walls follows the ASD approach, wherein all uncertainty in the variation of applied loads transferred to the foundation(s) and the ultimate geotechnical capacity of the soil and rock to support the loads are incorporated in a factor of safety, FS. Structural design of reinforcements is based on an allowable stress, σ_{all}, which accounts for uncertainty in the variation in applied loads and in the initial reinforcement strength, and for possible ultimate strength decreases which may occur due to corrosion aging, temperature changes, environmental conditions and construction damage. As a result, loads used for design, Q, consist of those actual forces estimated to be applied directly to the structure. In LRFD terminology, this process is equivalent to applying a load factor of 1.0 to the estimated forces. In ASD, eight primary performance and failure conditions are evaluated in the design of an MSE wall:

- Settlement and lateral movement
- Bearing capacity
- Location of base resultant force
- Sliding stability
- Overall stability
- Structural capacity of reinforcements and connections with wall facing
- Pullout of reinforcements
- Structural capacity of wall facing

The 1997 Interims to the AASHTO ASD Bridge Design Specifications (AASHTO, 1997b) incorporate a number of revisions to the design of MSE walls. The revisions are based primarily on the results of FHWA Demonstration Project 82, "Mechanically Stabilized Earth Walls and Reinforced Slopes Design and Construction Guidelines" (Elias and Christopher, 1996) and generally include the following:

1. *Revision of the specified variation of lateral earth pressure coefficient for internal stability evaluations (reinforcement tension and pullout) for all reinforcements.*

2. *Revision of internal stability calculations to include a simplified method for stress calculations which considers only overburden stress.*

3. *Revisions in the equations for pullout resistance using a unified normalized approach which considers internal loads and tensile/pullout resistance per unit width of wall.*

4. *Revised (less conservative) default values for strength reduction factors to account for installation damage (RF_{ID}), creep (RF_{CR}) and chemical/biological degradation (RF_D) of geosynthetic reinforcements in evaluation of reinforcement tension.*

5. *Inclusion of a requirement for evaluation of sliding resistance along the lowest reinforcement level when considering external stability of a wall with continuous or near continuous reinforcements.*

6. *The use of minimum average roll values (MARV) of ultimate strength from wide width tests for T_{ult}.*

7. *Elimination of the requirement for a serviceability limit state assessment of reinforcement tension for geosynthetic reinforcements.*

8. *Inclusion of requirements for evaluation of compound stability failure surfaces when considering overall external stability.*

The current AASHTO LRFD Specification for MSE walls (AASHTO, 1997a) is based on and calibrated to the former AASHTO ASD Specification (AASHTO, 1996). Therefore, discussions related to ASD criteria for MSE wall design in this chapter correspond to the AASHTO ASD Specifications and the FHWA recommendations on which the specifications are based. Efforts are currently underway as part of NCHRP 20-7, Task 88, to revise the AASHTO LRFD Specifications for MSE walls to reflect the changes in the 1997 AASHTO ASD Interim Specifications (AASHTO, 1997b) as developed based on the results of FHWA Demonstration Project 82 (Elias and Christopher, 1996), and to develop resistance factors for MSE wall design using a reliability-based calibration and the revised design procedures.

Deformation

The design of MSE walls by ASD can be controlled by deformation or settlement considerations. Thus, the design of MSE walls by ASD requires estimation of foundation settlement and wall movements under the applied loads, and comparison of estimated settlement and wall movement with deformation criteria using the following:

$$\delta_i \leq \delta_n \qquad \text{(Eq. 14-1)}$$

where:

δ_i = Estimated displacement or differential displacement (mm)

δ_n = Tolerable displacement or differential displacement established by the designer (mm)

The actual wall settlement will be controlled primarily by the wall height and foundation compressibility. Tolerable movement criteria are usually a function of the type of structure, and for abutments and other earth retaining structures, criteria for lateral deflection or tilt may govern the design, as described in Chapter 11. Acceptable vertical and horizontal wall displacement criteria for MSE walls are discussed in Section 14.3.1.

Geotechnical/Structural Resistance

Following selection of a preliminary width of the reinforced zone based on empirical relationships with the height of wall, the ultimate bearing capacity and sliding resistance, R_n, are estimated by available theoretical or semi-empirical procedures as described in Chapter 8. The suitability of the design with respect to bearing capacity and sliding are then evaluated by determining the allowable vertical and horizontal load components, Q_{all}, using:

$$Q \leq Q_{all} = \frac{R_n}{FS} \qquad \text{(Eq. 14-2)}$$

where:

Q = Design load (kN)
Q_{all} = Allowable design load (kN)
R_n = Ultimate geotechnical capacity of a foundation (kN)
FS = Factor of safety (dim)

The required FS with respect to bearing capacity and sliding are generally specified by the governing agency, and may be constant or variable. The AASHTO ASD (AASHTO, 1996) FS criteria for bearing capacity, sliding, overturning and overall stability are shown in Table 14-1. The AASHTO ASD FS values assume that designs are based on soil and rock properties determined through appropriate field and laboratory testing and not presumptive properties.

In addition to bearing capacity, sliding failure and overturning failure, the location of the bearing pressure resultant of MSE walls is checked with respect to the centroid of the reinforced soil zone. In ASD, the location of the bearing pressure resultant must be maintained within B/6 of the center of the reinforced soil zone.

The overall stability of the retained earth and foundation must also be evaluated using procedures described in Section 8.2.1.

Table 14-1
Factors of Safety for External Stability of MSE Walls
(AASHTO, 1996)

Failure Condition	Required Minimum Factor of Safety (FS)
Bearing Capacity	2.0 to 2.5
Sliding Resistance	1.5
Overturning	2.0
Overall Stability	Refer to Chapter 8

The internal stability or structural capacity of the reinforcements and connections with the wall facing against rupture, and the pullout resistance of the reinforcements must also be evaluated. The structural capacity of the reinforcement is checked as follows:

$$\sum Q \leq \sigma_{all} \times A = P_{all} \qquad \text{(Eq. 14-3)}$$

where:

σ_{all} = Allowable tensile stress in soil reinforcement (kPa)
P_{all} = Allowable tensile structural capacity of reinforcement (kN)
A = Cross-sectional area of soil reinforcement less any sacrificial thickness (m^2)

Table 14-2 summarizes the maximum allowable tensile force criteria in AASHTO (1996) for evaluating the internal stability of reinforcements and facing connections. Because polymeric materials exhibit time- and temperature-dependent creep behavior, these reinforcements are evaluated at a maximum load level obtained from creep tests (i.e., Limit State) and at a load representative of a total strain level of 5 percent (i.e., Serviceability State). *(As noted previously, the report on Demonstration Project 82 (Elias and Christopher, 1996) recommends elimination of the Serviceability Limit State tension check for polymeric reinforcements.)* In addition, the maximum allowable tensile load of polymeric reinforcements at both the Limit and Serviceability Limit States are adjusted to account for the effects of environmental and aging losses, FD, and for the effects of construction damage, FC. The overall factor of safety, FS, accounts for uncertainties in structure geometry, engineering properties of backfill soils, variations in reinforcement manufacturing, and externally applied loads.

Table 14-2
ASD Criteria for Internal Stability Evaluation of Reinforcements for MSE Walls
(AASHTO, 1996)

Reinforcement Type	Maximum Allowable Tensile Load [(1)] (kN)
Steel • Strip • Grid	 $0.55\ A_s\ F_y$ $0.48\ A_s\ F_y$
Polymeric • Limit State • Serviceability State [(2)]	 T_ℓ/FC FD FS T_w/FC FD

[(1)] A_s = Net cross-sectional area reduced for corrosion (m^2)
F_y = Minimum yield stress (kPa)
T_ℓ = Limit State tensile capacity from creep tests (kN)
T_w = Serviceability State long-term tensile capacity at 5% strain (kN)
FC = Construction damage FS ranging from 1.1 to 3.0, and taken as 3.0 w/o tests (dim)

FD = Durability FS ranging from 1.1 to 2.0, and taken as 2.0 w/o tests (dim)
FS = FS to account for load, geometry, and material property uncertainty (dim)

(2) Recent FHWA Demonstration Project 82 (Elias and Christopher, 1996) recommends elimination of the Serviceability Limit State check on polymeric reinforcement tension.

The value of FS is taken as 1.78 for the most recent version of AASHTO (1996) and 1.50 for earlier versions of AASHTO (1991) and FHWA Demonstration Project 82.

Pullout resistance is evaluated using Eq. 14-2. The minimum factor of safety against pullout of reinforcements is taken as 1.5.

Finally, after all geotechnical deformation and capacity criteria are met, the structural design of the facing is performed using service loads and allowable stresses (AASHTO, 1996), or by LFD or LRFD (AASHTO, 1996) or ACI.

14.2.3 LRFD Summary

Whereas ASD considers all uncertainty in the applied loads and ultimate geotechnical or structural capacity in factors of safety or allowable stresses, LRFD separates the variability of these design components by applying load and resistance factors to the load and material capacity, respectively. ***When properly developed and applied, the LRFD approach can provide a consistent level of safety for the design of all structure components.*** Thus, the probability that a structure component will fail or perform unacceptably is no different than any other component. For design, the resistance and deformation of supporting soil and rock materials and structure components must satisfy the LRFD equations below. For the Strength and Service Limit States:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 14-4) (}\mathit{A1.3.2.1\text{-}1}\text{)}$$

For the Service Limit States:

$$\sum \eta_i \gamma_i \delta_i \leq \phi \delta_n \qquad \text{(Eq. 14-5)}$$

where:

η_i = Factors to account for effects of ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_r = Factored resistance (kN or kPa)
R_n = Nominal (ultimate) resistance (kN or kPa)
δ_i = Estimated displacement (mm)
δ_n = Tolerable displacement (mm)

Relative to the external stability of the wall and the internal stability of system components, the suitability of an MSE wall with respect to the geotechnical and structural resistance can be obtained using Eq. 14-4, rewritten as:

$$\sum \eta_i \gamma_i Q_i \leq \phi Q_{ult} = Q_R \qquad \text{(Eq. 14-6)}$$

and

$$\sum \eta_i \gamma_i Q_i \leq \phi P_n = P_R \qquad \text{(Eq. 14-7)}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Factored load effect (kN)
ϕ = Resistance factor (dim)
Q_{ult} = Nominal (ultimate) geotechnical resistance (kN)
Q_R = Factored geotechnical resistance (kN)
P_n = P_{ult} = Nominal (ultimate) structural resistance of reinforcements, facing and connections (kN)
P_r = Factored structural resistance of reinforcements, facing and connections(kN)

The load factors and load factor combinations used for design were presented in Chapter 4. In general, values of $\gamma_i > 1.0$ are used to evaluate ultimate ground or structure capacity at the Strength Limit States, whereas the deformation performance of structures is evaluated at the Service I Limit State using $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads). In ASD (AASHTO, 1996), values of $\gamma_i = 1.0$ (or $\gamma_i = 0.3$ for wind loads) are used to evaluate structures for both strength (allowable stress) and serviceability (deflection). ***Because the AASHTO LRFD Specification for MSE walls was developed based on direct calibration with ASD, the results of analyses by LRFD and ASD to evaluate external wall stability and the internal stability of wall components are similar. Because the calibration used some discretion in the selection of safety factors and resulted in the selection of average resistance factors for the range of possible average load factors, some minor differences in walls designed by ASD and LRFD will occur due to variations in actual design loading conditions. For foundation and wall deformations, ASD and LRFD are identical.***

When using Eq. 14-4 for MSE wall design at the Strength Limit States, the following values of η can normally be used:

- $\eta_D = \eta_R = 1.00$

- $\eta_I = 1.05$ for structures deemed operationally important, 1.00 for typical structures and 0.95 for other structures

Determination of the operational importance of a structure (such as a bridge) is made by the facility owner as described in Chapter 4. Where an MSE wall supports a structure, the appropriate value of η_I is then applied throughout the superstructure design by the structural engineer. The value of η_I selected by the superstructure designer should then be applied in the MSE wall design. For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

When using Eq. 14-5 to evaluate an MSE wall at any Service Limit State, η_D, η_R, and $\eta_I = 1.0$.

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4 and Section 14.2.3.1, and loads should be developed as described in Chapter 4. The geotechnical design of MSE retaining walls with respect to external stability generally follows the LRFD approach for design of conventional retaining walls described in Chapter 11. The geotechnical and structural design of MSE walls with respect to internal design are described in Section 14.3. The ultimate resistance, R_n, should be determined for each type of resistance (e.g., bearing criteria, sliding, overall stability, and pullout and rupture of reinforcements) and the reinforced soil zone should be checked for overturning based on load eccentricity criteria as described in Section 8.3.4.

Values of $\phi \leq 1.0$ are applied when evaluating geotechnical resistance for any strength limit state using Eq. 14-4. Currently the value of $\phi = 1.0$ is applied when evaluating an MSE wall for any service limit state using Eq. 14-5, except for the evaluation of tensile resistance of polymeric reinforcement, for which a $\phi < 1.0$ is considered and Eq. 14-4 is applied.

14.2.3.1 Limit States (*A11.5*)

The design of MSE walls using LRFD requires evaluation of the suitability of the external stability of the wall, the internal stability of wall components and wall movements at various Performance Limit States (i.e., applicable Strength Limit States and the Service I Limit State). The selection of a Strength Limit State(s) depends on the type of applied loading (e.g., Strength I for design vehicle loading without wind or Strength II for permit vehicle loading). The design considerations which must be evaluated for MSE walls designed at the Strength and Service I Limit States are summarized in Table 14-3. As conditions warrant, it may also be necessary to evaluate wall and foundation performance at other limit states (e.g., Extreme Event I for loading from earthquakes).

Table 14-3
Strength and Service Limit States for Design of MSE Walls

Performance Limit	Strength Limit State(s)	Service I Limit State
Bearing Resistance	✔	
Sliding Resistance	✔	
Location of Base Resultant Force	✔	
Overall Stability	✔	
Rupture of Reinforcing Elements	✔	✔[1]
Pullout of Reinforcing Elements	✔	
Structural Resistance of Face Elements	✔	
Structural Resistance of Wall Connections	✔	
Settlement and Horizontal Movement		✔

[1] Checked for polymeric reinforcements only.

14.2.3.2 Resistance Factors (*A11.5.6*)

Resistance factors for the design of MSE walls are presented in Table 14-4. With the exception of resistance factors for bearing and sliding resistance, which were developed using a reliability-based calibration as discussed in Chapter 8, the resistance factors for MSE walls were developed based on direct calibration with ASD.

The current AASHTO LRFD Specification for MSE walls (AASHTO, 1997a) is based on and calibrated to the former AASHTO ASD Specification (AASHTO, 1996). Therefore, discussions related to ASD criteria for MSE wall design in this chapter correspond to the AASHTO ASD Specifications and the FHWA recommendations on which the specifications are based. Efforts are currently underway as part of NCHRP 20-7, Task 88, to revise the AASHTO LRFD Specifications for MSE walls to reflect the changes in the 1997 AASHTO ASD Interim Specifications (AASHTO, 1997b) as developed based on the results of FHWA Demonstration Project 82 (Elias and Christopher, 1996), and to develop resistance factors for MSE wall design using a reliability-based calibration and the revised design procedures.

The resistance factors for the Strength Limit tensile resistance of metallic reinforcements were developed using the following relationship:

$$\phi = \bar{\gamma}\frac{\sigma_{all}}{\sigma_n} \qquad \text{(Eq. 14-8)}$$

where:

σ_n = Ultimate tensile capacity, F_y, (kPa)
σ_{all} = Allowable tensile capacity from ASD (kPa)
$\bar{\gamma}$ = Assumed average load factor (dim)

The resistance factors in Table 14-4 were developed assuming an allowable stress of 0.55 F_y for strip reinforcements and 0.48 F_y for grid reinforcements and connectors for yielding of the gross section less sacrificial area. The reduction in resistance factors for fracture is consistent with a reduction of about 15 to 20 percent in the allowable tensile stress as required by AASHTO (1996) for steel members with holes for bolts or rivets. The average load factor, $\bar{\phi}$, used to develop these resistance factors was approximately 1.6.

The resistance factors for tensile resistance of geosynthetic or polymeric reinforcements for tensile resistance determined by a wide-width tensile test were developed using the following relationships:

Table 14-4
Resistance Factors for Strength Limit State of MSE Walls
(Modified after AASHTO, 1997a)

Condition		Resistance Factor[1]
Bearing Resistance		See Chapter 8
Sliding (Geosynthetic on soil)		1.00
Sliding (Soil on soil)		1.00
Tensile Resistance of Metallic Reinforcement	Strip reinforcements	
	• Yielding of gross section less sacrificial area	0.85
	• Fracture of net section less sacrificial area	0.70
	Grid reinforcements	
	• Yield of gross section less sacrificial area	0.75
	• Fracture of net section less sacrificial area	0.60
	Connectors	
	• Yielding of gross section less sacrificial area	0.75
	• Fracture of net section less sacrificial area	0.60
Strength Limit State Tensile Resistance of Polymeric Reinforcements	Applied to T_ℓ[2]	0.25
	Applied to T_n[3]	
	• Polyethylene	0.05
	• Polypropylene	0.05
	• Polyester	0.11
	• Polyamide	0.09
	• High Density Polyethylene	0.09
Service Limit State Tensile Resistance of Polymeric Reinforcements	Applied to T_w[2]	0.25
	Applied to T_5[3]	
	• Polyethylene	0.05
	• Polypropylene	0.05
	• Polyester	0.11
	• Polyamide	0.09
	• High Density Polyethylene	0.09
Ultimate Pullout Resistance in Soil		0.90

(1) Refer to Section 14.3 for description of design procedures for which φ factors have been calibrated.
(2) Based on 10,000 hour minimum creep test; (3) Based on wide-width tensile test.

$$\phi = \bar{\gamma}\frac{T_{all}}{T_n} \leq \bar{\gamma}\frac{T_{all}}{T_5} \quad \text{(Eq. 14-9)}$$

and

$$T_{all} = \frac{T_n \times CRF}{FD \times FC \times FS} \leq \frac{T_5 \times CRF}{FD \times FC} \qquad \text{(Eq. 14-10)}$$

where:

T_{all} = Maximum allowable tensile capacity (kN)
T_n = Ultimate wide strip tensile yield strength (kN)
T_5 = Tensile strength at 5% strain from wide-width tensile test (kN)
FS = Factor of safety to account for load, geometry and material property uncertainty taken as 1.5 (dim)
FC = Construction damage FS taken as 2.0 (dim)
FD = Durability FS taken as 2.0 (dim)
CRF = Creep reduction factor from creep tests or Table 14-5 (dim)

Table 14-5
Creep Reduction Factors for Polymeric Reinforcements
(AASHTO, 1991)

Reinforcement Type	CRF
Polyethylene	0.20
Polypropylene	0.20
Polyester	0.40
Polyamide	0.35
High-Density Polyethylene	0.35

The long-term tensile strengths are estimated from a wide-width tensile test as T_ℓ = (CRF) T_n and T_w = (CRF) T_5.

Combining Eq. 14-9 and 14-10 for application to T_n:

$$\phi = \frac{\bar{\phi} \times CRF}{FC \times FD \times FS} \qquad \text{(Eq. 14-11)}$$

and for application to T_5:

$$\phi = \frac{CRF}{FC \times FD} \qquad \text{(Eq. 14-12)}$$

The average load factor, $\bar{\phi}$, used to develop the resistance factors for polymeric reinforcements was approximately 1.6 for the strength limit and 1.0 for the service limit state.

Where the Strength Limit State Tensile resistance, T_ℓ, of polymeric reinforcements is obtained from laboratory creep tests, no reduction is made in the laboratory strength for creep (i.e., CRF = 1.0), and the resistance factor, ϕ, applied to T_ℓ is equal to 0.25 for all reinforcement types.

The resistance factors for the Serviceability Limit State tensile resistance of polymeric reinforcements obtained from laboratory creep tests is obtained from Eq. 14-11 and Eq. 14-12 without reductions for creep (CRF).

The resistance factor, ϕ, for reinforcement pullout in Table 14-4 provides an equivalent factor of safety of 1.5 for an average load factor, $\bar{\phi}$, of approximately 1.35 (because the live load surcharge is not included over the reinforced fill for pullout evaluation), based on the following:

$$\phi = \frac{\bar{\gamma}}{FS} \quad \text{(Eq.14-13)}$$

The resistance factors for tensile strength of polymeric reinforcements incorporate factors for the effects of creep, aging, environmental losses and construction damage. The resistance factors for the Strength Limit State in the table include default values of FC = 2.0 for the effects of construction damage and FD = 2.0 for the effects of post-construction environmental and aging strength losses, generally consistent with Table 14-2 for ASD. The LRFD Specification permits use of higher multipliers to increase the resistance factor if product-specific data is available that indicates that the effects of construction damage and/or environmental deterioration and aging are less severe than assumed in developing the table.

FHWA Demonstration Project 82 (Elias and Christopher, 1996) recommend a total default value of (FC FD)/CRF equal to 7 for preliminary design or applications not having severe consequences in the event of poor performance or failure. This default value would result in reinforcement tensile strength resistance factors of approximately 0.15 on T_n and T_5 for all polymeric reinforcement types, compared to a range of 0.05 to 0.11 in Table 14-4. Demonstration Project 82 also recommends elimination of the Service Limit State tensile resistance evaluation for polymeric reinforcements.

14.2.3.3 Comparison of Wall Design Using LRFD and ASD

The differences between the design of an MSE retaining wall by ASD and LRFD relate to differences in the foundation design as discussed in Chapter 8, and minor differences related to selection of average safety factors and resistance factors as described in Section 14.2.

Other aspects of MSE wall design such as identifying special considerations (e.g., potential for loss of support through scour), developing a design foundation and retained soil profile and determining requirements for construction control are inherent aspects of the design process required for both LRFD and ASD.

14.2.3.4 Modification of Resistance Factors

Resistance factors for MSE wall design using LRFD were developed based on direct calibration with ASD. In general, resistance factors for geosynthetic reinforcements were developed using default values for CRF, FD and FC. More appropriate resistance factors can be developed using product-specific data with regard to creep characteristics, durability and construction damage. Resistance factors for various aspects of MSE wall design can be modified for product-specific data, differing load factors and/or different safety factors using Eq. 14-11 and 14-12, and the procedures summarized in Chapters 7 and 9.

14.2.4 Summarized Comparison of ASD and LRFD

The process used to develop an MSE wall design using LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 14-1. Specific differences between the methods and other important issues are highlighted in Section 14.3.

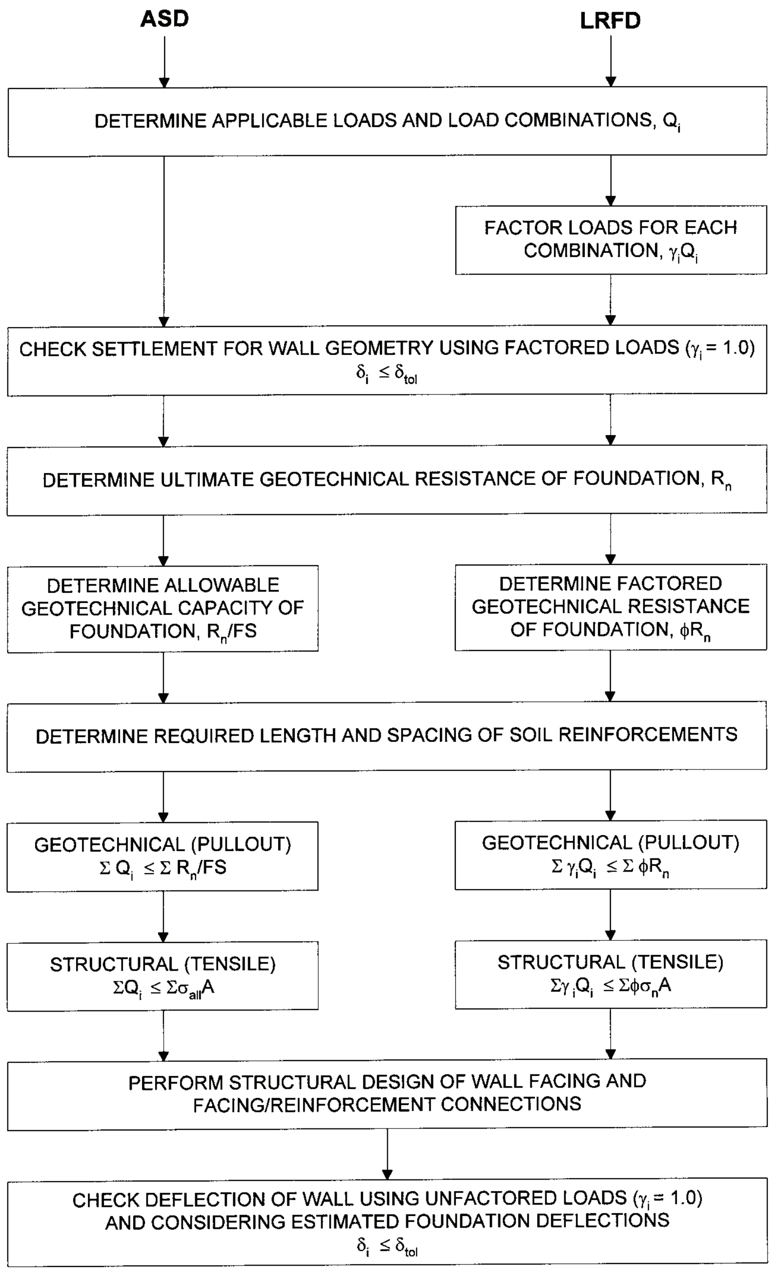


Figure 14-1
Generalized Flow Chart for MSE Wall Design by ASD and LRFD

14.3 Performance Limits

Design of an MSE retaining wall by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. In determining the wall geometry and reinforcement details, and in establishing a suitable bearing level to meet the criteria for vertical, inclined and/or moment loading, the design of these structures requires consideration of many factors which can affect wall performance, including:

- Bearing resistance to vertical and inclined loads and moments
- Sliding resistance to lateral loads
- Resistance to overturning forces and moments Resistance to rupture or pullout of reinforcing elements
- Resistance to effects of scour and frost
- Resistance to variable ground water levels, including the effect of seepage when footings support walls which do not provide adequate drainage
- Geometric constraints (e.g., nearby structures which could impose load on or be loaded by the wall)

14.3.1 Displacements and Tolerable Movements (*A11.9.3*)

The displacement of MSE walls and MSE abutments must be evaluated at the Service I Limit State. Vertical wall movements can be estimated and evaluated as described in Section 8.3.1. Differential movements along the base of the wall and lateral wall movements should be considered as applicable. Lateral wall displacements, which generally occur during construction, can be estimated by empirical methods (Christopher, et al., 1990).

The tolerable settlement of MSE walls is limited by the longitudinal deformability of the facing and the ultimate purpose of the structure. Limiting tolerable differential settlement to prevent open spaces in the joints for systems with panels less than 2.8 m^2 in area and a maximum joint width of 19 mm are presented in Table 14-6.

Table 14-6
Relationship Between Joint Width and Limiting Distortion of MSE Wall Facing
(AASHTO, 1996; AASHTO, 1997a)

Joint Width (mm)	Limiting Vertical Distortion
19.0	1/100
12.7	1/200
6.4	1/300

Where foundation conditions indicate the potential for large differential settlements over a short horizontal distance, a vertical full-height slip joint should be used. Tolerable horizontal wall displacement/alignment criteria are recommended by FHWA (Christopher, et al., 1990), as follows:

- 6.2 mm/m of wall height for precast wall facing
- 16.7 mm/m of wall height for flexible wrapped facing

14.3.2 Bearing Resistance (*A11.9.4.2*)

Bearing resistance along the base of the wall is evaluated as discussed in Chapter 8 for spread footings on soil. For the evaluation of bearing resistance, an equivalent footing having a width equal to the length of the reinforcing elements and a length equal to the length of the wall can be assumed. Bearing pressures should be computed using a uniform base pressure distribution over an effective width of footing in accordance with Section 8.3.2 for bearing resistance of footings on soil or rock. The effect of eccentricity and load inclination must be included in the evaluation of bearing resistance using an effective base width ($B' = B - 2\ e_B$).

14.3.3 Sliding Resistance (*A11.9.4.1*)

Sliding resistance along the base of the wall is evaluated using the procedures in Section 8.3.3 for spread footings on soil. For the evaluation of sliding resistance, the coefficient of friction at the base of the reinforced soil zone should be evaluated using the angle of friction the foundation material, or a maximum value of 30^{o} in the absence of specific strength data for the foundation materials.

14.3.4 Overall Stability (*A11.9.4.4*)

Overall stability of the wall is evaluated using the limit equilibrium procedures as described in 8.3.5.

14.3.5 Pullout of Reinforcing Elements (*A11.9.5.3*)

MSE walls must be evaluated using Eq. 14-6 for internal failure by pullout of reinforcements at each reinforcement level. Only the effective pullout length, L_e, which extends beyond the theoretical failure surface defining the boundary between the active and resistant zones within the reinforced mass may be considered. The length of reinforcements must meet the following criteria:

- The length of the resistant zone $\geq$ 0.90 m
- The total length of reinforcement $\geq$ 2.40 m
- The reinforcement length should be equal at all levels

For ribbed or smooth steel reinforcing strips, the ultimate pullout resistance, P_{fs}, is determined as:

$$P_{fs} = f - (\gamma_s Z A_s) \qquad \text{(Eq. 14-14) (\textit{A11.9.5.3-1})}$$

where:

P_{fs} = Nominal pullout capacity of ribbed or smooth steel reinforcing strips (kN)
f^{*} = Apparent coefficient of friction at each reinforcement level (dim)
γ_s = Unfactored soil density (kN/m^3)
Z = Depth below effective top of wall to reinforcement (m)
A_s = Total top and bottom surface area of reinforcement along , L_{eff},, less any sacrificial thickness (m^2)

In the absence of pullout data for ribbed reinforcement strips in backfill materials conforming to Division II for MSE backfills, the LRFD Specification imposes the following limitations on the value of f^{*}:

- f^* should not exceed 2.0 at ground level, and can be assumed to decrease linearly to $\tan\phi_f$ at (and below) a depth of 6.00 m

- f^* should not exceed 1.2 if the uniformity coefficient, D_{60}/D_{10}, of backfill is less than 4 but otherwise meets Division II requirements for MSE backfills

For smooth steel reinforcing strips, f^* is assumed to be constant at all depths and is determined using:

$$f^* = \tan\psi \leq 0.4 \qquad \text{(Eq. 14-15) } (A11.9.5.3\text{-}2)$$

where:

ψ = Soil-reinforcement angle of friction (deg)

For steel grid reinforcing systems with transverse bar spacings of 0.15 m or greater, the generalized relationship for the nominal pullout capacity, P_{fg}, is:

$$P_{fg} = N_p \times \gamma_s \times Z \times n \times A_b \qquad \text{(Eq. 14-16) } (A11.9.5.3\text{-}3)$$

where:

N_p = Passive resistance factor based on site-specific pullout tests, or as defined by Figure 14-2 (dim)
γ_s = Soil density (kN/m^3)
Z = Depth below effective top of wall to reinforcement (m)
n = Number of transverse bearing members behind failure plane (dim)
A_b = Surface area of transverse reinforcement in bearing, less sacrificial thickness of cross bars (m^2)

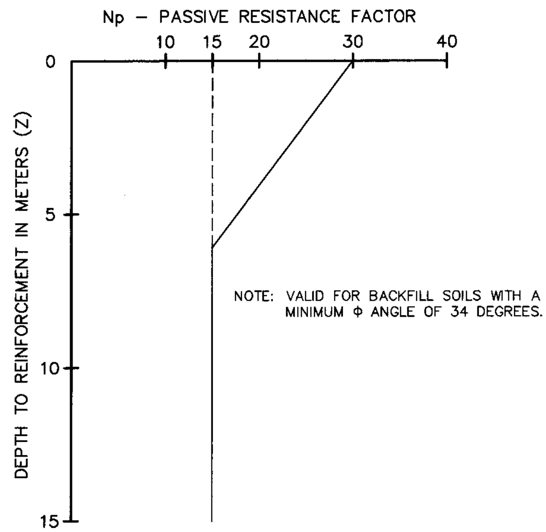


Figure 14-2 (*A11.9.5.3-1*)
Pullout Factors for Inextensible Mesh and Grid Reinforcement

For grid reinforcements with transverse spacing less than 0.15 m, the nominal pullout capacity, P_{fg}, is calculated using:

$$P_{fg} = 2 \times w \times l \times \gamma_s \times Z \times f_d \times \tan\phi_f$$ (Eq. 14-17) (*A11.9.5.3-4*)

where:

- P_{fg} = Nominal pullout resistance (kN)
- w = Width of grid reinforcement mat (m)
- l = Length of mat beyond failure plane (m)
- γ_s = Soil density (kN/m^3)
- Z = Depth below effective top of wall to reinforcement (m)
- f_d = Coefficient of resistance to direct sliding of reinforcement (dim)
- ϕ_f = Internal angle of friction of reinforced soil zone (dim)

The range of values of f_d applicable for various types of steel reinforcements are presented in Table 14-7.

Table 14-7
Coefficient of Resistance for Steel Reinforcements
(AASHTO, 1997a)

Reinforcement Type	f_d
Continuous sheets	0.45
Bar mats (transverse spacing = 0.15 m)	0.80

The appropriate values of f_d must be determined experimentally for each grid geometry. For polymeric reinforcements, Eq. 14-17 is applicable where f_d is developed for a range of normal stresses in accordance with GRI-GG-5 (GRI, 1994). Experimental values of f_d may be limited by the Strength Limit State Tensile Resistance at ultimate per Table 14-4**.**

For evaluation of pullout, the factored pullout resistance, ϕP_{fs} or ϕP_{fg}, is compared to the factored horizontal force acting on the reinforcement at any level, P_i:

$$P_i = \sigma_H \times H_i \qquad \text{(Eq. 14-18) } (\textit{A11.9.5.2.1-1})$$

where:

P_i = Factored horizontal force on soil reinforcement at level i (kN/m)
σ_H = Factored horizontal stress at layer i (kPa) as described in Section 14.3.7
h_i = Height of reinforced zone contributing horizontal load to the reinforcement at level i (m)

The value of h_i is the vertical distance from the mid-point between layer i and the next overlying layer to the mid-point between layer i and the next underlying layer.

FHWA Demonstration Project 82 (Elias and Christopher, 1996) recommends a unified normalized approach to evaluation of pullout resistance which would result in a modification to Eq. 14-14 through 14-17, but would not appear to substantially affect the magnitude of estimated ultimate pullout resistance.

14.3.6 Rupture of Reinforcing Elements (*A11.9.5.1*)
MSE walls must also be evaluated for internal failure by rupture of the reinforcements at any level. The factored tensile resistance of the reinforcement at each level is compared to the factored horizontal force, P_i, from Eq. 14-18 using Eq. 14-7.

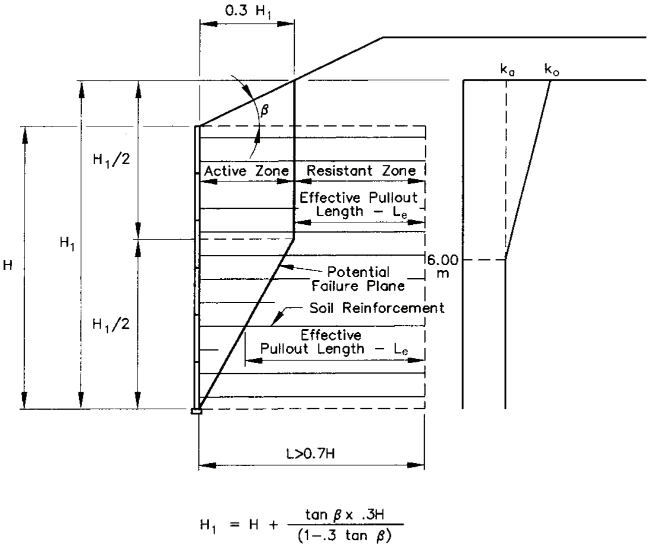
14.3.7 Horizontal Forces for Internal Stability Calculations (*A11.9.5.2*)
Traditionally, the computation of horizontal forces on inextensible (metallic) and extensible (polymeric) reinforcements are treated somewhat differently as follows.

FHWA Demonstration Project 82 (Elias and Christopher, 1996) recommends a modified approach to the determination of internal forces applicable to walls reinforced with either inextensible or extensible reinforcements. The simplified approach may result in some differences in the internal forces from the procedures described below.

Inextensible Reinforcements (*A11.9.5.2.2*)
The internal stability of structures constructed with metallic strip or grid (i.e., inextensible) reinforcements is analyzed by considering that the reinforced zone is divided into active and resistant zones by assuming a bilinear failure surface as shown in Figure 14-3.

Figure 14-3 (*A11.9.5.2.2-1*)
Determination of Failure Plane and Earth Pressure Coefficients
for MSE Wall with Inextensible Reinforcements

The factored horizontal stress, σ_H, at each reinforcement level is:



$$\sigma_H = \gamma_p \times \sigma_v \times k \qquad \text{(Eq. 14-19) (}\textit{A11.9.5.2.2-1}\text{)}$$

where:

γ_P = Load factor for vertical earth loads from Table 4-11 in Chapter 4 (dim)
k = Horizontal pressure coefficient (dim)
σ_V = Pressure due to the resultant vertical forces at reinforcement level (kPa)

The resultant vertical forces at each reinforcement level are determined considering only the forces acting at that level and by assuming a uniform pressure distribution over the effective width (B - 2 e_B) of the reinforced soil zone.

In the LRFD Specification, MSE wall structures are designed assuming k = k_o at H_1 above the top of the leveling pad, and that the value of k decreases linearly to k = k_a at a depth of 6.00 m as shown in Figure 14-3. Below a depth of 6.00 m, k = k_a. The earth pressure coefficients k_a and k_o remain the same regardless of the external loading conditions. *FHWA Demonstration Project 82 recommends a revised distribution of k with depth in which k is a function of k_a and depth, and for which somewhat greater stresses are predicted.*

The value of ϕ_f used to determine the horizontal force within the reinforced soil zone should not exceed 34° unless the specific project backfill is tested for frictional strength by triaxial or direct shear testing. Live loads must be positioned for extreme force effects within the zone subjected to live loads following the requirements for surcharge loads described in Chapter 4.

Extensible Reinforcements (*A11.9.5.2.3*)
The internal stability for MSE structures constructed with polymeric (i.e., extensible) reinforcements are analyzed using a tie-back wedge approach which assumes that the shear strength of the reinforced fill is fully mobilized and active lateral earth pressures based on k_a, are developed. Therefore, the assumed failure plane for both horizontal and sloping backfill conditions can be represented by the Rankine active earth pressure zone defined by a straight line passing through the wall toe and oriented at an angle of $45^o + \phi_f'/2$ from the horizontal.
The tensile force at each level of reinforcement is a function of the vertical stress induced by gravity, uniform normal surcharge and active thrust multiplied by k_a. Tension in the reinforcement induced by vertical or horizontal line or point loads should be added by superposition to the tensile forces induced by the reinforced soil mass and the retained backfill.

The value of k_a in the reinforced soil mass is assumed to be independent of all external loads except sloping fills. If testing of the site-specific select backfill is not available, the LRFD Specification prescribes that the value of ϕ_f used to compute the horizontal stress within the reinforced soil mass not exceed 34^o. Where site-specific tests are performed, the soil strength can be evaluated at residual stress levels.

14.3.8 Design Life (*A11.9.8*)
The long-term durability of steel and polymeric reinforcements must be considered in the design of MSE walls to ensure suitable performance throughout the design life of the structure. The structural design of galvanized steel soil reinforcements and connections is made on the basis of a thickness, E_c, defined as:

$$E_c = E_n - E_s \qquad \text{(Eq. 14-20) } (\textit{A11.9.8.1-1})$$

where:

E_c = Thickness of metal reinforcement at the end of service life (mm)
E_n = Nominal thickness of steel reinforcement at construction (mm)
E_s = Sacrificial metal thickness expected to be lost by uniform corrosion during the service life of the structure (mm)

Based on the results of performance testing and monitoring of constructed MSE walls, the following sacrificial thicknesses should be computed for each exposed surface:

- Loss of galvanizing = 0.015 mm/year for the first 2 years and 0.004 mm/year thereafter

- Loss of carbon steel = 0.012 mm/year after zinc depletion

If other corrosion-resistant coatings are specified, they must be electrostatically applied, resin-bonded epoxy type, with a minimum application thickness of 0.40 mm.

In determining the requirements for corrosion protection, the following should be considered:

- Galvanizing is preferred to the use of epoxy-bonded coatings because the long-term performance of epoxy-bonded coatings buried in soil is unknown

- Epoxy coatings should have a minimum thickness of 0.38 mm to 0.46 mm when placed in granular fills with sharp angular fragments

- The sacrificial thickness should be provided in addition to the epoxy coating

- Alloys such as aluminum and stainless steel should not be used for reinforcements

The durability of polymeric reinforcements is influenced by time, temperature, mechanical damage, stress levels, microbiological attack and changes in the molecular structure by radiation or chemical exposure. The long-term, stress-strain-time behavior of these materials must be determined from the results of controlled laboratory creep tests conducted for a minimum duration of 10,000 hours for a range of load levels on samples of the finished product in accordance with ASTM D 5262 (1997). These samples must be tested in the direction in which the load will be applied, and the results extrapolated to the required design life using procedures outlined in ASTM D 2837 (1997). From this testing, the reinforcement tensile strength should be the lesser of:

- T_{ℓ} - The highest load level at which the log time-creep-strain rate continues to decrease with time within the required lifetime without either brittle or ductile failure; or

- T_w - The tension level at which total strain is not expected to exceed 5% within the design life of the structure.

The effects of aging, chemical and microbiological attack, environmental stress cracking, stress relaxation, hydrolysis and variations in the manufacturing process, as well as the effects of construction damage, must be evaluated and extrapolated to the required design life.

14.4 Design Example 1: MSE Wall Design by LRFD

Problem: The following example problem presents the design of the mechanically-stabilized earth (MSE) retaining wall with polyester geogrid reinforcements as shown in Figure 14-4 to illustrate geotechnical aspects of the wall design by LRFD. The wall supports a roadway embankment and is to be designed for a life of 100 years.

During the subsurface exploration, it was determined that the foundation soils consist predominantly of hard clay to a depth of 10 m beneath the base of the proposed MSE wall. Bedrock underlies the hard clay. Field vane shear tests indicate that the hard clay has an average undrained shear strength, S_u, equal to 230 kPa and consolidated undrained triaxial compression testing indicates that the clay has no effective cohesion intercept, c'_f, and an effective stress friction angle, ϕ'_f, equal to 28 degrees. Similar triaxial testing of the reinforced backfill and embankment fill material indicates an effective stress friction angle of 38 degrees. The seasonal high groundwater table is located at a depth of 6 m below the bottom of the MSE wall.

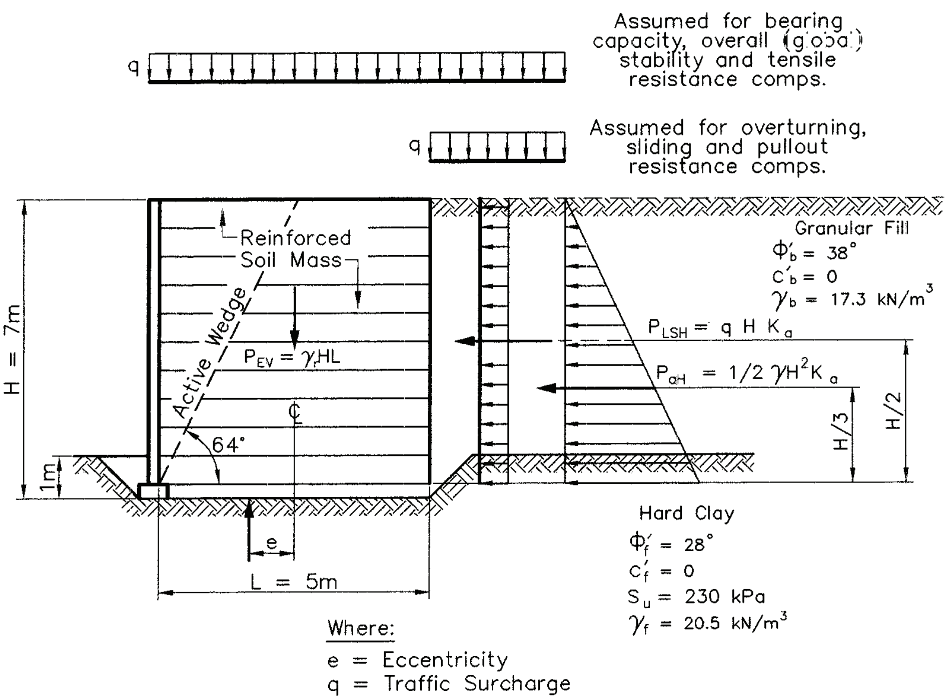


Figure 14-4 – Problem Geometry

The leveling pad will be constructed using precast concrete. The wall will be constructed using a polyester geogrid reinforcement.

Approach: To perform the design of the wall, the following steps are taken:

- Compute and tabulate the unfactored loads and moments required for design at the applicable limit states

- Determine and tabulate the factored loads and moments required for design at the applicable limit states

- Evaluate wall settlement and horizontal displacement at the Service Limit State

- Compare eccentricity of factored loads with eccentricity acceptance criteria

- Check for adequate resistance against sliding failure

- Check for adequate resistance against bearing failure

- Check for adequacy of internal stability including calculation of loads and resistances for evaluation of geogrid tensile and pullout resistance

Solution:

Step 1: Unfactored Loads

(A) Length of Soil Reinforcement
The minimum soil reinforcement length, L, is the greater of either 70% of the wall height, H, measured from the leveling pad (i.e., 0.7 x 7.0 m = 4.9 m) or 2.40 m (*A11.9.5.1.4*). However, for many problems, longer reinforced lengths are needed. As a preliminary estimate, use L = 5.0 m.

(B) Vertical Earth Pressure (EV)
The weight of the reinforced soil backfill is (*A3.5.1*):

$P_{EV} = H\ L\ \gamma_r$
$P_{EV} = (7\text{ m})(5.0\text{ m})(17.3\text{ kN/m}^3)$
$P_{EV} = 605.5$ kN/m length of wall

(C) Live Load Surcharge (LS)
The live load surcharge, LS, is applied where vehicular load is expected to act on the backfill within a distance equal to the wall height behind the wall. Because LS is applied above the reinforcing strips, both the vertical and horizontal forces will be considered.

Interpolating from Table 14-8, the equivalent height, h_{eq}, of soil for the effect of vehicular loading from the AASHTO LRFD Specification (1997a) equals 0.71 m for a wall height of 7.0 m.

Table 14-8 (*A3.11.6.2-1*)
Equivalent Height of Soil for Vehicular Loading
(AASHTO, 1997a)

Wall Height (m)	h_{eq} (m)
≤1.5	1.70
3.0	1.20
6.0	0.76
≥9.0	0.61

The vertical force (weight) of the reinforced soil mass is:

$P_{LSV} = (\gamma_r)(h_{eq})(L)$
$P_{LSV} = 17.3\ \text{kN/m}^3 \times 0.71\ \text{m} \times 5.0\ \text{m} = 61.4\ \text{kN/m}$ length of wall

Note that the live load surcharge over the reinforced zone is not considered for checks on sliding, overturning (eccentricity) or reinforcement pullout, but is considered in evaluation of bearing capacity, overall stability and reinforcement rupture.

(D) Horizontal Earth Pressure (EH)
The active earth pressure coefficient, k_a is:

$$k_a = \frac{\sin^2(\theta + \phi')}{\Gamma \sin^2\theta \sin(\theta - \delta)} \qquad (A3.11.5.3\text{-}1)$$

in which:

$$\Gamma = \left[1 + \sqrt{\frac{\sin(\phi' + \delta)\sin(\phi' - \beta)}{\sin(\theta - \delta)\sin(\theta + \beta)}}\right]^2 \qquad (A3.11.5.3\text{-}2)$$

where:

$\beta =$ Nominal slope of backfill behind wall (deg) = 0^o
$\delta =$ Angle of wall friction (deg) = 0^o
$\phi' = \phi'_b =$ 38°
$\theta =$ 90^o for vertical wall

from which:

$k_a = 0.238$

The uniform increase in horizontal earth pressure due to a live load surcharge is:

$\Delta p = (k)(\gamma_s)(h_{eq}) = (k_a)(\gamma_b)(h_{eq})$ (*A3.11.6.2-1*)
$\Delta p = (0.238)(17.3\ kN/m^3)(0.71\ m) = 2.92\ kPa$

The resultant of the live load surcharge horizontal (lateral) earth pressure, P_{LSH}, acting on the reinforced soil mass (assuming a uniform or rectangular distribution) is:

$P_{LSH} = (\Delta p)(H)$
$P_{LSH} = 2.92\ kPa \times 7.00\ m = 20.4\ kN/m$ length of wall

The basic earth pressure is assumed to vary linearly with depth in a triangular distribution. The resultant, P_a, of the triangular distribution can be determined (*A3.11.5.1*) using:

$P_{EH} = P_{aH} = 0.5\ (\gamma_b)(H^2)k_a$
$P_{EH} = (0.5)(17.3\ kN/m^3)(7\ m)^2(0.238) = 100.9\ kN/m$ length of wall

(E) Summary of Unfactored Loads

Table 14-9
Unfactored Vertical Loads/Moments

Item	V (kN/m)	Moment Arm About Toe (m)	Moment about Toe (kN-m/m)
P_{EV}			
P_{LSV}			
Total			

Table 14-10
Unfactored Horizontal Loads/Moments

Item	H kN/m	Moment Arm About Toe (m)	Moment about Toe (kN-m/m)
$P_{EH} = P_{aH}$			
P_{LSH}			
Total			

Step 2: Load Factors

From Tables 4-10 and 4-11, the applicable load combinations and load factors are:

Table 14-11
Load Factors and Load Combinations

GROUP	γ_{EV}	γ_{EH} (Active)	γ_{LS}
Strength I-a	1.00	1.50	1.75
Strength I-b	1.35	1.50	1.75
Service I	1.00	1.00	1.00

Step 3: Factored Loads and Moments

Table 14-12
Factored Vertical Loads

GROUP	P_{EV} (kN/m)	P_{LSV} (kN/m)	$V_{TOT} = P_{EV1} + P_{LSV}$ (kN/m)
Unfactored			
Strength I-a			
Strength I-b			
Service I			

Table 14-13
Factored Horizontal Loads

GROUP	P_{EH} (kN/m)	P_{LSH} (kN/m)	$H_{TOT} = P_{EH} + P_{LSH}$ (kN/m)
Unfactored			
Strength I-a			
Strength I-b			
Service I			

Table 14-14
Factored Moments from Vertical Loads

GROUP	M_{EV} (kN-m/m)	M_{LSV} (kN-m/m)	$M_{vTOT} = M_{EV1} + M_{LSV}$ (kN-m/m)
Unfactored			
Strength I-a			
Strength I-b			
Service I			

Table 14-15
Factored Moments from Horizontal Loads

GROUP	M_{EH} (kN-m/m)	M_{LSH} (kN-m/m)	M_{hTOT} = M_{EH} + M_{LS} (kN-m/m)
Unfactored			
Strength I-a			
Strength I-b			
Service I			

Step 4: MSE Wall Settlement/Lateral Displacement

For this example, it is assumed that the MSE wall will not experience unacceptable settlements or lateral displacements due to the relative stiffness of the hard clay foundation soil, adequate construction control and sufficient reinforcement length.

Step 5: Eccentricity

Table 14-16
Summary for Eccentricity Check

Group/Item Units	P_{EV} (kN/m)	M_{EV} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o m	e_B m	$q_{uniform}$ (kPa)
Strength I-a	605.5	1513.8	478.8	1.71	0.79	177
Strength I-b	817.4	2043.6	478.8	1.91	0.59	214
Service I	605.5	1513.8	307.2	1.99	0.51	152

where:

X_o = Location of the resultant from toe of wall = $(M_{EV} - M_{hTOT})/P_{EV}$
B = Base width = Length of reinforcement strips
e_B = Eccentricity = $B/2 - X_o = 2.5\text{ m} - X_o$
$q_{uniform} = P_{EV}/(B - 2\,e_B) = P_{EV}/2X_0$

The location of the resultant must be in the middle half of the base (*A11.9.4.3*, *A11.6.3.3*).

$e_{max} = B/4 = 5.0\text{ m}/4 = 1.25\text{ m}$

For all cases, $e_B < e_{max}$; therefore, the design is adequate with regard to eccentricity.

Step 6: Sliding

From Chapter 8, the factored resistance against failure by sliding, Q_R, is:

$$Q_R = \phi_\tau\, Q_\tau + \phi_{ep}\, Q_{ep} \qquad \text{(Eq. 8-15)}$$

Because of the potential for loss of soil in front of the wall, neglect the passive resistance of the foundation material so that $\phi_{ep}\, Q_{ep} = 0$ (*A10.6.3.3*). Then from Table 14-4, $\phi_\tau = 1.0$ for sliding of soil against soil.

Nominal shear resistance is:

$$Q_\tau = V \tan \delta \qquad (A10.6.3.3\text{-}2)$$

where:

$\tan \delta = \tan \phi_f$
$V = P_{EV}$

Therefore:

$$Q_\tau = P_{EV} \tan \phi_f = P_{EV} \tan \_\_\_\_\_^{o} = \_\_\_\_\_\ P_{EV}$$

From Table 14-12, the minimum value of P_{EV} = ____ kN/m length of wall for the Strength I-a Limit State. From Table 14-13, the maximum value of H_{TOT} = _____ kN/m length of wall for the Strength 1-a Limit State.

$$Q_\tau = (\_\_\_\_\_)\ (\_\_\_\_\_\ \text{kN/m}) = \_\_\_\_\_\ \text{kN/m length of wall}$$

Applying the resistance factor φ_τ to Q_τ, the factored sliding resistance is:

$$Q_R = (\_\_\_\_\_)(\_\_\_\_\_\ \text{kN/m}) = \_\_\_\_\_\ \text{kN/m length of wall}$$

Because $Q_R >> H_{TOT}$ (max) from Table 14-13 (i.e., ______ kN/m < = > ______ kN/m) sliding resistance is/is not adequate. **(Cross out incorrect options in sentence.)**

Step 7: Bearing Resistance

From Chapter 8, the factored unit bearing resistance, q_R, is:

$$q_R = \varphi\, q_{ult} \qquad \text{(Eq. 8-11)}$$
$$q_{ult} = c\, N_{cm} + \gamma_s\, D_f\, N_{qm} \qquad (A10.6.3.1.2b\text{-}1)$$

where:

$c = S_u = 230$ kPa
$\gamma_s = \gamma_f = 20.5$ kN/m^3
$D_f = 0$ m (i.e., embedment neglected)

Therefore:

$q_{ult} = c\ N_{cm}$

The bearing capacity factor, N_{cm}, for $D_f/B \leq 2.5$, $B/L \leq 1.0$ and $H/V \leq 0.4$ is obtained from A10.6.3.1.2b:

$N_{cm} = 3.8$

and

$q_{ult} = (3.8)(230\ kPa) = 874\ kPa$

The resistance factor, ϕ, for bearing resistance of clay using the rational method and shear resistance measured in laboratory tests is obtained from Table 8-8, from which $\phi = 0.60$. The factored bearing resistance is then:

$q_R = \phi\ q_{ult} = 0.60\ (874\ kPa) = 524\ kPa$

Table 14-17
Summary for Bearing Check

Group/Item Units	V_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o m	e_B m	$q_{uniform}$ (kPa)
Strength I-a	713.0	1782.4	478.8	1.83	0.67	194
Strength I-b	924.9	2132.2	478.8	1.79	0.71	258
Service I	666.9	1667.3	307.2	2.04	0.46	163

Because $e < B/6 = 0.83$ m, for Strength I-a and Strength I-b, the actual base pressure distribution for these cases will be trapezoidal. For design of MSE walls, an equivalent uniform base pressure is considered (*A11.9.4.2*). The uniform pressure, $q_{uniform}$, is taken as the average pressure of an assumed rectangular distribution.

$q_{uniform} = V_{TOT}/(B - 2\ e_B) = V_{TOT}/2\ X_o$

From Table 14-17, the maximum value of $q_{uniform} = 258$ kPa for Strength I-b. Because $q_R > q_{uniform}$ (i.e., 524 kPa > 258 kPa), the bearing resistance is adequate.

Step 8: Overall Stability

Verify overall stability against a deep-seated soil failure using a limit equilibrium method of analyses. For this example, it is assumed that the overall stability is adequate.

Step 9: Internal Stability - Tensile Resistance Reinforcement

(A) Factored Horizontal Force Acting on the Reinforcement
The factored horizontal force acting on any single layer of reinforcement is:

$P_i = \sigma_H h_i = (\gamma_p \sigma_v k) h_i = [\gamma_{EV} (P_{EVi}) + \gamma_{LS} (P_{LSV})] k h_i$ (Eq. 14-18 and 14-19)

where:

$h_i =$ Height of reinforced soil zone at Level i (m)
$\sigma_H =$ Horizontal stress at Layer i = $\gamma_p \sigma_v k$ (kPa)
$\gamma_p =$ Load factor applied to the unfactored σ_v (dim)

The appropriate load factors are applied to the components comprising P_{VT} used to calculate σ_v and the moments used to calculate X_o (and e_B).

$\gamma_p\sigma_v =$ Pressure due to resultant of factored vertical forces at Level i (kPa)
$\gamma_p\sigma_v = q_{uniform} = P_{VT}/(B - 2 e_B) (1 m) = P_{VT}/2 X_o$ (kPa)
$P_{VT} =$ Resultant of factored vertical forces at the base of Layer i: (kN/m)
$k =$ Lateral earth pressure coefficient = k_a for polymeric reinforcement (dim)

The value of k_a from Step 1D is:

$k_a = 0.238$

Based on Table 14-17, the load case which produces the largest uniform pressure when live load surcharge is included over the reinforced fill is Strength I-b ($\gamma_{EV} = 1.35$, $\gamma_{LS} = 1.75$ and $\gamma_{EH} = 1.5$). The factored uniform pressure distribution $q_{uniform}$ is calculated at each layer of reinforcing. For this example, reinforcing strips are placed at a vertical spacing of 0.66 m beginning at 0.33 m below the top of the wall. The uniform pressure for each layer is shown in Table 14-18 and sample calculations for layer 1 are presented below where the values in Table 14-18 are:

$P_{VT} =$ Calculated based on the weight of the soil and uniform live load surcharge overlying the strip using $\gamma_{EV} = 1.35$ and $\gamma_{LS} = 1.75$

$M_v =$ Calculated considering the same loads and load factors used to calculate P_{VT}

$M_h =$ Calculated based on the horizontal earth pressures with a load factor of $\gamma_{EH} = 1.5$

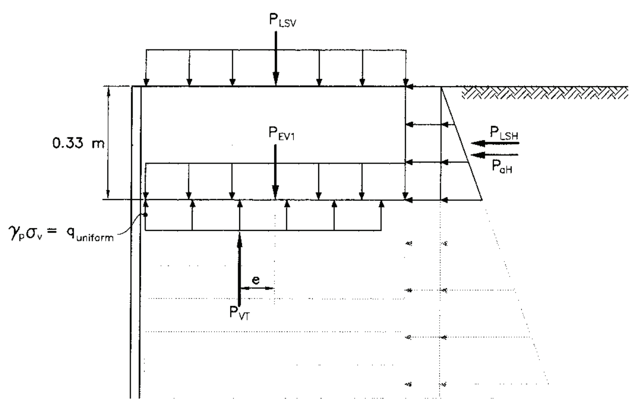
$$e = \frac{B}{2} - X_O = \frac{B}{2} - \frac{M_V - M_h}{P_{VT}} = \frac{L}{2} - M_V - M_h \mathrm{over} P_{VT}$$

$\gamma_p\sigma_v = q_{uniform} = P_{VT}/(B - 2 e_B) = P_{VT}/(L - 2 e)$

Table 14-18
Factored Vertical Load and Moment at Reinforcement Levels for Reinforcement Tensile Resistance Evaluation

Layer	Z (m)	P_{VT} (kN/m)	M_v (kN-m/m)	M_h (kN-m/m)	$e^{(1)}$ (m)	$q_{uniform}$ (kPa)
1	0.33	146.0	365.0	0.3	0.002	29.2
2	1.00	224.3	560.8	3.6	0.02	45.2
3	1.67	302.5	756.2	11.9	0.04	61.5
4	2.33	379.5	948.8	26.9	0.07	78.1
5	3.00	457.8	1144.5	50.8	0.11	95.8
6	3.67	536.1	1340.2	85.3	0.16	114.6
7	4.33	613.1	1532.8	131.5	0.21	133.9
8	5.00	691.4	1728.5	192.5	0.28	155.7
9	5.67	769.6	1924.0	269.8	0.35	179.0
10	6.33	846.7	2116.8	363.5	0.43	204.5

(1) For determination of $q_{uniform}$, eccentricity is assumed as zero for calculated e < 0.



Sample Calculations for Layer 1 follow (See Figure 14-5 for pressure diagram at Layer 1).

Figure 14-5
Pressure Diagram for Computation of $q_{uniform}$ for Reinforcement Tensile Resistance Evaluation of Layer 1

<u>Resultant of Vertical Forces, P_{VT}</u>

P_{VT} = $P_{EV1} + P_{LS}$ = Factored weight of soil overlying reinforcement below top of wall plus factored surface surcharge over reinforcement (kN/m)

$P_{EV1} = \gamma_p W$

$P_{EV1} =$ 1.35 (17.3 kN/m^3) (0.33 m) (5.0 m) = 38.54 kN/m length of wall

$P_{LSV} =$ 107.5 kN/m length of wall (from Table 14-12)

$P_{VT} =$ $P_{EV1} + P_{LSV}$ = 38.5 kN/m + 107.5 kN/m = 146.0 kN/m

Moments About Front Face of Wall at Connection to Strip, M_v and M_h

$M_v =$ (146.0 kN/m) (5.0 m/2) = 365.0 kN-m/m length of wall

$M_h =$ 1.5 (17.3 kN/m^3) (0.238) (0.33 m)^3/6 + 1.75 (2.92 kPa) (0.33 m)^2/2

$M_h =$ 0.3 kN-m/m length of wall

Uniform Pressure on Effective Base Width

$e = L/2 - [(M_v - M_h)/P_{VT}]$
e = 5.0 m /2 - [(365.0 kN-m/m - 0.3 kN-m/m)/146.0 kN-m/m] = 0.002 m
$q_{uniform} = P_{VT}/(L - 2e)$ = (146.0 kN-m/m)/[5.0 m - 2(0.002 m)] = 29.2 kPa

Values of σ_H can be calculated by multiplying the factored $\gamma_p \sigma_v$ values ($q_{uniform}$ from Table 14-18) by the active earth pressure coefficient, $k_a = 0.238$. P_i can then be calculated by multiplying σ_H by h_i (which was set at about 0.66 m). The AASHTO LRFD procedure (AASHTO, 1997a) requires a check of the Strength Limit State and Service Limit State tensile resistance for polymeric reinforcement, with different resistance factors given for each condition. Therefore, Table 14-19 includes the factored horizontal force acting on each layer for both the Strength I-b Limit State, P_{iA}, and the Service I Limit State, P_{iB}.

Table 14-19
Summary of Factored Horizontal Loads
for Reinforcement Tensile Resistance Evaluation

Layer	Z (m)	$\gamma_p \sigma_V$ (kPa)	σ_{HA} (kPa)	σ_{HB} (kPa)	h_i (m)	P_{iA} (kN/m)	P_{iB} (kN/m)
1	0.33	29.2	7.0	4.3	0.66	4.6	2.8
2	1.00	45.2	10.8	7.1	0.66	7.1	4.7
3	1.67	61.5	14.6	9.9	0.66	9.6	6.5
4	2.33	78.1	18.6	12.8	0.66	12.3	8.4
5	3.00	95.8	22.8	15.9	0.66	15.0	10.5
6	3.67	114.6	27.3	19.1	0.66	18.0	12.6
7	4.33	133.9	31.9	22.5	0.66	21.0	14.8
8	5.00	155.7	37.1	26.1	0.66	24.5	17.2
9	5.67	179.0	42.6	30.0	0.66	28.1	19.8
10	6.33	204.5	48.7	34.2	0.66	32.1	22.6

The tensile loading at the level of each grid must then be compared to the geogrid tensile resistance.

(B) Reinforcement Tensile Resistance
The vertical geogrid spacing is 0.66 m. From Table 14-19, for the strength limit state the maximum value of σ_{HA} = 48.7 kPa. Therefore, the maximum load per m length of wall, P_{iA}, is:

$$P_i = \sigma_{HA}\, h_i\,(1\text{ m}) = (48.7\text{ kPa})\,(0.66\text{ m})\,(1\text{ m}) = 32.1\text{ kN/m}$$

For this problem, laboratory creep tests have been performed which indicate a minimum limit state reinforcement tensile strength, T_ℓ, of 130 kN/m and a tensile strength at 5 percent strain, T_w, of 91 kN/m. Applying ϕ = 0.25 from Table 14-4 to T_ℓ, results in a factored resistance of 33.6 kN/m, which is adequate for this example from Eq. 14-8 as follows:

$$\gamma_i\, Q_i \le \varphi\, P_n$$
$$P_{iA} \le \varphi\, T_\ell$$
$$32.1\text{ kN/m} \le 0.25\,(130\text{ kN/m}) = 32.5\text{ kN/m}$$

For the service limit state, the maximum value of σ_{HB} = 34.2 kPa. Therefore, the maximum load per m length of wall, P_{iB}, is:

$$P_{iB} = \sigma_{HB}\, h_i\,(1\text{ m}) = (34.2\text{ kPa})(0.66\text{ m})(1\text{ m}) = 22.6\text{ kN/m}$$

The resistance factor of 0.25 from Table 14-4 for the service limit state will result in an acceptable design from Eq. 14-8 for this example, as follows:

$$\gamma\, Q_i \le \phi\, P_n$$
$$P_{iB} \le \phi\, T_n$$
$$22.6\text{ kN/m} \le 0.25\,(91\text{ kN/m}) = 22.8\text{ kN/m}$$

Step 10: Internal Stability - Facing Connection

For this problem, it is assumed that the geogrid to wall facing connection is adequate.

Step 11: Internal Stability - Reinforcement Pullout

(A) Factored Horizontal Force Acting on the Reinforcement
The factored horizontal force acting on any single layer of reinforcement is:

$$P_i = \sigma_H\, h_i = (\gamma_p\, \sigma_v\, k)\, h_i = [\gamma_{EV}\,(P_{EVi})]\, k\, h_i \qquad \text{(Eq. 14-18 and 14-19)}$$

where:

h_i = Height of reinforced soil zone at Level i (m)
σ_H = Horizontal stress at Layer i = $\gamma_p\, \sigma_v\, k$ (kPa)
γ_p = Load factor applied to the unfactored σ_v (dim)

The appropriate load factors are applied to the components comprising P_{VT} used to calculate σ_v and the moments used to calculate X_o (and e).

$\gamma_p \sigma_v$ = Pressure due to resultant of factored vertical forces at Level i (kPa)
$\gamma_p \sigma_v$ = $q_{uniform} = P_{VT}/(B - 2\ e_B)\ (1\ m) = P_{VT}/2\ X_o$ (kPa)
P_{VT} = Resultant of factored vertical forces at the base of Layer i: (kN/m)
k = Lateral earth pressure coefficient = k_a for polymeric reinforcement (dim)

The value of k_a determined from Step 1D is:

$k_a = 0.238$

Based on Table 14-16, the load case which produces the largest uniform pressure is Strength I-b (γ_{EV} = 1.35 and γ_{EH} = 1.5). Note that the live load surcharge is not included in the calculations for pullout resistance. The factored uniform pressure distribution $q_{uniform}$ is calculated at each layer of reinforcing. For this example, reinforcing strips are placed at a vertical spacing of 0.66 m beginning at 0.33 m below the top of the wall. The uniform pressure for each layer is shown in Table 14-20 and sample calculations for layer 1 follow.

Table 14-20
Factored Vertical Load and Moment at Reinforcement Levels for Pullout Resistance Evaluation

Layer	Z (m)	P_{VT} (kN/m)	M_v (kN-m/m)	M_h (kN-m/m)	e[1] (m)	$q_{uniform}$ (kPa)
1	0.33	38.5	96.3	0.3	0.006	7.7
2						
3	1.67	195.0	487.5	11.9	0.06	40.0
4	2.33	272.0	680.2	26.9	0.10	56.7
5	3.00	350.3	875.8	50.8	0.14	74.4
6	3.67	428.6	1071.4	85.3	0.20	93.1
7	4.33	505.6	1264.1	131.5	0.26	112.9
8	5.00	583.9	1459.7	192.5	0.33	134.5
9	5.67	662.1	1655.3	269.8	0.41	158.2
10						

[1] For determination of $q_{uniform}$, eccentricity is assumed as zero for calculated e < 0.

The values in Table 14-20 are:

P_{VT} = P_{EV} - Calculated based on the weight of the soil overlying strip using γ_{EV} = 1.35
M_v - Calculated considering the same loads and load factors used to calculate P_{VT}
M_h - Calculated based on the horizontal earth pressures with a load factor of γ_{EH} = 1.5

$$e = \frac{B}{2} - X_0 = \frac{B}{2} - \frac{M_v - M_h}{P_{VT}} = \frac{L}{2} - \frac{M_v - M_h}{P_{VT}}$$

$\gamma_p \sigma_v = q_{uniform} = P_{VT}/(B - 2\ e_B) = P_{VT}/(L - 2\ e)$

Sample Calculations for Layer 1 follow (See Figure 14-6 for pressure diagram at Layer 1).

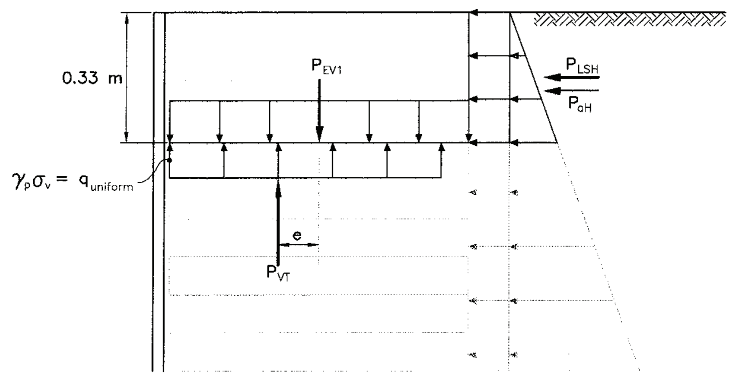


Figure 14-6
Pressure Diagram for Computation of $q_{uniform}$
for Reinforcement Pullout Resistance Evaluation of Layer 1

<u>Resultant of Vertical Forces, P_{VT}</u>

P_{VT} = P_{EV1} = Factored weight of soil overlying reinforcement below top of wall (kN/m)
P_{EV1} = γ_p W = 1.35 (17.3 kN/m^3) (0.33 m) (5.0 m) = 38.5 kN/m length of wall
P_{VT} = P_{EV1} = 38.5 kN/m

<u>Moments About Front Face of Wall at Connection to Strip, M_v and M_h</u>

M_v = (38.5 kN/m) (5.0 m/2)= 96.3 kN-m/m length of wall
M_h = 1.5 (17.3 kN/m^3) (0.238) (0.33 m)^3/6 + 1.75 (2.92 kPa) (0.33 m)^2/2
M_h = 0.3 kN-m/m length of wall

<u>Uniform Pressure on Effective Base Width</u>

e = L/2 - [(M_v - M_h)/P_{VT}]
e = 5.0 m /2 - [(96.3 kN-m/m - 0.3 kN-m/m)/38.5 kN-m/m] = 0.006 m
$q_{uniform}$ = P_{VT}/(L - 2 e) = (38.5 kN-m/m)/[5.0 m - 2(0.006 m)] = 7.7 kPa

Values of σ_H can be calculated by multiplying the factored σ_v values ($q_{uniform}$ from Table 14-20) by the active earth pressure coefficient, k_a = 0.238. P_i can then be calculated by multiplying σ_H by h_i

(which was set at about 0.66 m).

Table 14-21
Summary of Factored Horizontal Loads for Reinforcement Pullout Evaluation

Layer	Z (m)	$\gamma_p\ \sigma_V$ (kPa)	σ_H (kPa)	h_i (m)	P_i (kN/m)
1	0.33	7.7	1.84	0.66	1.2
2					
3	1.67	40.0	9.52	0.66	6.3
4	2.33	56.7	13.49	0.66	8.9
5	3.00	74.4	17.71	0.66	11.7
6	3.67	93.1	22.16	0.66	14.6
7	4.33	112.9	26.87	0.66	17.7
8	5.00	134.5	32.01	0.66	21.1
9	5.67	158.2	37.65	0.66	24.8
10					

The tension loading at the level of each grid must then be compared to the geogrid pullout resistance.

(B) Reinforcement Pullout Resistance
The pullout resistance of the geogrid reinforcement, P_{fg}, is:

$$P_{fg} = 2w\ell f_d\ \gamma_s\ Z\ \tan\phi_f \qquad \text{(Eq. 14-17)}$$

where:

- f_d = Coefficient of resistance to direct sliding of reinforcement, assumed to be 0.80 for this example (dim)
- $\tan\phi_f$ = $\tan 38^o = 0.78$
- γ_s = γ_r = reinforced soil mass unit weight = 17.3 kN/m^3
- Z = Depth below effective top of wall (m)
- w = Width of geogrid (m)
- ℓ = L_e= Length of geogrid beyond failure plane (m)

From Table 14-4, ϕ = 0.90 for reinforcement pullout resistance in soil. The factored pullout resistance $\phi\ P_{fg}$ for each layer is calculated in Table 14-20 and sample calculations for Layer 1 are presented below.

<u>Sample Calculation for Layer 1</u>:
The pullout resistance, P_{fg}, at any layer is:

$$P_{fg} = 2w\,\ell\,f_d\,\gamma_s\,Z\tan\phi_f \qquad \text{(Eq. 14-17)}$$

From Figure 14-4

$Z =$ 0.33 m
$\ell =$ $L_{eff} = L - (H - Z)/\tan 64^o = 5.0\text{ m} - (7.0\text{ m} - 0.33\text{ m})/\tan 64^o = 1.75\text{ m}$
$f_d =$ 0.80 (assumed typical value)
$\tan\phi_r =$ $\tan\phi_b = \tan 38^o = 0.78$
$\gamma_r =$ 17.3 kN/m^3
$w =$ 1 m

Substituting:

$$P_{fg} = 2w\ell f_d\,\gamma_s\,Z\tan\phi_f = 2(1\text{ m})(1.75\text{ m})(0.8)\,(17.3\text{ kN/m}^3)(0.33\text{ m})(0.78) = 12.47\text{ kN/m}$$

and the factored pullout resistance is:

$$\phi\,P_{fg} = 0.90\,(12.47\text{ kN/m}) = 11.2\text{ kN/m}$$

Comparing the factored pullout resistance to the maximum factored horizontal stress at Level 1, P_i = 1.2 kN/m, (from Table 14-18) using Eq. 14-7:

$\gamma_i\,Q_i \leq \phi\,Q_{ult}$
$P_i \leq \phi P_{fg}$
$1.2\text{ kN/m} \leq 11.2\text{ kN/m}$

From a comparison of P_i with ϕP_{fg} in Table 14-22, it is apparent that pullout resistance is adequate at all reinforcement levels.

Table 14-22
Pullout Resistance in Reinforcement Levels

Layer	Z (m)	L_{eff} (m)	ϕP_{fg} (kN/m)	P_i (kN/m)
1	0.33	1.75	11.2	1.2
2				
3	1.67	2.40	78.0	6.3
4	2.33	2.72	123.4	8.9
5	3.00	3.05	178.0	11.7
6	3.67	3.38	241.1	14.6
7	4.33	3.70	311.5	17.7
8	5.00	4.02	391.5	21.1
9	5.67	4.35	480.0	24.8
10				

Summary
This example illustrates the design of an MSE retaining wall by LRFD. ***The AASHTO LRFD Specification (pre 2002) for MSE walls was calibrated directly to ASD procedures in effect at the time the calibration was performed (AASHTO, 1996; Christopher, et al., 1990). A reliability-based calibration for MSE wall design incorporating the procedures specified in the current AASHTO ASD Interims has been completed based on the results of FHWA Demonstration Project 82 (Elias and Christopher, 1996)***

14.5 Design Example 2:
Design of MSE Wall with Broken Backslope and Traffic Surcharge

Problem: The following example problem presents the design of an MSE retaining wall with metallic strip reinforcements as shown in Figure 14-7 to illustrate geotechnical aspects of the wall design by LRFD. The wall provides grade separation between a roadway above the wall and level ground below the wall. The design life of the wall is 100 years.

During the subsurface exploration, it was determined that the foundation soils consist predominantly of sand to a depth of 10 m beneath the base of the proposed MSE wall. Bedrock underlies the sand. After correcting for the effect of overburden pressure, the average N-value for the sand is 20 blows/0.300 m, and the seasonal high groundwater table is located at a depth of 2 m below the bottom of the MSE wall.

The leveling pad will be constructed using precast concrete. The wall will be constructed using steel ribbed strip reinforcement which is 50 mm wide, 4 mm thick and has a center-to-center spacing of 0.70 m and a vertical spacing of 0.75 m. The steel reinforcing is galvanized.

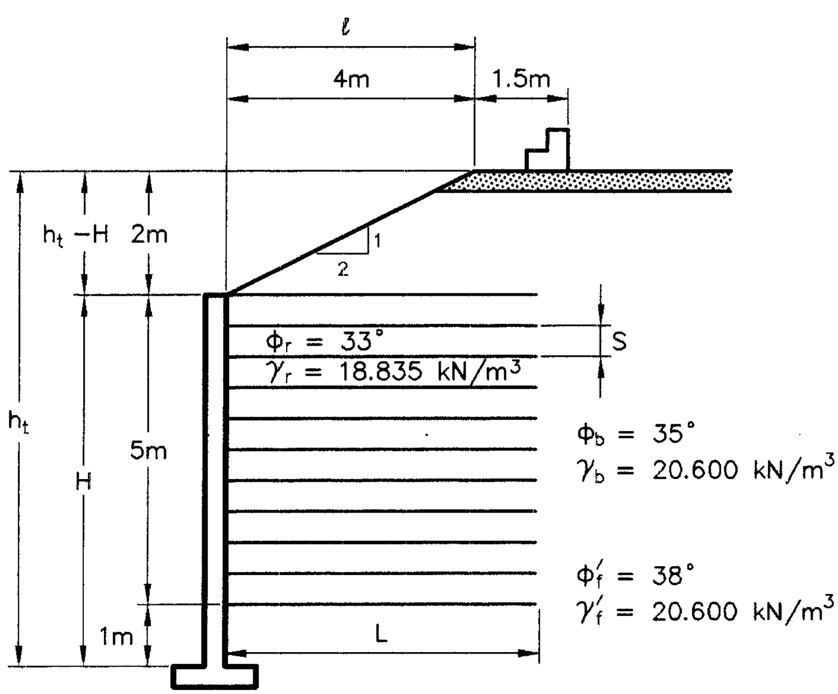


Figure 14-7 - Problem Geometry

Approach: To perform the geotechnical design of the wall, the following steps are taken:

- Compute and tabulate the unfactored loads and moments required for design at the applicable limit states
- Determine and tabulate the factored loads and moments required for design at the applicable limit states
- Estimate wall settlement and lateral displacement at the Service I Limit State
- Compare eccentricity of factored loads with eccentricity acceptance criteria
- Check for adequate for adequate resistance against sliding failure
- Check for adequate resistance against bearing failure
- Check for adequacy of internal stability including calculation of loads and corrosion losses with regard to breakage of strips, adequacy of panel connection and requirements for pullout resistance

Solution:

Step 1: Unfactored Loads

(A) Length of Soil Reinforcement
The minimum soil reinforcement length, L, is the greater of either 70% of the wall height, H, measured from the leveling pad (i.e., 0.7 × 6.0 m = 4.2 m) or 2.40 m (*A11.9.5.1.4*). However, for many problems, longer reinforced lengths are needed. As a preliminary estimate, use L = 5.4 m.

(B) Vertical Earth Pressure (EV)
The weight of the reinforced soil backfill is (*A3.5.1*):

$$P_{EV1} = H\ L\ \gamma_r = (6\text{ m})(5.4\text{ m})(18.835\text{ kN/m}^3) = 610.3\text{ kN/m length of wall}$$

The weight of the fill above the reinforced soil backfill is:

$$P_{EV2} = [\tfrac{1}{2}\,\ell\,(h_t - H) + (L - \ell)(h_t - H)]\ \gamma_b$$
$$P_{EV2} = [(\tfrac{1}{2})(4\text{ m})(2\text{ m}) + (5.4\text{ m} - 4\text{ m})(2\text{ m})](20.6\text{ kN/m}^3) = 140.1\text{ kN/m length of wall}$$

(C) Live Load Surcharge (LS)
The live load surcharge, LS, is applied where vehicular load is expected to act on the backfill within a distance equal to the wall height behind the wall. Because LS is not applied above the reinforcing strips in this example, only the horizontal forces will be considered.

From Table 14-23, the equivalent height, h_{eq}, of soil for the effect of vehicular loading equals 0.76 m for a wall height of 6.0 m.

Table 14-23 (*A3.11.6.2-1*)
Equivalent Height of Soil for Vehicular Loading
(AASHTO, 1997a)

Wall Height (m)	h_{eq} (m)
≤1.5	1.70
3.0	1.20
6.0	0.76
≥9.0	0.61

It is assumed for this problem that no live load surcharge acts over the reinforced soil mass due to the backfill slope above the reinforcement.

(D) Horizontal Earth Pressure (EH)
The active earth pressure coefficient, k_a for the broken back slope as shown on Figure 14-8 is:

$$k_a = \frac{\sin^2(\theta + \phi')}{\Gamma \sin^2 \theta \sin(\theta - \delta)} \qquad \text{(A3.11.5.3-1)}$$

in which:

$$\Gamma = \left[1 + \sqrt{\frac{\sin(\phi' + \delta)\sin(\phi' - \beta)}{\sin(\theta - \delta)\sin(\theta + \beta)}}\right] \qquad \text{(A3.11.5.3-2)}$$

where:

β = B = Nominal slope of backfill behind wall (deg)
δ = Angle of wall friction (deg) = 0°
$\phi' = \phi'_b = 36°$
θ = 90° for vertical wall

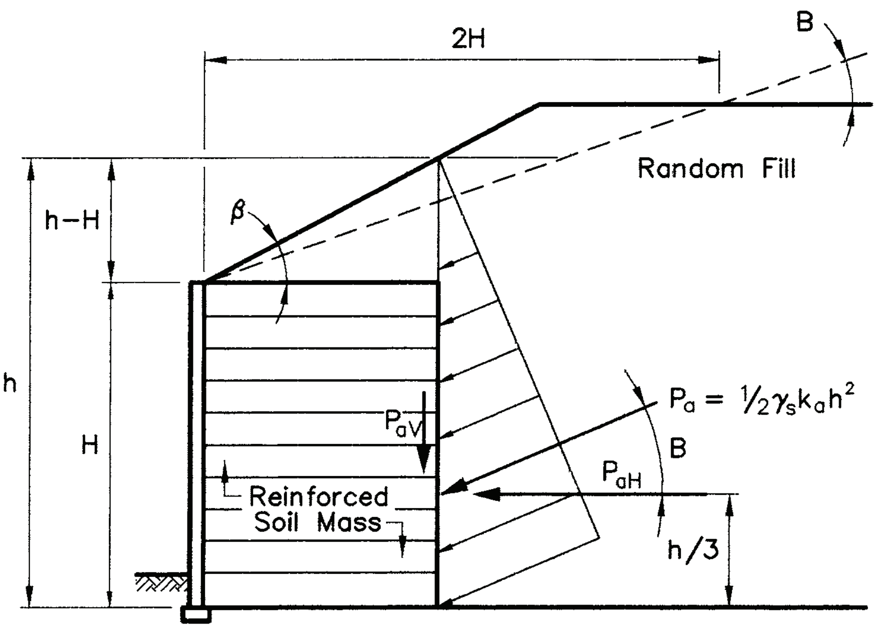


Figure 14-8 (*A3.11.5.7-3*)
Earth Pressure Distribution for MSE Wall with Broken Back Backfill Surface

By geometry (referring to Figures 14-7 and 14-8), the slope of the backfill, B, is:

$$B = \tan^{-1}\frac{h_1 - H}{2H} = \tan^{-1}\left(\frac{8\,\text{m} - 6\,\text{m}}{2(6\,\text{m})}\right) = 9.5°$$

Substituting:

$\Gamma = 2.31$; and
$k_a = 0.28$

The uniform increase in horizontal earth pressure due to a live load surcharge is:

$\Delta p = (k)(\gamma_s)(h_{eq}) = (k_a)(\gamma_b)(h_{eq})$ (*A3.11.6.2-1*)
$\Delta p = (0.28)(20.6\ \text{kN/m}^3)(0.76\ \text{m}) = 4.38\ \text{kPa}$

The sum of the live load surcharge horizontal earth pressure, P_{LSH}, acting over the entire wall (assuming a rectangular distribution) is:

$P_{LSH} = (\Delta p)(h) = (4.38\ \text{kPa})(8.00\ \text{m}) = 35.0$ kN/m length of wall

The basic earth pressure is assumed to vary linearly with depth in a triangular distribution. The resultant, P_a, and the horizontal, P_{aH}, and vertical, P_{aV}, force components of the triangular distribution can be determined (*A3.11.5.1*) using:

$P_a =$ $0.5\ (_b)(h^2)k_a$
$P_a =$ $(0.5)(20.6\ kN/m^3)(8\ m)^2(0.28) = 184.6$ kN/m length of wall
$P_{aH} =$ $P_a \cos B = (184.6)(0.99) = 182.7$ kN/m length of wall
$P_{aV} =$ $P_a \sin B\ (184.6)(0.16) = 29.5$ kN/m length of wall

(E) Summary of Unfactored Loads

Table 14-24
Unfactored Vertical Loads/Moments

Item	V (kN/m)	Moment Arm About Toe (m)	Moment about Toe (kN-m/m)
P_{EV1}	610.3	2.7	1648
P_{EV2}	140.1	3.5	490
P_{aV}	29.5	5.4	159
Total	779.9		

Table 14-25
Unfactored Horizontal Loads/Moments

Item	H kN/m	Moment Arm About Toe (m)	Moment about Toe (kN-m/m)
$P_{EH} = P_{aH}$	182.7	2.7	493
P_{LSH}	35.0	4.0	140
Total	217.7		

Step 2 - Load Factors
From Tables 4-10 and 4-11, the applicable load combinations and load factors are:

Table 14-26
Load Factors and Load Combinations

GROUP	γ_{EV}	γ_{EH} (Active)	γ_{LS}
Strength I-a	1.00	1.50	1.75
Strength I-b	1.35	1.50	1.75
Strength IV	1.35	1.50	0
Service I	1.00	1.00	1.00

Step 3: Factored Loads and Moments

Table 14-27
Factored Vertical Loads

GROUP	P_{EV1} (kN/m)	P_{EV2} (kN/m)	P_{aV} (kN/m)	$V_{TOT} = P_{EV1} + P_{EV2} + P_{aV}$ (kN/m)
Unfactored	610.3	140.1	25.5	779.9
Strength I-a	610.3	140.1	38.2	788.6
Strength I-b	823.9	189.1	38.2	1051.2
Strength IV	823.9	189.1	38.2	1051.2
Service I	610.3	140.1	29.5	779.9

Table 14-28
Factored Horizontal Loads

GROUP	P_{EH} (kN/m)	P_{LS} (kN/m)	$H_{TOT} = P_{EH} + P_{LS}$ (kN/m)
Unfactored	182.7	35.0	217.7
Strength I-a	274.0	61.2	335.2
Strength I-b	274.0	61.2	335.2
Strength IV	274.0	0	274.0
Service I	182.7	35.0	217.7

Table 14-29
Factored Moments from Vertical Loads

GROUP	M_{EV1} (kN-m/m)	M_{EV2} (kN-m/m)	M_{aV} (kN-m/m)	$M_{vTOT} = M_{EV1} + M_{EV2} + M_{aV}$ (kN-m/m)
Unfactored	1648	490	159	2297
Strength I-a	1648	490	238	2376
Strength I-b	2225	662	238	3125
Strength IV	2225	662	238	3125
Service I	1648	490	159	2297

Table 14-30
Factored Moments from Horizontal Loads

GROUP	M_{EH} (kN-m/m)	M_{LS} (kN-m/m)	M_{hTOT} = $M_{EH} + M_{LS}$ (kN-m/m)
Unfactored	493	140	633
Strength I-a	740	245	985
Strength I-b	740	245	985
Strength IV	740	0	740
Service I	493	140	633

Step 4: MSE Wall Settlement/Lateral Displacement

Settlement

Assume embankment construction behind the MSE wall has been performed previously and that any settlement from the embankment has already occurred. All loadings used in the settlement analysis will be Service I Loads.

The average elastic settlement can be estimated (*A10.6.2.2.3a*, *A10.6.2.2.3b*) using the D'Appolonia Method (Gifford, et al., 1987) as:

$$\rho = q\ B\ \mu_o\ \mu_1/M$$

where:

$q = V_{TOT}/L_E$ = (779.9 kN/m)(5.4 m) - 144.4 kPa
$B = L_E$ = 5.4 m

Estimating the modulus of compressibility of sand, M, based on the average SPT blow count:

M = 33.5 MPa = 33 500 kPa

μ_o and μ_i are correction factors for the settlement equations based on D, H and B.

for D/B = 0.19; μ_o = 0.9
for H/B = 1.9 and L/B >10; μ_1 = 0.7

Substituting:

ρ = (144.4 kPa)(5.4)(0.9)(0.7)/(33 500 kPa) = 0.015 m
$\rho < \rho_{tol}$ = 0.022 m; $\therefore$ settlement is acceptable.

Lateral Displacment
Using empirical procedures developed by FHWA (Christopher, et al., 1990), the lateral wall facing displacment during construction is estimated at 5.2 mm/m of height which is within an acceptable displacement limit of 6.2 mm/m from Section 14.3.1.

Step 5: Eccentricity

Table 14-31
Summary for Eccentricity Check

Group/Item Units	V_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o m	e_B m	$q_{uniform}$ (kPa)
Strength I-a	788.6	2376	985	1.76	0.94	224.0
Strength I-b	1051.2	3125	985	2.04	0.66	257.6
Strength IV	1051.2	3125	740	2.27	0.43	231.5
Service I	779.9	2297	633	2.14	0.56	182.2

where:

X_o = Location of the resultant from toe of wall = $(M_{vTOT} - M_{hTOT})/V_{TOT}$
e_B = Eccentricity = $B/2 - X_o = 2.7 - X_o$

The location of the resultant must be in the middle half of the base.

$e_{max} = B/4 = 5.4 \text{ m}/4 = 1.35 \text{ m}$

For all cases, $e < e_{max}$; therefore, the design is adequate with regard to eccentricity (*A11.9.4.3*, *A11.6.3.3*).

Step 6: Sliding

From Chapter 8, the factored resistance against failure by sliding, Q_R, is:

$Q_R = \phi_\tau Q_\tau + \phi_{ep} Q_{ep}$ (Eq. 8-15)

Because of the potential for loss of soil in front of the wall, neglect the passive resistance of the foundation material so that $\phi_{ep} Q_{ep} = 0$ (*A10.6.3.3*). Then from Table 14-4, $\phi_\tau = 1.0$ for sliding of soil against soil.

Nominal shear resistance is:

$Q_\tau = V \tan \delta \, Q_{ep}$ (*A10.6.3.3-2*)

where:

$\tan\delta = \tan\phi_f \leq \tan\phi_r$
$V = V_{TOT}$

Therefore:

$Q_\tau = V_{TOT}\tan\phi_r = V_{TOT}\tan 33^o = 0.65\, V_{TOT}$

From Table 14-27, the minimum value of V_{TOT} = 788.6 kN/m length of wall (i.e., Strength 1-a).

Q_τ = (0.65) (788.6 kN/m) = 512.6 kN/m length of wall

Applying the resistance factor ϕ_τ to Q_τ, the factored sliding resistance is:

Q_R = (1.0)(512.6 kN/m) = 512.6 kN/m length of wall

Because $Q_R >> H_{TOT}$ (max) from Table 14-28 (i.e., 512.6 kN/m > 335.2 kN/m) sliding resistance is adequate, and a shorter reinforcement length may be adequate.

Step 7: Bearing Resistance

From Chapter 8, the factored unit bearing resistance, q_R, is:

$q_R = \phi\, q_{ult}$ (Eq. 8-11)
$q_{ult} = (0.5)(\gamma_f)(B)(C_{w1})(N_{\gamma m}) + (\gamma_f)(C_{w2})(D_f)(N_{qm})$ (*A10.6.3.1.2c-1*)

Using values of C_{w1}, $C_{w2,}$ N_{qm} and $N_{\gamma m}$ from A10.6.3.1.2c, and substituting $\gamma_f = 20.6$ kN/m^3, B = 5.4 m and D_f =1 m:

$$q_{ult} = \left[(0.5)\left(20.6\frac{kN}{m^3}\right)(5.4\,m)(0.62)(19.4) + \left(20.6\frac{kN}{m^3}\right)(1.0)(1.0\,m)(19.3)\right] = 1066\,kPa$$

The resistance factor, ϕ, for bearing resistance of cohesionless soil using the rational method and shear resistance based on SPT data is obtained from Table 8-8, from which ϕ = 0.35. The factored resistance is then:

$q_R = \phi\, q_{ult}$ = (0.35)(1066 kPa) = 373 kPa

Because e < B/6 = 0.9 m, for Strength I-b and Strength IV, the actual base pressure distribution for these cases will be trapezoidal. Because e > B/6 for Strength I-a, the actual base pressure distribution for this case will be triangular. For design of MSE walls, an equivalent uniform base pressure is considered. The uniform pressure, $q_{uniform}$, is taken as the average pressure of an assumed rectangular distribution.

$q_{uniform} = V_{TOT}/(B - 2\, e_B) = V_{TOT}/(2\, X_o)$

From Step 5, the maximum value of $q_{uniform}$ = 257.6 kPa for Strength I-b. Because $q_R > q_{uniform}$ (i.e., 373 kPa > 258 kPa), the bearing resistance is adequate.

Step 8: Overall Stability

Verify overall stability against a deep-seated soil failure using a limit equilibrium method of analyses.

Step 9: Internal Stability - Reinforcement Tensile and Pullout Resistance

To evaluate internal stability, the slip or the rupture of the reinforcement strips must be considered.

(A) Factored Horizontal Force Acting on Reinforcement
The factored horizontal force acting on any single layer of reinforcement is:

$$P_i = \sigma_H h_i = (\gamma_p \sigma_V k) h_i \qquad \text{(Eq. 14-18 and 14-19)}$$

where:

h_i = Height of reinforced soil zone at Level i (m)
σ_H = Factored horizontal stress at Layer i = $\gamma_p \sigma_v k$ (kPa)
γ_p = Load factor applied to the unfactored σ_v (dim)

The appropriate load factors are applied to the components comprising P_{VT} to calculate γ_p and X_o (and e).

$\gamma_p \sigma_v$ = Pressure due to resultant of factored vertical forces at Level i (kPa)
$\gamma_p \sigma_v$ = $q_{uniform} = P_{VT}/(B - 2 e_B) (1 \text{ m}) = P_{VT}/2 X_o$ (kPa)
P_{VT} = Resultant of factored vertical forces at the base of Layer i (kN/m)
k = Lateral earth pressure coefficient from Figure 14-9 (dim)

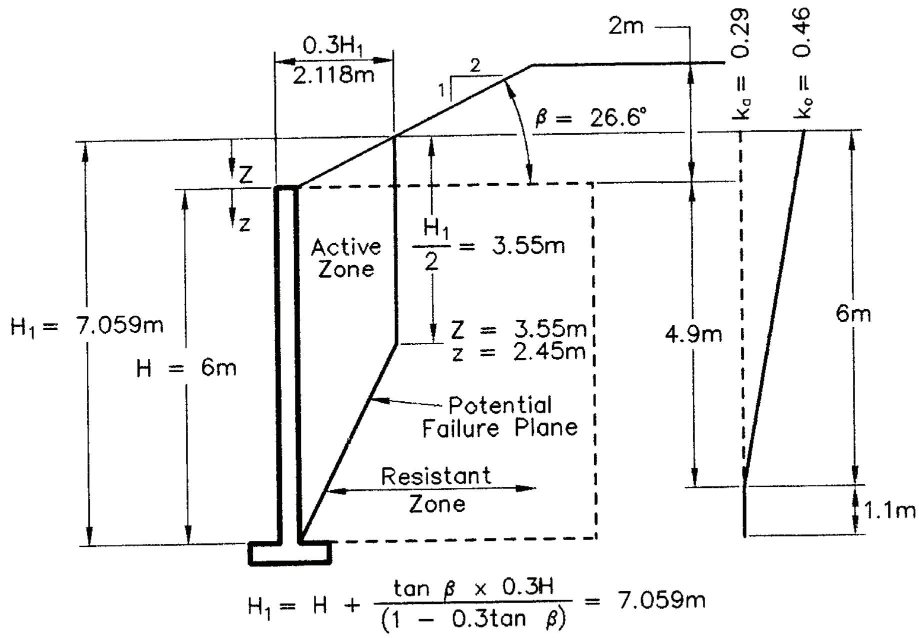


Figure 14-9
Geometry of Assumed Failure Surface for Example Problem

Use the geometry shown in Figure 14-9 to establish the distribution of the lateral earth pressure coefficient, k, with depth. From Figure 14-9, $k = k_o$ at 1.1 m (i.e., H_1 - H) above the top of the wall, and $k = k_a$ at 4.9 m below the top of the wall and lower (i.e., at 6.0 m below point where k_o is applied). From Eq. A3.11.5.7-5:

$$k_o = 1 - \sin(\phi_r) = 1 - \sin(33^o) = 0.46$$

From Eq. A3.11.5.3-1 and A3.11.5.3-2 with $\theta = 90^o$, $\delta = 0^o$ and β taken as 0^o, $k_a = 0.29$.

Based on Table 14-31, the load case which produces the largest eccentricity and the corresponding largest uniform pressure is Strength Ib (EV = 1.35 and EH = 1.5). Note that the live load surcharge is not included in the calculation of the vertical load resultant, P_{VT}, for internal stability, regardless of position, and is considered for vertical load calculations only for bearing capacity and global stability if the surcharge is positioned over the reinforced soil zone. The factored uniform pressure distribution is then calculated at each layer of reinforcing. For this example, reinforcing strips are placed at a vertical spacing of 0.75 m beginning at 0.375 m below the top of the wall. The uniform pressure for each layer is shown in Table 14-32 and sample calculations for layers 1 and 7 follow.

Table 14-32
Factored Vertical Load and Moment at Reinforcement Levels

Layer	z (m)	P_{VT} (kN/m)	M_v (kN-m/m)	M_h (kN-m/m)	$e^{(1)}$ (m)	$q_{uniform}$ (kPa)
1	0.38	206	566	40.7	0.15	40.3
2	1.12	312	860	80.8	0.20	62.3
3	1.88	418	1158	140.3	0.27	86.0
4	2.62	526	1461	222.7	0.35	111.7
5	3.38	634	1768	331.6	0.44	140.0
6	4.12	743	2080	470.6	0.54	171.8
7	4.88	853	2395	643.3	0.65	207.8
8	5.62	964	2715	853.3	0.77	249.6

(1) For determination of $q_{uniform}$, eccentricity is assumed as zero for calculated $e < 0$.

where:

P_{VT} = Calculated based on the weight of the soil overlying the strip and the vertical component of the lateral earth pressure, and includes γ = 1.35 and 1.50 as appropriate

M_v = Calculated in the same manner as P_{VT}

M_h = Calculated based on the horizontal earth pressures with a load factor of 1.5

$$e = \frac{B}{2} - X_0 = \frac{B}{2} - \frac{M_v - M_h}{P_{VT}}$$

$$\gamma_p \, \sigma_v = q_{uniform} = P_{VT}/(B - 2\,e_B)$$

Sample Calculations for Layers 1 and 7 are summarized below (See Figure 14-10 for pressure diagram at Layer 1).

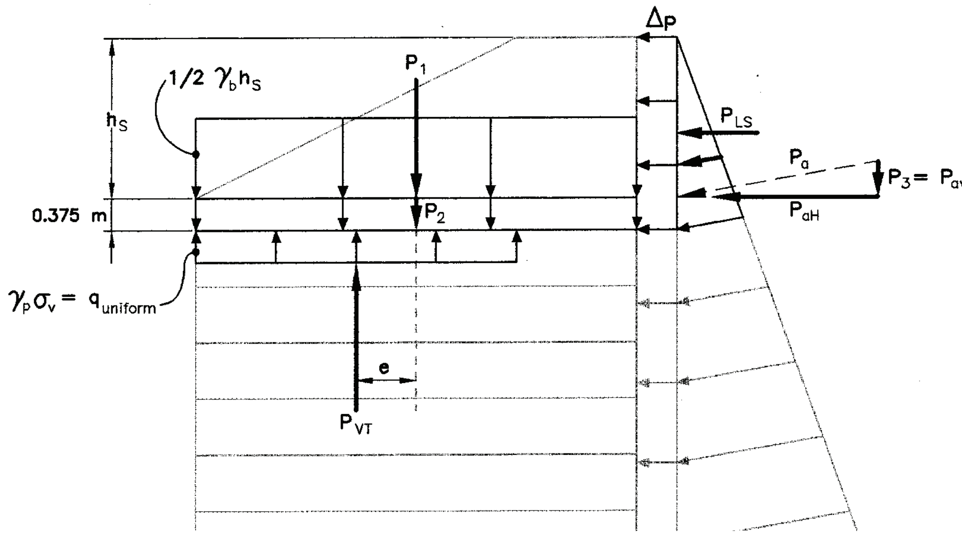


Figure 14-10
Pressure Diagram for Computation of $q_{uniform}$
for Internal Stability Evaluation of Layer 1

<u>**Layer 1**</u>

<u>Resultant of Vertical Forces, P_{VT}</u>

$P_{VT} = P_1 + P_2 + P_3$

$P_1 =$ Factored weight of soil overlying strip above wall (kN). For broken-back slopes, the weight of the soil wedge above the top of the reinforced soil zone is conventionally replaced in internal stability analysis by a uniform surcharge equal to 0.5 γ_b h_S where h_S is the height of the slope at the back of the wall.

$P_2 =$ Factored weight of soil overlying strip below top of wall (kN)

$P_3 =$ Vertical component of factored active earth pressure load and is calculated similar to Step 1 (D). The lateral earth pressure is calculated as a triangular distribution over the area of the wall above the reinforcing and has a vertical and a horizontal component.

$P_1 =$ γ_p (0.5 $\gamma_b h_s$) L = 1.35 (0.5) (20.6 kN/m^3) (2 m) (5.4 m) = 150.2 kN/m length of wall

$P_2 =$ γ_p W_2 = 1.35 (18.835 kN/m^3) (0.375 m) (5.4 m) = 51.5 kN/m length of wall

$P_3 =$ γ_p P_a sin B = 1.5[0.5(20.6 kN/m^3)(0.28)(2.0 m + 0.375 m)^2 sin 9.5$^{o\circ}$] = 4.0 kN/m length of wall

At Layer 1:

$P_{VT} = P_1 + P_2 + P_3 = 150.2 + 51.5 + 4.0 = 205.7$ kN/m length of wall

Moments About Front Face of Wall at Connection to Strip, M_r and M_h

$M_v =$ (150.2 kN/m)(5.4 m/2) + (51.5 kN/m)(5.4 m/2) + (4.0 kN/m)(5.4 m)

$M_v =$ 405.5 + 139.0 + 21.6 (kN-m/m) = 565.1 kN-m/m length of wall

$M_h =$ 1.5 cos (9.5°) (20.6 kN/m³) (0.28) (2.0 m + 0.375 m)³/6 + 1.75 (4.38 kPa) (2 m + 0.375 m)²/2

$M_h =$ 40.7 kN-m/m length of wall

Uniform Pressure on Effective Base Width

$e =$ $L/2 - [(M_v - M_h)/P_{VT}]$

$e =$ 5.4 m /2 - [(566.1 kN-m/m - 40.7 kN-m/m)/205.7 kN/m] = 0.146 m

$q_{uniform} =$ $P_{VT}/(L - 2\,e)$ = (205.7 kN/m)/[5.4 m - 2(0.146 m)] = 40.3 kPa

Layer 7

$P_1 =$ 150.2 kN/m length of wall (Same as Layer 1)

$P_2 =$ 1.35 (18.835 kN/m³) (4.875 m) (5.4 m) = 669.4 kN/m length of wall

$P_3 =$ 1.5[0.5 (20.6 kN/m³) (0.28) (2.0 m + 4.875 m)² sin 9.5°°] = 33.7 kN/m

$P_{VT} =$ $P_1 + P_2 + P_3$ = 150.2 + 669.4 + 33.7 = 853.3 kN/m length of wall

$M_v =$ (150.2 kN/m) (5.4 m/2)+ (669.4 kN/m) (5.4 m/2)+ (33.7 kN/m) (5.4 m)

$M_v =$ 405.5 + 1 807.4 + 182.0 = 2394.9 kN-m/m length of wall

$M_h =$ 1.5 cos (9.5°)(20.6 kN/m³)(0.28)(2.0 m + 4.875 m)³/6 + 1.75 (4.38 kPa)(2 m + 4.875 m)²/2

$M_h =$ 643.3 kN-m/m length of wall

$e =$ 5.4 m /2 - [(2394.9 kN-m/m - 643.3 kN-m/m)/853.3 kN/m] = 0.647 m

$q_{uniform} =$ $P_{VT}/(L - 2\,e)$ = (853.3 kN/m)/[5.4 m - 2(0.647 m)] = 207.8 kPa

By calculating the lateral earth pressure coefficients, k, at specific locations over the height of the wall, the value of k can be determined by interpolation at the levels of the various strips as indicated on Table 14-33. Values of σ_H can be calculated by multiplying these k values by the factored σ_v ($q_{uniform}$ from Table 14-32). P_i can then be calculated by multiplying σ_H by h_i (which was set at 0.75 m).

Table 14-33
Summary of Factored Horizontal Loads

Layer	z (m)	k	σ_V (kPa)	σ_H (kPa)	h_i (m)	P_i (kN/m)
1	0.375	0.42	40.3	16.9	0.75	12.7
2	1.125	0.40	62.3	24.9	0.75	18.7
3	1.875	0.38	86.0	32.7	0.75	24.5
4	2.625	0.36	111.7	40.2	0.75	30.2
5	3.375	0.33	140.0	46.2	0.75	34.6
6	4.125	0.31	171.8	53.3	0.75	39.9
7	4.875	0.29	207.8	60.3	0.75	45.2
8*	5.625	0.29	249.6	72.4	0.75	54.3

Having determined the loading at the level of the strip the loading on the strips must then be determined and compared to the allowable load. To determine the load on the strip, the area of the strip at the end of the design life (the reduced strip area to include corrosion loss) will be used.

(B) Corrosion Losses
The initial width, w_i, and thickness, t_i, of the reinforcing are:

w = 50 mm
t_i = 4 mm

Use a galvanizing thickness of 0.086 mm/side.

For structural design, requirements for sacrificial thickness for each exposed surface are (*A11.9.8.1*):

- Loss of galvanizing =0.015 mm/yr for first 2 years, and 0.004 mm/yr thereafter
- Loss of carbon steel = 0.012 mm/yr after zinc depletion

Thickness of galvanizing after first two years (per side):

0.086 - 2(0.015) = 0.056 mm

Service life for remaining zinc:

0.056/0.004 = 14 years

Therefore, service life for zinc = 2 years + 14 years = 16 years. After loss of galvanizing, the remaining design life is 100 years - 16 years = 84 years, and the loss of steel during the remaining design life is:

(84 years) 0.012 mm/yr/side = (1.01 mm) (2 sides)

Thickness after 100 years:

t_{100} = 4.00 mm - 2.02 mm = 1.98 mm = 0.00198 m

(C) Reinforcement Tensile Resistance
The center-to-center strip spacing is 0.70 m. From Table 14-33, the maximum value of σ_H = 72.4 kPa for Layer 8, for which the maximum load per meter length of wall, P_i, is:

$P_i = \sigma_H h_i$ 1 m = (72.4 kPa)(0.75 m) = 54.3 kN

The maximum factored load per strip is:

$P_{max} = P_i$ (strip spacing) = (54.3 kN/m) (0.7 m) = 38.0 kN

Therefore the maximum factored tensile stress, σ_{max}, adjusted for long-term corrosion effects is:

$$\sigma_{max} = \frac{P_{max}}{w \times t_{100}} = \frac{38.0\,\text{kN}}{0.05\,\text{m} \times 0.00198\,\text{m}} = 384\,000\,\text{kPa}$$

The factored tensile strength in the reinforcement, $\phi\ \sigma_n$, is

$\phi\ \sigma_n = \phi\ F_u$ = (0.85) 520 000 kPa = 442 000 kPa

Because $\phi\ \sigma_n \geq \sigma_{max}$, resistance against strip rupture is adequate.

Step 10: Internal Stability - Panel Connection

The horizontal force used to design the connection to the panels may be taken as not less than 85% of the maximum calculated force as determined above, except for the lower one-half of the structure (below Layer 4) where the connections must be designed for 100% of the maximum calculated force (*A11.9.5.1.2*).

Connection design forces are calculated by multiplying the factored horizontal force from Table 14-33 by the tributary width for the strip (= c/c spacing). For the upper half of the wall the horizontal force is reduced by 15%. The design force for the strips is summarized in Table 14-34 and sample calculations for the strips follow.

<u>Sample Calculation for Layer 1</u>:

P_i = 12.7 kN/m
Tributary Width (c/c spacing) = 0.7 m
Force/Strip = (12.7 kN/m) (0.7 m) = 8.9 kN
Connection Design Force =(0.85) (8.9 kN) = 7.6 kN

Table 14-34 - Summary of Maximum Factored Horizontal Forces for Connection of Strip to Facing

Layer	P_i (kN/m)	Force/Strip (kN)	Connection Design Force/Strip (kN)
1	12.7	8.9	7.6
2	18.7	13.1	11.1
3	24.5	17.2	14.6
4	30.2	21.1	17.9
5	34.6	24.2	24.2
6	39.9	27.9	27.9
7	45.2	31.6	31.6
8	54.3	38.0	38.0

Step 11: Internal Stability - Reinforcement Pullout

Calculate the pullout resistance of the reinforcement, P_{fs}, using:

$$P_{fs} = f^{*} \gamma_s Z A_s \qquad \text{(Eq. 14-14)}$$

where:

f^* = 2.0 at the ground level; and $f^* = \tan \phi_f = \tan 33^o = 0.65$ at 6 m
γ_s = 18.835 kN/m^3
Z = Depth below effective top of wall (H_1 on Figure 14-9)
A_s = Total top and bottom surface area of reinforcement along the effective pullout length beyond the failure plane less any sacrificial thickness = (L_{eff}) (50.0 mm) (2 sides)

From Table 14-4, ϕ = 0.90 for soil pullout. The factored pullout resistance ϕ P_{fs} for each layer is calculated in Table 14-35 and sample calculations for Layer 1 follow.

Table 14-35
Pullout Resistance in Reinforcement Levels

Layer	Z (m)	f*	L_{eff} (m)	A_s (mm^2)	P_{fs} (kN)	ϕ P_{fs} (kN)
1	1.43	1.68	3.28	0.3282	14.9	13.4
2	2.18	1.51	3.28	0.3282	20.4	18.3
3	2.93	1.34	3.28	0.3282	24.3	21.9
4	3.68	1.17	3.38	0.3375	27.4	24.7
5	4.43	1.00	3.82	0.3825	32.0	28.8
6	5.18	0.83	4.28	0.4275	34.8	31.3
7	5.93	0.67	4.72	0.4725	35.1	31.6
8	6.68	0.65	5.18	0.5175	42.3	38.1

By inspection, $L_{eff} \geq$ minimum length of reinforcement beyond the assumed failure surface of 0.9 m, and the minimum total required length of reinforcement of 2.4 m for all levels (*A11.9.5.1.4*).

Sample Calculation for Layer 1:
he pullout resistance, P_{fs}, at any layer is:

$$P_{fs} = f^* \gamma_s Z A_s \qquad \text{(Eq. 14-14)}$$

From Figure 14-9:

$$Z = H_1 - H + z = 7.059 \text{ m} - 6 \text{ m} + 0.375 \text{ m} = 1.434 \text{ m}$$
$$L_{eff} = L - 0.3 H_2 = 5.4 \text{ m} - 0.3 (7.059 \text{ m}) = 3.282 \text{ m}$$
$$A_s = 2 (3.282 \text{ m}) (0.05 \text{ m}) = 0.3282 \text{ m}^2$$
$$f^* = 2.0 - (2.0 - 0.65)\ 1.434/6.0 = 1.68$$

Substituting:

$$P_{fs} = f^* \gamma_s Z A_s = (1.68) (18.835 \text{ kN/m}^3) (1.434 \text{ m}) (0.3282 \text{ m}^2) = 14.9 \text{ kN}$$

and the factored pullout resistance is:

$$\phi\ P_{fs} = 0.90 (14.9 \text{ kN}) = 13.4 \text{ kN}$$

Because the factored horizontal force per strip from Table 14-34 is less than or equal to the factored pullout resistance at each level, ϕ P_{fs}, from Table 14-35, the pullout resistance is adequate.

CHAPTER 15
FLEXIBLE CULVERT DESIGN

15.1 Introduction

Culverts are produced with large variety of material properties, geometric wall sections, sizes and shapes. For highway applications, culverts are concrete, steel, aluminum, thermoplastic and composites of these materials, and can have diameters as small as 0.30 m and spans as large as 15 m. The scope of this training manual does not permit treatment of all the available culvert systems. Therefore, discussions herein are limited to small-diameter, round culverts.

Culverts are generally divided into two major classes, flexible and rigid. Flexible culverts respond to vertical loads by depending on a large strain capacity and interaction with the surrounding soil to hold their shape. If the backfill envelope is not constructed to develop adequate passive resistance and stiffness, the culvert will deflect beyond a tolerable limit. Conversely, the strain capacity of rigid culverts is limited. Therefore, these structures must develop significant ring stiffness and strength to support the vertical pressures imposed upon them. This is the basis of the D-Load (i.e., Three-Edge Bearing) design method.

The differentiation between flexible and rigid culverts is becoming blurred. New culvert products being advanced that employ a flexible structural shell with a rigid liner to resist abrasion and corrosion. The strain capacity of such a composite is more limited than the flexible shell. Alternatively, the latest direct design method being advanced for rigid culverts (ASCE, 1993) provides for consideration of side and haunch soil support in excess of the active or at-rest cases. Hence, rigid culvert designs are becoming more dependent upon soil support than designs based solely upon the rigid body three-edge bearing model. This developing trend is not yet recognized in the current AASHTO specifications. Consistent with AASHTO and common current practice, the treatment of culverts in this training manual is divided into two parts. This chapter focuses with comparisons between Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD) for flexible culverts. Rigid culverts are addressed in Chapter 16.

Culverts, like other transportation systems, require input from multiple engineering disciplines during design. Successful performance of the total system requires application of the principles of hydraulics, hydrology, economics, roadway geometric design, traffic and safety. All but the structural and geotechnical aspects of culvert design are beyond the scope of this manual. Relative to geotechnical and structural design of flexible culverts, this chapter:

- Describes primary differences between LRFD and ASD of flexible culverts
- Identifies the strength and serviceability performance limits that must be considered for flexible culvert design by LRFD
- Enumerates the design basis assumed in current practice
- Presents an example of a flexible culvert design by LRFD

15.2 Design Methods

The design of flexible culverts by ASD or LRFD follows virtually the same process and utilizes the same design procedures. The primary difference between ASD and LRFD for flexible culverts is that the ASD design use a safety factor to evaluate a specified structural response while LRFD assigns load and resistance factors to evaluate the specified structural response.

Structurally, culverts perform as highly redundant composite members comprised of the culvert structure and the soil continuum. As a result, structural and geotechnical design of the culvert-soil system are performed concurrently, generally using empirical procedures. Notwithstanding the tremendous understanding gained over the last 80 years through experimental and analytical investigations, many aspects of current practice do not yet attempt to rationalize and optimize all of the performance parameters that could be considered. The current state-of-the-practice is comprised of numerous procedures that have developed along separate, but parallel paths. The designer is faced with the need to synthesize divergent and difficult to access data to make unbiased comparisons between the variety of culvert products available today.

15.2.1 ASD Summary

As described in Chapter 3, the ASD methodology assumes loads at a magnitude consistent with the service limit state (i.e., working loads). The working (or allowable) stress is determined as the ultimate resistance divided by a factor of safety, which accounts for the uncertainty in both load and resistance. The ASD methodology does not accommodate different levels of uncertainty for either the load or resistance components of the structural system.

Existing culvert design practice establishes the maximum and minimum soil cover height for a given design live load. The culvert cross sectional size and shape are generally controlled by hydraulic or other end-use considerations. As a result, the design is directed toward selection of the culvert structural wall section. For small-diameter culverts, the design process includes evaluation of:

- Wall compression (thrust)
- Wall buckling
- Seam strength (for culvert types with seams)
- Handling flexibility

Relative to loading of a flexible culvert, the ultimate load capacity, Q_{ult}, is estimated based on the culvert material strength and the suitability of the design is evaluated by determining the allowable axial design load, Q_{all}, using:

$$Q \leq Q_{all} = Q_{all}/FS \qquad \text{(Eq. 15-1)}$$

where:

$Q =$ Design load (kN)
$Q_{all} =$ Allowable design load (kN)

Q_{ult} = Nominal (ultimate) load capacity of culvert (kN)
FS = Factor of safety (dim)

For structural design based on allowable stress, Eq. 15-1 can be rewritten as:

$$Q \leq \sigma_{all} \times A = \frac{\sigma_{ult}}{FS} \times A = P_{all} \qquad \text{(Eq. 15-2)}$$

where:

σ_{all} = Allowable stress in culvert material (kPa)
P_{all} = Allowable structural resistance (kN)
σ_{ult} = Limit stress in culvert material (kPa)
A = Cross sectional area of pipe wall (m^2)

The required values of FS for wall compression or buckling and for longitudinal seam resistance to thrust are commonly taken as indicated in Table 15-1.

Table 15-1
Factor of Safety on Ultimate Structural Capacity of Flexible Culverts
(AASHTO, 1997b)

Performance Limit	Required Minimum Factor of Safety (FS)
Wall Compression (Thrust)	2.0
Wall Buckling	2.0
Seam Strength	3.0

The reduction of the structural capacity of a culvert from an ultimate value (based on the yield or buckling strength of the material) to an allowable value based on a Factory of Safety (FS) accounts for all uncertainty in the variation of applied loads and the ultimate structural capacity of the pipe. Therefore, the structural design of the pipe is performed by ASD using actual estimated loads.

In addition to structural capacity evaluation, the design of flexible culverts by ASD establishes limitations of culvert flexibility to avoid damage due to handling and construction-related deformations using:

$$FF \leq FF_{max} \qquad \text{(Eq. 15-3)}$$

where:

FF = Flexibility Factor (m/kN)
FF_{max} = Suggested maximum permissible Flexibility Factor (m/kN)

15.2.2 LRFD Summary

In ASD, the safety factor is applied to only one element of the design relationship. However, various elements of load and resistance often contain differing levels of uncertainty. ***The advantage of the LRFD method over ASD is that it provides a format capable of assigning both load and resistance elements unique "equivalent safety factors," based on the relative level of uncertainty associated with each****. Uncertainty is accounted for by quantifying the variability in both the load and resistance components of the design.* For culvert design, the resistance and deformation of the soil-structure interaction system must satisfy the general LRFD relationships below. For Strength and Construction Limit States:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r \qquad \text{(Eq. 15-3)}$$

For the Service Limit States:

$$\sum \eta_i \gamma_i \delta_i \leq \phi \delta_n \qquad \text{(Eq. 15-4)}$$

where:

η_i = Factors to account for ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)
R_r = Factored resistance (kN or kPa)
δ_i = Estimated deflection (mm)
δ_n = Tolerable deflection (mm)

For flexible culvert design at the Strength Limit States, the following values of η in Eq. 15-4 normally can be used:

$\eta_D = \eta_R = 1.0$

η_I = 1.05 for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures.

Assessments of the relative ductility and redundancy of flexible culverts and of the roles that ductility and redundancy play in culvert performance have not yet been made. Therefore, $\eta_D = \eta_R = 1.0$ is assumed for flexible culverts in this chapter.

Determination of the operational importance of a structure is made by the facility owner as described in Chapter 4. For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

At any Service or Construction Limit State, η_D, η_R and $\eta_I = 1.0$.

Relative to loading of a flexible culvert, the suitability of a culvert section with respect to structural resistance can be obtained using Eq. 15-4, rewritten as:

$$\sum \eta_i \gamma_i Q_i \leq \phi P_n = P_r \tag{Eq. 15-6}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Factored load effect (kN)
ϕ Resistance factor (dim)
P_n= P_{ult} = Nominal (ultimate) or structural resistance of culvert (kN)
P_r = Factored structural resistance of culvert (kN)

Values of load factor and load factor combinations for each applicable limit state must be developed using the guidelines described in Chapters 3 and 4 and in Section 15.2.2.1, and loads should be developed as described in Chapter 4. The ultimate resistance, R_n, should be determined for each type of resistance described in Section 15.3. Selection and modification of resistance factors, ϕ, are described in Section 15.2.2.3 through Section 15.2.2.5.

15.2.2.1 Limit States (*A12.5*)

Limit states, as implemented in the AASHTO LRFD Specification (AASHTO, 1997a), represent a more refined version of the concept of critical load combinations that have been part of the AASHTO specifications for nearly 30 years (AASHO, 1969). However, before limit state design concepts were introduced into the specification, various percentages of allowable stresses were specified for different critical load combinations.

The AASHTO LRFD Specification requires that Strength and Service I Limit States be considered for culvert design. In addition, the LRFD specification requires that construction loads be considered in the design. For buried culverts, construction loads may govern design under shallow covers, particularly when heavy earth moving equipment is considered. The Service I Limit State represents performance at working levels of the AASHTO design vehicle live load.

The performance limits and limit states that must be considered for design of small-diameter, round flexible culverts are listed in Tables 15-2 and 15-3.

Table 15-2
Limit States Applicable for Design of Metal Pipes and Arches

Performance Limits	Strength I Limit State	Service I Limit State	Construction Limit State
Wall Compression (Thrust)	✔		✔
Wall Buckling	✔		✔
Seam Strength	✔		✔
Handling Flexibility		✔	

Table 15-3
Limit States Applicable for Design of Thermoplastic Pipe

Performance Limits	Strength I Limit State	Service I Limit State	Construction Limit State
Wall Compression (Thrust)	✔		✔
Wall Buckling	✔		✔
Tensile Strain		✔	
Handling Flexibility		✔	

Although handling flexibility is not actually evaluated under Service loads, the consideration of flexibility is in effect a Service Limit State in that the intent is to limit deformations during handling and installation. Excessive deformations could induce bending stresses of unquantifiable magnitude in flexible culverts, and could negatively impact the structural capacity of a culvert.

From Tables 15-2 and 15-3 it is evident that deflection criteria (i.e., tensile strain) under service loads are applied only to thermoplastic pipe. This is not meant to imply that deflection limits are inherently more applicable to one type of flexible pipe than another. Rather, it recognizes that thermoplastics may experience limitations of ductile strain capacity while undergoing deflection. Some thermoplastic product designs must be limited to between 3.5 and 5 percent strain to avoid local buckling of the cross section and material rupture.

15.2.2.2 Loads and Load Factors (*A12.5.3, A12.5.4*)

Culvert design is influenced primarily by vertical earth pressures and, under shallow cover, by vehicular live loads. Chapter 4 points out that loads resulting from unbalanced horizontal earth pressures, compaction and water buoyancy can be important in the design of flexible culverts. However, loads from wind, temperature fluctuations, vehicle braking, and structure dead weight are insignificant compared to the vertical earth pressures and can be safely neglected. Also, historical earthquake records appear to support the fact that buried culverts are highly resistant to seismic damage, except in the vicinity of faulting or where ground instability or unbalanced loading occurs.

The magnitude of vertical earth pressure on a culvert can vary significantly, being not only dependent upon the height of cover and density of material above the culvert, but also upon the ring stiffness of the culvert relative to its adjacent side fill. The magnitude of vertical pressure exerted on flexible culverts is integrally associated with the passive pressure that the culvert develops in the adjacent side fill. Also, foundation stiffness can be a factor. If the culvert settles more than its adjacent side fill, the load carried by the culvert wall is reduced. When the opposite is true, the load on the culvert wall is increased. As in other structural systems, the stiffest element attracts the greatest load.

Because of the numerous factors affecting its magnitude, vertical earth pressure is highly variable relative to other loads on culverts. The load factors specified in the LRFD AASHTO Specification and given in Table 15-4 reflect this variability.

Table 15-4
Load Factors for Flexible Culvert Design
(after AASHTO, 1997a)

Load Type	Load Factor, γ		
	Limit State		
	Strength I	Service I	Construction
Earth Loads:			
• Max. Vertical Earth Press (EV)	1.95	1.00	1.00
• Min. Vertical Earth Press. (EV)	0.90	1.00	1.00
Vehicular Live Loads:			
• Design Truck or Tandem (LL)	1.75	1.00	1.50
• Dynamic Load Allowance (IM)	1.75	1.00	1.50

As seen in the table, $\gamma = 1.95$ is assigned for vertical earth pressure, EV, under the Strength I Limit State, whereas for vehicular live load or impact load, $\gamma = 1.75$ for the same limit state. This represents a reverse ratio compared to other structural systems, where live loads are generally considered to have more uncertainty than dead loads.

15.2.2.3 Resistance Factors (*A12.5.5*)

Resistance factors for culverts are summarized in Table 15-5. The factors are based on direct calibration with ASD.

Table 15-5
Resistance Factors for Design of Flexible Culverts
(after AASHTO, 1997a)

Structure Type and Design Consideration	Resistance Factor
Metal Pipe, Arch & Pipe Arch:	
Helical Pipe with Lock Seam or Fully-Welded Seam:	
• Wall compression (thrust) and buckling	1.00
Annular Pipe:	
• Wall compression (thrust) and buckling	0.67
• Seam strength	0.67
Structural Plate:	
• Wall compression (thrust) and buckling	0.67
• Seam Strength	0.67
Thermoplastic Pipe:	
PE and PVC pipe	
• Wall compression (thrust) and buckling	1.00

15.2.2.4 Comparison of Flexible Culvert Design Using ASD and LRFD

Very little quantitative test data exist on which to base a statistically meaningful calibration of flexible culvert limit states. Therefore, except as noted, the LRFD load and resistance factors for flexible culverts have been derived using both judgment and the fitting procedures described in Chapter 3.

Table 15-6 compares the ASD safety factors (from Table 15-1) and the equivalent safety factors (FS_{LRFD}) implied by the LRFD resistance factors (from Table 15-4) for flexible culverts for the Strength I Limit State. The equivalent safety factors for LRFD are calculated according to the following formula:

$$FS_{LRFD} = \frac{\sum \eta_i \gamma_i}{\phi} \qquad \text{(Eq. 15-7)}$$

where:

η_i = Factors to account for ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
ϕ = Resistance factor (dim)
FS_{LRFD} = LRFD equivalent factor of safety (dim)

Table 15-6 suggests that the LRFD load and resistance factors generally result in slightly less conservative design than ASD when earth loads predominate and a less conservative design than ASD when live loads predominate for a structure designed for the Strength I Limit State. However, due to the greater live loads incorporated in the LRFD Specification (AASHTO, 1994) as noted in Chapter 4, the LRFD method is in fact more conservative than ASD, even where live loads predominate.

Table 15-6
Comparison of Effective Factors of Safety for Flexible Culverts
Strength I Limit State

Loading Combinations	Load Factor γ_i	Maximum Resistance Factor ϕ	LRFD Equivalent Factor of Safety, FS_{LRFD} (1)	ASD Factor of Safety
EV:				
• Thrust & Buckling (no seam)	1.95	1.00	2.0	2.0
• Seam Strength	1.95	0.67	2.9	3.0
LL, IM:				
• Thrust & Buckling (no seam)	1.75	1.00	1.8	2.0
• Seam Strength	1.75	0.67	2.6	3.0

(1) Assumes η_i = 1.0 for typical structure.

15.2.2.5 Modification of Resistance Factors

As indicated in Section 15.2.2.3, the LRFD resistance factors for design of flexible culverts in Table 15-4 were developed based on calibration with ASD. As described in Section 15.2.2.4, application of the resistance factors in Table 15-6 in conjunction with typical values of $\eta\gamma_i$ for earth loads and live loads results in an "equivalent" factor of safety of 2.0 for thrust (wall compression) and buckling, and 2.9 for seam strength.

In ASD, the designer or owner might decide to increase or decrease required factors of safety or allowable design stresses in consideration of a number of factors, such as:

- The live load model utilized
- The potential consequences of a failure
- The extent or quality of information available from geotechnical exploration and testing
- Past experience with the soil conditions encountered and/or capacity prediction method used
- The level of construction control anticipated or specified
- The likelihood that the design loading conditions will be realized

When using LRFD, similar flexibility to vary the required level of safety should also be available and, in some cases, is inherent in the load and resistance factors used. Whereas the same factor of safety is generally used in ASD regardless of the source of loading, the equivalent factor of safety in LRFD (defined by Eq. 15-7) varies for a given resistance factor depending on the source of loading as a function of the available load factors.

To modify the resistance factors for design of flexible culverts to account for average load factors and equivalent factors of safety other than those identified in Table 15-6, the following equation may be used:

$$\phi_m = \phi_T \times \left(\frac{FS_T}{FS_D}\right) \times \left(\frac{(\eta_i \gamma_i)_T}{\left(\sum \eta_i \gamma_i Q_i\right) / \sum Q_i}\right) \qquad \text{(Eq. 15-8)}$$

where:

ϕ_m = Modified resistance factor (dim)
ϕ_T = Tabulated resistance factor (dim)
FS_T = Tabulated factor of safety (dim)
FS_D = Desired factor of safety (dim)
$(\eta_i\gamma_i)_T$ = Product of tabulated load factors (dim)
$\dfrac{\Sigma \eta_i \gamma_i Q_i}{\Sigma Q_i}$ = Actual average load factor (dim)

Modifying resistance factors may seem reasonable, but such modification may not be consistent with the goal of LRFD to achieve equal reliability against failure of structure components, unless the factor of safety accurately models the reliability of the capacity predictive method used. The resistance factors may be more appropriately modified through application of the probabilistic procedures described in Chapter 3 to achieve the desired level of reliability if sufficient data are available.

15.2.3 Summarized Comparison of ASD and LRFD

As noted before, the process used to develop a flexible culvert LRFD differs very little from the process used for ASD. The similarity is illustrated in the parallel flow charts in Figure 15-1. Specific differences between the methods and other important issues are highlighted in the following section.

Other aspects of the culvert design such as identifying special considerations (e.g., potential for loss of support through scour), developing a design foundation profile and determining requirements for construction control are inherent aspects of the design process required for both LRFD and ASD.

15.3 Performance Limits

Design of flexible culverts by either LRFD or ASD must provide adequate resistance against geotechnical and structural failure and limit deformations to within tolerable limits. Following selection of a culvert based on hydraulic considerations, the design of a flexible culvert requires consideration of many factors which can affect the culvert performance, including:

- Wall resistance to thrust or compression under applied earth and traffic loadings
- Wall resistance to buckling under applied earth and traffic loadings
- Resistance of longitudinal seams in the pipe wall to applied loads
- Resistance to stresses induced during culvert handling and installation

For these design factors, there is no difference between LRFD and ASD analysis procedures. The following sections highlight differences between LRFD and ASD in the performance criteria and application of design procedures.

15.3.1 Displacements and Tolerable Movements (*A12.6.2*)

Wall displacements are usually limited based on controls required during construction. However. short- or long- term deflection limits for small diameter round culverts are generally not specified explicitly.

Control of ring deformation of flexible culverts must be considered both during and after culvert installation. During handling and placement, the culvert wall stiffness and strength, as well as specified installation practices, serve to limit ring deformation. The handling flexibility factor described in Section 15.3.5 is intended to quantify this performance limit.

After culvert installation, the stiffness of the surrounding soil envelope totally controls long term ring deformations. This is true for any practical wall stiffness or strength associated with flexible culverts. These facts were established by early investigators such as Spangler (1941, 1950) and Watkins (1958) who found the long-term culvert deflections were a function not only of the ring stiffness of the culvert but also of the modulus of the surrounding soil envelope. This soil-structure

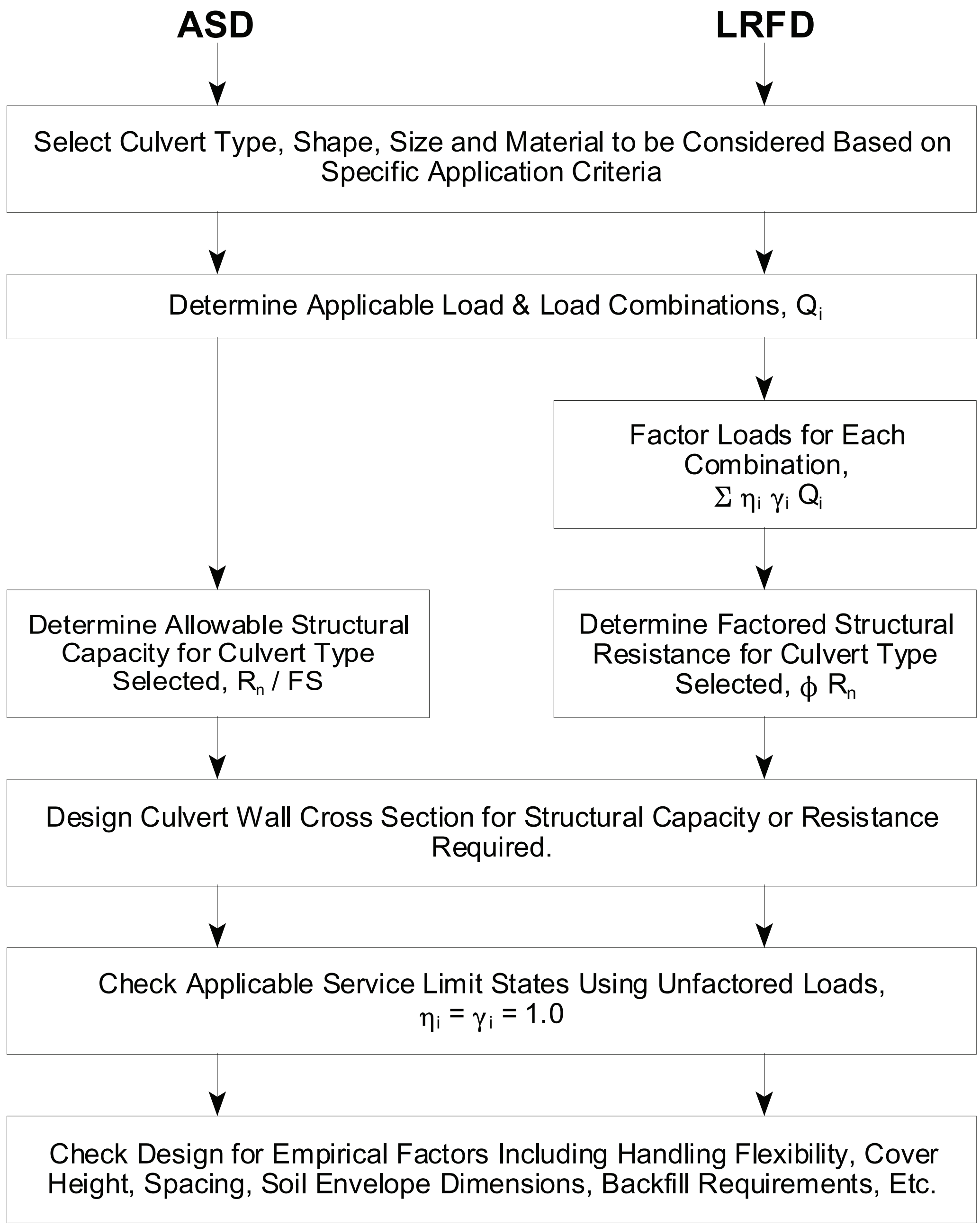


Figure 15-1
Generalized Flow Chart for Flexible Culvert Design by ASD and LRFD

interaction phenomena has been studied extensively using both empirical and analytical techniques in an effort to develop a generalized theory of performance. While great strides have been achieved in the understanding and quantification of culvert performance, displacements remain the most difficult performance limit to predict due to the lack of a consistent and reliable predictive model for the backfill stiffness modulus of soil surrounding a flexible culvert.

It is instructional to note that round flexible culverts generally show two types of ring deformation modes, depending upon the relative density of the adjacent side fill, as illustrated in Figure 15-2. For loosely placed backfill, the deformation mode is one of ovaling. As load is increased, the top arc eventually becomes so flat that a reverse curvature (snap-through buckling potential) occurs. On the other hand, for densely placed backfill, ovaling is not predominant. Rather, a local instability, either wall crushing or inward cusping (local bending/buckling) occurs at some point around the pipe periphery, generally over the top third.

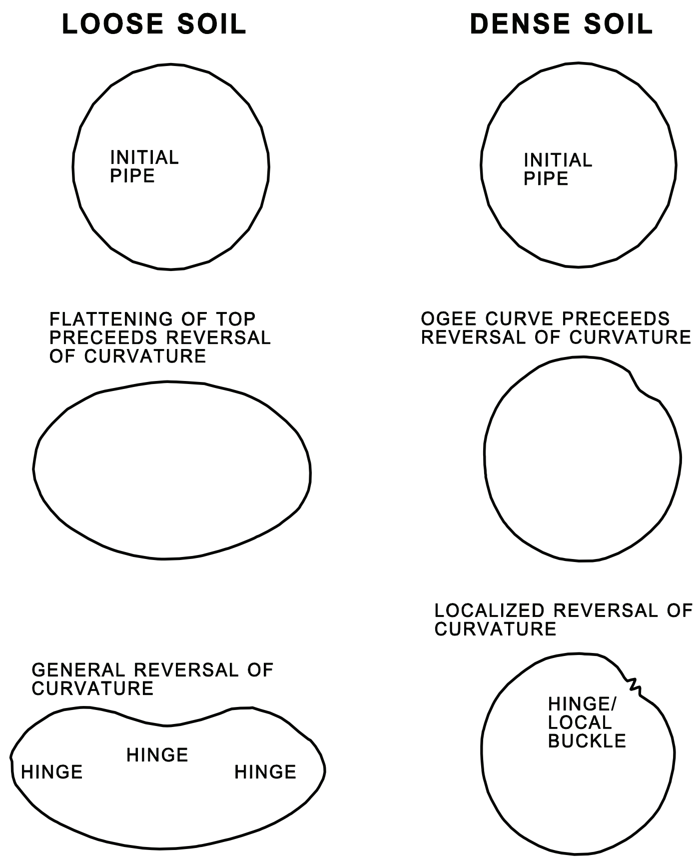


Figure 15-2
Comparison of Flexible Culvert Deflection Modes in Loose and Dense Soils
(After Watkins and Moser, 1969)

Tests on over 100 corrugated steel pipe specimens ranging in diameter from 0.90 m to 1.50 m performed in the Utah Test Cell produced the ovaling mode of deformation for soil that was either loosely placed or compacted to less than about 85 percent relative compaction (AASHTO T-180; Watkins and Moser, 1969). For this range of relative compaction levels, reverse curvature was reached at vertical deflections of between 10 and 20 percent. For relative compaction levels between 85 and 90 percent, a mix of ovaling and inward cusping was reached at vertical deflections of between 5 and 10 percent. For relative compaction levels greater than 90 percent, the performance limit was generally wall crushing and inward cusping, which occurred at vertical deflections of less than 5 percent. As the compactive effort and soil density increased, loss of deflection tolerance was offset by greater loads than could be supported.

This finding led to the conclusion that, for small-diameter, round culverts, adequate density of the backfill envelope eliminates the potential for catastrophic, snap-through buckling due to excessive ovaling and reverse curvature. Because snap-through buckling has been analytically and empirically challenging to quantify for soil-structure interaction systems, the focus became field control of deflections through a combination of backfill material quality and construction procedures.

While this approach is not fully rigorous from a theoretical viewpoint, its implementation has been highly successful statistically. However, the approach is not intended to be applied to large diameter and multi-radius culverts and long spans that can experience more complicated failure mechanisms.

15.3.2 Wall Compression (Thrust) (*A12.7.2.2, A12.12.3.4*)

Flexible culvert walls must be designed to resist earth and live loads without a compression failure. The factored resistance of a culvert wall to compression failure can be determined as:

$$R_r = \phi R_n = \phi A f_L \qquad \text{(Eq. 15-9) } (\textit{A12.7.2.3-1}) \ (\textit{A12.12.3.5-1})$$

and compared to the applicable loads using:

$$R_r \geq T_L = P_L\left(\frac{S}{2}\right) \qquad \text{(Eq. 15-10) } (\textit{A12.7.2.2-1}) \ (\textit{A12.12.3.4-1})$$

where:

R_n = Nominal resistance (kN/m)
R_r = Factored resistance (kN/m)
ϕ = Resistance factor from Table 15-4 (dim)
S = Span or diameter of culvert (m)
P_L = Factored crown pressure = $\Sigma\eta_i\gamma_i Q_i$ (kPa)
T_L = Factored thrust in culvert wall (kN/m)
A = Wall area per unit length (m^2/m)
f_L = Material limit stress (kPa)

Mechanical properties for common flexible culvert materials are presented in Tables 15-7 and 15-8. Note that the material limit stress for aluminum and steel is the yield point stress. For thermoplastics, the limit stress is the 50-year ultimate tensile stress. Wall section properties for culvert types can be found in Section 12, Appendix A of the LRFD Specification.

Table 15-7 (*A.A12-9*)
Strength and Elastic Properties of Aluminum and Steel Culverts
(after AASHTO, 1997a)

Culvert Material	$f_L = f_y$ (kPa)	E_m (kPa)
Aluminum	165 000	69 000 000
Steel	228 000	200 000 000

Table 15-8 (*A12.12.3.3-1*)
Strength and Elastic Properties of PE and PVC Culverts
(after AASHTO, 1997a)

Culvert Material	Material Cell Class	$f_L^{(3)} = f_y$ (kPa)	$E_m^{(3)}$ (kPa)
PE:[1]			
Solid Wall	335434C	9 930	152 000
Corrugated Wall	315420C	6 210	152 000
Profile Wall	334433C	7 720	138 000
Profile Wall	335434C	9 930	152 000
PVC:[2]			
Solid Wall	12454C	25 500	965 000
Solid Wall	12364C	17 900	1 090 000
Profile Wall	12454C	25 500	965 000
Profile Wall	12364C	17 900	1 090 000

(1) ASTM D3350 Cell Class Specification; (2) ASTM D1784 Cell Class Specification
(3) 50-year properties

Several sources of variability exist on both the load and resistance sides of Eq. 15-10. First, the design pressure is dependent upon assumptions of live and dead load magnitude and distribution through the soil. Undoubtedly, load estimation is the source of greatest uncertainty in this relationship. Typically, provisions for load and load distribution have been structured with very conservative assumptions for most applications. Moreover, loads and load distribution are considered to be the same for all flexible culvert types. AASHTO LRFD specifies use of the prism load described in Chapter 4. Load relief through positive arching is not currently recognized.

Another significant source of uncertainty relative to the determination of resistance in Eq. 15-10 is the calculation of wall area. For linear elastic wall sections that are fully effective, such as for corrugated metal, the variability is low. On the other hand, the metal spiral rib wall section is not fully effective. Whereas it is known that the degree of cross-section effectiveness is load dependent, current design criteria only recognize the effective area at ultimate load, based on tests. Hence, at working loads an unquantified "safety factor" or reliability index is present, which is greater than that which exists for the corrugated profile at the same percentage of load.

The variation in diameter due to manufacturing processes is very small. However, flexible culverts may deflect during or after installation, which increases their horizontal diameter and, hence, the load they must support. Failure to account for this increase is unconservative. But, for small diameter culverts, the effects are small and readily quantifiable. A five percent deflection generally results in less than a five percent increase in load for circular culverts with adequate backfill density. However, this simplicity does not apply to larger multi-radius culverts and long spans, where shape changes can have major nonlinear effects on both load and resistance.

Thermoplastic pipe exhibits viscoelastic behavior. Therefore, its material limit stress is both load and time dependent. Because of this complex performance and the relatively recent introduction of thermoplastics as a culvert material, provisions base the thermoplastic limit stress on a 50-year, continuous tensile loading. This is generally believed to be very conservative for burial depths greater than two diameters (assuming a stable wall section under compression). More research is currently being conducted to better define the compressive strength of thermoplastic culvert wall sections.

15.3.3 Wall Buckling (*A12.7.2.4, A12.12.3.6*)

Flexible culvert walls must be designed to resist earth and live loads without a buckling failure. The resistance of a culvert wall section to buckling failure can be determined as:

$$R_r = \phi R_n = \phi A f_{cr} \qquad \text{(Eq. 15-11)}$$

and compared to the applicable loads using:

$$R_r \geq T_L = P_L \left(\frac{S}{2}\right) \qquad \text{(Eq. 15-12) } (\textit{A12.7.2.2-1}) \ (\textit{A12.12.3.4-1})$$

where:

R_n = Nominal resistance (kN/m)
R_r = Factored resistance (kN/m)
ϕ = Resistance factor from Table 15-4 (dim)
S = Culvert span or diameter (m)
P_L = Factored crown pressure = $\Sigma\eta_i\gamma_i Q_i$ (kPa)
T_L = Factored thrust in culvert wall (kN/m)
A = Wall area per unit length (m^2/m)
f_{cr} = Buckling strength of culvert wall (kPa)

Two wall buckling relationships have commonly been adopted for flexible culverts. The first, applicable to metal culverts, has been credited to Watkins, although details of its development have never been published. This relationship was first published as a design criterion in the "Handbook of Steel Drainage and Highway Construction Products (AISI, 1967), and was later adopted by AASHTO (1969). The relationship is based on the theoretical buckling of thin circular tubes under hydrostatic pressure (Timoshenko and Gere 1961), but was modified for application to buried structures by an empirical soil correlation factor.

The AISI handbook cited model tests by Meyerhof and Baikie (1963) and a discussion by Watkins to support the empirical soil correlation. However, rather than develop the buckling formulation using plate or shell theory and Winkler springs as used by most investigators at that time, its developer chose to follow the format of the more basic Euler buckling relationship typically employed for discrete linear columns. This process resulted in the following for small diameter round culverts:

$$f_{cr} == \frac{12\,E_m}{\left(\frac{kS}{r}\right)^2} \qquad \text{(Eq. 15-13) } (\textit{A12.7.2.4-2})$$

where:

f_{cr} = Buckling strength of culvert wall (kPa)
E_m = Modulus of elasticity of culvert material (kPa)
k = Soil correlation factor (dim), k = 0.22 for well-compacted backfill
S = Span or diameter (m)
r = Radius of gyration of culvert wall (m)

An interaction equation is also used to check metal culverts for behavior in a transition between the above critical buckling stress and yield stress. However, this interactive check applies only to small diameter culverts and is not discussed herein.

The thermoplastic culvert industry chose to use the buckling equation originated by Luscher (1966) and adopted by the American Water Works Association. Luscher assumed an elastically supported thin plate with an empirical correction for soil stiffness based on his tests with model tubes, and also recognized that shallow cover buckling was less sensitive to side fill stiffness than to the relationship between cover height ring stiffness. This equation was further modified by Glascock (1980), who performed further calibrations based on additional soil box tests and some full scale observations. The resulting equation was adopted by AASHTO (1997a, 1997b):

$$f_{cr} = 0.77\,\frac{R}{A}\left(\frac{B\,M_s\,E_m\,I}{0.149\,R^3}\right)^{1/2} \qquad \text{(Eq. 15-14) } (\textit{A12.12.3.6-1})$$

where:

f_{cr} = Buckling strength of culvert wall (kPa)
R = Radius of the culvert cross section at the neutral axis (m)
A = Area of the culvert wall (m^2/m)
B = $1 - 0.33\,\frac{h_w}{h}$ (dim) (*A12.12.3.6-2*)
M_s = Confined modulus of the soil (kPa)

E_m = Long-term (50 year) modulus of elasticity of thermoplastic culvert material (kPa)
I = Moment of inertia of the culvert wall (m^4/m)
h_w = Height of water surface above culvert (m)
h = Height of ground surface above culvert (m)

A complete treatment of culvert buckling is beyond the scope of this chapter. In addition to the criteria described, the buckling of buried conduits has been addressed by many other investigators. Some other studies were summarized by Leonards and Stetkar (1978), which may serve as a further reference to the workshop participant.

15.3.4 Seam Strength (*A12.7.2.5*)

Specification provisions for seam strength are applicable only to culverts with bolted, riveted or spot welded seams. Bolted seams are found in both aluminum and steel versions of field-assembled corrugated plate, known as structural plate. These plates are fabricated in a wide range of sizes and shapes, with diameters or spans ranging from 1.50 m to over 15 m. The corrugations of structural plate are typically deeper and the metal thickness greater than used for factory-made pipe. Two primary reasons for using-field assembled pipe are when size restrictions preclude shipping of factory made pipe and for applications requiring heavier wall sections than can be fabricated from the spiral wound coil stock used in factory made pipe.

Some small diameter factory made pipe is still being fabricated from sheets that are riveted or spot welded together. However, this practice is decreasing dramatically in the United States, being supplanted by the more efficient spiral corrugation process. In addition to most factory made corrugated profiles, all aluminum and steel box rib profiles are spiral wound. Also, all thermoplastic pipes employed as culverts are fabricated without seams.

Longitudinal seams carry the primary ring compression forces and are the focus of the design provisions. Circumferential seams are secondary. They serve to clamp the plates together but generally do not carry significant forces.

The resistance of longitudinal culvert wall seams, SS, to the applied load is checked using:

$$SS \geq \frac{T_L}{\phi} = \frac{P_L\,S}{2\,\phi} \qquad \text{(Eq. 15-15)}$$

where:

P_L = Factored crown pressure from all loads = $\Sigma\eta_i\gamma_iQ_i$ (kPa)
ϕ = Resistance factor from Table 15-4 (dim)
S = Span or diameter of culvert (m)
T_L = Factored thrust in culvert wall (kN/m)
SS = Seam strength (kN/m)

Values of SS for bolted, riveted and spot-welded corrugated steel and aluminum pipe are provided in the Section 12, Appendix of the AASHTO LRFD Specification. The values for SS tabulated in AASHTO are based on the results of short column compression tests using average rather than minimum material properties. These numbers represent two very different failure modes. The most catastrophic mode is brittle shear failure of the fasteners, which could result in collapse of the culvert. This mode is generally associated with high wall stiffness relative to fastener stiffness and strength.

A less catastrophic failure mode is ductile local yielding and buckling of the wall material at the fastener location. This results from high fastener stiffness and strength relative to wall stiffness. The local deformations associated with this type of failure results in clamping of the wall section, at which point the fasteners are partially or fully unloaded. Also, the local buckling that occurs at the fastener location shortens the culvert wall axially, which induces positive arching, further reducing the load on the culvert wall under deep burial conditions (greater than one diameter of cover).

The specifications have long recognized the greater uncertainty associated with seam capacity and have assigned a higher safety factor for wall compression of culverts with seams compared to those without. However, wall sections having ductile failure modes have grater levels of safety against collapse in extreme events than those with brittle failure modes. Nevertheless, this difference is not currently recognized in the specification provisions.

15.3.5 Handling Flexibility (*A12.7.2.5, A12.12.3.7*)

Loads applied during construction often represent the most significant source of bending stress in flexible culverts. However, these loads are extremely difficult to quantify. As a result, empirical measures are employed to define minimum wall sectional properties that offer adequate bending resistance to handling and construction-related deformations. These measures are formulated in terms of ring flexibility (the inverse of ring stiffness) using:

$$FF = \frac{S^2}{E_m I} \qquad \text{(Eq. 15-16) } (\mathit{A12.7.2.6\text{-}1}) \ (\mathit{A12.12.3.7\text{-}1})$$

where:

- FF = Flexibility factor (m/kN)
- S = Span or diameter (m)
- E_m = Modulus of elasticity of culvert material (kPa)
- I = Moment of inertia of culvert wall (m^4/m)

FF is then compared to the maximum permissible flexibility factor, FF_{max}, using Eq. 15-3. Values of FF_{max} for various culvert materials are provided in Table 15-9.

Factors other than the parameters identified in Eq. 15-16 (e.g., material type, corrugation profile, cross-sectional shape and backfill characteristics) are known to contribute to the ability of culverts to resist handling and construction distress. Therefore, the AASHTO LRFD Specifications provide a number of tables that list maximum flexibility factors for various parameters.

Table 15-9 (*A12.5.6.1-1, A12.5.6.3*)
Flexibility Factor Limits for
Corrugated Metal, Structural Plate and Thermoplastic Pipe
(AASHTO, 1997a)

Material Type	Corrugation Size (mm)	Maximum Permissible Factor, FF_{max} (m/kN)
Steel Pipe	6.35 12.7 25.4	0.25 0.25 0.19
Aluminum Pipe	6.35 and 12.7 1.52 mm material thickness 1.90 mm material thickness All others 25.4	0.18 0.35 0.53 0.34
Steel Plate	150 × 50 Pipe Pipe-Arch Arch	0.11 0.17 0.17
Aluminum Plate	230 × 64 Pipe Pipe-Arch Arch	0.14 0.21 0.21
Thermoplastic Pipe		0.54

These tabulated flexibility limit values have been developed and promulgated by the industries that manufacture culvert structures. As experience factors, they implicitly include economic considerations as well. For instance, a contractor installing a flexible culvert may be willing to tolerate additional handling care as a tradeoff for lighter handling weight. Therefore, flexibility factors represent gross measures of successful installation experience. However, they are not sufficiently precise or rigorous to be used as a basis for comparison between culverts of different material types or wall sectional profiles.

Also note from Eq. 15-16 that no load combinations are explicitly specified and no resistance factors (or safety factors for ASD) are applied. *Hence, flexibility factors do not relate to either Strength or Serviceability Limit States, but are simply empirical factors that are applied in the same form to both ASD and LRFD methodologies.*

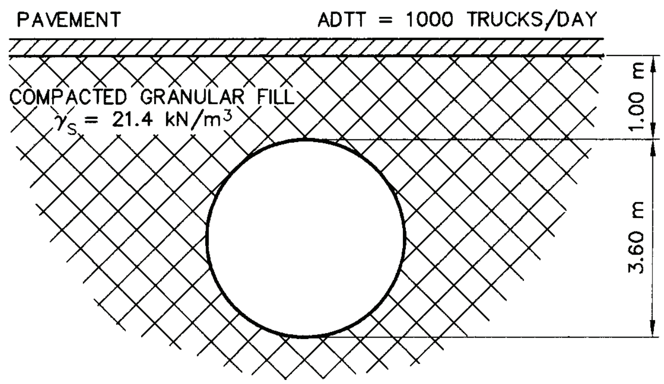
15.3.6 Other Considerations

To assure successful performance, a number of other factors must be considered as well. These include backfill material and density, differential foundation movement, construction procedures, end conditions and unsymmetric loading. Design procedures for these factors are generally independent of either ASD or LRFD methodologies.

15.4 Design Example

Problem: As shown in Figure 15-3, a 3.6-m diameter flexible metal culvert is required for support of 1.0 m of soil/pavement cover and vehicle loading. The unit weight of soil backfill and cover, γ_s = 21.4 kN/m^3. The number of trucks per day in one direction averaged over the design life of the structure, ADTT = 1000 vehicles per day. It is assumed that $\eta_I = \eta_D = \eta_R = 1.00$ so that $\eta_i = 1.00$.

Figure 15-3



Problem Definition

Objective: Develop of permanent and live loads for structural design of the culvert, and select structural plate CMP section to meet performance limits for LRFD.

Approach: To perform the culvert design, the following steps are taken:

- Develop the load factors, multiple presence factor, reduction factor, spreading factor, arching factor and structural resistance factor required for design.
- Determine the surface tire contact area for the design truck and design tandem.
- Determine the distribution areas of live load at the culvert crown for the design truck and design tandem.
- Determine the unfactored distributed live load pressure for the design truck, design tandem and design lane.
- Determine whether the design truck load or design tandem load is critical for design of the culvert.
- Calculate the factored vertical earth pressure and total factored vertical

pressure on the culvert.

- Select the pipe wall based on seam strength and check for wall buckling and handling flexibility.

Solution:

Step 1: Develop Design Parameters
From Table 15-4 for the Strength I Limit State, the load factors are:

$\gamma_{LL} = \gamma_{IM} = 1.75$
$\gamma_{EV} = 1.95$

As described in Section 4.5.2, a single lane contribution can be conservatively assumed for buried structures. From Table 4-13, the Multiple Presence Factor, m (*A3.6.1.1.2*), is:

$m = 1.2$

Due to the low volume of truck traffic (i.e., ADTT = 1000), the reduction factor, R = 0.95 (*AC3.6.1.1.2*).

Because the structure will be backfilled and covered using a compacted granular fill material, the spreading factor, $S_E = 1.15$ (*AC3.6.1.2.6*).

Using a prism load as a conservative estimate of the vertical earth pressure, the arching factor, $F_e = 1.0$.

From Table 15-5 for a structural plate alternative, the resistance factor, $\varphi = 0.67$

Step 2: Determine Surface Contact Area for Design Truck and Design Tandem
From Eq. 4-39, the dynamic load allowance, IM, at the ground surface ($D_E = 0$) is:

$IM = 40\,(1.0 - 0.41\,D_E) = 40\,(1.0 - 0)$
$IM = 40\%$

From Eq. 4-38, the length of tire contact surface, ℓ, for the design truck (P = 72.5 kN) is:

$$\ell = 0.00228\,\gamma\left(1+\frac{IM}{100}\right)P = 0.00228\,(1.75)\left(1+\frac{40}{100}\right)72.5$$

$\ell = 0.405$ m

For the design tandem (P = 55 kN), ℓ is:

$$\ell = 0.00228\,(1.75)\left(1+\frac{40}{100}\right)55 = 0.307\,\text{m}$$

Note the width dimension, w, of the tire contact area is constant for both design vehicles and equals:

$w = 0.51$ m

Step 3: Determine Dimensions and Distributed Area of Live Load at Crown (D_E = 1.0 m)
For the design truck, from Eq. 4-40 and from Steps 1 and 2:

$L_D = \ell + S_E\, D_E = 0.405 + 1.15\ (1.0)$
$L_D = 1.56$ m

Check applicability of Eq. 4-43 for $D_E = 1.0$ m:

$D_E \leq (1.80\text{ m} - w)/S = (1.80\text{ m} - 0.51\text{ m})/1.15 = 1.12$ m
$D_E = 1.0 \leq 1.12$ m (OK) (Therefore, use Eq. 4-43)

Using Eq. 4-43:

$W_D = 2\ (w + S_E\, D_E) = 2\ (0.51\text{ m} + 1.15\ (1.0\text{ m}))$
$W_D = 3.32$ m

From Eq. 4-45, the distributed area, A_D, for the design truck is:

$A_D = L_D\, W_D = (1.56\text{ m})\ (3.32\text{ m})$
$A_D = 5.18\text{ m}^2$

For the design tandem, check applicability of Eq. 4-41 for $D_E = 1.0$ m:

$D_E \leq (1.20\text{ m} - \ell)/S_E = (1.20\text{ m} - 0.309\text{ m})/1.15 = 0.78$ m
$D_E = 1.0 > 0.78$ m (NG) (Therefore, use Eq. 4-42)

Using Eq. 4-42:

$L_D = \ell + S_E\, D_E + 1.20\text{ m} = 0.307\text{ m} + 1.15\ (1.0\text{ m}) + 1.20$
$L_D = 2.66$ m

From Eq. 4-45, the distributed area for the design tandem is:
$A_D = L_D\, W_D = (2.66\text{ m})\ (3.32\text{ m})$
$A_D = 8.83\text{ m}^2$

Step 4: Calculate Distributed Live Load Pressures at Depth D_E = 1.0 m
From Eq. 4-39, the dynamic load allowance at $D_E = 1.0$ m is:

$IM = 40\ (1.0 - 0.41\ D_E) = 40\ (1.0 - (0.41)\ (1.0))$
$IM = 24\%$

Using Eq. 4-36, with AL = 145 kN; and values of m, R, γ and A_D from Steps 1 and 3, the pressure distribution for the Design Truck is:

$$DTP = \frac{m\,R\,\gamma_{LL}\,AL\left(1+\frac{IM}{100}\right)10^3}{A_D} = \frac{(1.2)\,(0.95)\,(1.75)\,(145\,kN)\,(1.24)}{5.18\,m^2} = 69.2\,kPa$$

DTP = 69.2 kPa

Using Eq. 4-36 and AL = 220 kN, the pressure distribution for the Design Tandem is:

$$DTP = \frac{m\,R\,\gamma_{LL}\,AL\left(1+\frac{IM}{100}\right)10^3}{A_D} = \frac{(1.2)\,(0.95)\,(1.75)\,(220\,kN)\,(1.24)}{8.83\,m^2} = 61.6\,kPa$$

Using Eq. 4-37, the factored pressure distribution due to the Uniform Lane Load is:

$$DLP = \frac{9.3\,m\,R\,\gamma_{LL}}{3.00} = \frac{9.3\,(1.2)\,(0.95)\,(1.75)}{3.00} = 6.18\,kPa$$

The distributed live load pressure distributions at the pipe crown for the Design Truck is presented in Figure 15-4. The corresponding figure for the Design Tandem is presented in Figure 15-5.

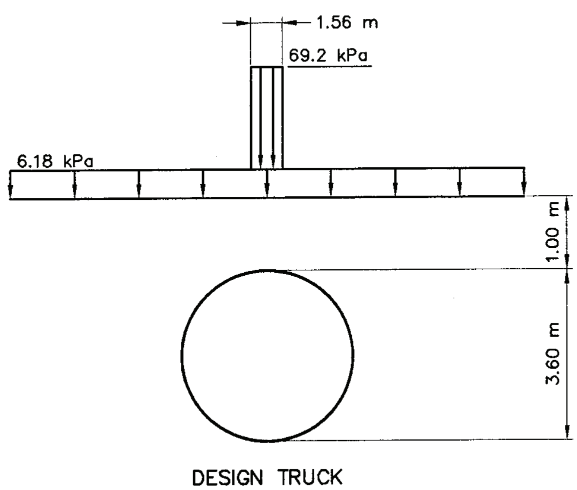


Figure 15-4
Pressure Distribution at Pipe Crown for Design Truck

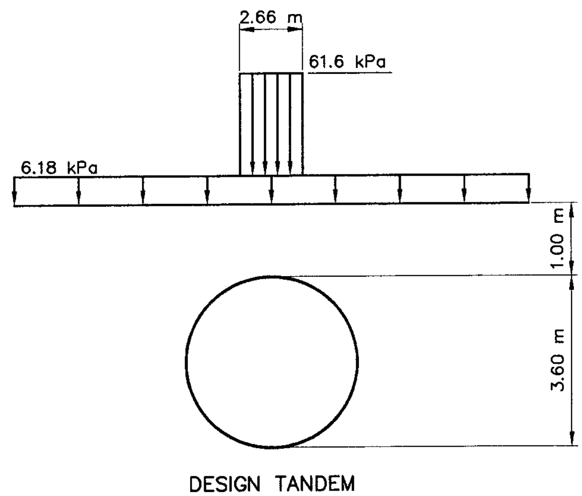


Figure 15-5
Pressure Distribution at Pipe Crown for Design Tandem

Step 5: Determine Whether Design Truck and Design Tandem Controls Design

Using Steps 3 and 4, the load for the design truck for $L_D \leq B_c$ is:

$$(DTP)\ (L_D) = (69.2\ \text{kPa})\ (1.56\ \text{m}) = 108\ \text{kN/m}$$

Using Steps 3 and 4, the load for the design tandem for $L_D \leq B_c$) is:

$$(DTP)\ (L_D) = (61.6\ \text{kPa})\ (2.66\ \text{m}) = 164\ \text{kN/m}$$

Load for design tandem > load for design truck; therefore, the design tandem governs.

Step 6: Calculate the Factored Load Due to Vertical Earth Pressure

Using Eq. 4-35, calculate the prism load, assuming an arching factor $F_e = 1.0$, which is conservative for flexible culverts under yielding foundations.

$$\gamma_{EV}\ W_E = \gamma_{EV}\ [F_e\ (\gamma_S\ B_c\ H)] = 1.95\ [(1.0)\ (21.4\ \text{kPa})\ (1\ \text{m})\ (3.6\ \text{m})]$$
$$\gamma_{EV}\ W_E = 1.95\ [77.0\ \text{kN/m}] = 150.2\ \text{kN/m}$$

Step 7: Calculate Total Factored Equivalent Pressure on Culvert
Using Steps 4, 5 and 6:

$$P_L = \Sigma\ \eta_i\ \gamma_i\ Q_i = [(DTP)\ (L_D) + DLP + \gamma\ W_E]/S_E$$
$$P_L = (164\ \text{kN/m} + 22.2\ \text{kN/m} + 150.2\ \text{kN/m})/3.6\ \text{m}$$
$$P_L = 93.4\ \text{kPa}$$

Step 8: Select Pipe Wall Section Based on Seam Strength

From Eq. 15-15:

$$SS = \frac{T_L}{\phi} = \frac{P_L}{\phi}\left(\frac{S}{2}\right) = \left(\frac{93.4\ \text{kPa}}{0.67}\right)\left(\frac{3.6\ \text{m}}{2}\right) = 251\,\text{kN/m}$$

From Table 15-10, a 2.77 mm thick steel plate section with a 152 x 50 mm corrugation is selected (i.e., SS = 251 kN/m < 628 kN/m).

Table 15-10
Minimum Longitudinal Seam Strength
for Steel and Aluminum Structural Plate Pipe
(AASHTO, 1997a)

152 × 50 mm Structural Plate Pipe				
Bolt Thickness (mm)	Bolt Diameter (mm)	4 Bolts/mm (kN/m)	6 Bolts/mm (kN/m)	8 Bolts/mm (kN/m)
2.77	19.1	628	-	-
3.51	19.1	905	-	-
4.27	19.1	1180	-	-
4.78	19.1	1360	-	-
5.54	19.1	1640	-	-
6.32	19.1	1930	-	-
7.11	19.1	2100	2630	2830

From AASHTO LRFD (1997a) Tables A12-3 and A12-10, the following properties are obtained:

$A =$ 3.29 mm^2/mm = 0.00329 m^2/m
$r =$ 17.3 mm = 0.0173 m
$I =$ 990 mm^4/mm = 9.9 x 10^{-7} m^4/m
$E_m =$ 200 000 MPa = 2 x 10^8 kPa
$F_y =$ 228 MPa = 228 000 kPa
$F_u =$ 310 MPa = 310 000 kPa

Step 9: Check Wall Buckling

From Eq. 15-11 and 15-12:

$$f_{cr(required)} = \left(\frac{P_L}{\phi A}\right)\left(\frac{S}{2}\right) = \left(\frac{93.4\ \text{kPa}}{0.67\ (0.00329\ \text{m}^2/\text{m})}\right)\left(\frac{3.6\ \text{m}}{2}\right) = 7.63 \times 10^4\ \text{kPa}$$

From Eq. 15-13 with k=0.22 (*A12.7.2.4*):

$$f_{cr} = \frac{12\,E_m}{\left(\frac{kS}{r}\right)^2} = \frac{12\,(2 \text{ x } 10^8 \text{ kPa})}{\left(\frac{0.22 \text{ x } 3.6 \text{ m}}{0.0173 \text{ m}}\right)^2} = 1.14 \text{ x } 10^6 \text{ kPa}$$

$$1.14 \text{ x } 10^6 \text{ kPa} >> 7.63 \times 10^4 \text{ kPa}$$

Therefore, wall buckling does not govern design.

Step 10: Check Handling Flexibility

From Eq. 15-16, the handling flexibility factor, FF, is:

$$FF = \frac{S^2}{E_m\,I} = \frac{(3.6 \text{ m})^2}{(2 \text{ x } 10^8 \text{ kPa})\,(9.9 \text{ x } 10^{-7} \text{ m}^4/\text{m})} = 0.55 \text{ m/kN}$$

From Table 15-9, the maximum permissible flexibility factor for steel plate pipe is:

$$FF_{max} = 0.110 \text{ m/kN} > 0.065 \text{ m/kN}$$

Therefore, handling flexibility does not govern design.

Note: Steps 8 through 10 could be repeated to optimize or compare alternative designs of other culvert materials.

CHAPTER 16
RIGID CULVERT DESIGN

16.1 Introduction

The design and analysis of rigid culverts represent complex soil-structure interaction problems in which the strength and deformation limits of the structure must be balanced with the reaction characteristics of the soil. Therefore, the development of methods for the design and analysis of these structures has involved efforts to quantify various aspects of the soil-structure interaction behavior controlling their performance.

Beginning with Marston and Anderson (1913), rational methods have been advanced to account for the important effects of positive and negative arching loads on conduits, trench and embankment construction methods, and bedding conditions (e.g., Schlick, 1920; Marston, 1930; Schlick, 1932; Spangler, 1933; Spangler, 1950). In addition, evolving refinements in design and analysis techniques for reinforced-concrete pipe have enabled greater precision and reliability in the placement of reinforcement elements to optimize both load capacity and economy (e.g., Paris, 1921; Olander, 1950; Heger, 1963 and 1982; and ASCE, 1993).

The focus of this chapter is limited to a comparison of ASD (AASHTO, 1997b) versus LRFD (AASHTO, 1997a) aspects of the design process of rigid (i.e., reinforced concrete) pipe. As such, the presentation assumes that the participants have some familiarity with the principals of soil-structure interaction and reinforced concrete design. This chapter:

- Describes primary differences between LRFD and ASD of rigid culverts
- Identifies the strength and serviceability performance limits that must be considered for rigid culvert design by LRFD
- Reviews the design basis inherently assumed in the current specification provisions

16.2 Design Methods

Both ASD (AASHTO, 1997b) and LRFD (AASHTO, 1997a) permit design of rigid culverts by either the *Indirect Design Method* or the *Direct Design Method*. The Indirect Design Method is based on empirical observations of both the structural capacity of a culvert and its installation condition. In the Indirect Design Method, no intrinsic structural response, other than an empirical measure of crushing capacity, is evaluated. Test data on crushing capacity has been compiled for many types of precast, circular, rigid pipe such as cast iron, vitrified clay, asbestos cement, unreinforced concrete and reinforced concrete pipe (RCP). Historically, all of these pipe types, except RCP, have been available only in small diameters not generally applicable for highway applications.

In the Direct Design Method, the loads and installation effects on the culvert are determined in the same way as for the indirect method. However, the structural responses of a culvert are evaluated using the principals of reinforced concrete design which take into account the geometry and constitutive material properties of the structural cross section. Direct Design represents a relatively recent innovation for RCP and was developed around modern limit state design concepts consistent with LFD and LRFD. Additionally, direct design is the only approach applicable for the design of special shapes such as reinforced concrete box culverts.

16.2.1 Indirect Design Method (*A12.10.4.3*)

In both ASD (AASHTO, 1997b) and LRFD (AASHTO, 1997a), the indirect design methodology assumes loads at a magnitude consistent with the service limit state (i.e., working loads). Likewise, structural capacities are taken at classical working stress performance limits. ***The Indirect Design methodology does not accommodate different levels of uncertainties for either the load or resistance components of a structural system.***

As for flexible culvert design, rigid culvert design practice specifies the maximum and minimum cover height and the design live load. Also, the cross sectional size and shape are usually controlled by hydraulic or other end use considerations. Thus the design process is directed toward selecting the structural wall section of the culvert. For reinforced concrete culverts, this process requires evaluation of:

- Flexure
- Thrust
- Shear
- Radial tension

The Indirect Design Method for reinforced concrete culverts is based on a comparison of the laboratory determined strength of a given pipe design, known as the Three Edge Bearing Strength (or D-Load), with an empirically derived field equivalent strength, which is calculated by multiplying the laboratory strength by a Bedding Factor. Thus, flexure, thrust, shear and radial tension are accounted for indirectly.

During development of the Indirect Design Method for rigid culverts early in this century, a number of laboratory test protocols were tried. Eventually, the Three-Edge Bearing Test (ASTM, 1997) became the standard. An illustration of the test setup is given in Figure 16-1. The result of a Three-Edge Bearing Test is reported as the D-Load. D-Load is calculated by dividing the total measured strength of the test specimen by its inside diameter as follows:

$$D = \frac{TEB}{S_i} \qquad \text{(Eq. 16-1)}$$

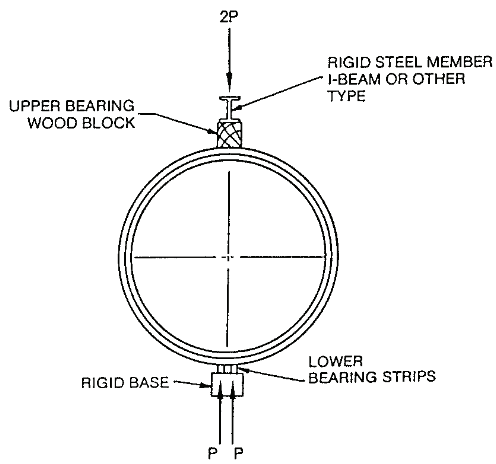


Figure 16-1
D-Load Test Setup
(ASTM, 1997)

where:

D = Normalized D-Load (kN/m)
TEB = Total strength of pipe specimen in three edge bearing test (kN)
S_i = Inside diameter of test specimen (m)

The laboratory tests generally measure two D-Load strengths. The first is the load required to produce a 0.25-mm wide crack which represents the limit of desirable performance based on considerations of corrosion and longevity. The second is the ultimate load capacity of the section, which for RCP is between 1.25 and 1.50 higher than the D-Load. The AASHTO ASD and LRFD Specifications for indirect design of rigid culverts are based on the D-Load strength representing the 0.25 mm crack.

For the Indirect Design Method, the nominal design load carrying capacity of the pipe, as represented by the D-Load, must equal or exceed the Service Limit State design load on the pipe, or:

$$\phi D = \sum \gamma_i Q_i \qquad \text{(Eq. 16-2)}$$

for which $\phi = \gamma_i = 1.0$.

For indirect design of rigid culverts, Eq. 16-2 can be rewritten as:

$$D = \left[\frac{1}{S_i}\right]\left[\frac{W_E + W_F}{B_{FE}}\right] \qquad \text{(Eq. 16-3) (A12.10.4.3.1-1)}$$

where:

B_{FE} = Earth load bedding factor (A12.10.4.3.2a or A12.10.4.3.2b) (dim)
B_{FLL} = Live load bedding factor (A12.10.4.3.2c) (dim)
S_i = Internal pipe diameter (m)
W_E = Total unfactored earth load (A12.10.2.1) (kN/m)
W_F = Total unfactored fluid load in pipe (A12.10.2.3)(kN/m)
W_L = Total unfactored live load (A12.10.2.4) (kN/m)

For AASHTO Standard Type I installations, the D-Load is modified by multiplying by an installation factor of 1.10.

16.2.2 Direct Design Method (*A12.10.4.2*)
As indicated in Section 16.2, the Direct Design Method employed in both ASD (AASHTO, 1997b) and LRFD (AASHTO, 1997a) incorporates limit state design concepts which account for uncertainty in both load and resistance. Although rigid culvert design represents a complex soil-structure interaction problem, the direct design of rigid culverts entails essentially a structural design of the culvert section.

Because the design of rigid (RCP) culverts involves a structural rather than geotechnical design procedure, the Direct Design Method in ASD follows the Load Factor Design (LFD) methodology used for structural design of all reinforced concrete structures in the AASHTO ASD Specification. In LFD, like LRFD, loads are modified by load factors and ultimate strengths are modified by strength-reduction factors to account for uncertainty.

In ASD, the load and resistance elements of RCP culverts must satisfy the general LFD relationship given by:

$$\gamma \sum \beta_i Q_i \leq \phi R_n \qquad \text{(Eq. 16-4)}$$

where:

R_n = Nominal (ultimate) resistance (kN or kPa)
ϕ = Strength-reduction factor (dim)
γ = Load factor (dim)
β_i = Load combination coefficient (dim)
Q_i = Load (kN or kPa)

Design of rigid culverts is influenced primarily by vertical earth pressures and, for the case of shallow cover, by vehicular live loads. The magnitude of vertical earth pressure on rigid culverts can vary significantly, depending on the degree of soil-structure interaction taking place. In general, rigid culverts are much stiffer than the surrounding soil and attract greater load than flexible culverts.

For rigid culvert design by ASD, the applicable load factor is $\gamma = 1.3$ and the applicable load coefficients are summarized in Table 16-1.

Table 16-1
Load Coefficients/Factors for LFD of Rigid Culverts
(after AASHTO, 1997b)

Load Type	Load Coefficient β	Effective Load Factor, $\gamma\beta$
Vertical Earth Pressure	1.00	1.30
Vehicular Live Load and Impact	1.67	2.17

In LRFD, the load and resistance elements of rigid culverts must satisfy the general LRFD relationship given by:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n \qquad \text{(Eq. 16-5)}$$

where:

η_i = Factors to account for ductility (η_D), redundancy (η_R) and operational importance (η_I) (dim)
γ_i = Load factor (dim)
Q_i = Force effect, stress or stress resultant (kN or kPa)
ϕ = Resistance factor (dim)
R_n = Nominal (ultimate) resistance (kN or kPa)

The load factors and load combinations used in the AASHTO LRFD Specification for design of rigid culverts were presented in Chapter 4 and are summarized in Table 16-2.

For rigid culvert design at the Strength Limit States, the following values of η normally can be used:

$\eta_D = \eta_R = 1.0$

η_I = 1.05 for structures deemed operationally important, 1.00 for typical structures and 0.95 for relatively less important structures

Table 16-2 (*excerpt from A3.4.1-1 and A3.4.1-2*)
Load Factors for LRFD of Rigid Culverts
(after AASHTO, 1997a)

Load Type	Load Factor, γ		
	Limit State		
	Strength I	Service I	Construction
Earth Loads:			
• Max. Vertical Earth Press (EV)	1.30	1.00	1.00
• Min. Vertical Earth Press. (EV)	0.90	1.00	1.00
Vehicular Live Loads:			
• Design Truck or Tandem (LL)	1.75	1.00	1.50
• Dynamic Load Allowance (IM)	1.75	1.00	1.50

Determination of the operational importance of a structure is made by the facility owner as described in Chapter 4. For the purpose of this chapter, the value of η_I is assumed equal to 1.0.

At any Service or Construction Limit State, η_D, η_R and η_I = 1.0.

In addition to the difference between load factors used in LFD (Table 16-1) and LRFD (Table 16-2), there is also a significant difference between the methods to compute live load pressures on a culvert due to revision of the live load model as discussed in Section 4.5.2. As discussed therein and shown on Figure 16-2, the net effect of the revised load factors and live load model is to increase the design live load by as much as 100 percent.

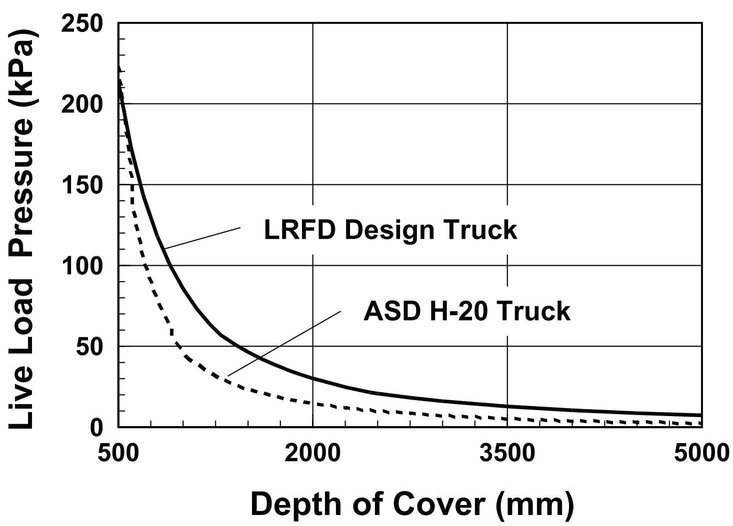


Figure 16-2
Comparison of AASHTO Live Load Pressures Through Earth Fills

16.2.2.1 Limit States (*A12.10.3, A12.10.4*)

Limit states, as implemented in the LRFD specification, represent a more refined version of the concept of critical load combinations included in the LFD provisions of the AASHTO ASD Specifications. The AASHTO LRFD Specification requires that Strength and Service I Limit States be considered for culvert design. In addition, the LRFD specification requires that construction loads be considered in the design. For buried culverts, construction loads may govern design under shallow covers, particularly when heavy earth moving equipment is considered. The Service I Limit State, represents performance at working levels of the AASHTO design vehicle live load.

The limit states that must be considered for design of small-diameter, round reinforced concrete pipe are listed in Table 16-3.

Table 16-3
Limit States Applicable for Design of Reinforced Concrete Pipe

Performance Limits	Strength I Limit State	Service I Limit State
Wall Flexure	✔	
Wall Thrust	✔	
Wall Shear	✔	
Radial Tension	✔	
Wall Cracking		✔

16.2.2.2 Resistance Factors (*A12.5.5*)

Resistance/strength-reduction factors specified in the AASHTO ASD and LRFD Specifications for the design of rigid culverts are presented in Table 16-4. For the most part, resistance factors for rigid culvert design have been established at the same values used for other reinforced concrete members. For flexure, the AASHTO LRFD Specification assumes the same level material variability for cast-in-place construction and in-plant manufacturing processes. Therefore, $\phi = 0.90$ is specified for both.

Table 16-4 (*excerpt from A12.5.5-1*)
Resistance/Strength-Reduction Factors for Precast Reinforced Concrete Culverts
(after AASHTO, 1997a; AASHTO, 1997b)

Performance Limit	Resistance/Strength – Reduction Factor
AASHTO Type I Installation	
• Flexure	0.90
• Shear	0.82
• Radial Tension	0.82
Other Type Installation	
• Flexure	1.00
• Shear	0.90
• Radial Tension	0.90

16.2.2.3 Comparison of Rigid Culvert Design Using ASD and LRFD

Both the AASHTO ASD and AASHTO LRFD Specifications permit design of rigid pipe culverts by either the Indirect Design Method of the Direct Design Method. The design procedures for these methods are identical in ASD and LRFD. Differences do exist however between the effective live load factors used in the AASHTO ASD LFD Direct Design Method (Table 16-1) and the live load factors used in the AASHTO LRFD Direct Design Method (Table 16-2). Significant differences also exist between the ASD and LRFD vehicular live load models as discussed in Sections 4.5.2 and 16.2.2. ***As a result, LRFD results in designs generally comparable to ASD (i.e., LFD) when earth loads predominate, and significantly more conservative designs than ASD when live loads predominate.***

16.3 Performance Limits

The following sections review the basic design aspects of the LRFD Direct Design Method and some of the variability associated with the method. However, to assure successful performance of rigid culverts, many factors must be considered beyond the design basis presented above for ASD and LRFD methodologies. Some of these include backfill material type and density, differential settlement, foundation consolidation, construction practices, and end treatments. Design procedures and specifications for these factors are generally independent of the design method.

16.3.1 Elastic Load Models (*A12.10.4.2.1, A12.10.2*)

The design load depends upon the assumptions used to develop the live and dead load contributions and their distribution through the soil. This load intensity can vary significantly due to a number of factors which apply to both flexible and rigid culverts. However, for rigid culverts, many more factors must be considered in the design including:

- Installation type (i.e., embankment versus trench construction)
- Installation conditions (i.e., negative or positive projecting)
- Trench width
- Bedding configuration
- Stiffness of the supporting soil

These factors significantly increase the generality and precision of the model. However, its accuracy is only as good as the individual assumptions associated with each of the parameters listed above. In addition to load intensity, some of the other significant uncertainties include installation related factors such as bedding type, trench width and relative soil stiffness. On the other hand, the variation in culvert diameter due to manufacturing processes is very small. Specifications limit variations in diameter to 1.5 percent. Also, variations in minimum concrete strength are limited to 10 percent.

Under the direct design methodology employed in the LRFD specification for rigid culverts, structural responses of the RCP are evaluated from an elastic analysis on an assumed culvert wall section. Once the moment, thrust and shear structural responses are obtained, the area of reinforcement steel is determined. Then, the assumed cross section is checked for adequacy. This process is iterated until convergence is reached.

The elastic analysis of the pipe ring is generally performed using an assumed pressure distribution such as proposed by Paris (1921) or Olander (1950). These are illustrated in Figure 16-3. It is noteworthy that between the two, Paris' distribution produces somewhat higher moments. However, Olander's distribution produces significantly higher thrusts and heaver designs for many cases. These distributions are only applicable to round shapes.

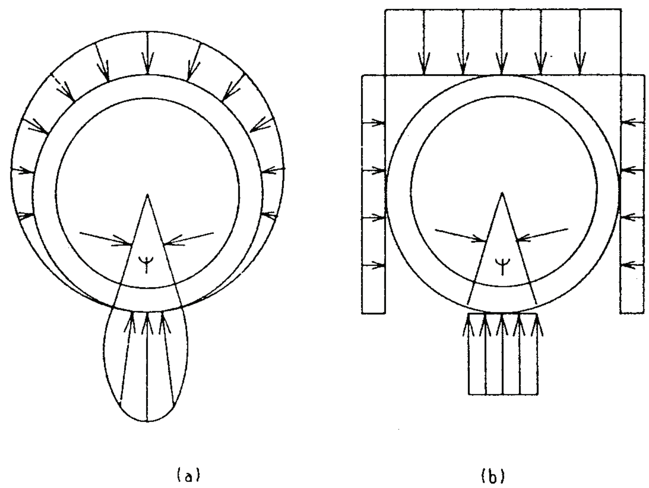


Figure 16-3 (*A12.10.4.2.1-1*)
Paris (a) and Olander (b) Pressure Distributions
(after AASHTO, 1997a)

Recent work with finite element analysis of soil-structure interaction has resulted in the development of generalized computer models that are applicable to any shape and relative stiffness and do not require an assumed pressure distribution. Heger has utilized such a computer model to derive a new assumed earth pressure distribution for round pipe (ASCE, 1993) shown in Figure 16-4.

As RCP culvert walls approach their ultimate limit state, they will try to shed their load into the surrounding soil. The surrounding soil picks up this load in a proportionate share to its relative stiffness. And, its stiffness decreases significantly as shear stress in the soil increases. This load transfer manifests itself by greater nonlinearity in wall bending moments than it does in wall thrusts. Therefore, failure to account for the ultimate load level and the nonlinearity in the analysis can result in at least two shortcomings, less accuracy and unknown and potentially unconservative factors of safety. This uncertainty is mitigated when the nonlinear effects of ultimate limit states and soil-structure interaction are carefully accounted for in the analysis, such as with nonlinear finite element models.

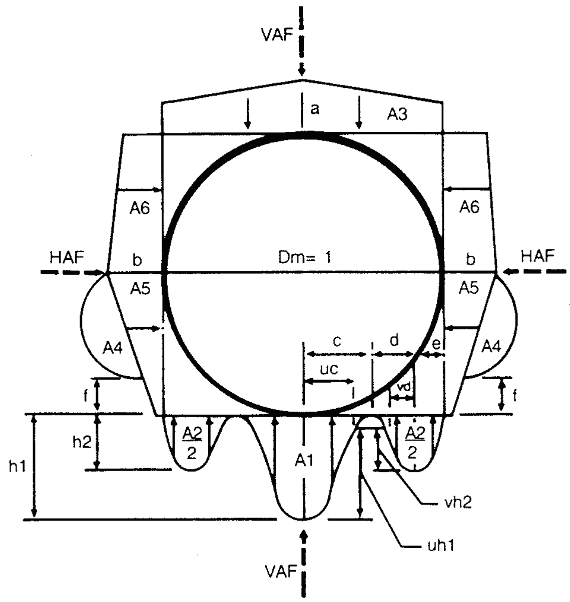


Figure 16-4 (*A12.10.2.1-1*)
Heger Pressure Distribution and Arching Factors
(AASHTO, 1997a)

Once the pipe wall responses are determined, the reinforced concrete wall section can then be designed using techniques that are well documented in other sources (AASHTO, 1997a and 1997b). However, these techniques and the force effects of the load distributions shown in Figures 16-3 and 16-4 cannot be evaluated readily by manual calculations. Hence, computer programs have been developed that perform both the load distribution and the wall section design (McGrath, et al., 1988).

A brief description of the reinforced concrete pipe design process is presented in the following sections.

16.3.2 Flexure and Thrust (*A12.10.4.2.4*)

The direct design equations developed by Heger (1982) utilized ultimate strength methodology. The primary criterion was flexure induced by the design loads, based on the Whitney stress block. To assure some level of ductility in pure bending, the section is proportioned such that the steel reinforcement reaches yield before the concrete reaches ultimate compression. However, under deep burial where compressive thrust is predominate, the section must be designed with compression reinforcement and radial ties, such as for a column that resists combined bending and axial loads.

Like flexible culverts, the governing load scenario for rigid culvert is sometimes based on installation stresses rather than permanent loads. This circumstance is accounted for by empirically derived relationships for minimum reinforcement that are based only on diameter and wall thickness. In addition, a lower limit is applied to these minimum reinforcement relationships to represent practical limits for manufacturing.

16.3.3 Shear (*A12.10.2.5*)

Heger and McGrath (1982) found that conventional ACI relationships for beam shear in pipe, box sections and slabs (also known as diagonal tension) were unconservative under concentrated loads and excessively conservative under distributed loads. They proposed new provisions to more accurately represent the shear capacity of such members. These provisions take into account such factors as:

- Crack depth
- Curvature
- Wall thrust

Wall sections that are found to be inadequate to resist beam shear must be increased in thickness, or radial ties must be utilized.

16.3.4 Radial Tension (*A12.10.4.2.6*)

Bending moments that produce tension on the inside of a pipe also produce radial tension (also known as bowstring tension) that tries to straighten out a curved section of reinforcement. The result of excessive radial tension is the delamination of the reinforced section, which is manifested by spalling of concrete to the depth of the inner layer of reinforcement. Based on curved slab and D-Load tests, Heger and McGrath (1982) developed design provisions to account for radial tension in the design. When radial tension exceeds the capacity of the section, the wall thickness must be increased or radial ties must be employed.

16.3.5 Crack Width (*A12.10.4.2.4d*)

The control of tension crack formation under service loads is required in both ASD and LRFD versions of the AASHTO specifications. Also, both versions limit the width to 0.25 mm extending over a maximum length of 305 mm. However, this control is accomplished in fundamentally different ways. Under ASD, the laboratory applied load that produces a 0.25 mm crack is experimentally determined. Under LRFD, sufficient reinforcement is specified to limit the crack width. An analytical check of crack width is an iterative process. The factors affecting the amount of reinforcement to control cracking include:

- Bending moment
- Axial thrust
- Cross section geometry
- Pipe diameter
- Reinforcement type and placement
- Concrete compressive strength

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CHAPTER 17
SUMMARY

17.1 Summary and Overview

In Chapter 1, the stated objectives of this manual were to provide a basis for developing an understanding of:

- Differences between ASD and LRFD for substructure design
- Benefits of LRFD for substructure design
- Importance of site characterization and selection of geotechnical design parameters
- Process for design of substructure elements by LRFD using the AASHTO LRFD Specifications as a guide
- Process for selection and application of load factors and load combinations
- Methods available for calibration of resistance factors
- Basis for calibration of the AASHTO LRFD resistance factors for substructure design
- Procedures available for modifying or developing resistance factors to achieve designs comparable to ASD

Except for the manner in which uncertainty is embodied in a factor of safety in ASD and by load and resistance factors in LRFD, very little difference exists between ASD and LRFD.

Where reliability-based calibrations exist, the level of safety in structure and substructure components is comparable, which should result in more efficient, and possibly cost effective designs. Where calibrations are based on direct correlation with ASD, LRFD should result in designs generally comparable to existing ASD practice.

The importance of site characterization and selection of geotechnical design parameters is critical regardless of whether substructure design is accomplished by ASD or LRFD. In Chapters 5 and 6, the reliability of various methods for developing geotechnical properties for design is demonstrated for various in-situ and laboratory test methods. The reliability of various semi-empirical methods used directly for design was commented upon in certain of the design chapters. In ASD, the reliability of the basis for development and selection of geotechnical properties is sometimes reflected in a variable factor of safety. In LRFD, each method of property estimation is assigned a resistance factor which reflects the relative reliability of the estimation method.
Methods used to calibrate the LRFD Specification were discussed in many of the chapters. The information provided in these chapters should help facilitate modification and development of

resistance factors by user agencies to reflect local design practices and to incorporate new design approaches.

Processes for design of substructure elements using LRFD principles and the AASHTO LRFD Specification in particular have been presented. Repeating an earlier point, very little difference exists between ASD and LRFD except for the manner in which uncertainty is embodied in a factor or safety in ASD and by load and resistance factors in LRFD.

Lastly, what does the future hold for LRFD? LRFD is used or under development in other parts of the world. Two examples include the Ontario Bridge Code and EuroCode 7. Also, the LRFD Specification should be considered as a "Work in Progress". As with any design specification, articles are revised and new are added to address mistakes, changes in design philosophy, and to introduce new technologies and design approaches. For the past two years, AASHTO has supported maintenance of the specification (i.e., NCHRP 12-42 - LRFD Bridge Design Specifications Support) to recommend changes and develop modifications to the LRFD Specification to assist the efforts of AASHTO Technical Committee T-15 on Foundations and Substructures in their charge to oversee efforts to update and revise the AASHTO specifications in this subject area. Other areas of sponsored research and study of LRFD include:

- NCHRP 20-5, Topic 28-02 - This project, to be completed by late 1998, will result in publication of a report synthesizing a survey of limit state design practice in Canada and Europe

- NCHRP 20-7, Task 88 - This project, to be completed in early 1999, will result in revision of provisions in the AASHTO LRFD Specification for the design of retaining wall, development of reliability-based resistance factors where sufficient performance data are available, in introduction of provisions for the design of flexible cantilever and segmental block systems as new wall types

- NCHRP 24-17 - Scheduled to begin in 1999, this project will entail incorporation of additional design methods, revision of provisions and recalibration of resistance factors for the design of driven piles and drilled shafts to reflect the level of quality control during construction

In addition, reliability-based resistance factors may be revised as more data regarding substructure design and performance are gathered. Another area which could be revised in the future is application of load and resistance factors for the Service Limit State (i.e., other than $\gamma = 1.0$ and $\phi = 1.0$) to reflect the reliability of methods used to estimate soil and rock deformation characteristics and substructure movements. However, the most useful input in this process is input from you, as a user of the document. By directing your constructive comments to the Chair of the AASHTO T-15 Technical Committee, necessary modifications and updates can be accomplished and incorporated into future editions of the LRFD Specifications.

APPENDIX A
METRIC STANDARDS AND UNIT CONVERSIONS

A.1 Introduction

Equations and calculations presented in this publication use the metric or International System of Units (SI). The following sections of this appendix provide:

- A description of the basic metric dimensional units
- Conversions between metric and English units
- Basic rules for expressing metric units in engineering text

This appendix describes the metric units considered standard by FHWA (1995) and ASTM (1993). In some cases, these standard units differ from standard units used in the AASHTO LRFD Bridge Design Specifications - SI Units (1997a).

A.2 Standard Metric Units and Conversions with English Units

Because a variety of metric units can be used in practice, care should be taken to assure that all calculations and equations are dimensionally consistent. Metric dimensions consist of base and derived units. The base and derived metric units used throughout this course are described in the following sections.

A.2.1 Base Units

Base units commonly used in geotechnical engineering consist of those dimensions used to define length, mass, time and temperature. Base units are defined as dimensionally independent units used to derive all other measurement units.

A.2.1.1 Length

The standard metric unit for length is the meter, m. Other commonly used metric units of length include:

- millimeter (mm) = 10^{-3} m
- kilometer (km) = 10^{3} m

The use of centimeter (cm) and decimeter (dm) is not common.

For geotechnical design, the mm is used as the unit for displacements such as foundation and wall movements which are typically small. However for structural design, use of mm is less convenient (i.e., involves a large number) when dimensions such as bridge span, pile length, footing dimensions and wall height must be evaluated or used in design computations.

A2.2.1.2 Mass

The standard metric unit for mass is the kilogram (kg). The kilogram is the only base unit with a prefix. Other metric units commonly used for mass include:

- milligram (mg) = 10^{-6} kg
- gram (g) = 10^{-3} kg

FHWA, ASTM and AASHTO discourage use of the term "metric ton" for engineering documents, but agree that the metric ton (t) is acceptable for common use.

A.2.1.3 Time
The standard metric unit for time is the second (s).

A.2.1.4 Temperature
The standard metric unit for thermodynamic temperature is the degree Kelvin (K). The degree (^{o}) symbol is not used with K.

A.2.2 Derived Units
Derived units commonly used in geotechnical engineering consist of those dimensions used to define force, pressure, mass density, unit weight, bending or overturning moment, and temperature. Derived units are defined as units formed by combining base units and/or other derived units by means of multiplication, division or the use of exponents.

A.2.2.1 Force
The standard metric unit for force, or mass (M) times acceleration of gravity (g), is the newton (N = kg-m/s^2). Other metric units commonly used include:

- kilonewton (kN) = 10^3 N
- meganewton (MN) = 10^6 N

A.2.2.2 Pressure
The standard metric unit for pressure is the pascal (Pa = N/m^2). Other metric units commonly used include:

- kilopascal (kPa) = kN/m^2
- megapascal (MPa) = MN/m^2

A.2.2.3 Mass Density and Unit Weight
The standard metric units for mass density (ρ) and unit weight ($\gamma = \rho\ g$), are kg/m^3 and kN/m^3, respectively.

A.2.2.4 Temperature
Wide use is made of the degree Celsius (^{o}C) to express temperature and temperature intervals in the metric system. Water freezes at 0 ^{o}C (273.15 K) and boils at 100 ^{o}C (373.15 K) at a pressure of one atmosphere. Therefore, a temperature interval of 1 ^{o}C is equal to a temperature interval of 1 K.

A.2.3 Unit Prefixes
Prefixes commonly used in geotechnical engineering are summarized in Table A-1. Consistent with ASTM E-380, only prefixes representing 1000 raised to an integral power (i.e., multiples of 10^3 or

10^{-3}) are recommended.

Table A-1
Prefixes Used with Metric Units

Unit	Symbol	Power
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	µ	10^{-6}

A.2.4 Summary of Standard and Preferred Metric Units
A summary of standard and preferred metric units for variables commonly used in geotechnical engineering is provided in Table A-2.

A.2.5 English to SI Unit Conversion
English units are converted to metric units by either a "soft" (exact or nearly exact) conversion or by a "hard" (conveniently rounded) conversion. A "soft" conversion is typically used when an exact metric equivalent is needed for accuracy, such as for physical dimensions of a structural element. As an example of a "soft" conversion:

$$6\text{ in x } \frac{1\text{ ft}}{12\text{ in}} \text{ x } \frac{0.3048\text{ m}}{1\text{ ft}} \_ \; 0.1524\text{ m}$$

A "hard" conversion is typically used when a convenient rounded metric equivalent is desired, such as for specifying requirements for minimum foundation embedment or structure dimensions. As an example of a "hard" conversion:

6 in = 0.1524 m = 0.15 m

English to metric conversions commonly required for geotechnical problems are provided in Table A-3.

Table A-2
Metric Units for Common Geotechnical Problems

Variable	Symbol	Base Unit Dimensions	SI Unit Symbol
Length	L, ℓ	L	m
Width	B, b	L	m
Height or Thickness	H, h	L	m
Depth	D, z	L	m
Diameter	D, d	L	m
Area	A	L^2	m^2
Volume	V	L^3	m^3
Time	t	T	s
Velocity	v	LT^{-1}	m/s
Acceleration	a	LT^{-2}	m/s^2
Acceleration Due to Gravity (g = 9.81 m/s^2)	g	LT^{-2}	m/s^2
Mass	m	M	kg
Mass Density	ρ	ML^{-3}	kg/m^3
Unit Weight	γ	$ML^{-2}T^{-2}$	kN/m^3
Settlement	Δ	L	mm
Pressure or Stress	varies	$ML^{-1}T^{-2}$	kPa
Elastic Modulus	varies	$ML^{-1}T^{-2}$	kPa
Grain Size	D, d	L	mm
Coefficient of Consolidation	c_v	L^2T^{-1}	m^2/s
Cohesion or Shear Strength	c, τ	$ML^{-1}T^{-2}$	kPa
Hydraulic Conductivity	k	LT^{-1}	m/s
Force or Resistance	varies	MLT^{-2}	kN
Moment of Force	varies	$ML^{-2}T^{-2}$	kN-m

Table A-3
Common English to Metric Unit Conversions

Quantity	English Unit	Metric Equivalent(1)
Length	1 in 1 ft	25.40 mm* 0.3048 m*
Area	1 in^2 1 ft^2	645.2 mm^2 0.092 m^2
Volume	1 ft^3 1 cy	0.028 m^3 0.764 m^3
Area/Unit Length	1 ft^2/ft	0.3048 m^2/m*
Mass	1 lbm	0.4536 kg
Mass Density	1 lbm/ft^3	16.02 kg/m^3
Acceleration Due to Gravity	1 g = 32.174 ft/sec^2	9.807 m/s^2
Unit Weight	1 lb/ft^3 1 kip/ft^3	157.1 N/m^3 157.1 kN/m^3
Force	1 lb 1 kip	4.448 N 4.448 kN
Moment	1 kip-ft	1.356 kN-m
Force/Unit Length	1 kip/ft	14.59 kN/m
Pressure, Stress	1 lb/in^2 1 kip/in^2 1 lb/ft^2 1 kip/ft^2	6.895 kPa 6.895 MPa 0.0479 kPa 47.88 kPa
Pressure/Unit Length	1 lb/ft^2/ft 1 kip/ft^2/ft	0.1571 kPa/m 157.1 kPa/m

(1) Asterisk indicates exact conversion; all other conversions are rounded to four significant figures.

A.2.6 SI Units in Engineering Documents

FHWA (1995) and ASTM (1993) provide guidelines for presentation of metric units and numbers in correspondence and engineering documents.

A.2.6.1 Variable Symbols

All variable symbols should be expressed as upright text, such as B for width, H for height, and ε for strain. Some variable symbols are italicized, however (as indicated in Table A-2), to avoid confusion with base units, derived units and unit prefixes. For example:

- *L* is used as the symbol for length to avoid confusion with the abbreviation for the unit symbol for liter “L”
- *g* is used as the symbol for the acceleration due to gravity to avoid confusion with the mass unit symbol for gram “g”
- *m* is used as the symbol for mass to avoid confusion with the length unit symbol for meter “m”

A.2.6.2 Base Dimensions

All measurable units can be expressed in base dimensions of length, mass and time. In the SI system, base dimensions are italicized.

L for length, *M* for mass and *T* for time:

- Velocity (v) LT^{-1}
- Acceleration (a) LT^{-2}
- Mass density (ρ) ML^{-3}
- Unit weight (γ) $ML^{-2}T^{-2}$

A.2.6.3 Unit Symbols

<u>Text Style</u>

Unit symbols are always presented as upright text, regardless of the type style used in the surrounding text. For example:

Incorrect:	*Bob is running in tomorrow's* ***10 km*** *race.*
Correct:	*Bob is running in tomorrow's* **10 km** *race.*

<u>Plural Expression</u>

SI symbols are unaltered in the plural. Therefore, SI unit symbols should be presented as singular and unit names written as plural. For example:

Incorrect:	To train for the 10 km race, Bob runs **3 kms** a day.
Correct:	To train for the 10 km race, Bob runs **3 km** a day.
Correct:	To train for the 10 km race, Bob runs **three kilometers** a day.

<u>Periods</u>

SI symbols and prefixes are **symbols** and <u>not</u> **abbreviations** and, therefore, should not have periods that follow except when they fall at the end of a sentence. For example:

Incorrect:	After the **10 km.** race, Bob will walk 1 km.
Correct:	After the **10 km** race, Bob will walk 1 km.

<u>Lower Case Unit Names and Symbols</u>

SI unit names and symbols should be set in lower case, as follows:

mm	millimeter	km	kilometer
mg	milligram	kg	kilogram

The exceptions to this rule are:

- Unit symbols derived from a proper name, in which case the unit symbol is presented in upper case and the unit name is written in lower case.

Incorrect		Correct	
n	Newton	N	newton
j	Joule	J	joule
w	Watt	W	watt

- The unit name "Celsius". Unlike the other units derived from proper names, the unit name Celsius must always be capitalized, Also, the degree symbol "^{o}" must always accompany the unit symbol for Celsius (^{o}C).

  Correct: ^{o}C degree Celsius

- The symbol for the liter unit. Because the letter l can be easily confused for the numeral 1, an upper case L for liter is recommended. The same rule for symbols derived from proper names applies to the symbol for liter.

  Correct: L liter

Space Between Digit Number and Unit Symbol

When a quantity is expressed as a numerical value and a symbol, a space should be left between the value and symbol, For example:

Incorrect	Correct
35m	35 m
250kg	250 kg
20^{o}C	20 ^{o}C

Exception: 45^{o} not 45 ^{o}.

Unit Prefixes

In the SI system, no spaces are placed between the prefix and unit name or the prefix and unit symbol. For example:

Incorrect		Correct	
kilo pascal	k Pa	kilopascal	kPa
kilo meter	k m	kilometer	km

Symbol prefixes should be presented in lower case except when they represent a unit quantity greater than a thousand.

G	giga	$= 10^9$	unity		= 1.0
M	mega	$= 10^6$	m	milli	$= 10^{-3}$
k	kilo	$= 10^3$	μ	micro	$= 10^{-6}$

Incorrect		Correct	
Mega Newton	mn	meganewton	MN
Kilo Newton	Kn	kilonewton	kN
Kilo Pascal	Kpa	kilopascal	kPa
Mega Pascal	mpa	megapascal	MPa

A.2.6.4 Combined Units and Symbols

Product Units

Product unit names are written with a space or hyphen between the individual factor unit names. The product unit symbols are presented as dot products of the individual factor units. For example:

Incorrect		Correct	
newtonmeter	N*m	newton meter, newton-meter	N-M
kilo newton meter	kN*m	kilonewton meter, kilonewton-meter	kN-m
kilogrammeter	kg*m	kilogram meter, kilogram-meter	kg-m

Quotient Units

Quotient units should only be written with the word "per" between each factor unit (i.e., unit per unit). Quotient unit symbols can be correctly presented with either a divisor (slash or straight bar) between individual factor unit symbols or as a dot product with the denominator unit to a negative power. For example:

Incorrect	Correct
meters/seconds, m|s	meters per second, m/s, m-s^{-1} or $\frac{m}{s}$
newtons-meter, N-m	newton per meter, N/m or N-m^{-1}

Digit Groups, Decimal Points and Commas

Unlike customary US practice, many countries use the comma as a decimal marker instead of a dot or period. To avoid confusion, recommended international correspondence practice calls for the elimination of commas. Instead of commas, digits should be separated into groups of three with a space between the 3-digit groups (ASTM, 1993). Examples of this correspondence are given below.

The numbers in the "international" column have been aligned with the location of the decimal point.

US	International (SI)
26,345.0	26 345.0
3.141596	3.141 596
2,123,987.23	2 123 987.23
34.34523	34.345 23
24,246,680.0	24 246 680.0

The exceptions to this practice are on engineering drawings and financial statements. If the numbers contain a maximum of four digits on either side of the decimal point, a space is not necessary except for uniformity in tables. Examples are given below:

Incorrect	Correct
0.133 5	0.1335
2 345	2345
8976.345 6	8976.3456
6 234.5678	6234.5678

The addition of numbers in tables and columns should be set up as follows:

Incorrect	Correct
2345	2 345
56 234.5678	+56 234.567 8
6056.123345	+6 056.123 345
64 635.691145	64 635.691 145

Numbers Less Than One

A zero should be written before the decimal marker for numbers less than one. For example:

Incorrect	Correct
.1234	0.1234

Fractions

Fractions are not used in the SI system. Non-integer numbers should be expressed as decimals.

Incorrect	Correct
2½	2.5
3⅛	3.125

A.3 Examples

1. Which soil has a higher unit weight, Soil A with $\gamma = 110\ lb/ft^3$ or Soil B with $\gamma = 17.3\ kN/m^3$?

   $\boldsymbol{\gamma = 110\ lb/ft^3 = 110\ lb/ft^3 \times 0.157\ kN/m^3 = 17.3\ kN/m^3}$

   $\therefore$ **the unit weight of Soil A and Soil B are equal**

2. If the total unit weight of a soil submerged below the water table is $20\ kN/m^3$, what is the vertical effective soil stress at a depth of 5 m?

   $\boldsymbol{\sigma' = \gamma' z = 20\ kN/m^3 \times 5\ m = 100\ kPa}$

3. What is the ultimate load capacity of a spread footing with an area, A, equal to $2.5\ m^2$ and a nominal bearing resistance, q_N equal to 420 kPa?

   $\boldsymbol{Q_R = q_R \times A = 420\ kPa \times 2.5\ m^2 = 1050\ kN = 1.050\ MN}$

APPENDIX B
SOLUTIONS TO STUDENT EXERCISES AND PROBLEMS

B.1 Chapter 3 Student Exercise

1. Define the term "limit state".

   A limit state is a condition beyond which a structural component, such as a foundation or other bridge component ceases to fulfill the function for which it is designed.

2. What is the fundamental equation governing ASD?

$$\frac{R_n}{FS} \geq \sum Q_i \qquad \text{(Eq. 3-2)}$$

where:

R_n = Nominal resistance
$\Sigma\, Q_i$ = Load effect
FS = Factor of safety (dim)

3. What is the fundamental equation governing LRFD?

$$\phi\, R_n \geq \sum \eta_i\, \gamma_i\, Q_i \qquad \text{(Eq. 3-3)}$$

where:

ϕ = Resistance factor (dim)
R_n = Nominal resistance
γ_i = Load factor (dim)
Q_i = Load effect
η = Factors to account for effects of ductility, redundancy and operational importance

4. What are the advantages and disadvantages of both LRFD and ASD?

   ASD Advantages: Simplicity and familiarity

   ASD Disadvantages:

   1) ASD does not adequately account for variability of loads and resistances. Dead, live and environmental loads are all considered to be have no variability (i.e., deterministic). The factor of safety is applied only to resistance.

2) ***ASD does not embody a reasonable measure of strength which is a more fundamental measure of resistance than allowable stress.***

3) ***Selection of a factor of safety is subjective and does not provide a uniform measure of reliability in terms of probability of failure.***

LRFD Advantages:

1) ***Accounts for variability in both resistance and load.***

2) ***Achieve relatively uniform levels of safety based on the strength of soil and rock for different limit states and foundation types.***

3) ***Provides more consistent margins of safety in the superstructure and substructure because they are designed using the same loads for known probabilities of failure.***

LRFD Disadvantages:

1) ***Requires a change in design procedures for engineers accustomed to ASD.***

2) ***Resistance factors vary with design methods and are not constant.***

3) ***The most rigorous method for developing and making adjustments in resistance factors to meet individual situations requires availability of statistical data and probabilistic design algorithms.***

5. What are the methods by which resistance factors can be calibrated?

   Reliability theory, fitting with ASD or judgment.

6. Determine the appropriate resistance factor through calibration with ASD for the following parameters:

   FS = 3.0
   γ_D = 1.25
   γ_L = 1.75
   Q_D/Q_L = 2.0

***Using Equation 3-6*:**

$$\phi = \frac{\gamma_D\, Q_D / Q_L + \gamma_L}{FS\,(Q_D / Q_L + 1)} = \frac{1.25 \times 2.0 + 1.75}{3.0 \times (2.0 + 1)} = 0.47 \qquad \text{(Eq. 3-6)}$$

7. Define the mean, standard deviation and coefficient of variation of a set of normally

distributed data.

1) ***mean, $\bar{x} = \Sigma x_i/N$***

where:
x_i = ***data set***
N = ***Number of data values (dim)***

2) ***Standard deviation,*** $\sigma = \sqrt{\Sigma (x_i - \bar{x})^2 / (N - 1)}$

3) ***Coefficient of variation,*** **$COV = \sigma/\bar{x}$**

8. Define the bias, λ, of a set of data.

λ = ***R_m/R_n***
λ = ***Bias factor (dim)***
R_m = ***Measured capacity***
R_n = ***Predicted (nominal) capacity***

9. Define the lognormal mean and lognormal standard deviation.

$$\text{lognormal mean}, \xi_m = \ln [\, \bar{x} / \sqrt{(1 + COV^2)} \quad \text{(Eq. 3-11)}$$

$$\text{lognormal std. dev.}, \zeta = \sqrt{\ln (1 + COV^2)} \quad \text{(Eq. 3-12)}$$

where:

$\bar{x}$ = ***Mean value defined by Eq. 3-7***
COV = ***Coefficient of variation defined by Eq. 3-9***
$\ln(\,)$ = ***Natural logarithm of the expression in parentheses***

10. Define the reliability index, β.

Reliability index, β, is defined as the number of standard deviations σ_g between the mean value, $\bar{g}$, and the origin (i.e., $\beta = \bar{g}/\sigma_g$.}

where:

$g(R, Q) = R - Q$

11. For lognormal distributions of load and resistance, recall that the reliability index, β, can be expressed as follows:

$$\beta = \frac{\ln\left[(\overline{R}/\overline{Q})\sqrt{(1+COV_Q^2)/(1+COV_R^2)}\right]}{\sqrt{\ln[(1+COV_R^2)(1+COV_Q^2)]}} \qquad \text{(Eq. 3-21)}$$

And, recall that the probability of failure can be estimated as:

$$p_f = 460\exp(-4.3\beta) \qquad 2<\beta<6 \qquad \text{(Eq. 3-22)}$$

A subsurface exploration and laboratory testing program including field vane shear tests (VST) and unconfined compression (UC) tests results in a mean ultimate pile capacity prediction, $\overline{R}$ = 1000 kN as shown below. The mean unfactored load on the pile, $\overline{Q}$ = 333 kN. The reliability of each test method is somewhat different, with a $COV_R = 0.25$ for the VST and a $COV_R = 0.35$ for the UC tests.

For these load and resistance characteristics, determine the ASD factor of safety and the LRFD-based probability of failure based on VST and UC testing.

$\overline{R}$ =	1000 kN
$\overline{Q}$ =	333 kN
COV_Q =	0.15
COV_R (VST) =	0.25
COV_R (UC) =	0.35

1) ***ASD: Factor of Safety.*** $FS = \frac{\overline{R}}{\overline{Q}} = \frac{1000\text{ kN}}{333\text{ kN}} = 3.0$

2) ***LRFD: φ VST:*** $\beta = \frac{\ln\left[(1000)\sqrt{(1+0.15^2)/(1+0.25^2)}\right]}{\sqrt{\ln[(1+0.25^2)(1+0.15^2)]}}$

β = 3.74   2 < β <6
p_f = 460 exp(-4.3β)
p_f = 4.6 x 10^{-5} ≈ 1 in 22,000

UC: $\beta = \frac{\ln\left[(1000)\sqrt{(1+0.15^2)/(1+0.35^2)}\right]}{\sqrt{\ln[(1+0.35^2)(1+0.15^2)]}}$

β = 2.83
p_f = 460 exp (-4.3β)
p_f = 2.4 x 10^{-3} ≈ 1 in 400

B.2 Chapter 4 Student Exercise

Strength I Limit State:

Sliding/Overturning - The minimum load factors are used for those load components which contribute to the resistance (DC, EV) and the maximum load factor is used for EH. Note that the live load surcharge, LS is not applied over the heel of the wall for these conditions.

Bearing - The maximum load factors are used for all components of load for bearing. LS is included over the heel of the wall for evaluation of bearing.

Service I Limit State:

Settlement - All the applicable loads have a load factor of 1.00 and the analysis results are compared to a project specific settlement tolerance.

The appropriate load factors for the critical load combinations for the performance limits indicated are tabulated below.

Load Effect	Limit State and Performance Limit			
	Strength I			Service I
	Sliding	Bearing	Overturning	Settlement
Vertical Live Load Surcharge, LS_V	***1.75***	***1.75***	***1.75***	***1.00***
Horizontal Live Load Surcharge , LS_H	***1.75***	***1.75***	***1.75***	***1.00***
Horizontal Earth Load, EH	***1.50***	***1.50***	***1.50***	***1.00***
Vertical Earth Load, EV	***1.00***	***1.35***	***1.00***	***1.00***
Concrete Dead Load, DC	***0.90***	***1.25***	***0.90***	***1.00***

Summary:

Consistent with the example in Section 4.6, this exercise illustrates the selection of load factors for geotechnical design of a reinforced concrete cantilever wall. For such walls, dead earth and concrete loads, lateral earth pressures and vertical and horizontal live load surcharge predominate. Load factor combinations resulting in the maximum vertical load generally control for bearing pressure. Load factor combinations which include minimum vertical loads and maximum horizontal loads result in the greatest net overturning moment (and therefore greatest base pressure resultant eccentricity) and the lowest vertical stress (and therefore sliding resistance) at the bearing level.

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B.3 Chapter 4 Student Problem: Load Combinations for Retaining Wall

Problem: The cantilever retaining wall in Figure 4-21 is being considered for a grade separation between roadway lanes in a non-seismic area. The wall will be backfilled with a free draining granular fill such that the seasonal high water table will be below the bottom of the footing. The vehicular live load surcharge, LS, on the backfill will be applied as shown in the figure.

Objective: You need to develop unfactored and factored loads and moments needed for the geotechnical design of the cantilever retaining wall.

Approach: You will perform the evaluation using the following steps:

- Calculate the unfactored loads and resulting moments due to structure components, earth pressures and live load surcharge
- Select the load factors and load combinations controlling geotechnical design
- Calculate the factored loads and moments by multiplying the unfactored loads and moments by the appropriate load factors and load combinations

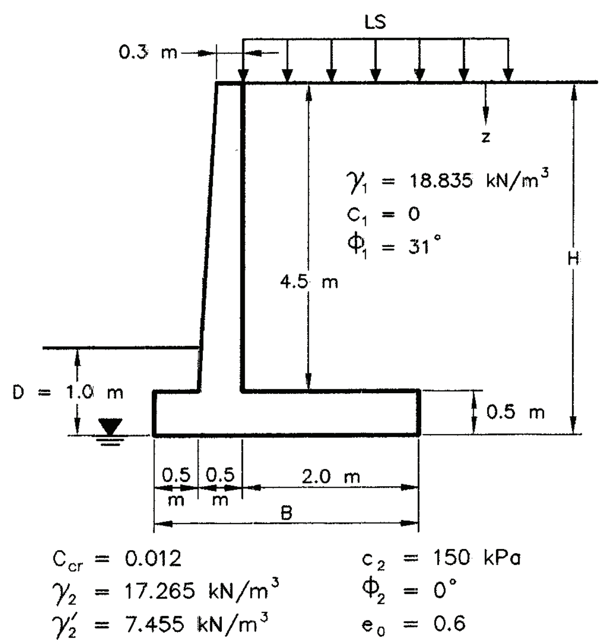

Figure 4-21
Schematic of Student Problem

Step 1: Calculate the Unfactored Loads

(A) Dead Load of Structural Components and Nonstructural Attachments (DC)
Referring to Figure 4-22 and assuming a unit weight of concrete, γ_C, equal to 23.544 kN/m^3:

$\mathbf{W_1} = B_1\,H_1\,\gamma_c = (0.3\text{ m})(4.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{31.8\ kN/m}$

$\mathbf{W_2} = 1/2\,B_2\,H_1\,\gamma_c = (0.5)(0.2\text{ m})(4.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{10.6\ kN/m}$

$\mathbf{W_3} = B\,H_2\,\gamma_c = (3.0\text{ m})(0.5\text{ m})(23.544\text{ kN/m}^3) = \mathbf{35.3\ kN/m}$

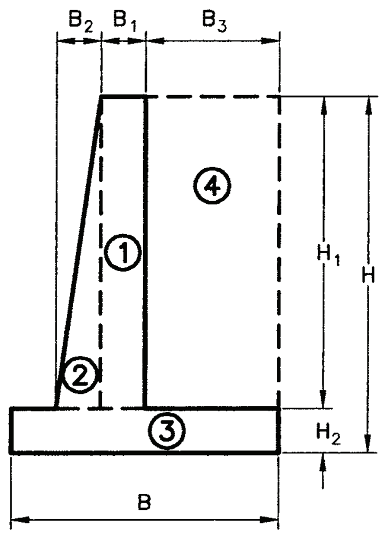

Figure 4-22
Retaining Wall Area Designation for Weight of Concrete

(B) Vertical Earth Pressure (EV)
Unit Weight of Soil γ_1 = 18.835 kN/m^3

Weight of Soil on Footing

$\mathbf{P_{EV}} = W_4 = B_3\,H_1\,\gamma_1 = (2.0\text{ m})(4.5\text{ m})(18.835\text{ kN/m}^3) = \mathbf{169.5\ kN/m}$

(C) Live Load Surcharge (LS)
A live load surcharge is applied when vehicle loads will be supported on the backfill within a distance equal to H. The live load surcharge is applied as an equivalent height of soil for the design vehicle loading (h_{eq}) using Table 4-2, and a wall height of 5 m.

By interpolation, $\mathbf{h_{eq}} = \mathbf{0.907\ m}$

Using the unit weight of the soil backfill (i.e., γ_1 = 18.835 kN/m^3), the unit vertical live load surcharge, LS, over the heel of the wall is:

$$p_{LSV} = \gamma_1 \, h_{eq} = (18.835 \text{ kN/m}^3)(0.907 \text{ m}) = 17.1 \text{ kPa (kN/m}^2)$$

For a heel width B_3 of 2 m:

$$\mathbf{P_{LSV}} = p_{LSV} \, B_3 = (\mathbf{17.1} \text{ kN/m}^2)(\mathbf{2} \text{ m}) = \mathbf{34.2\ kN/m\ of\ wall\ length}$$

The active earth pressure coefficient k_a for a wall friction angle, $\delta = \phi_f = 31^o$, and a horizontal backslope is, from AASHTO (1997a) *Table 3.11.5.3-1*:

$$k = k_a = 0.29$$

From Eq. 4.7, the lateral earth pressure due to the live load surcharge is:

$$\Delta p = k_s \, \gamma'_s \, h_{eq} = k_a \, \gamma'_1 \, h_{eq} = (0.29)(18.835 \text{ kN/m}^2)(0.907 \text{ m}) = 4.95 \text{ kPa}$$

Using a rectangular distribution, the live load lateral earth pressure resultant is:

$$P_{LS} = \Delta p \, H = (4.95 \text{ kN/m}^2)\ (5 \text{ m}) = 24.8 \text{ kN/m of wall length}$$

The horizontal and vertical components of the live load lateral earth pressure are:

$$\mathbf{\Delta P_{LSh}} = P_{LS} \cos\delta = (24.8 \text{ kN/m})(\cos 31^o) = \mathbf{21.3\ kN/m}$$
$$\mathbf{\Delta P_{LSv}} = P_{LS} \sin\delta = (24.8 \text{ kN/m})(\sin 31^o) = \mathbf{12.8\ kN/m}$$

(D) Lateral Earth Pressure (EH)
The lateral earth pressure is assumed to vary linearly with the depth of soil backfill as given by:

$$p = k_h \, \gamma_s \, z = k_a \, \gamma_s \, z = 0.29 \, \gamma_s \, z \qquad \text{(Eq. 4-2)}$$

At the base of the footing (i.e., @ z = H):

$$p = (0.29)(18.835 \text{ kN/m}^3)(5.00 \text{ m}) = 27.3 \text{ kPa (kN/m}^2)$$

The resultant of the basic lateral earth pressure (triangular distribution) acting on the wall is:

$P_{EH} = P_a = 0.5\ pH = (0.5)(27.3\ kN/m^2)(5.00\ m) = 68.3\ kN/m$ length of wall

The horizontal and vertical components of the lateral earth pressure are:

$\mathbf{P_{ah}} = P_a \cos\delta = (68.3\ kN/m)(\cos 31^o) = \mathbf{58.6\ kN/m}$
$\mathbf{P_{av}} = P_a \sin\delta = (68.3\ kN/m)(\sin 31^o) = \mathbf{35.2\ kN/m}$

(E) Summary of Unfactored Loads

Table 4-14
Unfactored Vertical Loads and Resisting Moments

Item	V kN/m	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
W_1	***31.8***	***0.85***	***27.0***
W_2	***10.6***	***0.63***	***6.7***
W_3	***35.3***	***1.50***	***53.0***
P_{EV}	***169.5***	***2.00***	***339.0***
P_{LSV}	***34.2***	***2.00***	***68.3***
ΔP_{LSv}	***12.8***	***3.00***	***38.4***
P_{av}	***35.2***	***3.00***	***105.6***
TOTAL	***329.4***		

Table 4-15
Unfactored Horizontal Loads and Overturning Moments

Item	H (kN/m)	Moment Arm About Toe (m)	Moment About Toe (kN-m/m)
ΔP_{LSh}	***21.3***	***2.50***	***53.3***
P_{ah}	***58.6***	***1.67***	***97.9***

Step 2: Determine the Appropriate Load Factors
In theory, structures could be evaluated for each of the limit states identified in Section 4.3. However, depending on the particular loading conditions and performance characteristics of a structure, only certain limit states need to be evaluated. For the classroom example problem, each limit state will be qualitatively assessed below relative to that limit state is applicable for the design problem:

- Strength I - Basic load combination related to the normal vehicular use of the bridge without wind. ***(Applicable as a standard load case)***.

- Strength II - Load combination relating to the use of the bridge by Owner-specified special design vehicles and/or evaluation permit vehicles, without wind. ***(Not applicable because special vehicle loading is not specified)***.

- Strength III - Load combination relating to the bridge exposed to wind velocity exceeding 90 km/hr without live loads. ***(Not applicable because wall is not subjected to other than standard wind loading)***.

- Strength IV - Load combination relating to very high dead load to live load force effect ratios exceeding about 7.0 (e.g., for spans greater than 75 m). ***(Applicable because dead loads predominate)***.

- Strength V - Load combination relating to normal vehicular use of the bridge with wind velocity of 90 km/hr ***(Not applicable because wind load not a design consideration)***.

- Extreme Event I - Load combination including earthquake. ***(Not applicable because problem does not include earthquake loading)***.

- Extreme Event II - Load combination relating to ice load or collision by vessels and vehicles. ***(Not applicable because problem does not include ice or collision loading)***.

- Service I - Load combination relating to the normal operational use of the bridge with 90 km/hr wind**.** ***(Applicable for design loading)***.

- Service II - Load Combination intended to control yielding of steel structures and slip of slip-critical connections due to vehicular live load. ***(Not applicable due to structure type.)***

- Service III - Load combination relating only to tension in prestressed concrete structures with the objective of crack control. ***(Not applicable due to structure type.)***

- Fatigue - Fatigue and fracture load combination relating to repetitive gravitational vehicular live load and dynamic responses under a single design truck. ***(Not applicable due to structure type.***

Consequently, only the Strength I, Strength IV and Service I Limit States apply to the retaining wall design. Therefore, from Tables 4-10 and 4-11, select the applicable load factors and combinations and present them in Table 4-16. ***(Note: Strength I-a and I-b***

represent the Strength I Limit State using minimum and maximum load factors, respectively, from Table 4-11.)

Table 4-16
Load Factors

Group	γ_{DC}	γ_{EV}	γ_{LS}	γ_{EH} (active)	Probable Use
Strength I-a	***0.90***	***1.00***	***1.75***	***1.50***	***EC/SL***
Strength I-b	***1.25***	***1.35***	***1.75***	***1.50***	***BC (max. value)***
Strength IV	***1.50***	***1.35***	***-***	***1.50***	***BC (max. value)***
Service I	***1.00***	***1.00***	***1.00***	***1.00***	***Settlement***

Notes: BC - Bearing Capacity; EC - Eccentricity; SL - Sliding

By inspection:

- Strength I-a (minimum vertical loads and maximum horizontal loads) will govern for the case of sliding and eccentricity (overturning)
- For the case of bearing capacity, maximum vertical loads will govern, and the factored loads must be compared for Strength I-b and Strength IV

Step 3: Calculate the Factored Loads and Factored Moments

Table 4-17
Factored Vertical Loads

Group/ Item Units	W_1 kN/m	W_2 kN/m	W_3 kN/m	P_{EV} kN/m	P_{LSV} kN/m	ΔP_{LSv} kN/m	P_{av} kN/m	V_{TOT} kN/m
V (Unf.)	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4
Strength I-a	***28.6***	***9.5***	***31.8***	***169.5***	***59.8***	***22.4***	***52.8***	***374.4***
Strength I-b	***39.7***	***13.3***	***44.2***	***228.8***	***59.8***	***22.4***	***52.8***	***461.0***
Strength IV	***47.7***	***15.9***	***53.0***	***228.8***	***0***	***0***	***52.8***	***398.2***
Service I	***31.8***	***10.6***	***35.3***	***169.5***	***34.2***	***12.8***	***35.2***	***329.4***

Table 4-18
Factored Horizontal Loads

Group/Item Units	ΔP_{LSh} kN/m	P_{ah} kN/m	H_{TOT} kN/m
H (Unf.)	21.3	58.6	79.9
Strength I-a	***37.3***	***87.9***	***125.2***
Strength I-b	***37.3***	***87.9***	***125.2***
Strength IV	***0***	***87.9***	***87.9***
Service I	***21.3***	***58.6***	***79.9***

Table 4-19
Factored Moments from Vertical Forces (M_v)

Group/ Item Units	W_1 kN-m/m	W_2 kN-m/m	W_3 kN-m/m	P_{EV} kN-m/m	P_{LSV} kN-m/m	ΔP_{LSv} kN-m/m	P_{av} kN-m/m	M_{vTOT} kN-m/m
M_v (Unf.)	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0
Strength I-a	***24.3***	***6.0***	***47.7***	***339.0***	***119.6***	***67.2***	***158.4***	***762.2***
Strength I-b	***33.8***	***8.4***	***66.2***	***457.7***	***119.6***	***67.2***	***158.4***	***911.3***
Strength IV	***40.5***	***10.1***	***79.5***	***457.7***	***0***	***0***	***158.4***	***746.2***
Service I	***27.0***	***6.7***	***53.0***	***339.0***	***68.3***	***38.4***	***105.6***	***638.0***

Table 4-20
Factored Moments from Horizontal Forces (M_h)

Group/Item Units	ΔP_{LSh} kN-m/m	P_{ah} kN-m/m	M_{hTOT} kN-m/m
M_h (Unf.)	53.3	117.2	170.5
Strength I-a	***93.3***	***175.8***	***269.1***
Strength I-b	***93.3***	***175.8***	***269.1***
Strength IV	***0***	***175.8***	***175.8***
Service I	***53.3***	***117.2***	***170.5***

Summary

This example illustrates:

- ***Selection of critical limit states (load combinations), load factors***
- ***Development of factored loads for geotechnical design of a reinforced cantilever retaining wall***

- ***For cantilever retaining walls, dead, earth and live load surcharge are the predominate loads***

- ***Because the load factor for active horizontal (or lateral) earth pressure in Table 4-11 is the same for all limit states, the controlling limit states for a typical cantilever retaining wall are generally those for which dead load and live load surcharge load factors in Table 4-11 are the greatest (i.e., Strength IV and Strength I, respectively)***

- ***Minimum load factors typically control for sliding and eccentricity criteria, because the lower factored soil and concrete dead weights provide less resistance to sliding and overturning***

- ***Maximum load factors typically control for bearing as the higher factored soil and concrete dead weights exert a higher bearing pressure***

The geotechnical (foundation) design for the retaining wall in this example is presented in the Classroom Example in Chapter 8, Section 8.4.

B.4 Chapter 7 Student Exercise

1. Name and briefly describe the two general methods of calibrating resistance factors (Refer to Section 7.2)?

   1) ***Use of reliability equations with statistics in soil strengths and loads; that is, computing resistance factors that will, on average, result in the same reliability (or probability of satisfactory performance) as achieved under ASD.***

   2) ***Fitting to ASD Specifications; that is, choosing resistance factors that will, on average, give the same size foundations as would result from design using ASD.***

2. The "six sigma rule" can be used to provide rule-of-thumb estimates of the standard deviation, σ, and coefficient of variation, COV_R. Describe the "six sigma rule" (Refer to Section 7.3.1).

   The "six sigma" rule involves three steps using strength values that can be estimated by an experienced engineer to estimate the statistical parameters σ and COV_R.

   1) ***Estimate the most likely value of the property (***V_{est}***), the lowest conceivable value (***V_{min}***), and the largest conceivable value (***V_{max}***)***

   2) ***Use the following equation to estimate the value of the standard deviation (σ):***

$$\sigma = \frac{V_{max} - V_{min}}{6}$$

   3) ***Calculate the coefficient of variation (***COV_R***):***

$$COV_R = \frac{\sigma}{V_{est}}$$

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B.5 Chapter 8 Student Exercise: Bearing Resistance of Spread Footings on Sand

The ultimate unit bearing resistance of a spread footing foundation in dense sand, q_{ult}, has been estimated at 1000 kPa. The load on the footing at the Strength I Limit State is composed of a dead load, Q_D, of 2000 kN (for which $\gamma_D = 1.25$) and a live load, Q_L, of 1000 kN (for which $\gamma_L = 1.75$).

Assuming a typical structure ($\eta_i = 1.0$), determine the minimum square footing size needed to satisfy bearing resistance requirements for the following:

1. LRFD (using Eq. 8-12), if the ultimate resistance was estimated:

   a. By the semi-empirical procedure using SPT data ($\phi = 0.45$)
   b. By the semi-empirical procedure using CPT data ($\phi = 0.55$)
   c. By the rational method using shear resistance (ϕ_f) estimated from SPT data ($\phi = 0.35$)

$$q_R \geq = \frac{\Sigma\, \eta_i\, \gamma_i\, Q_i}{A'} \qquad \text{(Eq. 8-12)}$$

2. ASD (using Eq. 8-2), if the minimum required factor of safety against a bearing capacity failure[1] is:

   a. 3.0 [Soil Strength (ϕ_f) based on SPT]
   b. 2.5 [Soil Strength (ϕ_f) based on Laboratory/Field Strength Tests]

$$Q \leq Q_{all} = R_n / FS \_ \; \Sigma\, Q \leq (q_{ult}\, A)/FS \qquad \text{(Eq. 8-2)}$$

[1] Minimum factors of safety using FHWA criteria (Cheney and Chassie, 1993); AASHTO ASD minimum factor of safety is 3.0 for all cases.

Determination of Required Footing Size

Q_D (kN)	$\gamma_D\, Q_D$ (kN)	Q_L (kN)	$\gamma_L\, Q_L$ (kN)	ASD $\sum Q$ (kN)	LRFD $\sum \gamma_i Q_i$ (kN)	q_{ult} (kPa)	LRFD $q_R = \phi\, q_{ult}$ (kPa)	ASD $q_{all} = q_{ult}/FS$ (kPa)	Minimum Required Footing Size, A_{req} (m × m)
LRFD: SEMI-EMPIRICAL PROCEDURE USING SPT DATA ($\phi = 0.45$)									
2000	2500	1000	1750	---	***4250***	1000	***450***	---	***3.07 × 3.07***
LRFD: SEMI-EMPIRICAL PROCEDURE USING CPT DATA ($\phi = 0.55$)									
2000	2500	1000	1750	---	***4250***	1000	***550***	---	***2.78 × 2.78***
LRFD: RATIONAL METHOD USING ϕ_f ESTIMATED FROM SPT ($\phi = 0.35$)									
2000	2500	1000	1750	---	***4250***	1000	***350***	---	***3.48 × 3.48***
ASD: MINIMUM FACTOR OF SAFETY = 3.0									
2000	---	1000	---	***3000***	---	1000	---	***330***	***3.03 × 3.03***
ASD: MINIMUM FACTOR OF SAFETY = 2.5									
2000	---	1000	---	***3000***	---	1000	---	***400***	***2.74 × 2.74***

LRFD: $\mathbf{A_{req}} = \dfrac{\sum \gamma_i Q_i}{q_R}$

ASD: $\mathbf{A_{req}} = \dfrac{\sum Q}{q_{all}}$

Summary

This exercise illustrates the similarities and differences in spread footing design by LRFD and ASD. Whereas LRFD and ASD result in similar footing dimensions, LRFD considers and accounts for the reliability of the bearing resistance prediction models. Of note is the greater reliability of semi-empirical procedures (which estimate bearing resistance directly from field test results) as compared to the rational method (which requires first an estimate of ϕ_f from a field test and then an estimate of resistance from ϕ_f).

As evident from the example, use of a more reliable design procedure has the same effect as reducing the required factor of safety (e.g., from 3.0 specified in AASHTO ASD to about 2.5). *Although reduced factors of safety can be and often are used to reflect greater reliability of soil strength date (e.g., Cheney and Chassie, 1993), the reduction is generally subjective. Conversely, the LRFD procedure using reliability-based calibration of resistance factors provides a rational means for considering reliability in design.*

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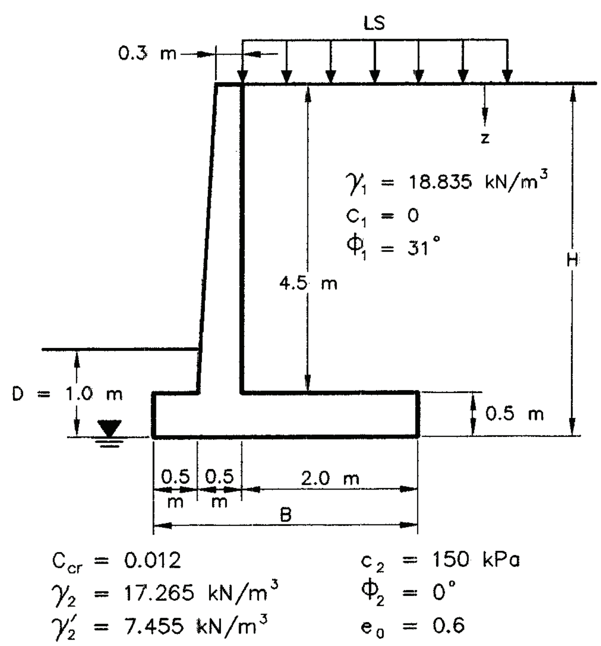
B.6 Chapter 8 Student Problem: Footing Design on Soil Using ASD and LRFD

Problem: In the Student Problem in Chapter 4, Section 4.6, you developed unfactored and factored loads and moments for the design of a cantilever retaining wall supported on a spread footing. You will use that information for this problem to perform the geotechnical design of the wall foundation by LRFD and you will compare these results with a design already completed using ASD.

You recall from Chapter 4 that the cantilever retaining wall in Figure 8-5 is being considered for a grade separation between roadway lanes. The wall will be backfilled with a free draining granular fill such that the seasonal high water table will be below the bottom of the footing. The vehicular live load surcharge (LS) on the backfill is applied as shown in the figure.

Figure 8-5



Schematic of Example Problem

During the subsurface exploration, it was determined that the foundation soils are predominantly clay to a depth of 6 m below the proposed bottom of footing; therefore, a 0.15-m thick blanket of compacted granular material will be placed below the footing to provide a uniform base for foundation construction and improve sliding resistance. In performing the wall design, the following are assumed:

- Dense sand and gravel underlies the clayey foundation soils so that the elastic settlement of the dense sand and gravel will be negligible

- The proposed wall backfill will consist of a free draining granular fill

- The seasonal high water table will be at the bottom of the footing

Objective: Demonstrate the procedure for geotechnical design of a spread footing by LRFD and compare the results to those obtained using ASD.

Approach: To perform the evaluation, the following steps are taken:

- Select footing length, L, and width, B, and determine unfactored and factored bearing pressure distribution

- Settlement: For ASD and LRFD, estimate footing settlement using unfactored loads and the applicable compression characteristics of the soil within the zone of influence and compare with tolerable movement criteria

- Bearing: For ASD, ensure that unfactored ultimate geotechnical bearing resistance, q_{ult}, of the footing divided by the factor of safety, FS, is greater than or equal to the design bearing stress due to unfactored load components, $\overline{q}$ or q_{max}, and for LRFD, ensure that the maximum bearing stress due to the factored load components, $\sum \eta_i \gamma_i Q_i$, is less than or equal to the factored geotechnical bearing resistance, $q_R = \phi\ q_{ult}$

- Sliding: For ASD, ensure that unfactored ultimate geotechnical lateral load capacity, Q_n, of the footing divided by the factor of safety, FS, is greater than or equal to the design load due to lateral load components, Q, and for LRFD, ensure that the sum of the factored lateral load components, $\sum \eta_i \gamma_i Q_i$, is less than or equal to the factored geotechnical lateral load resistance, $Q_R = \phi\ Q_n$

- Overturning: For ASD, ensure that the resultant unfactored vertical load component is located within L/6 and B/6 of the footing centroid, and for LRFD ensure that the factored resultant vertical load component is located within L/4 and B/4 of the footing centroid.

The factored and unfactored loads and moments for critical load combinations from the example problem in Section 4.6 of Chapter 4 are presented in Tables 8-10, 8-11, 8-12 and 8-13.

Table 8-10 (4-17)
Unfactored and Factored Vertical Loads

Group/ Item Units	W_1 kN/m	W_2 kN/m	W_3 kN/m	P_{EV} kN/m	P_{LSV} kN/m	ΔP_{LSv} kN/m	P_{av} kN/m	V_{TOT} kN/m
V (Unf.)	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4
Strength I-a	28.6	9.5	31.8	169.5	59.8	22.4	52.8	374.4
Strength I-b	39.7	13.3	44.2	228.8	59.8	22.4	52.8	461.0
Strength IV	47.7	15.9	53.0	228.8	0	0	52.8	398.2
Service I	31.8	10.6	35.3	169.5	34.2	12.8	35.2	329.4

Table 8-11 (4-18)
Unfactored and Factored Horizontal Loads

Group/Item Units	ΔP_{LSh} kN/m	P_{ah} kN/m	H_{TOT} kN/m
H (Unf.)	21.3	58.6	79.9
Strength I-a	37.3	87.9	125.2
Strength I-b	37.3	87.9	125.2
Strength IV	0	87.9	87.9
Service I	21.3	58.6	79.9

Table 8-12 (4-19)
Unfactored and Factored Moments from Vertical Forces (M_v)

Group/ Item Units	W_1 kN-m/m	W_2 kN-m/m	W_3 kN-m/m	P_{EV} kN-m/m	P_{LSV} kN-m/m	ΔP_{LSv} kN-m/m	P_{av} kN-m/m	M_{vTOT} kN-m/m
M_v (Unf.)	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0
Strength I-a	24.3	6.0	47.7	339.0	119.6	67.2	158.4	762.2
Strength I-b	33.8	8.4	66.2	457.7	119.6	67.2	158.4	911.3
Strength IV	40.5	10.1	79.5	457.7	0	0	158.4	746.2
Service I	27.0	6.7	53.0	339.0	68.3	38.4	105.6	638.0

Table 8-13 (4-20)
Unfactored and Factored Moments from Horizontal Forces (M_h)

Group/Item Units	ΔP_{LSh} kN-m/m	P_{ah} kN-m/m	M_{hTOT} kN-m/m
M_h (Unf.)	53.3	117.2	170.5
Strength I-a	93.3	175.8	269.1
Strength I-b	93.3	175.8	269.1
Strength IV	0	175.8	175.8
Service I	53.3	117.2	170.5

Step 1: Calculate the Settlement of the Retaining Wall on its Cohesive Foundation

Assume embankment construction has been performed earlier and that consolidation settlement from the embankment loading has already occurred. The original ground surface is located 1 m above the footing level.

Divide the 6-m thickness of cohesive soil below the wall foundation into four layers as follows:

- $H_1 = 1.0$ m
- $H_2 = 1.0$ m
- $H_3 = 2.0$ m
- $H_4 = 2.0$ m

The depth of the footing below the existing ground surface is 1 m, so the depth to the center of each layer from the final ground surface in front of the wall is:

- $d_1 = 1.5$ m
- $d_2 = 2.5$ m
- $d_3 = 4.0$ m
- $d_4 = 6.0$ m

(A) Calculate the effective overburden pressure at the center of each layer before wall construction
The depth of footing below the existing ground surface is 1 m so that the footing is located at the annual high water level. The effective overburden stress at the center of each layer is:

$$\sigma'_{oi} = d_i \gamma'_2$$

$$\sigma'_{o1} = (1.0\text{ m})(17.265\text{ kN/m}^3) + (0.5\text{ m})(7.455\text{ kN/m}^3) = 21.0\text{ kPa}$$

$$\sigma'_{o2} = (1.0\text{ m})(17.265\text{ kN/m}^3) + (1.5\text{ m})(7.455\text{ kN/m}^3) = 28.4\text{ kPa}$$

σ'_{o3} = (1.0 m)(17.265 kN/m^3) +(3.0 m)(7.455 kN/m^3) = 39.6 kPa

σ'_{o4} = (1.0 m)(17.265 kN/m^3) +(5.0 m)(7.455 kN/m^3) = 54.5 kPa

(B) Calculate Increase in vertical pressure resulting from loading of the wall
Estimate the wall settlement at the Service I Limit State.

Using Service I loading from Table 8-10, the total vertical unfactored load (V_{TOT}) for calculation of settlement is 329.4 kN/m length of wall. Assuming the vertical load is uniformly distributed over the base width of the wall foundation (i.e., B = 3 m), the increase in pressure at the base of the footing per unit length of wall is:

$$q_o = (V_{TOT})/(B) = \frac{329.4\ \text{kN/m}}{3\ \text{m}} = 109.8\ \text{kPa}$$

(C) Estimate the consolidation settlement:
Because the cohesive foundation soil is assumed to have consolidated under embankment loading (i.e., $\sigma'_p = \sigma'_o$), the only consolidation settlement that will occur will be due to recompression under the foundation loading, after removal of excess embankment soil and wall construction. The recompression settlement can be computed from AASHTO LRFD A10.6.2.2.3c as:

$$S_c = \left(\frac{H_c}{1+e_o}\right)\left(C_{cr}\log\frac{\sigma_f}{\sigma_o}\right) = \frac{C_{cr}}{1+e_o}\Sigma\left[H_i\log\left(\frac{\sigma_o+\Delta\sigma}{\sigma_o}\right)\right] \qquad (A10.6.2.2.3c\text{-}1)$$

where:

C_{cr} = 0.012 and e_o = 0.6

Table 8-14
Consolidation Settlement Calculation

Layer I	$(d_i - 1)/B$	$\Delta\sigma'/q_o$	$\Delta\sigma'$ (kPa)	σ'_{oi} (kPa)	H_i (m)	$\left[H_i\log\left(\frac{\sigma'_{oi}+\Delta\sigma}{\sigma'_{oi}}\right)\right]$
1	0.17	0.95	104.3	21.0	1.0	0.78 m
2	0.50	0.80	87.8	28.4	1.0	0.61 m
3	1.00	0.55	60.4	39.6	2.0	0.80 m
4	1.67	0.34	37.3	54.5	2.0	0.45 m
						Σ = 2.64 m

where:

d_i - 1 = Depth to the center of layer i below the footing from finished grade (m)
$\Delta\sigma'/q_o$ = Boussinesq stress contour at center of a continuous foundation (dim) (*A10.6.2.2.3a*)

From Table 8-14, the estimated total consolidation settlement is:

$$S_c - \frac{0.012}{1+0.6}(2.64\,\text{m}) = 0.020\text{ m} = \mathbf{20\ mm}$$

Assuming the maximum tolerable settlement, $\mathbf{S_{max} = 25\ mm}$, the total estimated settlement is acceptable.

Step 2: Eccentricity
The eccentricity of the retaining wall is checked in Table 8-15 as described in Section 8.3.4 by comparing the calculated eccentricity, e, for each loading group to the maximum allowed eccentricity (e_{max}) using the relationship:

$e_B =$ B/2 - X_o

where:

B/2 = ***1.5*** m
X_o = Location of the resultant from the toe = $(M_{vTOT} - M_{hTOT})/V_{TOT}$
e_{max} = B/4 = ***3.0*** m/4 = ***0.75*** m

For each load group, the total vertical forces (V_{TOT}), horizontal forces (H_{TOT}), moments due to vertical forces (M_{vTOT}) and moments due to horizontal forces (M_{hTOT}) are obtained from Tables 8-10, 8-11, 8-12 and 8-13, respectively. **Note that the force and moment due to live load surcharge over the heel (P_{LSV}) are not included in the eccentricity (i.e., overturning) evaluation (i.e., $V_{TOT} = V_{TOT(Table\ 8\text{-}10)} - P_{LSV(Table\ 8\text{-}10)}$ and $M_{vTOT} = M_{vTOT(Table\ 8\text{-}12)} - P_{LSV(Table\ 8\text{-}12)}$).**

Table 8-15
Summary of Eccentricity Check

Group/Item Units	V_{TOT} (kN/m)	H_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o (m)	e_B (m)	e_{max} (m)
Strength I-a	***314.6***	125.2	***642.6***	240.2	***1.28***	***0.22***	***0.75***
Strength I-b	***401.2***	125.2	***791.7***	240.2	***1.37***	***0.13***	***0.75***
Strength IV	***398.2***	87.9	***746.2***	146.9	***1.50***	***0.00***	***0.75***
Service I	***295.2***	79.9	***569.7***	151.2	***1.42***	***0.08***	0.50

For all cases, e_B is ≤, ~~= or ≥~~ e_{max} **(underline correct answer)**; therefore, the design is ***adequate***/~~inadequate~~ **(underline correct answer)** in regard to eccentricity.

Note that for ASD, e_{max} = B/6, or 0.5 m for a 3.0-m wide footing. Because ASD is equivalent to the Service I Limit State in table 8-15, the design also meets the ASD eccentricity requirements.

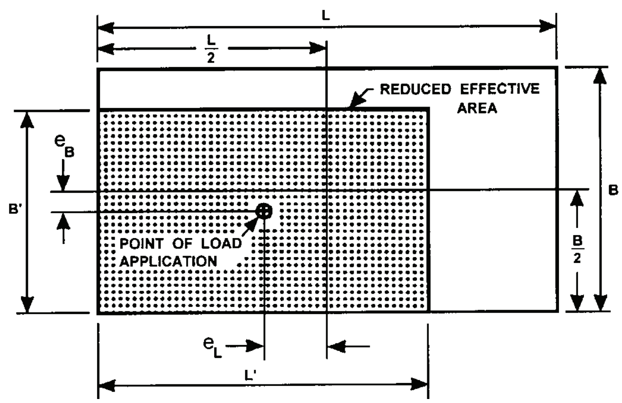
Step 3: Bearing Resistance

(A) Estimate the Bearing Pressures

From Section 8.3.2, the adequacy for bearing capacity is developed based on a rectangular distribution of soil pressure ($\bar{q}$) over the reduced effective area as indicated in Figure 8-6. For a rectangular distribution:

$L' =$ 1 m (i.e., unit length of wall)
$B' = B - 2\,e_B$
$e_B = B/2 - X_o$
$X_o = (M_{vTOT} - M_{hTOT})/V_{TOT}$
$\gamma\bar{q} = V_{TOT}/L'\,B' = V_{TOT}/[B - (B/2 - X_o)] = V_{TOT}/2X_o$

Note that the force and moment due to live load surcharge over the heel (P_{LSV}) <u>are</u> included in the bearing resistance evaluation.



Complete Table 8-16 using information from Tables 8-10, 8-12 and 8-13 for each applicable limit state.

Table 8-16
Summary of Factored Bearing Pressures

Group/Item Units	V_{TOT} (kN/m)	M_{vTOT} (kN-m/m)	M_{hTOT} (kN-m/m)	X_o (m)	$\gamma\bar{q}$ (kPa)
Strength I-a	<u>***374.4***</u>	<u>***762.2***</u>	240.2	<u>***1.39***</u>	<u>***135***</u>
Strength I-b	<u>***461.0***</u>	<u>***911.3***</u>	240.2	<u>***1.46***</u>	<u>***158***</u>
Strength IV	<u>***398.2***</u>	<u>***746.2***</u>	146.9	<u>***1.50***</u>	<u>***133***</u>
Service I	<u>***329.4***</u>	<u>***638.0***</u>	151.2	<u>***1.48***</u>	<u>***112***</u>

(B) Evaluate Adequacy of Bearing Resistance
The factored bearing resistance, q_R, at the Strength Limit State is determined, based on LRFD (AASHTO, 1997a) (*A10.6.3.1.2b*) using:

$q_R = \phi\, q_{ult} = \phi\, (c\, N_{cm} + \gamma_2\, D_f\, N_{qm})$ (*A10.6.3.1.1-1 and A10.6.3.1.2b-1*)
ϕ = ***0.60*** from Table 8-8 for undrained strength based on lab UU tests

AASHTO LRFD (1997a) Article 10.6.3.1.2b provides guidance regarding the values of N_{cm} and N_{qm}.

q_R= ***0.60*** [(***150*** kPa)(3.08) + (***17.265*** kN/m^3)(***1.00*** m)(0.45)]
q_R = (***0.60***) (***469.8*** kPa) = ***282*** kPa > $\gamma\bar{q}$ from Table 8-16

Because the factored bearing resistance, q_R, ~~is less than~~/***exceeds*** **(underline correct answer)** the maximum factored uniform bearing stress, $\gamma\bar{q}$ = 158 kPa, the bearing resistance is ***adequate***/~~inadequate~~ **(underline correct answer)**.

Relative to ASD, as represented by the Service I Limit State, the factor of safety against bearing capacity failure is q_{ult}/q_{max} = 470 kPa/112 kPa = 4.2. Because FS = 4.2 > 3.0 as required for ASD from Tables 8-1 and 8-2, the design is also acceptable with respect to bearing capacity by ASD.

Step 4: Sliding

Sliding of walls on clay is checked under AASHTO LRFD (1997a) using Figure 8-7.

From Section 8.3.3, the factored resistance, Q_R, against failure by sliding is:

$Q_R = \phi_\tau\, Q_\tau + \phi_{ep}\, Q_{ep}$ (Eq. 8-15)

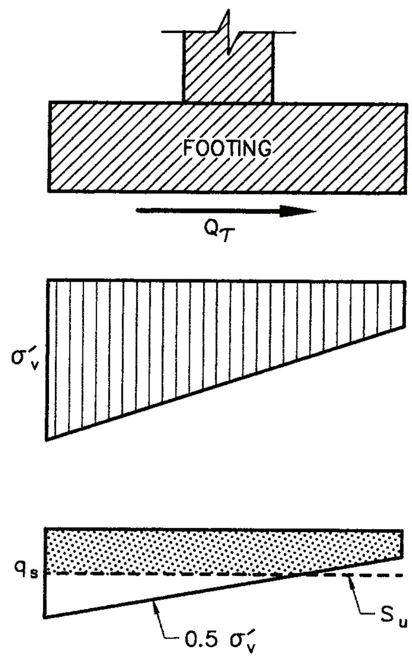
where:

ϕ_τ = ***0.85*** from Table 8-8
Q_τ = Nominal shear resistance between footing and foundation material (kN)
Q_{ep} = Nominal passive resistance of foundation material available throughout the design life of the footing (kN)

From Figure 8-7, $Q_\tau$is the lesser of:

- undrained shear strength or cohesion of the clay; or
- one-half the normal stress on the interface between the footing and the soil.

where the cohesion of the foundation soil (c_2) = 150.0 kPa and one-half the factored normal stress (γq_{max}) is given in Table 8-17. **As for the eccentricity check, the force and moment due to the live load surcharge over the heel (P_{LSV}) are <u>not</u> included in the sliding evaluation.**



q_s = UNIT SHEAR RESISTANCE, EQUAL TO S_u OR 0.5 σ'_v, WHICHEVER IS LESS (kPa)
Q_T = SHADED AREA UNDER q_s DIAGRAM (kN)
σ'_v = VERTICAL EFFECTIVE STRESS kPa
S_u = UNDRAINED SHEAR STRENGTH kPa

Figure 8-7 (*A10.6.3.3-1*)
Procedure for Estimating Sliding Resistance of Footings on Clay
(AASHTO, 1997a)

The actual base pressure (i.e., normal stress at the foundation/soil contact) will have a trapezoidal shape except when the eccentricity is greater than B/6 (i.e., 0.5 m), at which point the base pressure distribution becomes triangular and acts over a reduced base width. From Table 8-15, $e_B < 0.5$ m for all limit states. The values of γq_{max} and γq_{min} in Table 8-17 are calculated, therefore, for a trapezoidal base pressure distribution as follows:

$$\gamma q_{max} = (V_{TOT}/B) + [(6)(V_{TOT})(e_B)/B^2]$$
$$\gamma q_{min} = (V_{TOT}/B) - [(6)(V_{TOT})(e_B)/B^2]$$

where V_{TOT} and e_B are obtained from Table 8-15.

Because $c_2 > \gamma q_{max}/2$ in all cases, the normal stress at the footing/soil interface is used in the calculation of Q_τ.

For comparison with the total factored horizontal forces (H_{TOT}) from Table 8-11, Q_τ can be computed as follows:

$$Q_\tau = \frac{V_{TOT}}{2} = \frac{1}{2}\left(\frac{\gamma q_{max} + \gamma q_{min}}{2}\right) B$$

Using the relationships for γq_{max} and γq_{min}, the equation above for Q_τ, ϕ_τ from Table 8-8, and H_{TOT} from Table 8-11, complete Table 8-16 for each applicable limit state.

Table 8-17
Summary of Sliding Resistance

Group/Item Units	γq_{max} (kPa)	γq_{min} (kPa)	$\gamma q_{max}/2$ (kPa)	Q_τ (kN/m)	$\phi_\tau\ Q_\tau$ (kN/m)	H_{TOT} (kN/m)
Strength I-a	***151.0***	***58.8***	***75.5***	***157.4***	***133.8***	***125.2***
Strength I-b	***168.5***	***98.9***	***84.2***	***200.6***	***170.5***	***125.2***
Strength IV	***132.7***	***132.7***	***66.4***	***199.0***	***169.2***	***87.9***
Service I	***114.1***	***82.7***	***57.0***	***147.6***	***147.6***	***79.9***

Because the factored sliding resistance ($\phi_\tau\ Q_\tau$) calculated is ***greater***/less **(underline correct answer)** than the factored horizontal loading for all Strength Limit States, the sliding resistance is ***satisfactory***/unsatisfactory. By ASD (as represented by the Service I Limit State), the factor of safety against sliding = Q_τ/H_{TOT} = 147.5 kN/m ÷ 79.9 kN/m = 1.85, which is acceptable as indicated in Table 8-1.

Summary

Table 8-18
Summary of Spread Footing Design by LRFD and ASD

Performance Limit	LRFD			ASD		
	Factored Resistance/ Eccentricity Limit	Factored Load/ Eccentricity	✔	Required FS/ Eccentricity Limit	Actual FS/ Eccentricity	✔
Eccentricity	0.25 B	0.07 B	✔	0.167 B	0.03 B	✔
Bearing Resistance	282 kPa	158 kPa	✔	3.0	4.2	✔
Sliding Resistance	134 kN/m	125 kN/m	✔	1.5	1.8	✔

As summarized in Table 8-18, a comparable design is achieved by LRFD and ASD. *Whereas the ASD factors of safety for bearing resistance and sliding are fixed, however, the LRFD resistance factors could possibly be increased with additional data accumulation and reliability-based* *calibration for similar soils and loading conditions.* ***As such, the LRFD provisions reflect the reliability of the soil strength estimates and capacity prediction models and provide a more rational basis for design than the ASD provisions. Therefore, with further data accumulation and calibration, more reliable and economical designs might be achieved using LRFD.***

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B.7 Chapter 9 Student Problem: Comparison of Pile Designs Using ASD and LRFD

Problem: You are to design an axially loaded pile group to support the bridge pier illustrated in the following problem. This problem presents SPT, CPT and instrumented pile load test data for closed-end steel pipe piles driven to a depth of 12 m into a deep sand deposit. In situ and load test data presented in the example provide a basis for comparing ASD and LRFD concepts for a driven pile foundation. (Note: T his problem has been simplified for classroom purposes from the typical design case which would include both horizontal and axial loads. A more typical problem is presented in Sections 9.6 and 9.7).

Objectives: To demonstrate the procedures for driven pile design by LRFD, and to compare the results with those obtained using ASD.

Approach: To perform the designs, you should take the following steps:

- Establish unfactored design loads due to structure components, wearing surfaces and utilities, and vehicular live load
- Determine appropriate load factors and load combinations and calculate the total factored load effects
- Estimate the unfactored (ASD) and factored (LRFD) axial resistance of a single pile based on correlation with the results of SPTs
- Estimate the unfactored and factored axial resistance of a single pile based on correlation with the results of CPTs
- Estimate the unfactored and factored axial resistance of a single pile based on the results of full-scale load tests
- Establish the allowable (ASD) and factored (LRFD) structural capacity of a single pile
- Determine the required number of piles in the group based on geotechnical and structural criteria

Step 1: Establish Unfactored Loads
The unfactored vertical loads on the pile group shown in Figure 9-2 are:

DC = Dead load of structural components and non-structural attachments = 4600 kN

DW = Dead load of wearing surfaces and utilities = 3900 kN

LL = Vehicular live load = 3450 kN

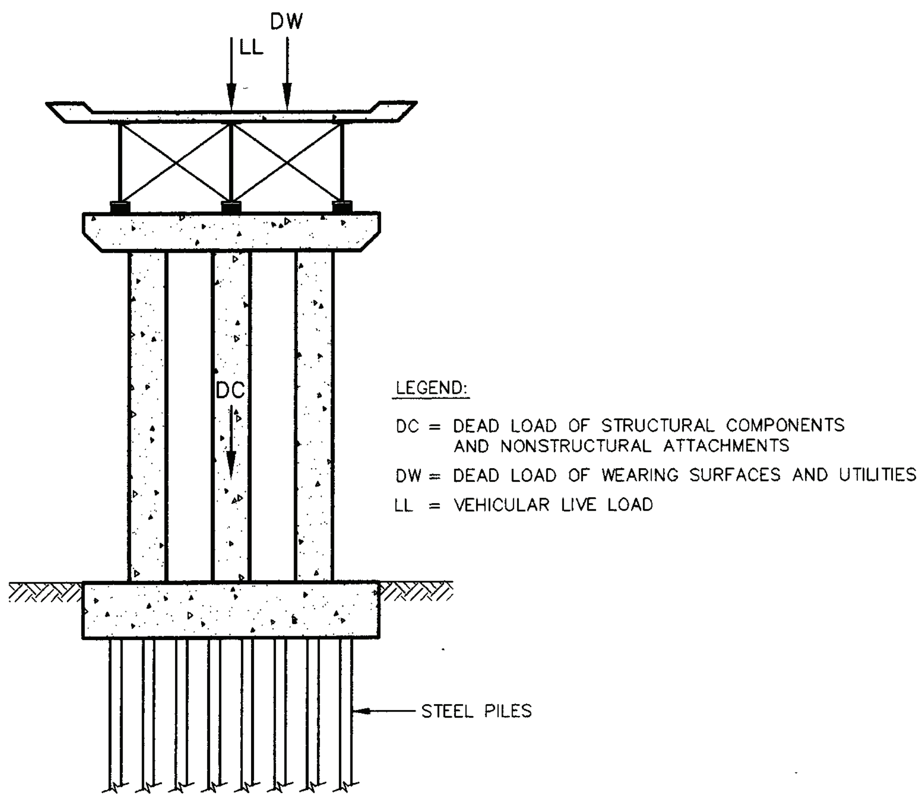


Figure 9-2
Pile Group Loading

From which the total unfactored load, Q, is:

$Q = DC + DW + LL$
$Q = \underline{\textbf{\textit{4600}}}$ kN + $\underline{\textbf{\textit{3900}}}$ kN + $\underline{\textbf{\textit{3450}}}$ kN
$Q = \underline{\textbf{\textit{11 950}}}$ kN

Step 2: Determine Load Factors and Factored Loads
For this design example, consider only Strength I and Service I Limit States.

For the Strength I and Service I Limit States, Eq. 9-6 is used to compute the factored load effect:

$$\sum \eta_i \gamma_i Q_i$$

Assume a typical structure such that $\eta_i = 1.0$.

Complete Table 9-10 by selecting load factors for the Strength I and Service I Limit States from Table 4-10 in Chapter 4:

Table 9-10
Load Factors

Limit State	γ_{DC}	γ_{DW}	γ_{LL}
Strength I	***1.25***	***1.50***	***1.75***
Service I	***1.00***	***1.00***	***1.00***

The total factored load effects are then calculated as follows. For the Strength I Limit State:

$$\sum \eta_i \gamma_i Q_i = \eta_i [\gamma_{DC} DC + \gamma_{DW} DW + \gamma_{LL} LL]$$
$$\sum \eta_i \gamma_i Q_i = \underline{\mathbf{\mathit{1.00}}}\ [(\underline{\mathit{1.25}})(\underline{\mathbf{\mathit{4600}}}\ \text{kN}) + (\underline{\mathbf{\mathit{1.50}}})(\underline{\mathbf{\mathit{3900}}}\ \text{kN}) + (\underline{\mathbf{\mathit{1.75}}})(\underline{\mathbf{\mathit{3450}}}\ \text{kN})]$$
$$\sum \eta_i \gamma_i Q_i = \underline{\mathbf{\mathit{17\ 638}}}\ \text{kN}$$

For the Service I Limit State:

$$\sum \eta_i \gamma_i Q_i = \eta_i [\gamma_{DC} DC + \gamma_{DW} DW + \gamma_{LL} LL]$$
$$\sum \eta_i \gamma_i Q_i = \underline{\mathbf{\mathit{1.00}}}\ [(\underline{\mathbf{\mathit{1.00}}})(\underline{\mathbf{\mathit{4600}}}\ \text{kN}) + (\underline{\mathbf{\mathit{1.00}}})(\underline{\mathbf{\mathit{3900}}}\ \text{kN}) + (\underline{\mathbf{\mathit{1.00}}})(\underline{\mathbf{\mathit{3450}}}\ \text{kN})]$$
$$\sum \eta_i \gamma_i Q_i = \underline{\mathbf{\mathit{11\ 950}}}\ \text{kN}$$

Step 3: Estimate Axial Capacity of Single Pile from SPTs
Figure 9-3 shows the generalized soil profile, pile geometry and idealized SPT blow count profile for use in design based on several borings performed near the pier location. Using Meyerhof (1976) (*A10.7.3.4.2*), the estimated ultimate axial capacity of the pile driven into the soil profile in Figure 9-3 is:

$$Q_p = q_p A_p = (8800 \text{ kPa})\ \pi\ (0.46 \text{ m}/2)^2 = 1460 \text{ kN} \qquad \text{(Eq. 9-19)}$$

and

$$Q_s = q_s A_s = (28 \text{ kPa})\ 2\pi\ (0.23 \text{ m})\ (11.00 \text{ m}) = 445 \text{ kN} \qquad \text{(Eq. 9-20)}$$

where:

Q_{ult}, Q_p, Q_s = Ultimate total, tip and side resistance (kN)
q_p, q_s = Unit tip and side resistance (kPa)
A_p, A_s = Area of pile tip and side (m^2)

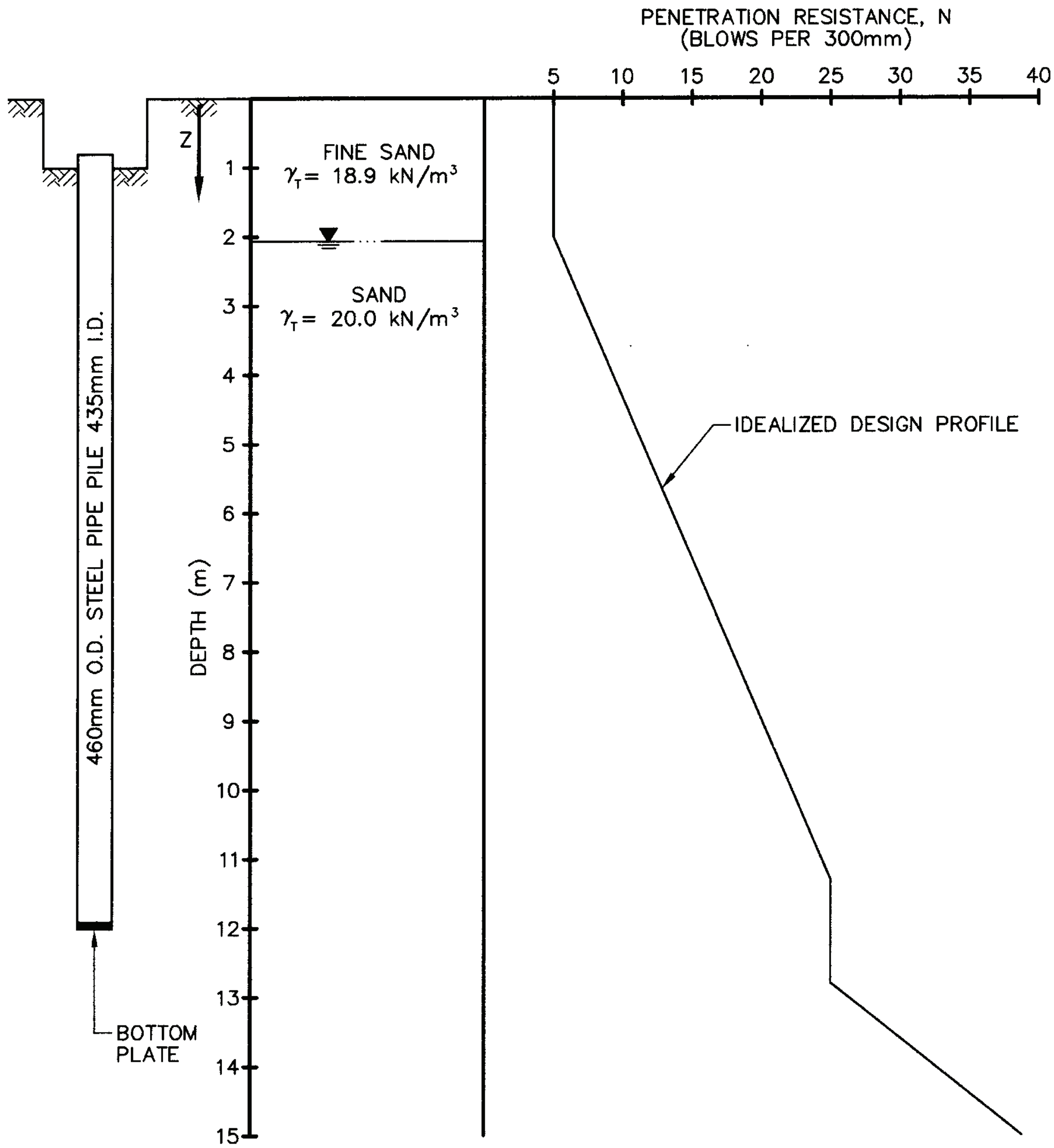


Figure 9-3
Generalized Problem Geometry and SPT Design Envelope

Using Eq. 9-17, the ultimate axial resistance of a single pile is:

$$Q_{ult} = Q_p + Q_s = \underline{\textbf{\textit{1460}}}\ \text{kN} + \underline{\textbf{\textit{445}}}\ \text{kN} \qquad \text{(Eq. 9-17)}$$
$$Q_{ult} = \underline{\textbf{\textit{1905}}}\ \text{kN}$$

From Eq. 9-18, the factored axial resistance of a single pile is:

$$Q_R = \phi\ Q_{ult} = \phi_{qp}\ Q_p + \phi_{qs}\ Q_s \qquad \text{(Eq. 9-18)}$$

From Table 9-5, the resistance factors for the SPT method are:

$$\phi_{qp} = \underline{\textbf{\textit{0.45}}}$$
$$\phi_{qs} = \underline{\textbf{\textit{0.45}}}$$

The factored bearing resistance is:

$$Q_R = \phi_{qp}\ Q_p + \phi_{qs}\ Q_s = \underline{\textbf{\textit{0.45}}}\ (\underline{\textbf{\textit{1460}}}\ \text{kN}) + \underline{\textbf{\textit{0.45}}}\ (\underline{\textbf{\textit{445}}}\ \text{kN}) = \underline{\textbf{\textit{857}}}\ \text{kN}$$

Step 4: Estimate Axial Capacity of Single Pile from CPTs

Figures 9-4 and 9-5 show a representative pile and idealized tip and sleeve friction profiles used for design from a typical mechanical cone penetration sounding performed near the pier location referred to in Step 3.

Using Nottingham and Schmertmann (1975) (A10.7.3.4.3), estimate ultimate axial capacity of the pile driven into the soil profile in Figures 9-4 and 9-5.

$$Q_p = q_p\ A_p = (11\ 300\ \text{kPa})\ \pi\ (0.46\ \text{m}/2)^2 = 1880\ \text{kN} \qquad \text{(Eq. 9-19)}$$

The ultimate unit side resistance of the pile is estimated using the CPT sleeve resistance values from Figure 9-4 and the following relationship:

$$Q_s = K_s \left[\sum_{i=1}^{N_1} \left(\frac{L_i}{8 D_i} \right) f_{si}\ a_{si}\ h_i + \sum_{i=N_i}^{N_2} f_{si}\ a_{si}\ h_i \right] \qquad (\textit{A10.7.3.4.3c-1})$$

For a steel pile with a length to diameter (z/D) ratio of 11.00 m/0.46 m = 23.9, the correction factor, K_s for a mechanical cone (from Figure A10.7.3.4.3c-1) is 0.39. The incremental values of Q_{si} along the pile are presented in Table 9-11.

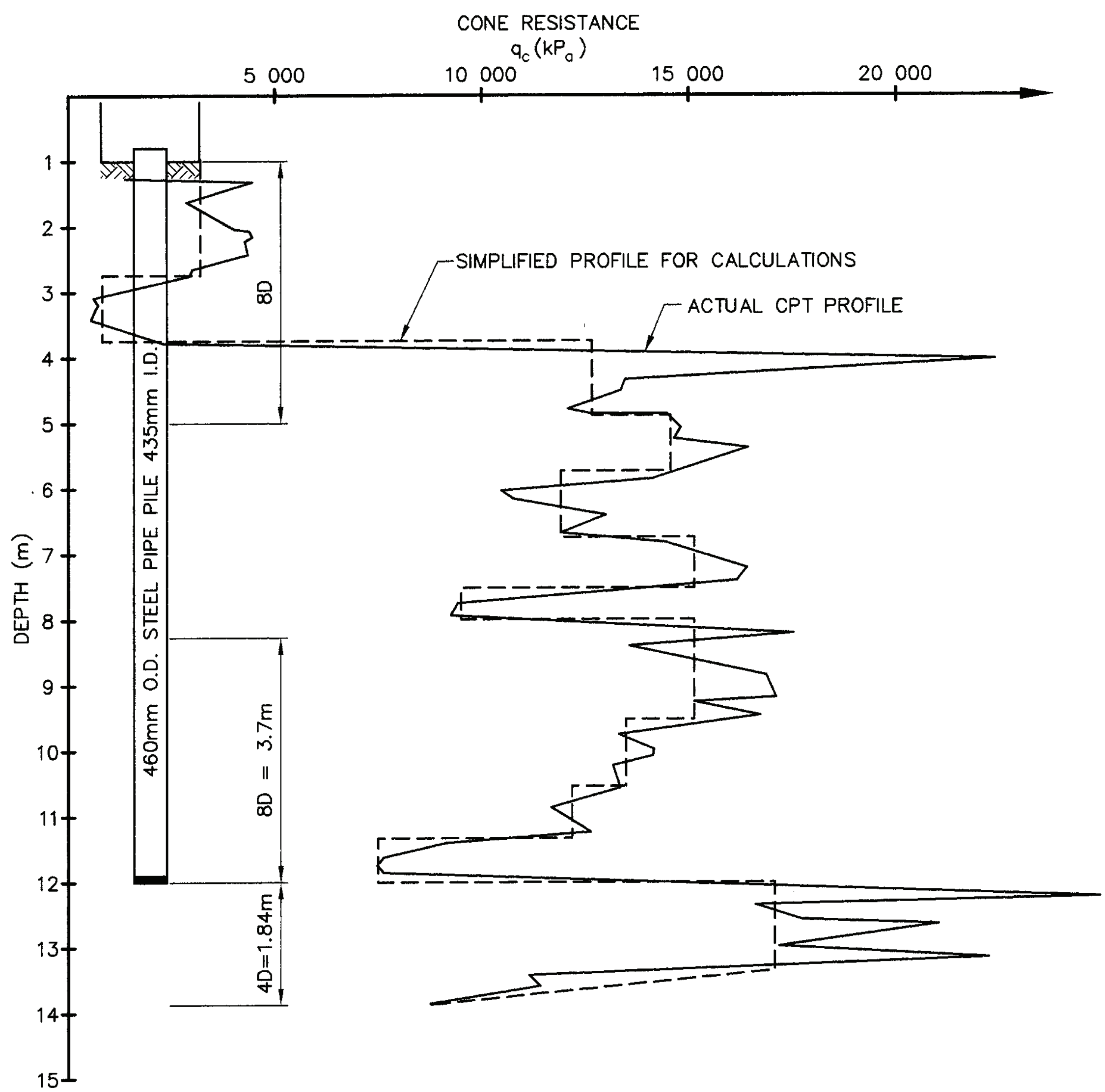


Figure 9-4
Representative Pile and CPT Tip Resistance Profile Used in Example

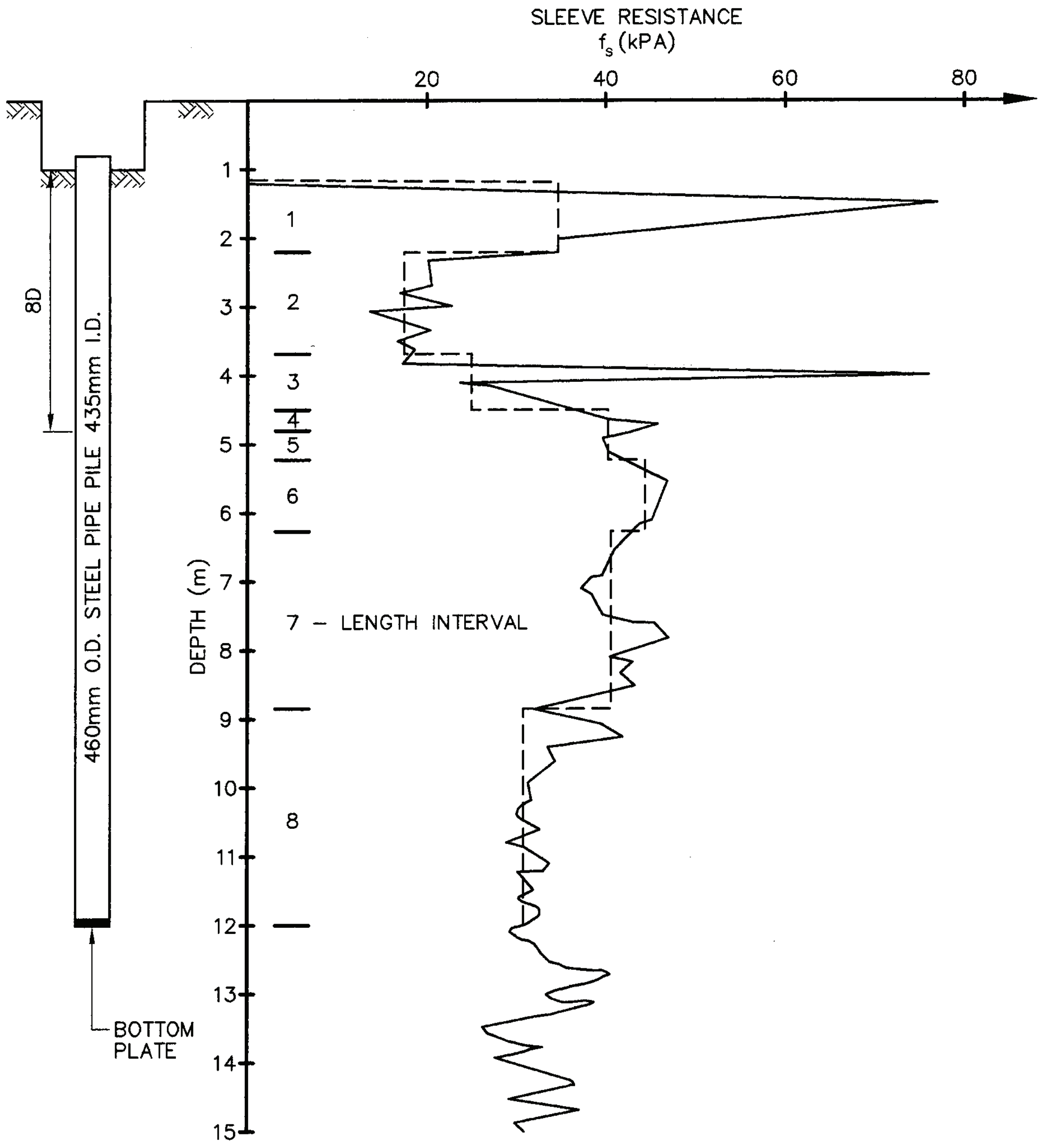


Figure 9-5
Representative Pile and CPT Side Resistance Profile Used in Example

Table 9-11
Tabulation of Side Resistance Along Pile Length

Pile Segment, I	L_i (m)	D_i (m)	f_{si} (kPa)	a_{si} (m^2/m)	h_i (m)	Q_{si} (kN)
1	0.643	0.46	345	1445	1.105	4
2	1.885	0.46	173	1445	1.380	7
3	2.970	0.46	249	1445	0.790	9
4	3.523	0.46	403	1445	0.315	7
5	NA	NA	403	1445	0.480	11
6	NA	NA	441	1445	1.000	25
7	NA	NA	403	1445	2.650	60
8	NA	NA	307	1445	3.190	55
					TOTAL Q_s:	178

where the variables are the same as those in Step 3.

From Eq. 9-17, the ultimate axial resistance of a single pile is:

$$Q_{ult} = Q_p + Q_s = \underline{\mathbf{\mathit{1880}}} \text{ kN} + \underline{\mathbf{\mathit{178}}} \text{ kN} = \underline{\mathbf{\mathit{2058}}} \text{ kN} \qquad \text{(Eq. 9-17)}$$

The factored axial resistance of a single pile is determined from Eq. 9-18 as:

$$Q_R = \phi\, Q_{ult} = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s \qquad \text{(Eq. 9-18)}$$

From Table 9-5, the resistance factors for the CPT method are:

- $\phi_{qp} = \underline{\mathbf{\mathit{0.55}}}$
- $\phi_{qs} = \underline{\mathbf{\mathit{0.55}}}$

The factored bearing resistance is then:

$$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s = \underline{\mathbf{\mathit{0.55}}}\,(\underline{\mathbf{\mathit{1880}}} \text{ kN}) + \underline{\mathbf{\mathit{0.55}}}\,(\underline{\mathbf{\mathit{178}}} \text{ kN}) = \underline{\mathbf{\mathit{1130}}} \text{ kN}$$

Step 5: Estimate Axial Capacity of Single Pile from Full-Scale Load Tests
From a full-scale axial load test on a pile at the site (Vesic, 1970), the ultimate tip and side resistance of a pile driven to a depth of 12 m at the site are:

$Q_s = 1180$ kN
$Q_p = 1900$ kN

from which the ultimate axial resistance based on Eq. 9-17 is:

$Q_{ult} = Q_p + Q_s =$ ***1900*** kN + ***1180*** kN = ***3080*** kN (Eq. 9-17)

From Eq. 9-18, the factored axial resistance of a single pile is:

$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s$ (Eq. 9-18)

From Table 9-5, the resistance factors for bearing resistance of a single pile based on a load test are as follows:

- $\phi_{qp} =$ ***0.80***
- $\phi_{qs} =$ ***0.80***

Therefore, the factored bearing resistance is:

$Q_R = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s =$ ***0.80*** (***1900*** kN) + ***0.80*** (***1180*** kN) = ***2464*** kN

Step 6: Check the Axial Structural Capacity of a Single Pile
From Table 9-2, the maximum allowable stress, σ_{all} (AASHTO, 1997b), for axial compression loading of an unfilled steel pipe pile is:

- For severe driving:
  $\sigma_{all} = 0.25\ F_y$

- For good driving:
  $\sigma_{all} = 0.33\ F_y$

For ASTM A709M, Grade 250, F_y = 250 000 kPa, the allowable axial stress is:

- For severe driving:
  $\sigma_{all} =$ ***0.25*** F_y = ***0.25*** (***250 000*** kPa)
  $\sigma_{all} =$ ***62 500*** kPa

- For good driving:
  $\sigma_{all} =$ ***0.33*** F_y = ***0.33*** (***250 000*** kPa)σ_{all} = ***82 500*** kPa

The maximum allowable axial load, P_{all}, is:

- For severe driving:
  $P_{all} = \sigma_{all}\ A_s$
  P_{all} = (***62 500*** kPa)(0.0176 m^2)= ***1098*** kN

- For good driving:
  $P_{all} = \sigma_{all} A_s$
  P_{all} = (***82 500*** kPa)(0.0176 m^2) = ***1450*** kN

For LRFD, the factored axial resistance, P_r, of an unfilled steel pipe pile from Eq. 9-21 and Eq. 9-22 is:

$$P_r = \phi\, P_n = \phi\, \sigma_n\, A_s = \phi\, F_y\, A_s$$

From Table 9-6, the resistance factors for axial compression loading of an unfilled steel pipe pile are:

- For severe driving:
  ϕ = ***0.35***
- For good driving:
  ϕ = ***0.45***

For ASTM A709M, Grade 250 steel:

- For severe driving:
  $P_r = \phi\, F_y\, A_s$
  P_r = (***0.35***)(***250 000*** kPa)(0.0176 m^2) = ***1538*** kN

- For good driving:
  $P_r = \phi\, F_y\, A_s$
  P_r = (***0.45***)(***250 000*** kPa)(0.0176 m^2) = ***1977*** kN

Step 7: Determine the Required Pile Group Size

For ASD, the number of piles, ×, required to support the total (unfactored) design load, Q, can be computed based on Eqs. 9-1 and 9-2 from the following:

$$\times \geq \frac{Q}{(Q_{ult}/FS)} \quad ; \quad \times \geq \frac{Q}{P_{all}}$$

where:

Q_{ult}/FS = Allowable geotechnical resistance (kN)
P_{all} = Allowable structural resistance (kN)

From Table 9-1, the required factors of safety (FS) for axial load resistance based on various predictive methods are:

- Static Calculation (SPT or CPT): FS = 3.5
- Static Load Test: FS = 2.0

Based on these safety factors and the results of Steps 1 through 6, complete Table 9-12 to determine the number of piles need to resist the design load based on geotechnical and structural resistance by ASD.

Table 9-12
Determination of Required Pile Group Size Based on ASD

Resistance	FS or σ_{all}	Design Group Load Q (kN)	Single Pile Ultimate Resistance Q_{ult} (kN)	Single Pile Allowable Resistance Q_{ult}/FS or P_{all} (kN)	Required Number of Piles, x
Geotechnical:					
• SPT Method	3.5	11 950	***1900***	***542.9***	***22***
• CPT Method	3.5	11 950	***2057***	***587.7***	***21***
• Static Load Test	2.0	11 950	***3080***	***1540***	***8***
Structural:					
• Severe Driving	0.25 F_y	11 950	---	***1098***	***11***
• Good Driving	0.33 F_y	11 950	---	***1450***	***9***

As shown in Table 9-12, a geotechnical design based on the SPT or CPT methods controls the size of the pile group, while a pile group size based on the results of a pile load test would not be as critical as the structural design.

For LRFD, the number of piles, ×, required to support the factored design load, Q, can be computed based on Eqs. 9-7 and 9-8 from the following:

$$\times \geq \frac{\sum \eta_i \gamma_i Q_i}{Q_R} \quad ; \quad \times \geq \frac{\sum \eta_i \gamma_i Q_i}{P_r}$$

where:

$\sum \eta_i \gamma_i Q_i$ = Sum of the factored axial loads (kN)
Q_R = Factored geotechnical resistance (kN)
P_r = Factored structural resistance (kN)

Based on the results of Steps 1 through 6, complete Table 9-13 to determine the number of piles need to resist the design load based on geotechnical and structural resistance by LRFD.

Table 9-13
Determination of Required Pile Group Size Based on LRFD

Resistance	Resistance Factor ϕ	Factored[1] Design Group Load $\sum \eta_i \gamma_i Q_i$ (kN)	Single Pile Resistance		Required Number of Piles, ×
			Ultimate Q_{ult} (kN)	Factored Q_R or P_r (kN)	
Geotechnical: • SPT Method	0.45	17 638	***1900***	***855***	***21***
• CPT Method	0.55	17 638	***2057***	***1131***	***16***
• Static Load Test	0.80	17 638	***3080***	***2464***	***8***
Structural: • Severe Driving	0.35	17 638	-	***1538***	***12***
• Good Driving	0.45	17 638	-	***1977***	***9***

(1) For the Strength I Limit State

Summary

This example illustrates the design of axially loaded pile groups by LRFD and compares the results to design by ASD. As shown on Tables 9-12 and 9-13:

- *The geotechnical resistance controls the design for static methods of axial capacity prediction and the structural resistance controls the design based on static load testing for both the ASD and LRFD procedures*

- *Comparison with Table 9-12 shows that the LRFD procedures based on the reliability-based calibration (i.e., SPT and CPT) are somewhat less conservative than the ASD procedure for the conditions analyzed*

- *LRFD design procedures based on direct calibration with ASD (i.e., static load test and structural resistance) provide essentially identical results with ASD (minor differences occur due to differences in the actual average load factor and the load factor assumed in the resistance factor calibration*)

As indicated in Section 9.2.2.2, the AASHTO LRFD Specifications do not yet satisfactorily incorporate the impact of field capacity verification (e.g., PDA or load tests) on the reliability of pile installations for which design capacities are initially predicted using static design methods. Therefore, this example assumes that the piles are designed on the basis of static methods without field capacity verification.

B.8 Chapter 9 Student Exercise: Pile Capacity Evaluation by Nordlund Method

Tables from the Student Exercise with cells completed for Nordlund's method are shown below.

Determination of Required Pile Group Size Based on ASD

Resistance	FS or σ_{all}	Design Load Q (kN)	Single Pile Ultimate Resistance Q_{ult} (kN)	Single Pile Allowable Resistance Q_{ult}/FS or P_{all} (kN)	Required Number of Piles, ×
Geotechnical:					
• SPT Method	3.5	11 950	1900	542.9	22
• CPT Method	3.5	11 950	2057	587.7	21
• Static Load Test	2.0	11 950	3080	1540	8
• Nordlund Method	3.5	11 950	1385	***<u>395.7</u>***	***<u>31</u>***
Structural:					
• Severe Driving	0.25 F_y	11 950	---	1098	11
• Good Driving	0.33 F_y	11 950	---	1450	9

Determination of Required Pile Group Size Based on LRFD

Resistance	Resistance Factor ϕ	Factored[1] Design Load $\sum \eta_i \gamma_i Q_i$ (kN)	Single Pile Resistance		Required Number of Piles, ×
			Ultimate Q_{ult} (kN)	Factored Q_R or P_r (kN)	
Geotechnical:					
• SPT Method	0.45	17 638	1900	855	21
• CPT Method	0.55	17 638	2057	1131	16
• Static Load Test	0.80	17 638	3080	2464	8
• Nordlund Method	0.60	17 638	1385	***<u>831</u>***	***<u>22</u>***
Structural: • Severe Driving	0.35	17 638	-	1538	12
• Good Driving	0.45	17 638	-	1977	9

[1] For the Strength I Limit State

Summary

This exercise illustrates the application of LRFD in the design of an axially loaded pile group, and compares the results of LRFD with ASD. From a review of the completed tables above, the following can be concluded:

1. ***The required number of piles is the same by ASD and LRFD for capacity estimation based on a Static Load Test, as the resistance factor for this method is based on direct calibration with ASD.***

2. ***For the static (SPT, CPT and Nordlund's) methods of capacity prediction (for which resistance factors were developed using a reliability-based calibration), LRFD results in fewer required piles (as much as 1/3 less for Nordlund's method) than ASD.*** *The resistance factors for these methods considers both their variability and bias, where the bias is a measure of the inherent conservatism, or lack thereof, of the method. As such, the "Equivalent LRFD Factor of Safety" for each method varies somewhat from the typical ASD Factor of Safety (equal to 3.5) as follows:*

	Method	FS_{LRFD}
•	*SPT Method*	*3.33*
•	*CPT Method*	*2.75*
•	*Nordlund's Method*	*2.55*

where:

$$FS_{LRFD} = \frac{\times Q_{ult}}{Q}$$

and $\times$ = Number of piles required by LRFD

3. ***The inherent conservatism of each method of capacity prediction is measured by the bias factor, λ_R, which can be computed (from Chapter 7) as the ratio of the measured resistance to the predicted resistance. If it is assumed that the Static Load Test measures the true resistance, the bias factors indicated in this example problem by the predicted resistances (Table 9-12a) are as follows:***

Method		λ_R
•	***SPT Method***	***1.62***
•	***CPT Method***	***1.50***
•	***Static Load Test***	***1.00***
•	***Nordlund's Method***	***2.22***

where:

$$\lambda = \frac{Q_{ult}\,(\text{static load test})}{Q_{ult}\,(\text{static design method})}$$

As described in Chapter 7, a bias factor greater than 1.00 indicates that the predictive method tends to result in conservatively low values of predicted resistance. *(Note that the bias factors tabulated above are applicable only for this* *example.)*

4. ***The required number of piles is essentially the same by ASD and LRFD based on structural requirements, as the resistance factors for structural design in Table 9-6 are based on direct calibration with ASD.*** *(Note that minor differences do occur due to a difference between the average load factor used in the exercise of 17 638/11 950 = 1.48, and the average load factor of 1.45 used for calibration of the resistance factors.)*

B.9 Student Exercise:
Development of Resistance Factor for Design of Drilled Shafts in Sand

Problem: The AASHTO LRFD Specification does not include specific Strength Limit State resistance factors for the various methods available to estimate drilled shaft capacity in sand. Determine the ASD calibrated resistance factor for these methods for dead to live load ratios, Q_D/Q_L, of 1.0, 2.0 and 3.0 (which correspond to approximate span lengths of 17 m, 34 m and 51 m), average dead load factors, γ_D, of 1.25 and 1.50 and a live load factor, γ_L, of 1.75 (i.e., the Strength I Limit State). Recall that the equation for this calibration (Eq. 7-7, Chapter 7) is:

$$\phi = \frac{\gamma_D Q_D / Q_L + \gamma_L}{FS(1 + Q_D / Q_L)} \qquad \text{(Eq. 7-7)}$$

From Section 10.2.1, the AASHTO ASD factor of safety for these type of predictive methods is 2.5.

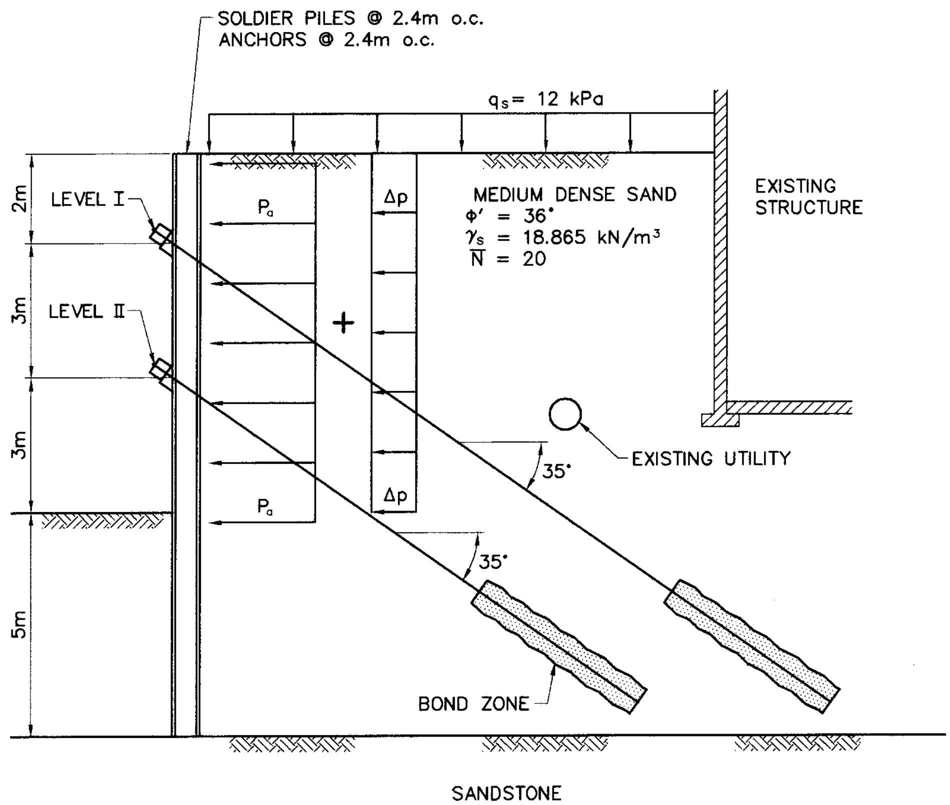
Solution: ***The table below shows the variation in the calibrated resistance factor with φ variations in the dead and live load factors as well as the ratio of dead to live load.***

From Chapter 4, Table 4-11, it can be seen that, for typical bridge structure and retaining wall foundations, dead load (including earth load) load factors range from 1.25 to 1.50. From the table below, it appears reasonable to select a resistance factor of about 0.60 for bearing resistance of drilled shafts in sand for design of most structures.

Q_D/Q_L	Average Dead Load Factor, γ_D	Live Load Factor, γ_L	Resistance Factor, φ	Average Load Factor, $\bar{\gamma}$
1.00	1.25	1.75	***0.60***	***1.50***
2.00	1.25	1.75	***0.57***	***1.42***
3.00	1.25	1.75	***0.55***	***1.38***
1.00	1.50	1.75	***0.65***	***1.63***
2.00	1.50	1.75	***0.63***	***1.58***
3.00	1.50	1.75	***0.63***	***1.56***

B.10 Student Problem:
Anchored Soldier Pile Wall Design by LRFD

Problem: You are to design an anchored retaining wall by LRFD. Figure 13-4 shows the problem geometry, in which a cut-and-cover excavation is required for construction of a roadway tunnel. The final excavation depth will be 8 m and the excavation will be supported by a soldier pile and timber lagging wall incorporating two levels of anchors. The high water table is below the bottom of the excavation, and the vehicular live load surcharge (LS = q_s) on the backfill is applied as shown in the figure.



γ =	Unit weight of retained soil (kN/m^3)
q_s =	Vehicular live load surcharge (kPa)
p_a =	Lateral earth pressure (kPa)
Δp =	Lateral earth pressure due to vehicular live load (kPa)

Figure 13-4
Schematic of Example Problem

During the subsurface exploration, it was determined that the foundation soils consist of medium dense sand to a depth of 13 m below the ground surface, underlain by hard, sandstone bedrock. In performing the wall design, you can assume the following:

- The anchor derive their capacity wholly within the medium dense sand

- Only the end of excavation stage needs to be checked for this problem (Note: For a complete design, each stage of the excavation should be checked)

- The high water table will be below the bedrock surface

Objective: To demonstrate the procedure for anchored retaining wall design by LRFD

Approach: To perform the anchor wall design, you should take the following steps:

- Compute and tabulate the unfactored loads and moments required for design at the applicable limits states
- Determine and tabulate the factored loads and moments required for design at the applicable limits states
- Estimate wall movements at the Service I Limit State
- Evaluate the global stability of the excavation
- Evaluate the earth pressures on the excavation support system
- Estimate the required anchor bond zone length
- Determine the required soldier pile section using the applicable factored loads and simplified structural analysis procedures
- Determine the required section for the timber lagging
- Evaluate the vertical stability of the wall

Solution:

Step 1: Calculate Loads

The first step in determining the loading conditions is to assess the earth pressures on the wall. The apparent earth pressure diagram shown in Figure 13-4 is selected in accordance with recommendations made by Peck, et. al (1974) and AASHTO (1997a, 1997b) for wall systems with multiple levels of support in cohesionless soils (*A3.11.5.6*). A live load surcharge of 12 kPa (equivalent to a uniform soil surcharge of about 0.63 m) is used for design. The calculations for development of the lateral earth pressure and lateral pressure due to vehicular live load surcharge are shown below.

Unfactored Apparent Earth Pressure (EH):

$P_a = 0.65\, k_a\, \gamma_s\, H = 0.65 \tan^2(45^\circ - \phi_f/2)\, \gamma_s\, H$ (*A3.11.5.6-1*)

$P_a = 0.65 \tan^2 (45^\circ - \underline{\mathbf{\mathit{36}}}^\circ/2)\ (\underline{\mathbf{\mathit{18.865}}}\ \text{kN/m}^3)\ (\underline{\mathbf{8}}\ \text{m})$

$P_a = \underline{\mathbf{\mathit{25.5}}}$ kPa

Unfactored Surcharge Pressure (LS):

$\Delta p = k_a\, q_s = \tan^2 (45^\circ - \underline{\mathbf{36}}^\circ/2)\ \underline{\mathbf{\mathit{12}}}$ kPa $= \underline{\mathbf{\mathit{3.1}}}$ kPa (*A3.11.6.1-1*)

The apparent earth pressure and surcharge pressure diagrams are then distributed to the various members of the system assuming tributary areas extending equidistant between the support levels as shown below.

The unfactored horizontal force to be supported at Level I is the sum of the apparent earth and vehicular surcharge lateral pressures from the ground surface down to a depth of 3.5 m (z_I = 3.5 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:

$P_1 = P_a z_I = (\underline{\mathbf{\mathit{25.5}}}$ kPa$)(\underline{\mathbf{\mathit{3.5}}}$ m$) = \underline{\mathbf{\mathit{89.25}}}$ kN/m

Unfactored horizontal load from vehicular surcharge lateral pressure:

$P_2 = \Delta p\, z_I = (\underline{\mathbf{\mathit{3.1}}}$ kPa$)(\underline{\mathbf{\mathit{3.5}}}$ m$) = \underline{\mathbf{\mathit{10.85}}}$ kN/m

Total unfactored horizontal load:

$P_a = P_1 + P_2 = \underline{\mathbf{\mathit{89.25}}}$ kN/m + $\underline{\mathbf{\mathit{10.85}}}$ kN/m = $\underline{\mathbf{\mathit{100.1}}}$ kN/m

The load factors from Chapter 4 (Tables 4-10 and 4-11) for horizontal earth pressure, EH, and live load surcharge, LS, are:

$\gamma_{EH} = \underline{\mathbf{\mathit{1.50}}}$
$\gamma_{LS} = \underline{\mathbf{\mathit{1.75}}}$

Therefore, the total factored horizontal load is:

$\gamma\, P = \gamma_{EH}\, P_1 + \gamma_{EH}\, P_2 = \underline{\mathbf{\mathit{1.50}}}\ (\underline{\mathbf{\mathit{89.25}}}$ kN/m$) + \underline{\mathbf{\mathit{1.75}}}\ (\underline{\mathbf{\mathit{10.85}}}$ kN/m$) = \underline{\mathbf{\mathit{152.9}}}$ kN/m

The unfactored horizontal force to be supported at Level II is the sum of the apparent earth and vehicular surcharge lateral pressures from a depth of 3.5 m to a depth of 6.5 m (z_{II} = 3.0 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:

$P_1 = P_a z_{II} = (\underline{\mathbf{\mathit{25.5}}}$ kPa$)(\underline{\mathbf{\mathit{3.0}}}$ m$) = \underline{\mathbf{\mathit{76.50}}}$ kN/m

Unfactored horizontal load from vehicular surcharge lateral pressure:

$P_2 = \Delta p\, z_{II} = (\underline{\mathbf{\mathit{3.1}}}$ kPa$)(\underline{\mathbf{\mathit{3.0}}}$ m$) = \underline{\mathbf{\mathit{9.30}}}$ kN/m

Total unfactored horizontal load:

$P_a = P_1 + P_2 = \underline{\mathbf{\mathit{76.50}}}$ kN/m + $\underline{\mathbf{\mathit{9.30}}}$ kN/m = $\underline{\mathbf{\mathit{85.8}}}$ kN/m

Applying the load factors for EH and LS, the total factored horizontal load is:

$\gamma\, P = \gamma_{EH}\, P_1 + \gamma_{LS}\, P_2 = \underline{\mathbf{\mathit{1.50}}}\ (\underline{\mathbf{\mathit{76.6}}}$ kN/m$) + \underline{\mathbf{\mathit{1.75}}}\ (\underline{\mathbf{\mathit{9.30}}}$ kN/m$) = \underline{\mathbf{\mathit{131.0}}}$ kN/m

The unfactored horizontal force to be supported at the base of the wall for design of the soldier pile embedment is the sum of the apparent earth and vehicular surcharge lateral pressures from a depth of 6.5 m to the base of the wall (z_{III} = 1.5 m) as shown below:

Unfactored horizontal load from apparent earth pressure diagram:
$P_1 = P_a\, z_{III} = (\underline{\mathbf{\textit{25.5}}}$ kPa$)(\underline{\mathbf{\textit{1.5}}}$ m$) = \underline{\mathbf{\textit{38.25}}}$ kN/m

Unfactored horizontal load from vehicular surcharge lateral pressure:
$P_2 = \Delta p\, z_{III} = (\underline{\mathbf{\textit{3.1}}}$ kPa$)(\underline{\mathbf{\textit{1.5}}}$ m$) = \underline{\mathbf{\textit{4.65}}}$ kN/m

Total unfactored horizontal load:
$P_a = P_1 + P_2 = \underline{\mathbf{\textit{38.25}}}$ kN/m + $\underline{\mathbf{\textit{4.65}}}$ kN/m = $\underline{\mathbf{\textit{42.9}}}$ kN/m

Applying the load factors for EH and LS, the total factored horizontal load at the wall base is:

$\gamma P_I = \gamma_{EH} P_1 + \gamma_{LS} P_2 = \underline{\mathbf{\textit{1.50}}}\ (\underline{\mathbf{\textit{38.25}}}$ kN/m$) + \underline{\mathbf{\textit{1.75}}}\ (\underline{\mathbf{\textit{4.65}}}$ kN/m$) = \underline{\mathbf{\textit{65.5}}}$ kN/m

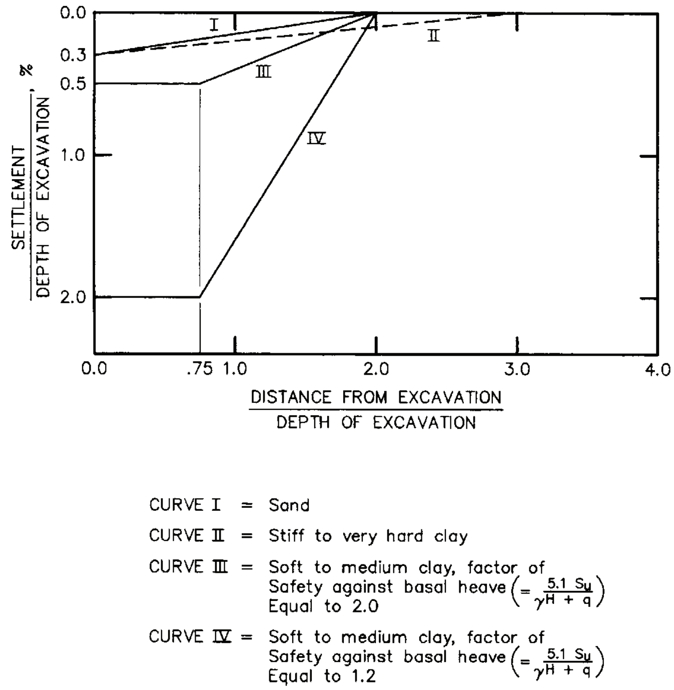
These calculations are summarized in the table below.

Load	Unfactored EH (kN/m)	Unfactored LS (kN/m)	Unfactored Horizontal Load (kN/m)	Factored Horizontal Load (kN/m)
Level I Anchor	***89.25***	***10.85***	***100.1***	***152.9***
Level II Anchor	***76.50***	***9.30***	***85.8***	***131.0***
Excavation Base	***38.25***	***4.65***	***42.9***	***65.5***

In addition to anchor design and soldier pile embedment, it is necessary to consider load distribution and load factoring for structural design of the soldier piles and timber lagging. The distribution and factoring of the apparent earth pressure and vehicular surcharge lateral pressures to the anchors provides the necessary loading information for analysis of the vertical stability of each pile.

Step 2: Estimate Lateral Wall Deflection and Settlement Profile Behind Wall
Figure 13-3 shows the typical settlement profiles behind a wall of this type assuming good construction practices are followed. For this problem, it is evident from Curve I in Figure 13-3 that the maximum settlement at the wall will be about 0.3 percent of the excavation depth or 0.024 m. The lateral wall deformations should be similar in magnitude to this settlement. It is assumed that this level of deformation is acceptable for this problem.

Figure 13-3 (*AC3.11.5.6-1*)



Settlement Profiles Behind Anchored Walls
(Modified after Clough and O'Rourke 1990)

Step 3: Global Stability

An analysis of the global stability of the excavation is not presented herein, but would be checked using a limit equilibrium analysis (*A11.8.4.3*) and is assumed to be adequate for this example.

Step 4: Preliminary Geotechnical Design of Anchors

As with ASD, design of an anchored wall includes preliminary estimation of anchor capacity to permit selection of a reasonable anchor spacing and design of facing and other structural components of the wall. For this problem, you can assume that the anchors will develop their capacity within the medium dense sand deposit. In accordance with AASHTO (1997a) and FHWA (Cheney, 1988) recommendations presumptive ultimate anchor capacity, Q_a, as provided in Table 13-7 for estimating the length of the bond zone in sand for each anchor. The bond zone lengths are estimated as shown in the following calculations. The adequacy of anchor capacity is confirmed during construction in the same manner as for ASD, by means of proof and performance tests. For the purposes of this example, the structural capacity of the anchor strands is assumed to be adequate.

Table 13-7 (*A11.8.4.2-1 Excerpt*)
Ultimate Unit Resistance of Anchors in Soil
(AASHTO, 1997a; AASHTO, 1997b; Cheney, 1988)

Soil Type	Compactness or SPT Resistance (Blows per 0.30 m)		Ultimate Unit Anchor Resistance, Q_a (kN/m)
Sand	Loose	4-10	100
	Medium	**10-30**	**145**
	Dense	30-50	190

Anchor Level I
Using the anchor inclination, i, of 35° from the horizontal, unfactored and factored horizontal loads for Anchor Level I from Step 1, the total unfactored and factored anchor loads for a spacing, s, of 2.4 meters along the length of the wall between anchors are:

$Q = P_{aI}$ s/(cos i) = (***100.1*** kN/m)(***2.4*** m)/(cos ***35***°) = ***293.3*** kN
γ Q= γ P_I s/(cos i) = (***152.9*** kN/m)(***2.4*** m)/(cos ***35***°) = ***448.0*** kN

From Table 13-4, the resistance factor for anchor pullout in sand based on correlation with Standard Penetration Test is:

ϕ = ***0.65***

For preliminary design, the ultimate anchor capacity, Q_a, for estimation of anchor bond length is:

Q_a = ***145*** kN/m (from Table 13-7 for medium dense sand)

Using Eq. 13-13, the estimated anchor bond length, L, is:

$L \geq \gamma Q/\phi Q_a$ (*Eq. 13-13*)
L = ***448.0*** kN/(***0.65***)(***145*** kN/m) = ***4.8*** m

Anchor Level II
Using the anchor inclination, i, of 35° from the horizontal, the unfactored and factored horizontal loads for Anchor Level II from Step 1, the total unfactored and factored anchor loads for a spacing, s, of 2.4 meters along the length of the wall between anchors are:

$Q = P_{aII}$ s/(cos i) = (**85.8** kN/m)(**2.4** m)/(cos **35**°) = **251.4** kN
γ Q = γ P_{II} s/(cos i) = (***131.0*** kN/m)(***2.4*** m)/(cos ***35***°) = ***383.8*** kN

Then using Eq. 13-13, the estimated anchor bond length, L, is:

$L \geq \gamma Q/\phi Q_a$ (*Eq. 13-13*)

$L = \underline{\boldsymbol{383.8}}$ kN/($\underline{\boldsymbol{0.65}}$)($\underline{\boldsymbol{145}}$ kN/m) = $\underline{\boldsymbol{4.1}}$ m

The table below summarizes the bond zone lengths for Anchors at Levels I and II.

Anchor Level	LRFD Bond Zone Length (m)
Level I	***4.8***
Level II	***4.1***

Step 5: Soldier Pile Embedment
The passive resistance of the embedded section of soldier pile must resist the lateral earth load at the excavation base. From Step 1, the unfactored and factored horizontal loads at the excavation base are 42.9 kN/m and 65.5 kN/m. For a pile spacing, s, of 2.4 m, the total unfactored and factored loads on each embedded pile section are:

$Q = \quad P_{aIII}\, s = (42.9 \text{ kN/m})(2.4 \text{ m}) = 103.0 \text{ kN}$
$\gamma\, Q = \quad \gamma\, P_{III}\, s = (65.5 \text{ kN/m})(2.4 \text{ m}) = 157.2 \text{ kN}$

As depicted in Figure 13-5, the net passive resistance of each embedded pile is obtained assuming that passive pressure on the front of the pile acts over a width equal to three times the pile flange width and that the active pressure on the back of the pile acts over the actual pile flange width.

From Step 1, the active earth pressure coefficient is:

$$k_a = \tan^2(45^\circ - \phi_f/2) = \tan^2(45^\circ - 36^\circ/2) = 0.26$$

(Note that $k_a = 0.26$ is also obtained using AASHTO Eq. A3.11.5.3-1 and A3.11.5.3-2 for a vertical wall, level backfill and no wall friction.)

The passive earth pressure coefficient, k_p, for a vertical wall, horizontal ground surface and no wall friction ($\delta = 0^\circ$) and $\phi_f = 36^\circ$ is obtained from AASHTO (1997a) Figure A3.11.5.4-1 as:

$$k_p = (0.36)(11.6) = 4.18$$

From Table 13-4, the resistance factor, ϕ, for passive resistance of embedded vertical wall elements in soil is 0.60.

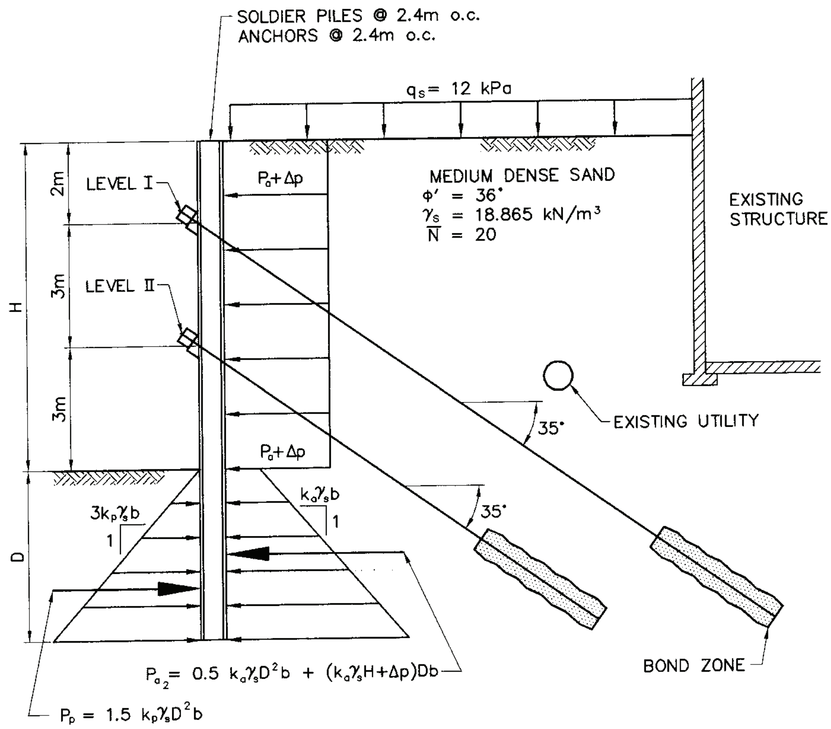


Figure 13-5
Lateral Earth Pressure Diagram
for Soldier Pile Embedment into Cohesionless Soil

From Step 1, the load factor for active earth pressure, γ_{EH}, is 1.50 and for live load surcharge, γ_{LS}, is 1.75. For a 310 mm HP soldier pile (b = 0.31 m), the factored active pressure from Figure 13-5 is:

$$\gamma P_{a2} = \gamma_{EH}\ 0.5\ k_a\ \gamma_s\ D^2\ b + (\gamma_{EH}\ k_a\ \gamma_s\ H + \gamma_{LS}\ \Delta p)\ D\ b$$
$$\gamma P_{a2} = 1.5(0.5)(0.26)(18.865\ \text{kN/m}^3)D^2(0.31\ \text{m})$$
$$+ [1.5(0.26)(18.865\ \text{kN/m}^3)(8\ \text{m}) + 1.75(3.1\ \text{kPa})]D(0.31\ \text{m})$$
$$\gamma P_{a2} = 1.14\ D^2 + 19.9\ D$$

and the factored passive pressure from Figure 13-5 is:

$$\phi\ P_p = \phi\ (1.5\ k_p\ \gamma_s\ D^2\ b)$$
$$\phi\ P_p = 0.60\ (1.5)(4.18)(18.865\ \text{kN/m}^3)(D^2)(0.31\ \text{m})$$
$$\phi\ P_p = 22.0\ D^2$$

Each soldier pile must resist the factored lateral load, γ Q, computed as the unit factored horizontal load at the excavation base from Step 1 times the pile spacing, s, of 2.4 m so that:

$$\gamma\ Q = (65.5\ \text{kN/m})(2.4\ \text{m}) = 157.2\ \text{kN}$$

Equating the net factored passive resistance of the embedded vertical element to the load at the excavation base:

$$\gamma\ Q = \phi\ P_p - \gamma\ P_{a2}$$
$$157.2\ kN = 22.0\ D^2 - 1.14\ D^2 - 19.9\ D$$

or

$$0 = D^2 - 0.95\ D - 7.54$$

from which the required depth of embedment is:

$$D = 3.3\ m$$

Step 6: Vertical Stability of Wall

The soldier piles must support the vertical component of anchor load by side and base resistance of the soldier pile within the depth of embedment.

Tip Resistance
The nominal unit tip resistance, q_p, of a pile in cohesionless soil is:

$$q_p = \frac{38 N_{corr} D_b}{D} < q_\ell \qquad (A10.7.3.4.2a\text{-}1)$$

for which:

$$N_{corr} = 0.77 \log_{10}\left(\frac{1920}{\sigma'_v}\right) N \qquad (A10.7.3.4.2b\text{-}1)$$

and:

N_{corr} = Representative SPT blow count near the pile tip corrected for σ'_v (blows/0.3 m)
N = Measured SPT blow count (blows/0.3 m)
D = Pile width or diameter (m) D_b = Depth of penetration in bearing stratum (m)
q_ℓ = Limiting tip resistance taken as 400 N_{corr} for sands and 300 N_{corr} for silt (kPa)
σ'_v = Effective overburden pressure at pile tip (kPa)

For a pile tip at 3.3 m below the excavation base (or 3.3 m + 8.0 = 11.3 m) below the original ground surface:

$$N_{corr} = 0.77 \log_{10}\left(\frac{1920}{(18.865\ kN/m^2)(11.3\ m)}\right) 20 = 14.7$$

and

$$q_p = \frac{(38)(14.7)(3.3\,\text{m})}{0.31\,\text{m}} = 5946\,\text{kPa} < q_\ell = (400)(14.7) = 5880\,\text{kPa}$$

Use q_p = 5880 kPa

For a typical pile tip plug area of 0.30 m × 0.31 m = 0.093 m^2 for an HP 310 pile section, the nominal bearing resistance for a 3.3 m embedment is:

$Q_p = q_p A_p$ (Eq. 9-19)
$Q_p = (\underline{\boldsymbol{5880}}\ \text{kPa})(\underline{\boldsymbol{0.093}}\ \text{m}^2) = \underline{\boldsymbol{547}}\ \text{kN}$

Side Resistance
The nominal unit side resistance, q_s, of an H-pile in cohesionless soil is:

$$q_s = 0.96\ \overline{N}$$

where:

q_s = Unit skin friction for driven piles (kPa)
$\overline{N}$ = Average (uncorrected) SPT blow count along pile shaft (blows/0.3 m) from which:

q_s = 0.96 (20) = 19.2 kPa

For a typical pile perimeter area of 2 (0.30 m) + 2 (0.31 m) = 1.22 m^2/m for an HP 310 pile section, the nominal side resistance for a 3.3 m embedment is:

$Q_s = q_s A_s$ (Eq. 9-20)
$Q_s = (\underline{\boldsymbol{19.2}}\ \text{kPa})(\underline{\boldsymbol{1.22}}\ \text{m}^2/\text{m})(\underline{\boldsymbol{3.3}}\ \text{m}) = \underline{\boldsymbol{77}}\ \text{kN}$

Factored Bearing Resistance
The total factored bearing resistance of each soldier pile is:

$Q_R = \phi\, Q_n = \phi_{qp}\, Q_p + \phi_{qs}\, Q_s$ (Eq. 9-18)

The resistance factor for a single pile for capacity estimation based directly on SPT results is obtained from Table 9-5 in Chapter 9 as:

$\phi_{qp} = \phi_{qs} = \phi = \underline{\boldsymbol{0.45}}$

from which:

$Q_R = \underline{\boldsymbol{0.45}}\ (\underline{\boldsymbol{547}}\ \text{kN}) + \underline{\boldsymbol{0.45}}\ (\underline{\boldsymbol{77}}\ \text{kN})$
$Q_R = \underline{\boldsymbol{281}}\ \text{kN}$

Factored Load
The total factored vertical load on an individual soldier pile is calculated as follows:

Factored vertical load:

Level I factored vertical load, $\gamma\ V_I$ = (***152.9*** kN/m) (tan ***35***°) (***2.4*** m) = ***256.9*** kN
Level II factored vertical load, $\gamma\ V_{II}$ = (***131.0*** kN/m) (tan ***35***°) (***2.4*** m) = ***220.1*** kN
Total factored vertical load, $\gamma\ V = \gamma\ V_I + \gamma\ V_{II}$= ***256.9*** kN + ***220.1*** kN = ***477.0*** kN

For the final LRFD check, the factored resistance effect, Q_R, must be greater than or equal to the factored load effect, γ Q using Eq. 13-7, as shown below.

$\gamma\ Q \leq Q_R$
477.0 kN > ***281*** kN NG

Therefore, a soldier pile embedment of 3.3 m is insufficient to adequately resist the vertical components of the anchor forces. By inspection, the additional skin friction available through the remaining 1.7 m depth of soil would not increase the resistance sufficiently to resist the anchor forces. Therefore, drive soldier piles to the top of rock at a depth of 5 m below the excavation base. For the purpose of this example, it is presumed that the pile bearing resistance on the sandstone bedrock is adequate.

Step 7: Soldier Pile Structural Design
The soldier piles considered in this problem are subjected to flexure due to the imposed earth pressures and to axial load from the two levels of anchors. Therefore, the structural analysis of these soldier piles must consider the interaction effects of combined axial load and flexure. As specified in the LRFD Specification (AASHTO, 1997a) Article A6.9.2.2, Eq. A6.9.2.2-2 applies for $P_u/P_r \geq 0.2$:

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1 \qquad (A6.9.2.2\text{-}2)$$

where:

P_u = Factored axial load (kN)
P_r = Factored compressive resistance (kN)
M_{ux} = Factored flexural moment about the x axis (kN-m)
M_{rx} = Factored flexural resistance about the x axis (kN-m)
M_{uy} = Factored flexural moment about the y axis (kN-m)
M_{ry} = Factored flexural resistance about the y axis (kN-m)

The AASHTO LRFD Specification (1997a) provides simplified design equations for flexural analysis of the vertical supporting elements dependent upon the average factored lateral pressure, distance between vertical elements and height of the section of vertical element being considered. Because the load and resistance factors in LRFD are calibrated for this approach, this example problem uses these simplified design equations. Using these simplified equations, the maximum moment, M_{max}, will occur below the bottommost row of anchors as:

$$M_{max} = 0.333\ p\ L\ x^2 \qquad (\textit{AC11.8.5.2-3})$$

where:

$p =$ Average lateral pressure (kPa)
$L =$ Spacing between vertical elements (m)
$x =$ Height of the section for the vertical element being considered (m)

In the AASHTO LRFD Specification (1997a), the average lateral pressure is taken as the average factored lateral pressure and the maximum flexural moment is the maximum factored flexural moment. The factored maximum moment below the Level II anchors is:

$$M_{max} = 0.333\ p\ L\ x^2 \qquad (\textit{AC11.8.5.2-3})$$

where:

$p = (1.5)(25.5\ \text{kPa}) + (1.75)(3.1\ \text{kPa}) = 43.7\ \text{kPa}$
$M_{max} = 0.333\ (43.7\ \text{kPa})(2.4\ \text{m})(3\ \text{m})^2 = 314.3\ \text{kN-m}$

From Section 6 of the AASHTO LRFD Specification (Steel Structures), structural design of the soldier piles must also include consideration of the interaction effects from combined axial load and flexure. From the calculations in Step 6, the factored axial load is 477.0 kN.

From the AASHTO LRFD Specification (1997a), Article 6.5.4.2, the resistance factors for flexure and compression in LRFD are:

$\phi_f = 1.00$ (flexure)
$\phi_c = 0.90$ (compression)

First trying an HP 310 × 94 pile, the nominal moment capacity, M_n, of the HP soldier piles is determined from the AASHTO LRFD Specification (1997a) Articles 6.12.2.2.1, 6.10.6.2 and 6.10.5.2.1-1 and Metric Properties of Structural Shapes (AISC, 1992) as:

$M_n = M_p$
$M_n = Z_x\ F_y$
$M_n = (14.5 \times 10^{-4}\ \text{m}^3)(250\ 000\ \text{kPa})$
$M_n = 362.5\ \text{kN-m}$

where:

$M_p =$ Plastic Moment resistance kN-m

$Z_x =$ Plastic section modulus = 14.5×10^{-4} m^3
$F_y =$ Steel yield stress = 250 000 kPa

Therefore:

$$M_r = \phi_f M_n = 1.00 \ (362.5 \text{ kN - m}) = 362.5 \text{ kN - m}$$

From AASHTO LRFD (1997a) Article 6.9.4.1, the nominal axial capacity, P_n, of the soldier pile is:

$$P_n = 0.66^{\lambda} F_y A_s$$

which is applicable for $\lambda \leq 2.25$, where λ is the normalized column slenderness factor defined as:

$$\lambda = \left(\frac{K\,\ell}{r_s\,\pi}\right)^2 \frac{F_y}{E} = 0.045$$

for:

$A_s =$ Gross cross-sectional area = 11.9×10^{-3} m^2
$F_y =$ Yield strength = 250 000 kPa
$E =$ Modulus of elasticity = 200×10^6 kPa
$K =$ Effective length factor = 0.8 (from Table *AC4.6.2.5-1*)
$\ell =$ Unbraced length = 3 m
$r_s =$ Radius of gyration about the plane of buckling = 0.128 m

Through substitution back into the equation for the nominal axial capacity we find that:

$$P_n = 0.66^{0.045} F_y A_s = 0.98 F_y A_s = 0.98 \ (250\,000 \text{ kPa})(11.9 \times 10^{-3} \text{ m}^2) = 2916 \text{ kN}$$

Then the factored axial capacity, P_r is:

$$P_r = \phi_c P_n = 0.90 P_n = 0.90 \ (2916 \text{ kN}) = 2624 \text{ kN}$$

At this point, we can evaluate the axial resistance component of the interaction equation as:

$$\frac{P_u}{P_r} = \frac{477.0 \text{ kN}}{2624 \text{ kN}} = 0.18 < 0.2$$

For this problem, it is reasonable to expect that there will be no significant bending in the plane of the wall and therefore we can assume that M_{uy} is zero. The factored flexural resistance about the x axis is:

$$M_{rx} = \phi M_n = (1.00) \ 362.5 \text{ kN-m} = 362.5 \text{ kN-m}$$

Therefore, the moment resistance component of the interaction equation is:

$$\frac{M_{ux}}{M_{rx}} = \frac{314.3\ \text{kN-m}}{362.5\ \text{kN-m}} = 0.87$$

Substituting into the interaction equation, we find the following:

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = 0.18 + \frac{8}{9}(0.87) = 0.95 \leq 1 \qquad (A6.9.2.2\text{-}2)$$

Therefore, this soldier pile provides a section which is adequate for the design loading conditions for LRFD. In addition to this combined stress check, the slenderness of the soldier beam should be checked to assure that buckling will not control design. For this example, slenderness is not a controlling factor.

Step 8: Timber Lagging Design

As for the structural design of the soldier piles, in this problem you can use the simplified design equations to select a timber lagging section. The maximum factored moment is:

$M_{max} = 0.083\ p\ L^2$ (*AC11.8.5.3-2*)

where:

p = Average factored lateral pressure = (1.50)(25.5 kPa) + (1.75)(3.1 kPa) = 43.7 kPa
L = Lagging span =2.4 m - 0.306 m = 2.094 m (Note: Pile flange width = 306 mm)

Substituting into Eq. AC11.8.5.3-2, the maximum factored moment is:

$M_{max} = 0.083\ (43.7\ \text{kPa})(2.094\ \text{m})^2 = (15.9\ \text{kN-m/m})$

The factored flexural capacity, M_r, of the lagging is determined by multiplying the resistance factor of 0.85 for flexure of wood from Article 8.5.2.2 of the AASHTO LRFD Specification (1997a) by the nominal flexural resistance, M_n, as follows:

$M_r = \phi\ M_n = 0.85\ M_n$ (*A8.6.1-1*)

where the nominal flexural resistance of wood, M_n, from Article 8.6.2 of the AASHTO LRFD Specification (1997a) is:

$M_n = F_b\ S\ C_s$ (*A8.6.2-1*)

and:

F_b = Specified nominal resistance in flexure = 29 000 kPa (*Table AC8.4.1.1.4-1*)
S = Section modulus (m^3)

C_s = Size effect factor (dim)

Evaluation of the size effect factor and the specified resistance in flexure both require calculations which are not presented herein, although the results shown are consistent with the material and geometry specified. For this problem, use Select Structural Southern Pine 102 mm thick and 254 mm wide for the timber lagging. For this material, F_b = 29 000 kPa, S = 4.4 × 10^{-4} m^3, C_s = 1.0 and the nominal flexural resistance of the lagging is:.

$$M_n = (29\ 000\ \text{kPa})(4.4 \times 10^{-4}\ \text{m}^3)(1.0) = 12.8\ \text{kN-m}$$

The factored flexural resistance, M_r, is then:

$$M_r = 0.85\ (12.8\ \text{kN-m}) = 10.8\ \text{kN-m}$$

The maximum factored moment on a 254 mm wide lagging section is:

$$M_{max} = (15.9\ \text{kN-m/m})\ (0.254\ \text{m}) = 4.0\ \text{kN-m}$$

Therefore, the lagging is acceptable.

As indicated in Section 13.3.6.2, the thickness of timber lagging for temporary support is generally selected based on empirical relationships with excavation depth (Goldberg, et al., 1975). For an excavation depth of 8 m, a lagging thickness of 76 mm to 102 mm is prescribed for soldier piles spaced at 2.5 m in medium dense sand. It is apparent from the above calculation that thinner lagging would be acceptable.

Summary

- ***This example problem illustrates the design of an anchored soldier pile and lagging retaining wall by LRFD***
- ***The wall consists of driven steel H-piles supported by two rows of anchors and spanned using timber lagging***
- ***This example is intended to illustrate the LRFD process and procedures and may not represent the conditions the optimal design for the conditions identified (an actual design would be optimized to identify the most economical combination of pile size and spacing; anchor location, spacing and capacity; and lagging thickness)***

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