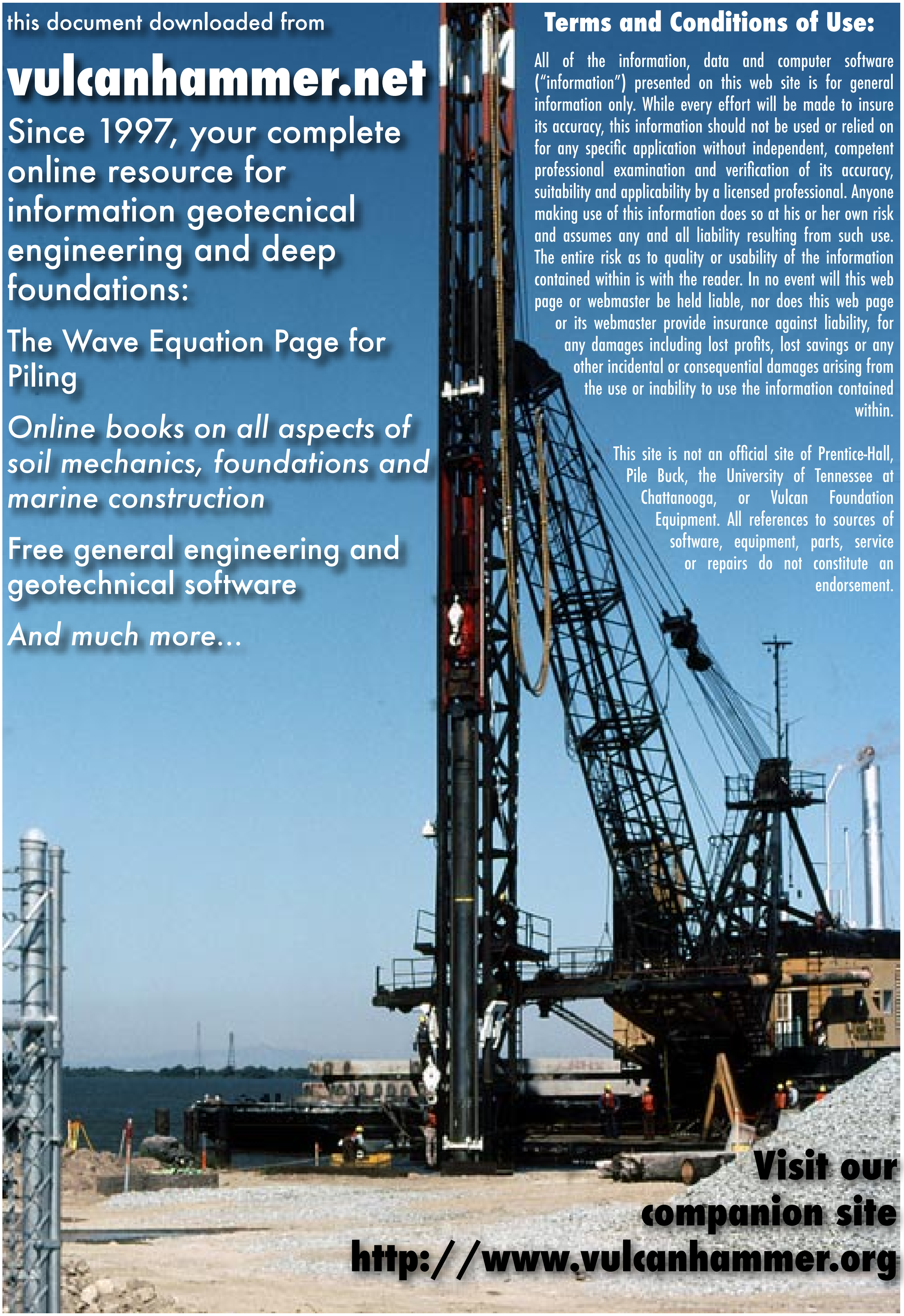

DEPARTMENT OF THE ARMY
U.S. Army Corps of Engineers

CECW-EW Washington, D.C. 20314-1000 ETL 1110-2-561

Engineer Technical Letter
No. 1110-2-561 31 January 2006

Engineering and Design
RELIABILITY ANALYSIS AND RISK ASSESSMENT
FOR SEEPAGE AND SLOPE STABILITY FAILURE MODES
FOR EMBANKMENT DAMS

Distribution Restriction Statement

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RELIABILITY ANALYSIS AND RISK ASSESSMENT FOR SEEPAGE AND SLOPE STABILITY FAILURE MODES FOR EMBANKMENT DAMS

1. Purpose

This document provides guidance for performance of risk assessment analyses of dam safety-related detrimental seepage (internal erosion, piping, under seepage, and heave) and slope stability problems. Detailed descriptions of reliability and risk analysis for seepage and slope stability problems are provided.

2. Applicability

This ELT is applicable to all USACE Commands having Civil Works Responsibilities. It applies to all studies for major rehabilitation projects.

3. References

See Appendix A.

4. Distribution

Approved for public release, distribution is unlimited.

5. Discussion

a. Risk assessment is performed to evaluate various parameters to assist in the decision making process. A risk analysis and assessment provides the total annualized consequences or risk with and without the proposed seepage/stability correction project. By comparing the with and without projects, the risk assessment process is used to guide the selection of the alternative that is most effective in reducing the risk of unsatisfactory performance.

b. Site characteristics and potential modes of failure are identified. An event tree is then used to describe the various modes of unsatisfactory performance, and weighted damages are determined by multiplying the probabilities of occurrence and the costs incurred to give expected risk. Once the risk is determined for the without-project condition, the process is repeated for each with-project alternative. The most feasible alternative can then be selected.

c. The above methodology is used to assess seepage risk analysis and slope stability risk analysis.

d. Practical examples and case histories on the application of reliability analysis and risk assessment for seepage and slope stability failure modes of embankment dams are presented in the appendices to this ETL as follows:

-Appendix A lists the references used in this document.
-Appendix B discusses Poisson distribution.
-Appendix C provides a discussion of the six-sigma rule.
-Appendix D is a step-by-step reliability analysis of a slope stability problem.
-Appendix E is guidance on performing expert elicitation.
-Appendices F and G are case histories on applying risk analysis to projects with seepage problems.
-Appendix H provides information on using a historic data model to analysis piping in an embankment dam.
-Appendix I is a case history of a slope stability reliability analysis.
-Appendix J is a case history of historical frequency of occurrence model for pipes.
-Appendix K discusses Monte Carlo simulation.

FOR THE DIRECTOR OF CIVIL WORKS:

11 Appendices
(See Table of Contents)

DONALD L. BASHAM, PE
Chief, Engineering and Construction
Directorate of Civil Works

RELIABILITY ANALYSIS AND RISK ASSESSMENT FOR SEEPAGE AND SLOPE STABILITY FAILURE MODES FOR EMBANKMENT DAMS

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RELIABILITY ANALYSIS AND RISK ASSESSMENT FOR SEEPAGE AND SLOPE STABILITY FAILURE MODES FOR EMBANKMENT DAMS

1. Introduction to Geotechnical Reliability Procedures

a. Requirement For Risk Assessment.

(1) The Office of the Assistant Secretary of the Army for Civil Works directed that the Dam Safety Seepage and Stability Correction Program comply with policy and criteria of the Civil Works Major Rehabilitation Program. A Major Rehabilitation Evaluation Report is required to obtain "Construction, General" (CG) funding for correction of seepage/stability problems at existing dams. Upon approval of the Evaluation Report, a proposed project can be funded in accordance with EC 11-2-179, Section B-2-4.

(2) A risk assessment, as generalized in Figure 1, is required as part of that decision document. The risk analysis and assessment provides the total annual economic risk (for example, economic and environmental impact and loss of life) with and without the proposed seepage/stability correction project. The “without project” or “baseline” condition should demonstrate that without the proposed correction to the project that the expected probability of unsatisfactory performance is high for all loading conditions or increases over time. The "with project" condition should demonstrate a significant reduction in the conditional probability of unsatisfactory performance (and reduced annual economic risk) to show that the project provides an acceptable level of performance for all loading conditions. The risk assessment process should be used to guide the selection of the alternative that is most effective in reducing the annual economic risk of unsatisfactory performance.

(Annual Loading Probability) X (Probability of Unsatisfactory Performance) X (Conditional Performance Level) X (Consequences) = Annual Risk for Each Load

Figure 1 - Generic Concept to Determine Annual Risk for a Given Load

(3) Risk is defined as the probability of a loss occurring in a given time period (annually), where loss consists of all economic damages (measured in dollars) and the environmental loss of habitat (habitat units loss) as well as the potential for loss of life.

b. Risk Assessment Process. The required risk assessment process involves two steps, a thorough definition of the problem, and then development of an event tree to provide a framework for analyzing the annual economic risk of various alternatives. These two steps are further described below.

(1) Problem Definition. Risk assessment begins with a definition of the problem. The steps in defining the problem include: 1) Site characterization, and 2) Identification of potential modes of failure. (For the purpose of this discussion, the "site" is defined to include both the natural setting where the dam exists and the dam itself.)

(a) Site Characterization

1 Site characterization consists of identification of all of the site-related factors significant in the evaluation of embankment stability and detrimental seepage. Typical factors include characteristics of the constructed project, such as embankment geometry, zoning, materials, construction methods, and seepage cutoff/control features; and characteristics of the setting, such as site geology and stratigraphy, and foundation material characteristics. Additional site information is provided by documented performance history.

2 To the maximum extent practicable, existing data from design and construction records, performance history, and post-construction investigations are used to characterize the site. At preliminary stages of risk assessment, existing data supplemented by engineering judgment provide a sufficient basis for evaluation. If significant risk for the project is indicated, field investigations and additional analyses are warranted.

(b) Identification of the Potential Modes of Failure. The second step in defining the problem consists of identifying potential modes of failure. For example, with respect to piping, uncontrolled seepage through the embankment, through the foundation, or through the embankment into the foundation can all cause failure of a dam. With respect to embankment stability, sliding or deformation may be the result of insufficient strength of materials, may be induced by excessive pore pressures, or could be induced by an external loading. All potential modes of failure should be identified. Analysis and remediation of seismic or hydrologic deficiencies are covered by ER 1110-2-1155, Dam Safety Assurance Program and are not covered under the Major Rehabilitation program.

(2) Event Trees.

(a) After defining the problem through site study and determination of the modes of failure, risk analysis requires development of an event tree. An event tree is a graphical representation of the various events that could occur (each with estimated probabilities), and in turn, the various events that could follow from each preceding event. Construction of an event tree allows those performing the risk analysis to think through the various sequences of events that can cause unsatisfactory performance of the dam and the consequences resulting from the unsatisfactory performance. A simplified event tree is shown in Figure 2, which depicts four analytical components in risk assessment of a dam. The four components are annual loading frequency, probability of unsatisfactory performance, performance level, and consequences. Risk (in the last column) is the product of the four components.

(b) Traditional design in geotechnical engineering is based on allowable factors of safety developed by the profession as a whole from years of experience. Reliability analysis is based on the capacity-demand model, where the probability of unsatisfactory performance is defined as the probability that the demand on the system or component exceeds the capacity of the system or component. The capacity and demand can be combined into a single function and the event that the capacity equals the demand taken as the limit state. The probability of unsatisfactory performance is the probability that the limit state will be exceeded. The term “probability of unsatisfactory performance” is used instead of “failure” because failure is often thought of as the complete catastrophic failure of the dam with a complete release of pool. This is not necessarily the case. Failure of a dam slope as defined by a limit state analysis resulting in a factor of safety of one could be a massive slope failure causing the complete release of pool. But it could be a lesser slide not causing any pool release but which creates an unreliable situation; it could be a surface slump on the dam, or it could be transverse crack on the crown of the dam. Therefore, the term unsatisfactory performance is used instead of the term failure. The capacity-demand model is represented by Equation 1.

$$Factor\ of\ Safety = \frac{Capacity}{Demand} \tag{1}$$

In the capacity-demand model, unsatisfactory performance is based on the probability that the limit state is exceeded meaning the probability that the factor of safety is less than one. Then the different possible unsatisfactory performance outcomes (termed performance levels) are determined along with their probability of occurrence.

(c) The term “performance level” is used to indicate how the structure will physically perform due to an unsatisfactory performance event .

(d) The event tree facilitates the systematic calculation of conditional probabilities of various outcomes and provides the basis for calculating consequences and annual economic risk of proposed alternative actions. Based on this analysis, proposed alternative actions can be rationally compared. Detailed discussion on the terms, development and use of event trees is given in Paragraph 2.

c. Conventional Analysis. Reliability analysis is to be used for risk assessment of an embankment dam. Reliability analysis is not meant to be used in place of conventional analysis for design purposes. The basic premise of reliability models is that they are an investment tool for decision-making. They allow decision makers the ability to analyze the risk and associated consequences of multiple future scenarios by accounting for the uncertainties in the analysis and possible outcomes. Nowhere in geotechnical practice has probabilistic methods supplanted the use of conventional methods of design, such as the factor of safety approach. For routine design, deterministic methods have been proven to provide safe, adequate, and economical designs. Probabilistic design would require an additional work effort that normally would not be warranted. The use of probabilistic design would require a huge calibration effort, which has not been accomplished to date, to make sure that safe economical designs would be produced. This calibration effort would involve performing comparative deterministic and probabilistic analyses for a wide range of geotechnical problems.

2. Event Trees

a. Introduction. An initial step in performing a risk analysis of an embankment dam is to construct an event tree that includes all relevant failure modes. An event tree allows engineers to diagram the steps and conditions that will determine the risk associated with an embankment dam. It also allows them to think through the process of the events that can cause failure of the dam and the consequences resulting from the failure. Once the initial event tree is constructed, a review of the event tree is made to make sure it accurately portrays the events that can cause dam failure and to make sure that no failure modes or events are left out. A simplified example of an event tree is shown in Figure 2 that depicts the steps in the risk analysis of an embankment dam. This event tree is a good starting point in the development of the actual event tree for the dam being analyzed. The four components of the event tree are annual loading probability, probability of unsatisfactory performance, performance level, and consequences. Consequences may include economic and environmental impacts and loss of life. For this example, only economic impacts are considered. Risk is the combination of the probabilities of occurrence of the events and the adverse consequences of the events. Examining the event tree in Figure 2, the risk for each branch of the event tree is the product of the four components making up that branch (annual loading probability, probability of unsatisfactory performance, performance level, and consequences). The annual economic risk for the event tree is the sum of the risks for each branch. The annual economic risk calculated using the event tree is then used in the economical analysis for a Major Rehabilitation Report to determine the economical feasibility of the recommended rehabilitation measures.

b. Independent Failure Mode Versus Sequential Failure Mode Event Trees. A separate event tree should be created for each independent failure mode. This means a separate event tree is required for each failure mode that is not conditionally dependent on another failure mode. For example, where a slope stability failure mode is independent of a detrimental seepage failure mode, two separate event trees would be developed. However, if a slope stability failure caused detrimental seepage to develop, then only one event tree is used showing the sequential relationship of the failure modes. Figure I-4 in Appendix I shows an event tree using sequential failure modes. The sequential events are defined in terms of conditional probabilities.

c. Conditional Probabilities. Moving across the event tree shown in Figure 2, the components of the event tree represent the probability of a pool elevation occurring (P(Pool Elev)), the probability of unsatisfactory performance given that this pool elevation has occurred (P(u|Pool Elev)), the probability of a performance level occurring given that there has been unsatisfactory performance (P(Perf Level|u)), and the cost incurred if that performance level has occurred ($|P(Perf Level)).The last column in the event tree is the risk associated with each branch of the event tree. These risks are summed to get the annual economic risk for the event tree. All the probabilities under the annual loading probability category of the event tree must sum to one. The events in the second and third components of the event tree seen in Figure 2 represent conditional probabilities. Conditional probability is the probability of an event occurring given that another event has occurred. For example, say that there is a 10 percent probability of a slide given that a certain water level has occurred. Moving across the components of the event tree, values for probabilities and costs are multiplied together giving expected risk as the result. The risks in the final column of the event tree are summed to give the annual economic risk of unsatisfactory performance of the dam. Equation 2 represents the calculation of the annual economic risk.

$$\text{Annual Economic Risk} = \sum P(\text{Pool Elev}) \times P(u|\text{Pool Elev}) \times P(\text{Perf Level}|u) \times \$|P(\text{Perf Level}) \quad (2)$$

d. Annual Loading Probability.

(1) The main loading condition that a dam experiences during its life is caused by the water that is impounded by the dam. The water level acting on the dam varies daily. The probability of different water levels acting on the dam can be obtained from a pool elevation frequency curve. The pool elevation frequency curve represents the annual probability of a certain water level acting on the dam being equaled or exceeded. The pool elevation frequency curve that will be used for illustration purposes is shown in Figure 3. The water level on the dam is plotted on the y-axis. The annual probability that the water level equals or exceeds that elevation is plotted on the x-axis. Therefore, probabilities of unsatisfactory performance calculated from the pool elevation frequency curves are annual probabilities of unsatisfactory performance.

(2) The annual loading probability on the dam is obtained from the pool elevation frequency curve. This annual loading probability is represented in the event tree by the probability that the pool will be at various elevation ranges P(Pool Elev). The pool elevation frequency curve gives the annual probability that the water level will equal or exceed an elevation. But for the reliability analysis, the probability that the water level will be at a certain elevation is needed. The simplest method to obtain the probabilities that the water levels are at certain elevations is to represent each elevation as a range. For the example shown in Table 1, pool elevations 440.2, 435.2, 432.5, 429.4, 424.5, 420.7, and 416 feet are selected because they correspond to the return period of a 500, 100, 50, 25, 10, 5, and 2-year event, respectively. Elevations approximately midway on either side of each pool elevation are entered into the table except for the highest increment and lowest increment. Elevation 442.5 was chosen for the highest increment, as that is the top of the dam. Elevation 400 was chosen for the lowest increment, as that was the lowest point on the pool elevation frequency curve. For each elevation increment, the probability that the pool elevation equals or exceeds that elevation (P(exceedence)) is selected from the pool elevation frequency curve, converted from a percentage to a decimal, and entered into the table. The annual probability that the pool is at a certain elevation (P(Pool Elev)) represented by a range is the difference between the P(exceedence) at the top and bottom of the increment for that range.

TABLE 1
PROBABILITY OF POOL LEVELS OCCURRING

Pool Elev	Elevation Increment	P(exceedence)	P(Pool Elev)
	442.5	0	
440.2			0.005
	437.5	0.005	
435.2			0.011
	433.5	0.016	
432.5			0.012
	431.0	0.028	
429.4			0.032
	427.0	0.060	
424.5			0.08
	422.5	0.14	
420.7			0.2
	418.0	0.34	
416.0			0.66
	400.0	1.00	
		Σ	1.000

The probability that the pool is at Elevation 429.4 is determined using Equation 3.

$$P(\text{Pool Elev})_{429.4} = P(\text{equals or exceeds})_{427} - P(\text{equals or exceeds})_{431} \tag{3}$$

Applying Equation 3, the probability that the pool is at elevation 429.4 is determined to be 0.032:

$$P(\text{Pool Elev})_{429.4} = 0.06 - 0.028 = 0.032$$

The probability that the pool is at the other elevations in the table is determined in a similar manner.

The probability that the pool is at Elevation 429.4 is really the probability that the pool is in a range between elevations 431 and 427. If using this size range is not accurate enough, then more pool elevations should be analyzed and the range size reduced.

e. Probability of Unsatisfactory Performance. The next category of the event tree is the probability of unsatisfactory performance. Four methods are available to obtain the probability of unsatisfactory performance: hazard function, reliability index, expert elicitation, and historical frequency of occurrence. The probability of unsatisfactory performance must be determined for each pool level in Table 1 using the most appropriate of the above four methods. The probability of unsatisfactory performance can be determined using the hazard function or the reliability index method for failure modes for which there is an analytical method of analysis. For failure modes for which there is no analytical method of analysis, the probability of unsatisfactory performance can be obtained by either expert elicitation or historical frequency of occurrence for each pool elevation.

(1) Hazard Function. The hazard function h(t) is defined as the conditional probability of an event occurrence in the time interval $t+\Delta t$, given that no event has occurred prior to time t. To determine a hazard function using analytical methods, parameters must exist that vary with time in a known manner. Examples of parameters that vary with time are corrosion of a steel member, fatigue of a steel member due to cyclic loading, and scour around a foundation member. A hazard function can be calculated using a Monte Carlo analysis. This is most easily done for analyses that can be accomplished using a spreadsheet analysis in conjunction with Monte Carlo simulation software @RISK. A Monte Carlo simulation requires the analysis to be run thousands of times varying the random variables. The number of unsatisfactory performance events is counted in each year time increment allowing the hazard function to be calculated. , While some geotechnical parameters may be time dependent, only limited information exists on the mathematical functions defining how they vary with time. Thus, it is difficult to calculate a hazard function for use in Monte Carlo simulation. Therefore, a hazard function analysis is not typically done for seepage and slope stability problems using Monte Carlo simulation. Table 2 gives a discussion of various geotechnical parameters and information on how they vary with time. For most geotechnical parameters that vary with time, historic data is needed to define the time function. For this reason a historic frequency of occurrence model may be a better choice for the reliability analysis if there is historic data defining unsatisfactory performance.

TABLE 2
VARIATION OF GEOTECHNICAL PARAMETERS WITH TIME

GEOTECHNICAL PARAMETER	DISCUSSION
Shear Strength	The shear strength of sand is fairly constant with time. The shear strength of clay increases with time in a predictable fashion but that would give an increasing hazard function and would be of little use in justifying major rehabilitation of projects.
Settlement	The major amount of settlement of sands occurs fairly rapidly; however, we do know that additional settlement occurs with time that is somewhat predictable. Clay settles by a predictable rate over time; therefore, settlement of a structure on clay would be a perfect model for a hazard function analysis.
Scour/Erosion	Since some scour events are time dependent, a scour model would work well with a hazard function analysis. The only problem is that data is needed to predict how the scour will occur. Therefore, the scour event is predicted by historic data and the historic data could be used in a Monte Carlo simulation to predict the hazard function for a sliding model. Scour is predictable if the material being scoured is uniform throughout. But if there is layered material or discontinuous zones, it would be very difficult to predict

Scour/Erosion (continued)	the rate of scour with time. This method would work with erodable rock versus a known scouring agent, but not scour of a sand foundation at a bridge pier.
Permeability/Piping/and Underseepage	Permeability can change with time especially in the area of a known underseepage problem when foundation material is being removed with time. The problem with using a Monte Carlo simulation to analyze the underseepage problem is that no one knows how the permeability changes with time.
Relief Well Flow	Relief well flow is definitely time dependent as the screen of the well and the filter become plugged over time due to precipitation of chemical compounds or biological fouling. However the rate of reduced flow changes drastically depend on the location of the relief well. There is no mathematic model available to predict the change of relief well flow with time to use in a Monte Carlo simulation to calculate a hazard function.
Deformation of Pile Foundation	Lateral deformation of a pile foundation increases with the number of cycles of loading applied, which can be related to time. Data exists on the additional lateral deformation of a pile foundation that occurs with time due to cyclic loading and a time model can be developed.
Loading	Loading certainly can vary with time and for certain types of loading a time function can be developed. This would be very problem specific and would not apply to a large group of slope stability or seepage problems which is the topic of this ETL

(2) Reliability Index. The reliability index method is a practical and appropriate method for reliability analysis of geotechnical problems. Guidance for geotechnical reliability analysis is given in ETL 1110-2-547. When performing reliability analyses, the same analytical methods are used that are used in traditional geotechnical engineering. The only difference is in how the variables in the problem are represented. In reliability analysis, there are two types of variables: deterministic and random. In traditional geotechnical engineering, all variables are treated as deterministic. Deterministic variables are represented by a single number, implying the value of that variable is known exactly. A deterministic variable could be the unit weight of water or measured dimensions. Random variables are represented by a probability density function because their exact value is not known. Random variables could be shear strength, permeability, and earth pressure. The Corps of Engineers uses what is called a first order second moment method of analysis for the reliability index method. This means that only the first two moments (mean and variance) are used to represent the probability density function in the reliability analysis and all higher order terms are neglected in the Taylor Series expansion used to estimate the mean and variance of the performance function (natural log of the factor of safety). This first order second moment method of analysis greatly simplifies the reliability analysis procedure. This simplified reliability analysis is used to determine the reliability index beta (β). β is the number of standard deviations by which the expected value of the factor of safety is away from

the unsatisfactory performance condition, or the factor of safety equaling one. The larger the value of β, the safer the embankment dam. The smaller the value of β, the closer the embankment dam is to unsatisfactory performance. The value of beta can be used to calculate the probability of unsatisfactory performance. The reliability index provides a measure of the likelihood of unsatisfactory performance for a given performance mode and loading condition. If a distribution on P(u) is assumed, it can be used to calculate the probability of unsatisfactory performance for the considered loading condition. It does not, however, have a time basis, or directly provide the annual probability of failure for the considered mode.

Life-cycle economic simulations performed by the Corps of Engineers require annual probabilities of various unsatisfactory performance events. These probabilities are defined by hazard functions, which may be constant or vary with time. The hazard function or hazard rate h(t) is the conditional probability of failure in the current year, given that there was no failure in the previous years. It can be expressed as the ratio of the probability of failure distribution function at that time to one minus the cumulative failure probability at that time. A general discussion of the application of the hazard function in geotechnical engineering is presented in paragraph 2.e.(1).

(3) Expert Elicitation. Expert elicitation is a systematic process of posing questions to a panel of experts to get their knowledgeable opinion on the condition of the embankment dam so that the probability of unsatisfactory performance can be assessed. After the initial round of questions has been completed, the expert opinions are documented. The experts then explain their the initial documented opinions. Some experts may want to adjust their initial opinions based on this discussion. The probabilities are based on the panel's revised opinions. The expert elicitation process must be thoroughly documented. Further discussion is presented in Appendix E.

(4) Historical Frequency of Occurrence. Historical frequency of occurrence is the collection of data at the dam being investigated or at similar dams from which the probability of unsatisfactory performance is determined. The one disadvantage of the method is that it is usually very difficult to obtain this data. Sometimes the historical data has been collected and reduced by others, such as in the University of New South Wales procedure (Foster, 1998), which is used for the analysis of piping through embankment dams. A site-specific analysis can be performed using frequency-based methods fit to historical events. Ways to reduce the historical events into a usable form is through the use of common data distribution methods, such as, the Weibull, Exponential, or Poisson data distribution methods. Note, using Weibull, Exponential, or Poisson distribution methods to reduce historical data gives a hazard function. Appendix J gives an example for developing a historic data model using the Weibull and the Exponential distribution. An example using the University of New South Wales procedure is given in Appendix H.

f. Performance Level.

(1) After the probability of unsatisfactory performance is calculated, the engineer must assess what effect this would have on the embankment dam. Remember that what has been calculated is the probability of unsatisfactory performance not the probability of failure. So the

engineer must determine exactly what unsatisfactory performance means. Typically the degree of unsatisfactory performance can be represented by three performance levels; however, as many performance levels as needed to describe the problem being analyzed can be used. When using three performance levels the following can be used:

(a) Low Impact. The low impact performance level requires some increased maintenance effort on the embankment dam.

(b) Extreme Measures. The extreme measures performance level requires a major effort and/or expenditures of funds to prevent uncontrolled release of the pool, such as, pool restriction and extensive emergency construction measures to prevent failure of the dam.

(c) Catastrophic Failure. The catastrophic failure performance level is breaching of the dam and release of pool with or without warning with subsequent economic and environmental impact and loss of life.

(2) These are just three examples of what unsatisfactory performance can mean. There can be more than or less than these three outcomes of the unsatisfactory performance. These outcomes are referred to as the performance levels of the embankment dam assuming that the dam performs unsatisfactorily. The probability of each of these performance levels occurring must be determined by the engineer. The sum of these performance level probabilities must add to one. The engineer should determine performance level probabilities based on experience, historical data, or other expert opinion.

g. Consequences.

(1) The consequences are determined using a multi-disciplinary team of engineers, planners, economists, environmental resource specialists, and operating project managers for each performance level. The consequences (economic and environmental impacts both upstream and downstream and loss of life) must be determined for each performance level for each range of pool elevations where unsatisfactory performance will occur. This does not necessarily mean uncontrolled loss of the pool has to occur before unacceptable consequences would be incurred. Extreme consequences (economic and environmental impact) could occur without catastrophic loss of the pool just by restricting the pool elevation to prevent embankment dam failure. Impacts could be realized due to reduction in downstream flood protection (not being able to store inflow and causing flooding downstream), loss of hydropower benefits, damage to environmental habitat units, and impacts on operations of upstream projects. There would be system impacts if the project is part of a coordinated flood control, navigation, and hydropower system.

(2) Consequences for catastrophic failure should be determined based on the increase in damages over the base discharge flow at the time of dam breaching. The consequences should be determined for each representative pool elevation used in the event tree. The consequences should include loss of life; the cost of repair of the dam; damage to upstream and downstream property; environmental damage and repair; loss of benefits (hydropower, flood control,

navigation, recreation, water supply); cost of a monitoring program; and delays to transportation systems.

h. Annual Economic Risk.

(1) Once the impacts are determined, the risk portion of the event tree can be calculated. This is accomplished using Equation 2 and the event tree. The probabilities and costs are multiplied for the annual loading frequency, the probability of unsatisfactory performance, the performance level, and the consequences for each branch of the event tree. These values, which are in terms of dollars, are the risks. All of the individual branch risks are summed to give the annual economic risk for the without–project condition. This means the project as it is now without any repairs being made to the dam.

(2) This whole process must be repeated to get the annual economic risk for the with-project condition. This means the project after the repairs have been made to the dam. The main changes in the event tree between the without-project condition and the with-project condition will occur in the probability of unsatisfactory performance category of the event tree. The probabilities of unsatisfactory performance will be much lower after the embankment dam has been repaired. A simplifying assumption that is used is that when rehabilitations are completed (With Project), they are made to bring the structure up to current criteria. Therefore, there should be an insignificant chance of having a reliability problem in the future. In addition, if there is a problem it would most likely be towards the end of the study period where the present worth aspects of the economic analysis would make the effect insignificant to the overall analysis.

(3) The annual benefits portion of the economic analysis is then calculated using Equation 4.

Benefits = Annual Economic Risk(Without Project) – Annual Economic Risk(With Project) (4)

These benefits are used by economists to obtain the benefits-cost ratio.

i. Alternate Method of Calculating Benefits. The event tree would not change since the same pool elevations would be used, but instead of using the probability that the water level is at an elevation, the pool elevation frequency curve probability of exceedence would be used. This method of calculating the benefits is illustrated in Figure 4. Each pool elevation of interest is represented by a diamond on the curve. The exceedence probabilities are plotted on the x-axis. The weighted damages, which are calculated using Equation 5 for each pool elevation of interest, are plotted on the y-axis.

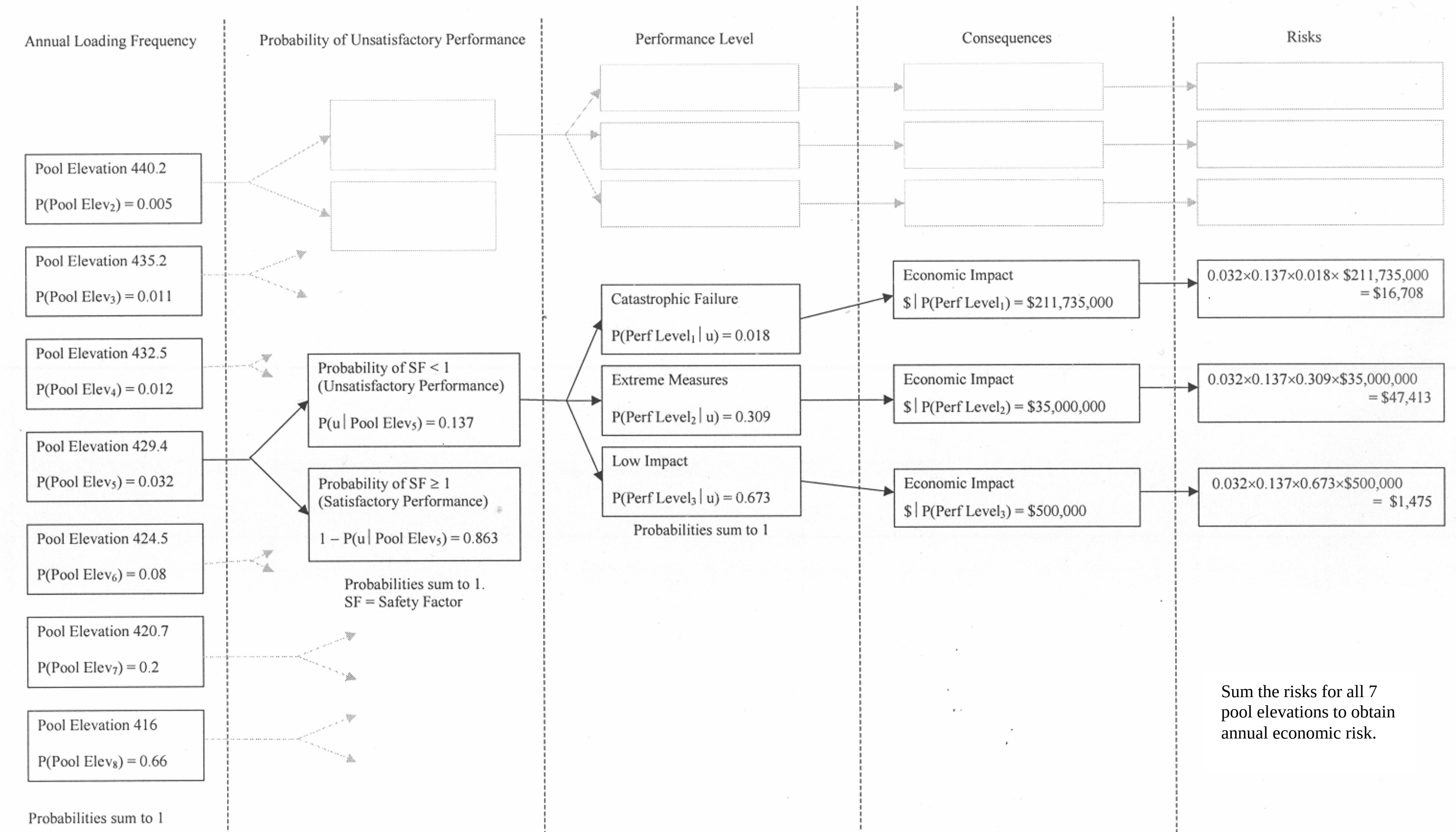
$$\text{Weighted Damages} = \sum P(u|\text{Pool Elev})\times P(\text{Perf Level}|u)\times \$|P(\text{Perf Level}) \qquad (5)$$

Note that Equation 5 does not contain the probability of a pool elevation occurring, as does Equation 2. The probability of a pool elevation occurring is taken into account in the area under the curve calculation, i.e., the weighted damages are multiplied by the exceedence probability of the pool.

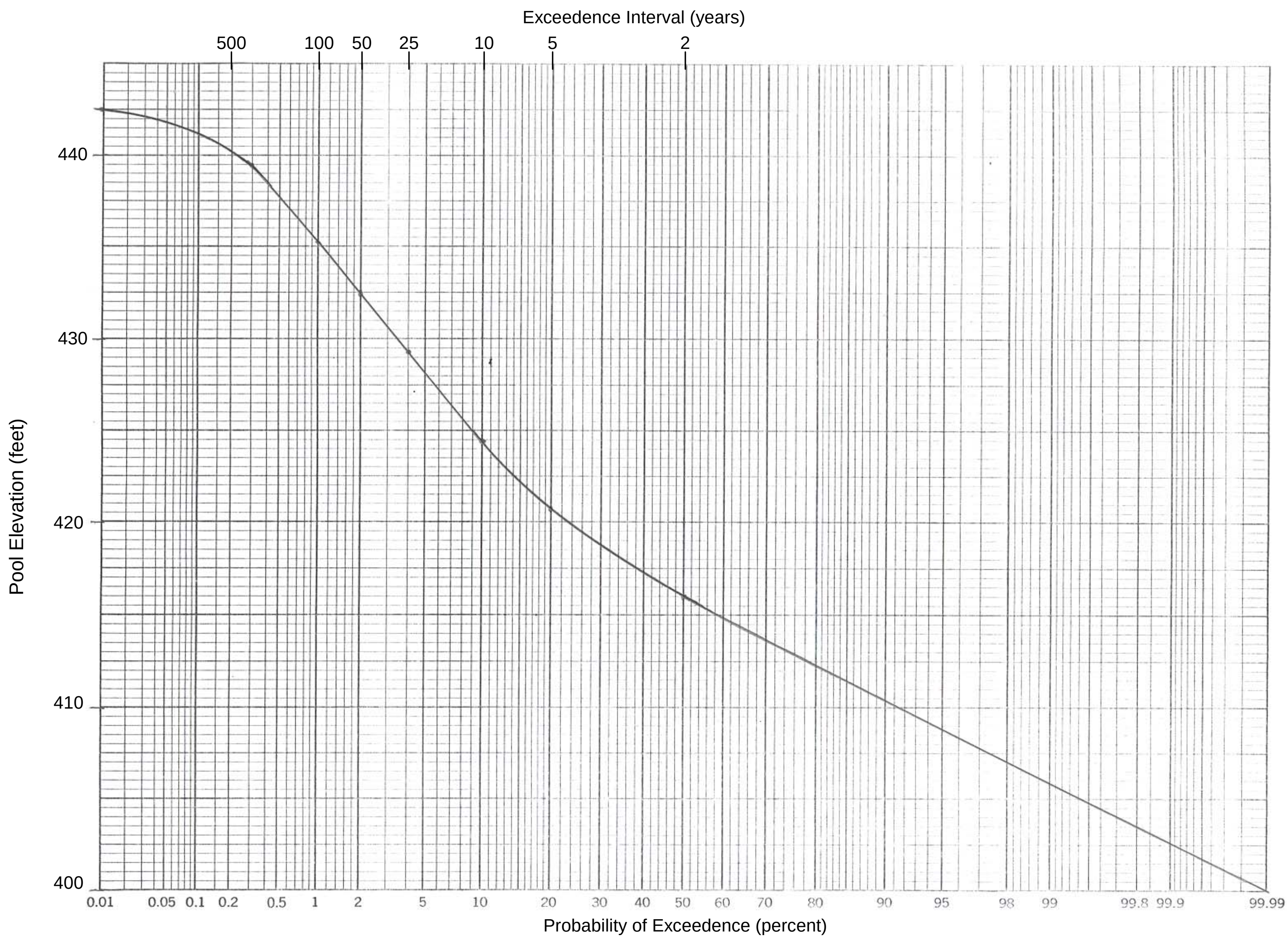
Using Equation 5, the weighted damages for pool Elevation 429.4 are calculated as follows using the data in the event tree, Figure 2:

Catastrophic Failure	$0.137 \times 0.018 \times \$211,735,000$	=	\$522,139
Extreme Measures	$0.137 \times 0.309 \times \$35,000,000$	=	1,481,655
Low Impact	$0.137 \times 0.673 \times \$500,000$	=	46,101
Weighted damages at Elevation 429.4		=	\$2,049,894

From Figure 3, Elevation 429.4 corresponds to a probability of exceedence of 0.04. The weighted damages of \$2,049,894 corresponding to a probability of exceedence of 0.04 or a 25-year return period are shown in Figure 4. This calculation gives one data point on the curve. Other pertinent pool elevations would be used to compute the rest of the data points on the curve in Figure 4. The area under the curve in Figure 4 is the annual economic risk. A curve similar to Figure 4 would be developed for both the without and with project conditions. The difference between the areas under the two curves gives the annual benefits.



Event Tree Example
Figure 2



Pool Elevation Frequency Curve

Figure 3

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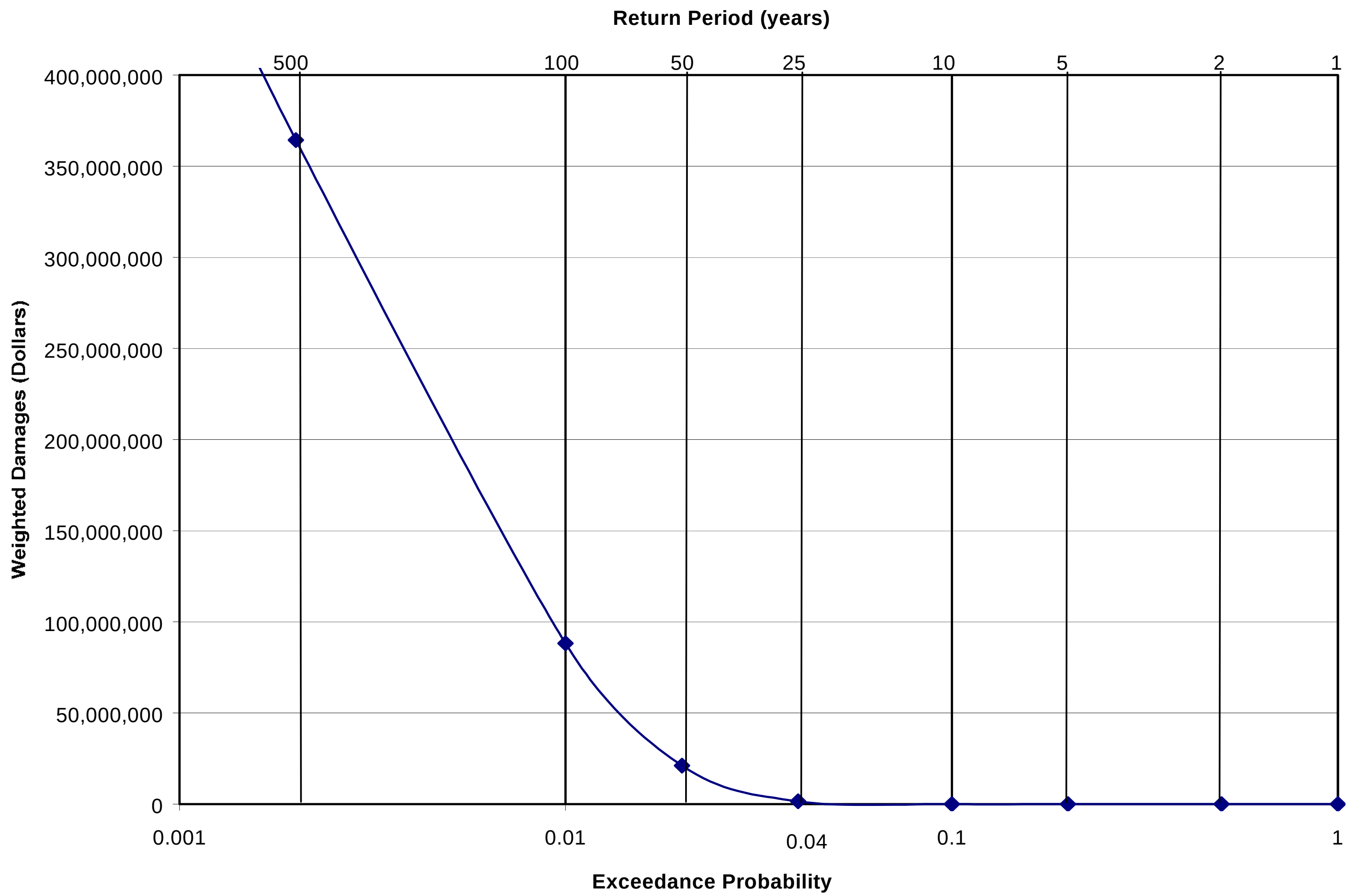

AVERAGE ANNUAL NED COST

Figure 4

3. Seepage Risk Analysis

a. Problem Definition.

(1) Seepage, through an embankment and foundation of a dam, can, if not properly controlled, cause failure of the dam. Direct observation of unsatisfactory performance such as excessive piezometric levels, sand boils, turbidity in seepage discharge, increases in seepage discharge over time, or changes in piezometric head for similar past pool elevations are all indicators of potentially serious seepage conditions. These conditions may be indicative of detrimental effects of seepage leading to piping and eventual breaching of an embankment dam or the foundation of a dam.

(2) Currently there are identified two different piping conditions requiring different methods of analysis: 1) piping directly related to pool elevation which can be analyzed using piezometric or seepage flow data, and 2) piping related to a complex seepage condition within the dam or its foundation that cannot be evaluated solely by analytical methods. Analysis of the first condition usually entails extrapolating observed piezometric data at various lower pool levels to predict if failure or unsatisfactory performance, defined as excessive seepage exit gradients, will exist at higher pool levels. This relationship is presented in Figure 5, Estimated Uplift vs. Pool Elevation - Performance Parameter Plot at Toe of a Dam. The second condition relies on the analysis of historical frequency of occurrence data and/or expert elicitation to determine the probability of failure by seepage and piping using either frequency-based methods to fit historical events and/or subjectively determine probability values based on expert elicitation. These generalized approaches are summarized in Figure 6.

b. Project Site Characterization. Site characterization consists of identifying those factors that are significant in the evaluation of through seepage, under seepage, and piping potential. Typical factors would include embankment zoning and material classifications, compliance with current filter and drain criteria, foundation materials and stratigraphy, seepage cutoff and control features, and performance history. The University of New South Wales (UNSW) method for conducting preliminary assessment of the probability of failure by piping provides a good example of factors considered in the evaluation process.

c. Define Mode(s) of Failure for the Site. As stated before, uncontrolled seepage through the embankment and foundation system of a dam can cause failure of a dam. Uncontrolled seepage through the embankment can lead to piping of the embankment with subsequent erosion of the embankment through the “pipe” with eventual breaching by the pipe at the upstream face allowing breaching of the embankment. Seepage through the foundation can cause excess uplift at the downstream toe of dam and “heave” the foundation material at the toe, which can lead to piping of the granular foundation material. Uncontrolled seepage through the embankment into the foundation can lead to movement of the embankment material into the foundation material creating a pipe or cavity in the embankment which will eventually collapse and cause loss of free board or a breach through the embankment. Internal erosion or movement of fine-grained material through the embankment can cause failure of the embankment (Talbot, 1985).

d. Methods to Determine Conditional Probability of Unsatisfactory Performance.

(1) Reliability Index Method. Where piping/seepage performance is correlated with pool level/elevation the First Order Second Moment Reliability Index method, presented in ETL 1110-2-547, can be used to determine the conditional probability of unsatisfactory performance. See Appendix G of this ETL for an example on how this method was used to determine the conditional probability of unsatisfactory performance.

(2) Historical Frequency of Occurrence and/or Expert Elicitation Methods. Factors leading to piping often may not be addressed by traditional seepage analysis. This is particularly true when dealing with the movement of fine-grained material through the embankment or foundation of a dam. Thus reliability analysis methods based on historical frequency of occurrence and/or expert elicitation are appropriate to obtain the average annual probability of failure by seepage and piping. One method based on historical frequency of occurrence is the UNSW method and another is the use of expert elicitation using available site specific information.

(a) UNSW Method. The UNSW method gives the probability of failure and per annum failure rate without any direct relationship to pool elevation. See Appendix H for an example using a historical data model. It is recommended that the normal operating pool (up to spillway crest for ungated spillways) be used to determine consequences of breaching failure for this method. The per annum failure rate should be directly multiplied by the consequences related to the normal operating pool elevation to obtain the annual economic risk.

(b) Expert Elicitation. A site-specific analysis can be performed using a combination of frequency-based methods fit to historical events and subjectively determined probability values based on expert elicitation. When using expert elicitation one needs to rigorously document the process used and provide that with the report. Appendix E contains detailed guidance on the use of expert elicitation in geological and geotechnical engineering. An example of this method used for Walter F. George Dam is presented in Appendix F. Walter F. George Dam, Mobile District, experienced seepage through the solutioned limestone foundation, and uncontrolled seepage events have occurred at seemingly random locations on random occasions unrelated to pool level. Having no situation readily amenable to analytical modeling, the risk assessment was performed using a combination of frequency-based reliability methods fit to historical events and subjectively determined probability values based on expert elicitation. Given the set of historical events, annual probabilities of new events were taken to be increasing with time due to the continued solutioning of the limestone. The expert panel estimated probability values for future seeps occurring at various locations, for locating the source of the seep in sufficient time, for being able to repair the seeps given that they are located, and for various structural consequences of uncontrolled seepage.

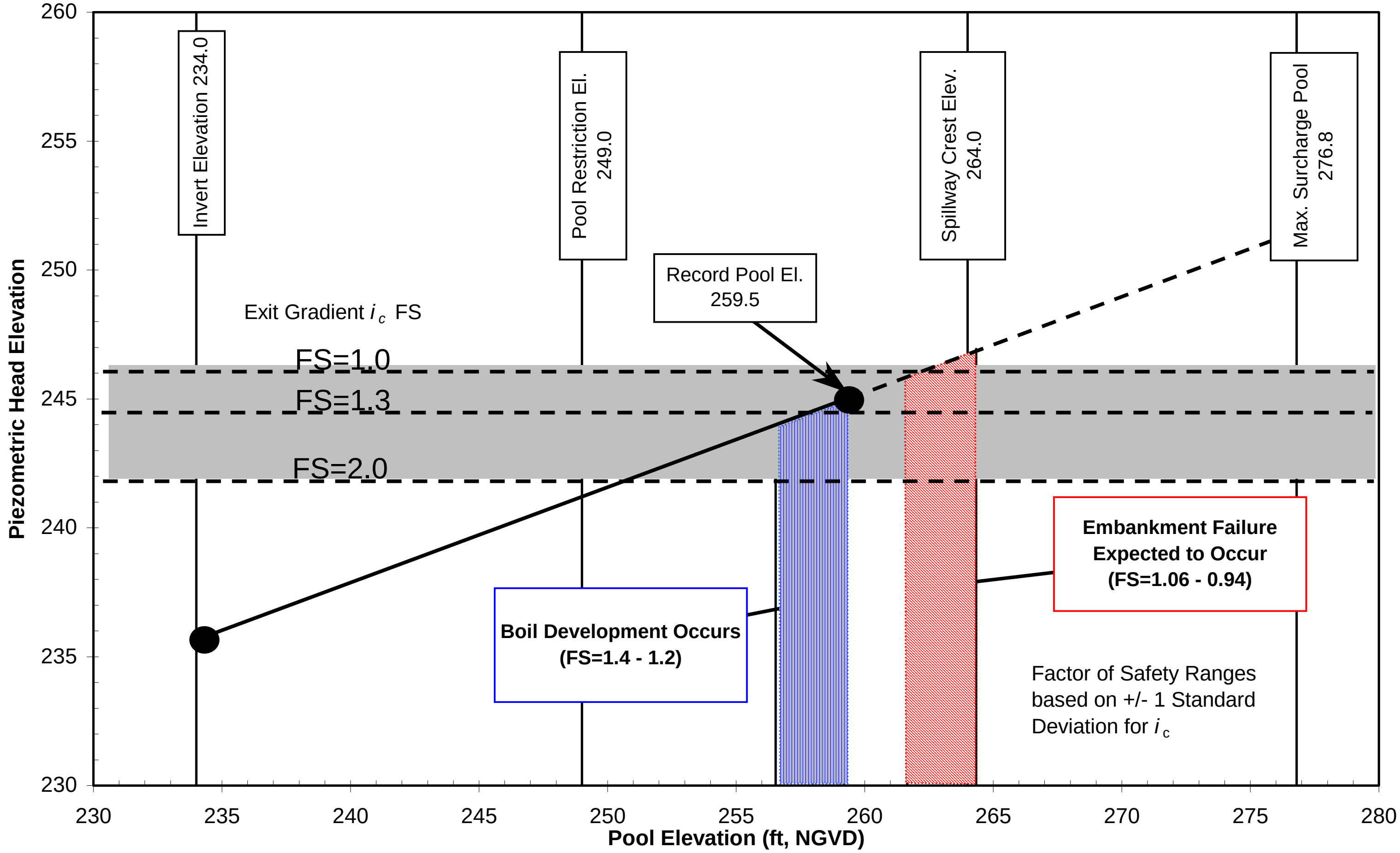

Figure 5. Estimated Uplift vs. Pool Elevation - Performance Parameter Plot for Conditions at Toe of a Dam

<table>
<tr><th>Analytical Based Method</th><th>Expert Elicitation Method and/or Historical Frequency of Occurrence Method</th></tr>
<tr><td>Piping/Seepage performance, i.e. exit gradient, can be directly related to pool elevation.</td><td>Piping/Seepage performance related to a change in conditions over time.</td></tr>
<tr><td colspan="2">(Annual Pool Level Probability)
X
(Conditional Probability of Unsatisfactory Performance)
X
(Performance Level)
X
(Consequences)
=
Annual Risk for Each Pool Level</td></tr>
<tr><td colspan="2">Sum Annual Risk for Each Load to Obtain the Annual Economic Risk for Seepage and Piping Failure.</td></tr>
</table>

Figure 6. Generalized Concept of the Different Piping/Seepage Risk Assessment Approaches.

4. Slope Stability Risk Analysis

a. Problem Definition.

A risk analysis or risk assessment of the slope stability of a project might be deemed appropriate due to changes in performance of the slope, changes in design criteria, or changes to the loading conditions.

(1) For example, a failure in a specific dam would suggest that all dams of similar design and construction are suspect and would require reanalysis and possible remediation.

(2) Changes in pore pressure or piezometric surface are indicators that potential stability problems might be developing, thus necessitating reanalysis.

(3) When measured deformations in the embankment and or foundation indicate that the embankment is not performing adequately, reanalysis and remediation may be required.

(4) Design criteria changes may indicate, after reanalysis, that remediation is required.

(5) When there is a change in the reservoir loading conditions, normally a change in the permanent pool, an increase in elevation of the normal pool range or operating range, reanalysis and possibly remediation will be required.

b. Project Site Characterization.

Site characterization consists of identifying those factors that are significant in the evaluation of slope stability.

(1) Typical factors would include embankment geometry, zoning and material properties, foundation geologic profile and material properties, pore pressures and piezometric profiles, and as-built conditions.

(2) Historical performance of the embankment or slope can provide evidence or indications of zones of movement, areas of high pore pressure, relationship of movement with pool elevation, rain fall, or groundwater levels.

(3) To reduce the level of uncertainty in the analysis the range of values for the significant input parameters need to be identified as well as possible and this may entail additional field exploration and laboratory testing.

c. Loading Conditions, Mode of Failure and Implications.

(1) As an existing project is being analyzed, there are two loading conditions that may be considered - the Steady State and Rapid Drawdown loading conditions.

(2) Shallow vs. Deep Seated Failure.

(a) Shallow shear failure, in which only a small shallow portion of the embankment moves, is designated as a surface slough. Such failures result when the slope is steeper than the soil shear strength can resist or when the seepage gradients are large enough to cause slope movement (for example, sudden drawdown loading condition). These failures are considered a maintenance problem and usually do not affect the structural capability of the embankment. However, if such failures are not repaired they can become progressively larger and may then represent a threat to the embankment safety. Implications are significant in instances where the shallow slides impact the intake tower or block the intake tower or spillway channel entrance.

(b) Deep shear failure, which involves the majority of the embankment cross-section or the embankment and the foundation, can reduce the crest elevation of the embankment thus increasing the potential for overtopping of the dam leading to uncontrolled release of the pool. Deep shear failure with large lateral movement can compromise the seepage control system of a zoned dam or dams with seepage control systems installed in the foundation at the downstream toe of an embankment. Where there is movement in the embankment and foundation in the area of the regulating outlet works the embankment tie in with the outlet works can be compromised and the outlet works could be damaged, which could lead to piping failure of the embankment along or into the outlet works conduit.

(3) Concrete Structure Sliding on Shear Surface in Rock. Concrete structures founded on weak rock or rock with layers of weak material may experience shear failure and the movement can cause loss of the pool. This movement could compromise an embankment tie-in with the concrete structure or an abutment contact with the concrete structure. Guidance for analysis of gravity structures on rock foundations can be found in Corps of Engineers Technical Report GL-83-13, Design of Gravity Dams on Rock Foundations: Sliding Stability Assessment by Limit Equilibrium and Selection of Shear Strength Parameters, October 1983.

(4) Process to Define Critical Shear Surface or Zone.

(a) For projects with observed movement or deformation there will normally be adequate data to locate the shear surface within reasonable bounds in the embankment and foundation. Use this surface to determine the probability of unsatisfactory performance using the reliability index method.

(b) For those projects where there is no clearly defined zone of movement a general discussion on determining critical slip surfaces is given on pages A-16 to A-17 of ETL 1110-2-556. However, we recommend that engineering judgment be used to guide the selection of the slip surfaces used in the reliability analysis.

1 Search for the slip surface corresponding to the conventional critical Factor of Safety slip surface and determine the reliability index for that slip surface.

2 Additionally, identify slip surfaces that would compromise critical features of the dam and determine the reliability index for those slip surfaces.

3 This set of slip surfaces would then be used to guide the selection of slip surfaces to be used in the risk assessment.

4 It should be noted that the slip surface that gives the minimum factor of safety does not necessarily correspond to the slip surface that gives the minimum reliability index (β). In fact the minimum β value may differ significantly from that calculated for the minimum factor of safety slip surface. Therefore it is imperative that the slip surface that gives the minimum β value be found. Presently there are no slope stability programs available that search for the slip surface that gives the minimum β value. An empirical algorithm to search for the critical probabilistic surface was published by Hassan and Wolff (1999).

(5) Two-Dimensional Versus Three-Dimensional Analysis Methods.

(a) For the vast majority of cases two dimensional analysis methods are satisfactory.

(b) Use of three-dimensional analysis methods is recommended when there is significant evidence indicating that the failure surface is constrained in such a manner that the plain strain assumption used in two-dimensional analysis is not valid, as in the case in which the potential failure surface is clearly constrained by physical boundaries.

(c) Typically this is only for translational failure masses where use of a sliding block with vertical sides is recommended, as it is simple to model and verify. Side forces are calculated as a function of at-rest earth pressure.

d. Conditional Probability Of Unsatisfactory Performance.

(1) Reliability Index. The conditional probability of unsatisfactory performance is determined by the reliability index method presented in ETL 1110-2-556, pages A-10 and A-11. The use of a well-documented and tested slope stability analysis program is recommended to compute the factors of safety used in the reliability analysis. Example problems are included in Appendices D and I to illustrate the use of the reliability index method as it relates to slope stability analysis.

(2) Reliability Index Factor versus Design Factor of Safety for Slope Stability.

(a) Comparisons of traditional factor of safety design methods to reliability index approaches generally concentrate on the ability of the probability-based methods to deal with reliability in a more comprehensive way than do the factor of safety methods. This view is stated well by Christian (1996):

> "The reliability index provided a better indication of how close the slope is to failure than does the factor of safety alone. This is because it incorporates more information – to wit, information on the uncertainty in the values of the factor of safety. Slopes with large values of β are farther from failure than slopes with small values of β regardless of the value of the best estimate of the factor of safety."

(b) However, the factor of safety entails more than a measure of reliability. Mechanically, the factor of safety is the ratio of forces that can be mobilized as strength to the forces tending to cause slope movement. Harr (1987) has described the factor of safety in terms of capacity and demand; for slope stability, the strength represents the capacity of the slope while the driving forces represent the demand. A factor of safety of one signifies that demand is just balanced by capacity and the system is at a limit state. If demand exceeds capacity, the slope is in a state of failure. A well-engineered structure requires a reserve in capacity implying a factor of safety greater than one. The particular choice of the factor of safety depends on both the type of loading and the type of structure. Recommended values are generally based on experience of structures that have, or have not, experienced satisfactory performance.

(c) The difficulty with incorporating the factor of safety into a reliability analysis is twofold. First, the experience with traditional factor of safety-based design method is based on a sequence of practices involving site investigation, laboratory technique, and data interpretation where each part of the sequence entails some level of conservatism. Second, the factor of safety conveys more than a level of reliability because failure of an earth structure is not a discrete event. The practical failure of a capacity-demand system generally occurs prior to extracting full capacity. In the case of an embankment slope, the capacity (strength) is extracted at the cost of deformation, which might have ramifications that would be considered unsatisfactory performance even before the factor of safety falls to one. Therefore, the reliability index method separates the issue of uncertainty from that of performance. The factor of safety is treated as a random variable through a procedure where the uncertainty of the computation process can be dealt with objectively. The question of acceptable factor of safety is dealt with through the performance level factor.

(3) Expected Value Versus the One Third/Two Thirds Rule.

(a) Corps practice has been to select the design strength such that it is less that two-thirds of measured strength values. For reliability analysis, this criterion is supplanted by the reliability index method, which is a process of quantifying the probability distribution of strength values thereby determining the probability distribution of the factor of safety. When the shear strength parameters are selected in accordance with the Corps of Engineers one third/two thirds rule, instead of the expected value, the β is inappropriately low. In its proper application, each step in computing the reliability index should be performed using an objective estimate of the expected value and variance.

(b) In reliability analysis, emphasis is on comparing the best estimate of actual prevailing or predicted conditions to those, which correspond to failure or unsatisfactory performance. Hence, it is essential that expected values be assigned in an unbiased manner that does not introduce additional conservatism, so that the reliability index β remains an accurate comparative measure of reliability.

(4) Tables of Coefficient of Variation. Often there is not adequate data to determine the coefficient of variation. When this is the case, it is recommended that typical values for the coefficient of variation be used. Typical values of tabulated data are presented in Table D-2 of Appendix D and Table 1, Appendix B, ETL 1110-2-556.

e. Performance Level.

(1) The traditional selection of factor of safety is supplanted by assigning a performance level factor. Given a low factor of safety, a number of levels of performance can be envisioned for that mechanism, with each having their own probability. For given loading cases and degree of performance a performance level value needs to be assigned for those values of the factor of safety where unsatisfactory performance is determined to occur. A factor of safety is tied to a mechanism (trial shear surface). It is generally implied that the mechanism having the lowest factor of safety approximates the geometry of the failure that would actually emerge. Several non-minimal surfaces might be considered unreliable because a high probability of failure is associated with both low factor of safety and level of uncertainty of either capacity or demand. Also, the minimal slip surface is often associated with shallow slips that are of limited consequence.

(2) The use of a performance level factor is incorporated into the risk calculations by assigning probabilities to the various possible outcomes associated with each factor of safety. The level of performance should take into account the susceptibility of the embankment to catastrophic failure by the various failure modes determined for the embankment under study. This may require an over topping analysis or a seepage piping and erosion analysis. Alternatively, catastrophic failure does not necessarily occur when an analysis shows the slope has reached the limit state defined by FS=1. Usually, levels of performance are associated with the amount of movement associated with failure. The probability of the discrete event of the factor of safety equaling one must be assessed in view of the reality that movement after failure is limited. Similarly, there might be considerable movement prior to reaching a factor of safety of one. Thus, an unsatisfactory performance level might be probable even if a full failure state is not reached, especially for slopes that have documented distress at levels of FS > 1. The general assumption of a slope stability calculation is that strength is mobilized along the entire potential sliding surface. The reality is that often movement is observed in an embankment or slope indicating that at least part of the slope has reached a limit state. In the particular case where much of the strength capacity is derived from passive resistance, significant movement might occur in the active block before FS=1 is obtained. This movement might or might not be indicative of poor performance. Typically some major change in reservoir operations can be enacted to reduce the load on the slope allowing some remedial action to be taken, which incurs a cost and impacts project purposes.

(3) The value for the performance level can be determined using case histories, expert elicitation, and engineering experience. Several options are available:

(a) Use the recommended factors of safety as the limit between satisfactory and unsatisfactory performance. Because conservative strength values were part of traditional design, the recommended safety factors were experience-based measures of satisfactory performance. (In this case the computation of performance level must be modified for FS>1 as described in Appendix D.)

(b) One alternative is a static deformation analysis. This needs to consider the available free board and the estimated total movement to determine the potential for breaching of the dam or impact on the seepage control system.

(c) Consider damage to drainage layers or structural components with low strain capacity. In the case where limited deformation cannot be tolerated, a poor performance level can be assigned without considering the actual magnitude of post-failure deformation.

f. Example Problems.

(1) A detailed explanation is provided in Appendix D on how to perform a reliability index analysis of a simple slope using the infinite slope analysis method.

(2) A steady state problem is presented in Appendix I to demonstrate how to use the method on a more complex, real world problem.

APPENDIX A

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APPENDIX B

POISSON DISTRIBUTION

B-1. Poisson Distribution. Events that occur randomly in time, and have an equally likely probability of occurrence in any unit time increment Δt (typically one year) can be described by Poisson's distribution. Events that have been assumed to follow Poisson's distribution include random arrivals (of floods, earthquakes, traffic, customers, phone calls, etc.). The Poisson distribution is a discrete distribution function that defines the probability that x events will occur in an interval of time t:

$$\text{Pr (x events in interval t)} = \Pr(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \qquad (1)$$

In Equation 1, λ is the mean rate of occurrence (or the expected number of events in a unit time period, Δt), the number of unit time increments is denoted by t, and the number of events is denoted by x.

$$\lambda = \frac{\textit{total number of events}}{\Delta t} \qquad (2)$$

B-2. Poisson Distribution Example. Routine maintenance is scheduled at one of the Corps of Engineers reservoir projects. One item of work is that the tainter gates in the gated spillway are to be painted. To complete this task, the upstream bulkheads need to be installed to keep the work area dry. The top of the bulkheads is Elevation 424.5. This work is to take place over a 6-month time period. Use the Poisson distribution to calculate the probability that water will not go over the bulkheads while the tainter gates are being painted. Also calculate the probability that water will go over the top of the bulkheads once or twice during painting of the tainter gates.

B-3. Solution to Problem.

Using the pool elevation frequency curve in Figure B-1, the top of the bulkheads (a pool Elevation of 424.5) corresponds to a 10-year exceedence interval for the lake level.

λ = 1/10 = 0.10 exceedence/yr

The duration of the event is one-half of a year. Note the units of t must be in term of Δt, where in this case Δt is one year.

t = 0.5 years

λt = (0.10)(0.5) = 0.05 exceedence (the expected number in the interval)

The probability of zero events (over topping of the bulkheads) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[0] = \frac{(0.05)^0 e^{-0.05}}{0!} = \frac{(1)(0.9512294)}{1} = 0.951$$

Pr[0] = 95.1 percent

The probability of one event (over topping of the bulkheads one time) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[1] = \frac{(0.05)^1 e^{-0.05}}{1!} = \frac{(0.05)(0.9512294)}{1} = 0.047$$

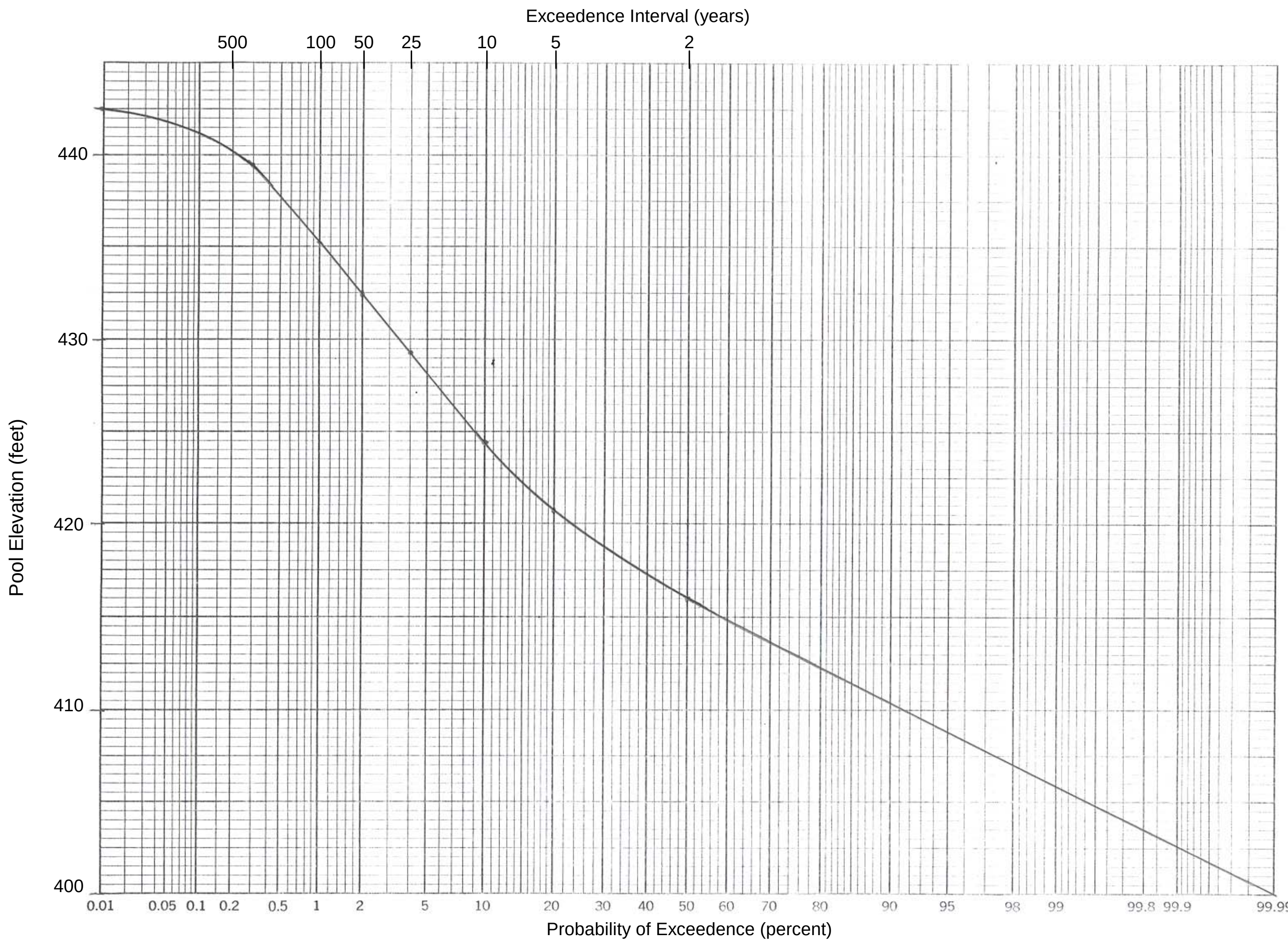
Pr[1] = 4.7 percent

The probability of two evens (over topping of the bulkheads two times) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[2] = \frac{(0.05)^2 e^{-0.05}}{2!} = \frac{(0.0025)(0.9512294)}{2} = 0.001$$

Pr[2] = 0.1 percent

From the above calculation it can be seen that there is a 95% probability that the work can be accomplished without delay to the contractor by water overtopping the bulkheads. Likewise, there is a 5% probability that the lake level will rise above the top of the bulkheads and the painting of bulkheads will be delayed. There is also a 0.1% chance that the water level will overtop the bulkheads twice during the six-month painting contract.



Pool Elevation Frequency Curve

Figure B-1

APPENDIX C

THE SIX SIGMA RULE

C-1. Introduction. In reliability analyses the random variables are represented by a probability density function. There are many probability density functions; however, for geotechnical reliability analyses the ones most commonly used are the uniform distribution, the triangular distribution, the normal distribution, and the lognormal distribution. One method of reliability analysis used by the Corps of Engineers is the reliability index method. This method is a first order second moment method of analysis. This means that only the first two moments (mean and variance) are used to represent the probability density function in the reliability analysis and all higher order terms are neglected in the Taylor Series expansion used to estimate the mean and variance of the performance function (natural log of the factor of safety). So when using the reliability index method no knowledge of the exact probability density function that represents a random variable is needed. Only the mean and the variance of the random variable are needed. The mean is typically called the expected value and the variance is equal to the standard deviation squared (σ^2). The expected value of a random variable is easily estimated by geotechnical engineers as that is the average value of the parameter or what one would expect the value of the parameter to be. The standard deviation is not as easy to estimate; however, several ways to determine the standard deviation are given in Appendix D. This discussion will deal with one of those methods for determining the standard deviation, the six sigma rule and variations of that procedure.

C-2. Six Sigma Rule. The six sigma rule is based on the normal distribution probability density function, see Figure C-1. The six sigma rule makes use of the experience of geotechnical engineers in estimating the value of the standard deviation. The six sigma rule is given by Equation 1.

$$\sigma = \frac{Largest\ \ Possible\ \ Value - Smallest\ \ Possible\ \ Value}{6} \tag{1}$$

By examining the normal probability density function in Figure C-1, the reason why the six sigma rule works becomes clear. Figure C-1 shows a normal probability density function with the percentage of the area under the curve for the mean plus or minus one standard deviation (a range of 2σ), plus or minus two standard deviations (a range of 4σ), plus or minus two and one half standard deviations (a range of 5σ), and plus or minus three standard deviations (a range of 6σ). For the mean plus or minus three standard deviations, 99.73 percent of the area under the normal distribution is included. Therefore 99.73 percent of all possible values of the random variable are included in this range. So, for a range of six sigma (the highest possible value to the lowest possible value of the random variable), essentially all the values represented by the normal distribution curve are included, thus the name the six sigma rule. In the literature, Duncan (1999) and Dai and Wang (1992) and in this document, the six sigma rule is sometimes called the three sigma rule because the six sigma range of the data is represented by a plus or minus three sigma.

C-3. Other Bounds of the Data. Other variation of the six sigma rule exist depending on your confidence in estimating the upper and lower limits of the values represented by the random variable. EC 1105-2-205, Risk-Based Analysis for Evaluation of Hydrology/Hydraulics and Economics in Flood Damage Reduction Studies, uses a four sigma rule and a two sigma rule to estimate the standard deviation.

a. The four sigma rule is given by Equation 2.

$$\sigma = \frac{E_{mean}}{4} \tag{2}$$

where: E_{mean} = the difference between reasonable upper and lower limits which bound 95 percent of all data.

b. The two sigma rule is given by Equation 3.

$$\sigma = \frac{E_{majority}}{2} \tag{3}$$

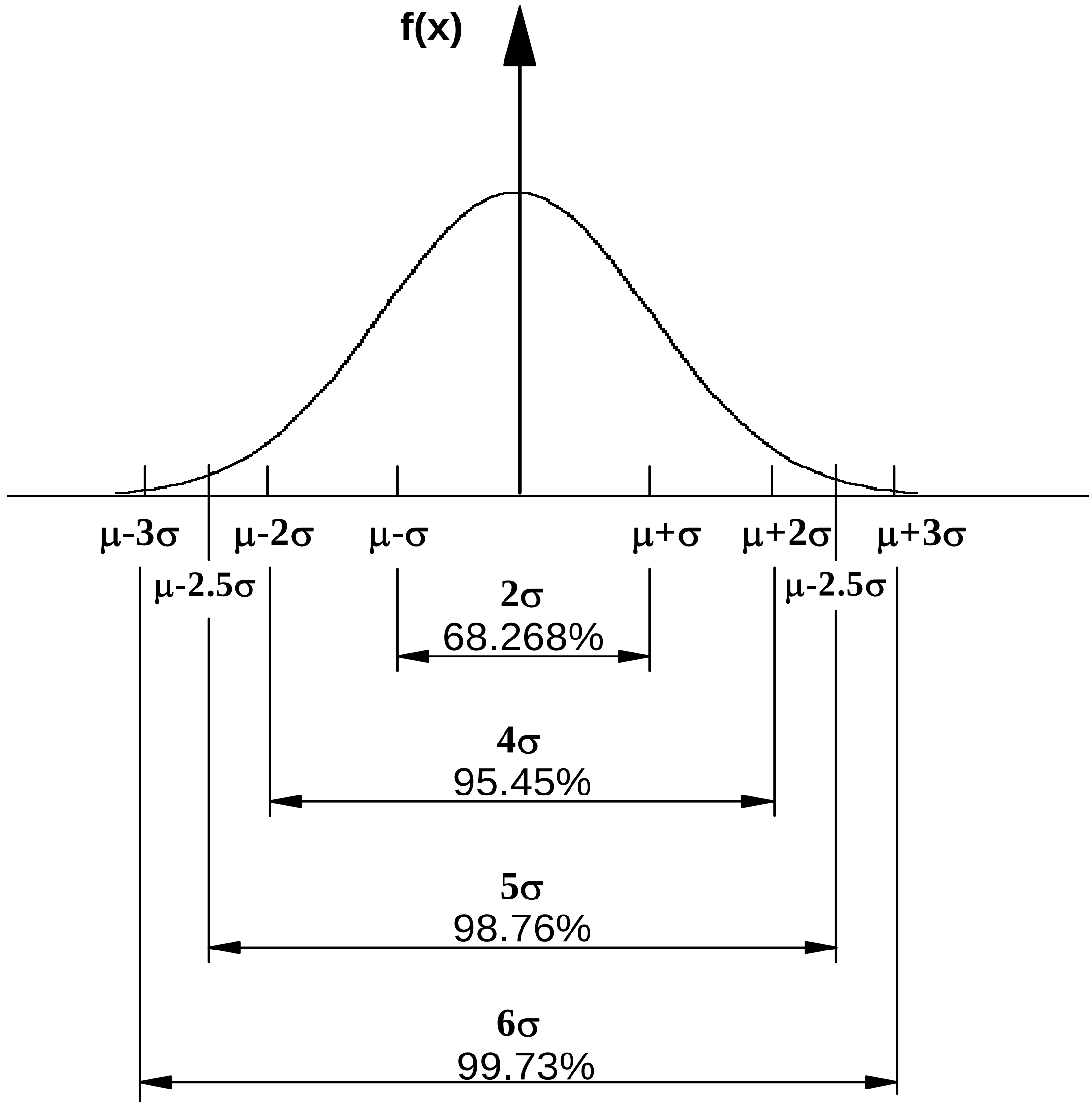
where: $E_{majority}$ = the difference between the upper and lower limits for the majority of the data bounding 68 percent of all data.

c. The five sigma rule is given by Equation 4.

$$\sigma = \frac{Largest\ Reasonable\ Value - Smallest\ Reasonable\ Value}{5} \tag{4}$$

where the difference between the largest reasonable value and the smallest reasonable value of the random variable includes 99 percent of all data.

C-4. Summary. All of the sigma rules are based on the normal distribution shown in Figure C-1 and the confidence the geotechnical engineer has in estimating the upper and lower limits of the data range representing the random variable. If there is 100 percent confidence in estimating the upper and lower limits of the random variable, use the six sigma rule. If there is a 99 percent confidence in the estimating the upper and lower limits of the random variable, use the five sigma rule. Likewise for 95 percent confidence in the estimating the range of the random variable, use the four sigma rule; and for 68 percent confident in the estimating the data range, use the two sigma rule. From a practical stand point, it would be difficult to justify using the two sigma rule since it would not be easy to estimate an upper and lower bound on 68 percent of the data. However, one could easily justify use of either the six, five, or four sigma rules and EC 1105-2-205 recommends hydrology/hydraulic engineers use the four sigma rule for calculating the standard deviation for water level data.



NORMAL DISTRIBUTION

Figure C-1

APPENDIX D

RELIABILITY ANALYSIS OF A SIMPLE SLOPE

D-1. Introduction.

a. The reliability analysis of a simple slope stability problem is presented and is intended to demonstrate the methodology of geotechnical reliability analysis using the Corps of Engineers reliability index method of analysis. The guidance for performing geotechnical reliability analysis is given in ETL 1110-2-547.

b. When performing reliability analyses, the same methods of analysis are used as are used in traditional geotechnical engineering. The only difference is in how the variables in the problem are represented.

c. In reliability analysis there are two types of variables: deterministic and random variables.

(1) Deterministic variables are represented by a single value because the value of that variable is known exactly. A deterministic variable could be the unit weight of water or measured dimensions.

(2) Random variables are represented by a probability density function, which defines the relative likelihood that the random variable assumes various ranges of values. Random variables could be shear strength, permeability, and earth pressure.

d. A probability density function is a plot of the value of the random variable on the x-axis and the likelihood of occurrence of that value on the y-axis. An important property of a probability density function is that the area under the curve is equal to one.

e. The field of statistics has defined the first four characteristics or moments of each probability density function as the mean (expected value), the variance, the coefficient of skewness, and the coefficient of kurtosis. The Corps of Engineers reliability index method of analysis uses what is called a first order second moment method of reliability analysis. This means that only the first term of the Taylor series expansion about the mean value (first order) is used and the first two moments (mean and variance) are used to represent the probability density function in the reliability analysis. This first order second moment method of analysis greatly simplifies the reliability analysis procedure.

D-2. Reliability Analysis Terms and Basic Equations.

a. The first two moments of the probability density function are the mean and variance. In engineering terms, the mean is the centroid of the probability density function and the variance is the central moment of inertia.

b. The mean μ_x is represented by Equation 1:

$$\mu_x = \sum \frac{x_i}{N} \tag{1}$$

where x_i is a data point and N is the number of data points.

c. In reliability analysis the mean is also referred to as the expected value *E(x)*.

d. The variance *Var[x]* is represented by Equation 2:

$$Var[x] = \frac{\sum [(x_i - \mu_x)^2]}{N-1} \tag{2}$$

e. The standard deviation σ_x is represented by Equation 3:

$$\sigma_x = \sqrt{Var(x)} \tag{3}$$

f. Inversely, *Var(x)* is equal to the square of the standard deviation of *x*. The variance and standard deviation are measures of the dispersion or variability of the random variable. The larger the standard deviation the larger the range of values the random variable may assume. The smaller the standard deviation the smaller the range of values the random variable may assume. As the value of the standard deviation gets smaller and smaller the random variable will approach a constant or a deterministic variable in the limit.

g. The expected value of *x* will be used to represent the first moment, and the standard deviation of *x* will be used to represent the second moment of the random variable.

h. The coefficient of variation V_x is represented by Equation 4:

$$V_x = \frac{\sigma_x}{E[x]} \times 100\% \tag{4}$$

i. As some of the variables in a geotechnical analysis are represented as random variables, i.e., a probability density function, it only stands to reason that the results of the analysis will be a probability density function. The results of many geotechnical analyses are expressed in terms of factors of safety (*FS*). In traditional geotechnical analyses, the *FS* is a single number. In reliability analyses, the *FS* is a random variable represented by a distribution (probability density function), not a single number.

j. The Corps of Engineers has chosen to represent the probability density function of the *FS* as a lognormal probability density function (see Figure D-1). The lognormal probability density function is shown at the bottom of Figure D-1. The transformed normal probability density function is shown at the top of Figure D-1. The lognormal probability density function is transformed to the normal probability density function by taking the natural log of the factor of safety. One reason for choosing the probability density function of the *FS* as a lognormal probability density function is that a lognormal function can never have a value less than zero. Thus, the FS will not have negative values. Another reason is that when one takes the natural log of the *FS*, the distribution of the *FS* becomes a normal distribution. A third reason is that a probability density function resulting from the product of many variables is lognormally distributed. In geotechnical analyses typically many numbers are multiplied together. For the lognormal distribution of the *FS* shown in Figure D-1, the hatched area under the curve and to the left of a *FS* = 1 gives the probability that the *FS* is less than one or the probability that the performance of the slope stability problem will not be satisfactory. *P(u)* is defined as the probability of unsatisfactory performance, i.e., the probability that the *FS* is less than 1. When the lognormal distribution is transformed to the normal distribution by taking the natural log of the *FS* it is noted that the ln(1) = 0. Thus the *P(u)* is represented by the hatched area under the normal distribution curve to the left of zero.

k. On the normal distribution curve in Figure D-1 the distance between zero and *E[ln(FS)]* is defined as $\beta\sigma_{ln(FS)}$. Beta (β) is the reliability index. The reliability index is a measure of the distance that the expected value of the FS is away from unsatisfactory performance. The larger the value of β, the more reliable the slope is. The smaller the value of β, the closer the slope condition is to unsatisfactory performance. Calculated values of β greater than three represent a stable slope whereas values of β less than 2 represent a poor performing slope. Values of β less than 1.5 represent unsatisfactory performance.

l. The reliability index β is represented by Equation 5:

$$\beta_{\log normal} = \frac{\ln\left[\dfrac{E(FS)}{\sqrt{1+V_{FS}^2}}\right]}{\sqrt{\ln(1+V_{FS}^2)}} \tag{5}$$

m. Once β is calculated, the probability of unsatisfactory performance can be calculated from Equation 6:

$$P(u) = \Phi(-\beta) \tag{6}$$

where $\Phi(-\beta)$ is obtained from a standard normal probability density function N(0,1). A standard normal probability density function has a mean of zero and a standard distribution of one. Table D-1 gives the area under the standard normal probability density function for values of *x* where *x* is given by Equation 7:

$$x = -\beta \tag{7}$$

n. For values of *x* to one decimal point the area under the standard normal probability distribution to the left of the *x* line (the shaded area) can be read from the table to the right of the *x* value in the first column. The second decimal of *x* is given across the top of the table. For example, the area under the standard normal probability distribution function to the left of *x* for a value of $x = 1.55$ is 0.9394. Note this area represents the probability of unsatisfactory performance for a beta of -1.55.

o. The reliability is the probability that the embankment slope will not experience unsatisfactory performance or that the slope will perform in a stable or reliable manner. The reliability R is represented by Equation 8:

$$R = 1 - P(u) \tag{8}$$

D-3. Taylor Series Method of Reliability Analysis.

a. The Taylor series method of reliability analysis will be used to compute the reliability index. This method is based on a Taylor series expansion of the performance function about some point. For this analysis the expansion is performed about the expected values of the random variables. Since this is a first order method, only the first order (linear) terms of the Taylor series expansion are retained. Equation 9 is obtained from the Taylor series expansion using the assumption that the first derivatives (slopes) are calculated at the expected value using an interval of plus or minus one standard deviation of the random variable.

$$Var(FS) = \sigma_{FS}^2 = \sum_{i=0}^{n} \left[\frac{FS(x_i + \sigma_{x_i}) - FS(x_i - \sigma_{x_i})}{2} \right]^2 \tag{9}$$

b. Equation 9 gives the variance of the factor of safety *(Var(FS))* where *x* is the random variable and there are *n* random variables. The Taylor series method of reliability analysis requires 2n+1 geotechnical analyses (slope stability analyses) to be performed. A benefit of using the Taylor series method of analyses is that the relative contribution of each random variable to the total uncertainty can be determined.

D-4. Determining Expected Values and Standard Deviation.

a. The advantage of using a first order second moment method of reliability analysis is that the probability density function of each random variable can be represented by only two values, the expected value and the standard deviation. The expected value of a random variable is fairly easy to obtain. There are a number of sources from which the expected values of soil properties can be estimated or obtained: testing (laboratory or in-situ), other projects in the area, back calculation of the value, experience, manuals and tables, or correlation with soil index properties.

b. Obtaining the expected value is the easy part, that is, using laboratory data, historical data, estimates based on field exploration and experience. But geotechnical engineers generally do not have a feel for the value of the standard deviation of soil properties. The standard deviation of soil properties is more difficult to obtain but there are three methods to obtain it.

(1) Data. If there is a substantial amount of test data for the soil properties, the expected value and the standard deviation can be calculated using Equations 1, 2, and 3. This is the best method to use. But a substantial amount of test data may not exist.

(2) Coefficient of Variation Tables. In the absence of test data values of coefficients of variation for soil properties can be found in the literature. Table D-2 gives coefficients of variation found in the literature or calculated from those values and the reference. From the expected value and the coefficient of variation, the standard deviation can be calculated from Equation 4.

(3) Six Sigma Rule. For random variables for which a coefficient of variation cannot be found in the literature, the six-sigma rule can be used. The six-sigma rule makes use of the experience of geotechnical engineers. The six-sigma rule is given by Equation 10.

$$\sigma = \frac{Largest\ Possible\ Value - Smallest\ \ Possible\ \ Value}{6} \tag{10}$$

(a) The six sigma rule is often called the three-sigma rule because it determines the value of the parameter at plus and minus three standard deviations from the mean or expected value. See Appendix C for more details on the six sigma rule.

(b) By examining the normal probability density function in Figure D-2, the justification for the six sigma rule can be demonstrated. Figure D-2 shows a normal probability density function with the percentage of the area under the curve for the mean plus or minus one standard deviation, two standard deviations, and three standard deviations (six sigma). For the mean plus or minus three standard deviations, 99.73% of the area under the normal distribution is included. So for a range of six sigma (the highest possible value to the lowest possible value), essentially all the values represented by the normal distribution curve are included, thus the name the six sigma rule. The shear strength of sand (ϕ) can be used to demonstrate the six sigma rule. The highest possible value of ϕ for a sand is 45°. The lowest possible value of ϕ is 25°. Applying the six sigma rule results in the following:

$\sigma = (45° - 25°)/6 = 3.33°$

(c) For a sand with an expected value for ϕ of 35°, using the coefficient of variation Table D-2, sigma is calculated using Equation 4 as

$\sigma = 0.10(35°) = 3.5°$

where 10% for the coefficient of variation (V_x) comes from Table D-2. The value of sigma of 3.33 from the six sigma rule compares well with the value of sigma of 3.5 from the coefficient of variation method.

D-5. <u>Example Problem – Infinite Slope Reliability Analysis.</u>

a. A simple slope stability problem can be examined to see how the reliability analysis presented above is performed. A simple embankment is shown in Figure D- 3. The embankment is constructed of sand. The embankment slope is shown as approximately 1 vertical on 1.5 horizontal. The goal is to determine how stable this slope is, that is, the probability of unsatisfactory performance of the embankment. As was stated before, the same methods of analysis will be used for the reliability analysis as are used in a traditional geotechnical slope stability analysis. There are several methods of analysis that can be used: force equilibrium slope stability computer programs, slope stability chart solutions, and the infinite slope equation. To simplify the reliability analysis, the infinite slope equation given by Equation 11 will be used.

$$FS = b \tan \phi \tag{11}$$

where *FS* is the factor of safety, *b* defines the slope of the embankment when the slope is 1 vertical on *b* horizontal, and ϕ is the angle of internal friction of the sand. There are two random variables in this reliability analysis, ϕ and *b*. The expected value and the standard deviation need to be determined for each random variable. Based on previous work at this site the expected value of the angle of internal friction of the sand ($E[\phi]$) is 38°. The standard deviation can be obtained from Equation 4 and Table D- 2. From Table D-2 the coefficient of variation of the angle of internal friction for sand is 10%. The standard deviation is calculated as

$\sigma_\phi = 0.1 \times 38° = 3.8°$

b. The expected value of *b* ($E[b]$) is 1.5. The six-sigma rule will be used to determine the standard deviation of *b*. From looking at this embankment the geotechnical engineer has determined that the embankment slope varies between a maximum value of b = 1.625 and a minimum value of b = 1.375. The standard deviation of *b* (σ_b) is calculated as follows using Equation 10

$\sigma_b = (1.625 - 1.375)/6 = 0.042$

c. Since a first order second moment method of analysis is being used, the two random variables are defined as follows:

Sand	$E[\phi] = 38°$	$\sigma_\phi = 3.8°$
Slope	$E[b] = 1.5$	$\sigma_b = 0.042$

d. The Taylor series method of reliability analysis will be used. Since there are two random variables, 2n+1 calculations will need to be accomplished.

$2n+1 = 2 \times 2 + 1 = 5$

e. The first calculation will be to determine the expected value of the factor of safety (*E[FS]*). Using the infinite slope equation (11), we get the following

$E[FS] = E[b]\tan(E[\phi]) = 1.5 \tan 38 = 1.17$

f. Next determine the expected values of the random variables at plus and minus one standard deviation.

$E[\phi]+\sigma_\phi = 38 + 3.8 = 41.8$

$E[\phi]-\sigma_\phi = 38 - 3.8 = 34.2$

$E[b]+\sigma_b = 1.5 + 0.042 = 1.542$

$E[b]-\sigma_b = 1.5 - 0.042 = 1.458$

g. Using these values, the infinite slope equation is used to calculate the factor of safety for ϕ plus and minus one standard deviation while holding *b* constant and for *b* plus and minus one standard deviation while holding ϕ constant.

$FS_{\phi+\sigma} = 1.5 \tan(41.8) = 1.34$

$FS_{\phi-\sigma} = 1.5 \tan(34.2) = 1.02$

$FS_{b+\sigma} = 1.542 \tan(38) = 1.20$

$FS_{b-\sigma} = 1.458 \tan(38) = 1.14$

h. The results of these calculations are presented in Table D- 3.

Table D-3. Summary of Taylor Series Method Reliability Calculations

Random Variable	ϕ	B	FS	Var(FS)	Percent	Remarks
$E[\phi]$, $E[b]$	38	1.5	1.17			
$E[\phi]+\sigma_\phi$, $E[b]$	41.8	1.5	1.34	0.026	96.7	Variable that controls analysis
$E[\phi]-\sigma_\phi$, $E[b]$	34.2	1.5	1.02			
$E[\phi]$, $E[b]+\sigma_b$	38	1.542	1.20	0.0009	3.3	Little effect on analysis
$E[\phi]$, $E[b]-\sigma_b$	38	1.458	1.14			
			$\sum$	0.0269	100	

i. Instead of using the infinite slope equation to calculate the factor of safety in each of the above five slope stability analyses, other methods of slope stability analysis could have been used: force equilibrium slope stability computer programs or slope stability chart solutions. No matter what method of analysis is used the end result is that 2n+1 factors of safety are calculated.

j. The next step in the reliability analysis is to use Equation 9 to calculate the variance of the factor of safety (*Var(FS)*) for each random variable. The variance of the factor of safety for the random variable ϕ is calculated using the factor of safety for ϕ plus one standard deviation minus the factor of safety for ϕ minus one standard deviation divided by two and the quantity squared.

$$Var(FS)_{\phi} = [(1.34 - 1.02)/2]^2 = 0.026$$

k. The variance of the factor of safety for the random variable *b* is calculated in the same manner.

$$Var(FS)_{b} = [(1.20 - 1.14)/2]^2 = 0.0009$$

l. The variance of the factor of safety (*Var(FS)*) is the sum of the variances for the factors of safety of all random variables.

$$Var(FS) = 0.026 + 0.0009 = 0.0269$$

m. One big advantage of using the Taylor series method of analysis is that the random variable that controls the reliability analysis can easily be determined. The variance of the factor of safety for each random variable divided by the sum of all the variances gives the percent that that random variable effects the analysis. In Table D- 3 under the column labeled "percent" we see that the random variable ϕ controls 96.7% of the analysis and that the random variable *b* controls 3.3% of the analysis. This indicates that a similar result would have been obtained if *b* had been chosen to be a deterministic variable.

n. Using Equation 3 the standard deviation of the factor of safety can be calculated from the variance of the factor of safety as follows:

$$\sigma_{FS} = (0.0269)^{0.5} = 0.16$$

o. The coefficient of variation of the factor of safety (V_{FS}) can then be calculated from the expected value of the factor of safety (*E[FS]*) and the standard deviation of the factor of safety (σ_{FS}) as follows using Equation 4.

$$V_{FS} = 0.16/1.17 = 0.14$$

p. Using the values of the expected value of the factor of safety and the coefficient of variation of the factor of safety, the value of $\beta_{lognormal}$ can be calculated using Equation 5.

$$\beta_{\log normal} = \frac{\ln\left[\dfrac{1.17}{\sqrt{1+0.14^2}}\right]}{\sqrt{\ln(1+0.14^2)}} = 1.06$$

For the embankment shown in Figure D-3 the stability of the slope is calculated to have a value of beta equal to 1.06.

q. The probability of unsatisfactory performance can be calculated using Equations 6 and 7 and Table D-1 as follows. Table D-1 gives the area under the standard normal distribution curve for a value of x. In this case x = -1.06 (from Equation 7). The area under the curve in Table D-1 for x = -1.06 is 0.1446. This area is the probability of unsatisfactory performance.

$P(u) = \Phi(-\beta) = \Phi(-1.06) = 0.1446$

which is rounded to two significant decimal places to

$P(u) = 0.14$

Note that previously it was indicated that values of beta less that 1.5 represent unsatisfactory performance.

r. One way to comprehend or communicate 14% probability of unsatisfactory performance is the concept that of 100 embankments constructed exactly like this one, 14 of them would have unsatisfactory performance. This probability of unsatisfactory performance is then used with the other parameters from the event tree to calculate risk.

D-6. <u>When the FS is Greater Than 1.</u>

a. When unsatisfactory performance is associated with FS>1, the associated probability cannot be determined from the reliability index β. A more general index can be derived from the definition of the normalized parameter z,

$$z(FS_u) = \frac{E[\ln(FS)] - \ln(FS_u)}{\sigma_{\ln(FS)}} \qquad (12)$$

b. The probability of unsatisfactory performance is $P(u) = P(FS<FS_u) = \beta(-z)$. Using the relationships between the mean and variance of the factor of safety and the parameters for the log normal distribution as given by

$$E[\ln(FS)] = \ln(E[FS]) - \frac{\sigma^2_{\ln(FS)}}{2} \tag{13}$$

and

$$\sigma_{\ln(FS)} = \sqrt{\ln(1+V_{FS}^2)} \tag{14}$$

where,

$$V_{FS} = \frac{\sigma_{FS}}{E[FS]}, \tag{15}$$

$z(FS_u)$ is found to be

$$z(FS_u) = \frac{\ln(\frac{E[FS]/FS_u}{\sqrt{1+V_{FS}^2}})}{\sqrt{\ln(1+V_{FS}^2)}}\,. \tag{16}$$

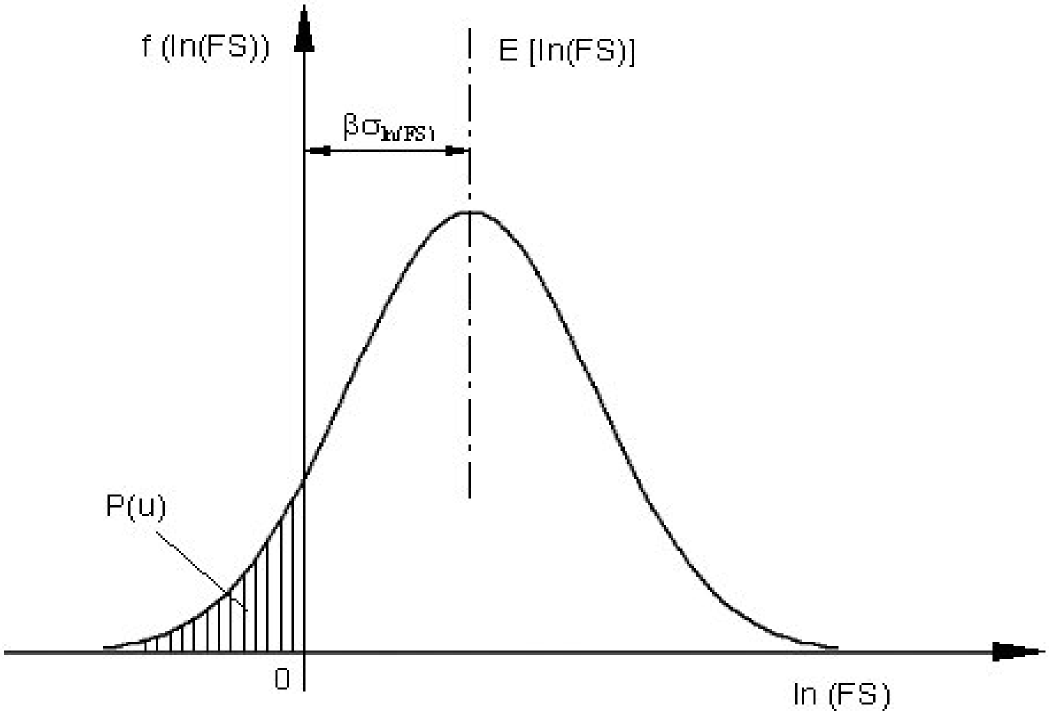
c. Consider a case where experience show that excessive deformation occurs when FS<1.1. Using the data from the previous example

$$z(FS_u\ ) = \frac{\ln\left[\frac{1.17/1.1}{\sqrt{1+0.14^2}}\right]}{\sqrt{\ln(1+0.14^2)}} = 0.37$$

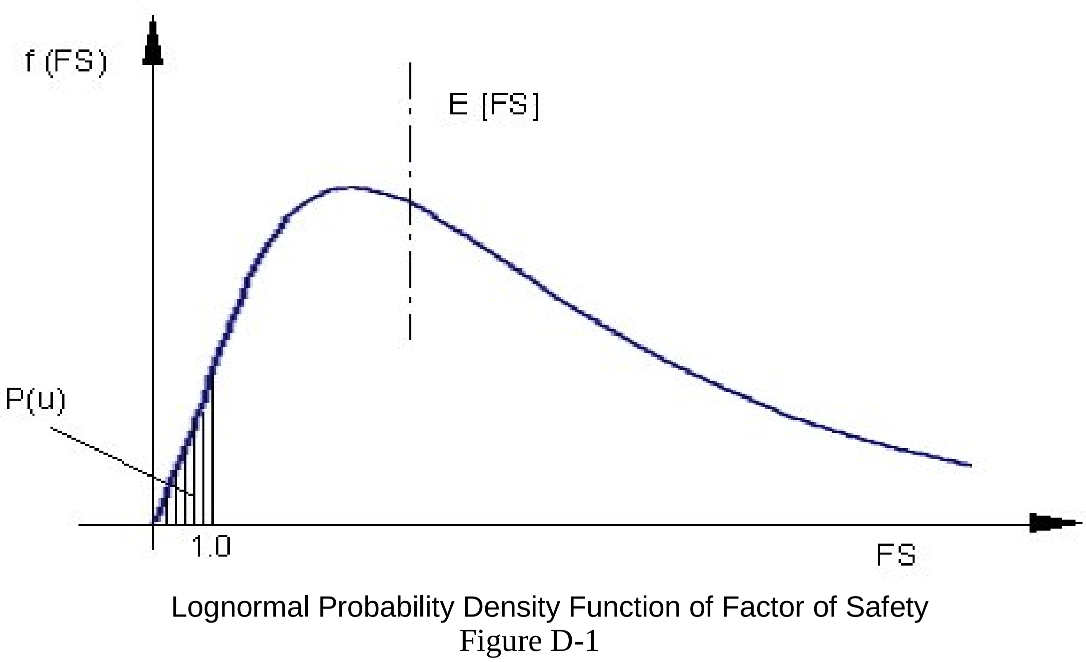
The probability of unsatisfactory performance is therefore

$P(FS<FS_u) = \Phi(-0.37) = 0.36$

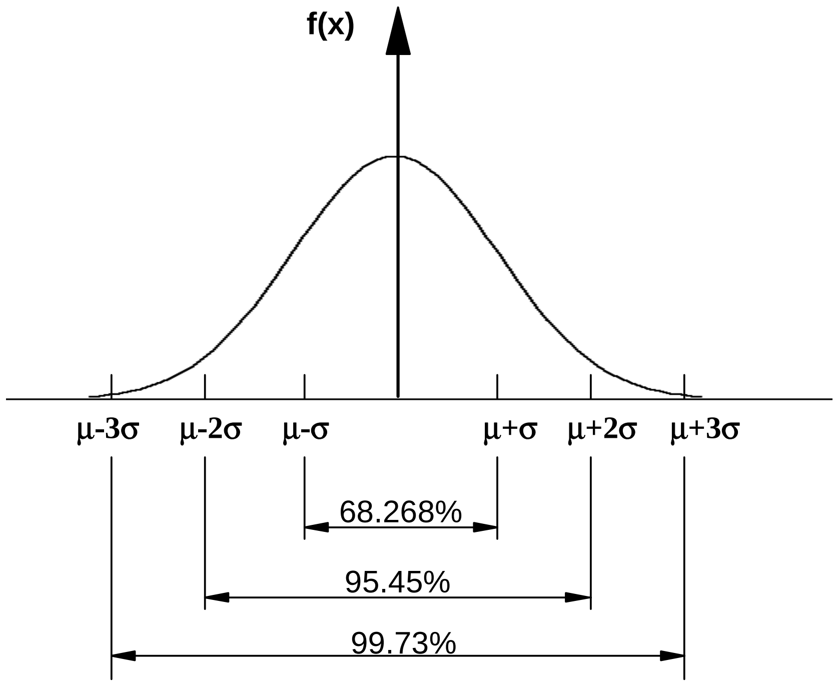
d. Note that this probability includes both unsatisfactory performance by slope failure and unsatisfactory performance by excessive deformation. If these two risks imply different consequences, the probabilities would have to be separated. In this case the probabilities of unsatisfactory performance by failure is a subset of unsatisfactory performance by excessive deformation. Therefore, the probability for slope failure is 0.14 while the probability for unsatisfactory performance by excessive deformation excluding slope failure is 0.36-0.14=0.22.



Transformed Probability Density Function of Factor of Safety



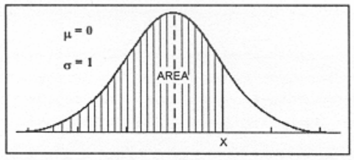
Lognormal Probability Density Function of Factor of Safety
Figure D-1



Normal Density Function
Figure D - 2

Embankment
Figure D - 3

Table D-1. Area Under the Standard Normal Probability Density Function



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table D-2
COEFFICIENT OF VARIATION

Parameter	Coefficient of Variation (%)	Reference
Unit Weight	3	Harr (1987)
Coefficient of Permeability (k)	90	Harr (1987)
Angle of Internal Friction (Sand) (ϕ)	10	LD25 Rehab Report (1992)
Cohesion (Undrained shear strength) (c)	40	Harr (1987)
Wall Friction Angle (δ)	20	S&W Report (1994)
Wall Friction Angle (δ)	25	ETL 1110-2-354
Earth Pressure Coefficient Sand (K)	15	Calculated
Downdrag Coefficient Sand (K_v)	25	Calculated
Standard Penetration Test	26	Harr (1987)
Standard Cone Test	37	Harr (1987)
Lag Factor (Silt and Sand Levees)	20	Clough (1966)
Dead Load	10	Harr (1987)
Live Load	25	Harr (1987)
Wind Load	37	Harr (1987)
Impact Load	30	ETL 1110-2-354
Hawser Pull Load	20	ETL 1110-2-354
Seismic Force	30	ETL 1110-2-354
Constant of Horizontal Subgrade Reaction (n_h)	25	ETL 1110-2-354
Group Reduction Factor (R_g)	8	ETL 1110-2-354

Table D-2
COEFFICIENT OF VARIATION

Parameter	Coefficient of Variation (%)	Reference
Cyclic Reduction Factor (R_c)	25	ETL 1110-2-354
Timber Piles, Sand Foundation		
Ultimate Compression Capacity	25	
$\phi = 35°$	22	Calculated
$\phi = 40°$	38	Calculated
Ultimate Tension Capacity	18	Calculated
Pile Diameter	8	Historic Data

APPENDIX E

EXPERT ELICITATION IN GEOLOGICAL AND GEOTECHNICAL ENGINEERING APPLICATIONS

E-1. Background. Many engineering evaluations are not amenable to quantitative analytical methods to determine the probability of unsatisfactory performance. Expert elicitation is one of several methods acceptable in USACE guidance documents for use in reliability analyses and risk assessments and has been used for various Corps projects. Expert elicitation is the formal quantification of expert opinion into judgmental probabilities. This document discusses how to structure a process of expert elicitation such that defensible probabilities result.

Risk analysis involves a large number of considerations - only a fraction of which are amenable to modeling and analysis. An analysis of modes of geotechnical failure (Wolff, 1998) indicated that expert elicitation was an appropriate probabilistic approach for analysis of seepage and piping through embankments, seepage through rock foundations, rock foundation stability, and erosion of soil and rock. The use of expert opinion in risk analysis allows the inclusion of uncertainties that might otherwise be difficult to calculate or quantify. Experienced engineers have long been required to evaluate opinions on many of these uncertainties. Judgmental probability is one way to quantitatively incorporate such evaluations into risk analysis.

The mathematical theory of probability is satisfied as long as the probabilities of exclusive and exhaustive events sum to 1.0. Thus, in some applications, probability is taken to mean the relative frequency of an event in a large number of trials; whereas, in others it is taken to mean the degree of belief that some event will occur or is true. Both interpretations are scientifically valid; judgmental probability is based on the latter.

On a basic level, judgmental probability is related to ones willingness to take action in the face of uncertainty. In practice, the magnitude of judgmental uncertainty can be compared to uncertainties in other situations, which may involve repetitive events, such as simple games of chance. If there is a greater willingness to bet on drawing a heart from a deck of cards than on the potential existence of piping within the foundation of a dam, then the judgmental probability of that adverse condition must be less than 1/4.

E-2. A Systematic Process to Elicit Quantified Judgmental Probabilities. The elicitation process needs to help experts think about uncertainty, to instruct and clarify common errors in how they quantify uncertainty, and to establish checks and balances to help improve the consistency with which probabilities are assessed. The process should not be approached as a ‘cookbook’ procedure; however, it is important that a systematic process be used to obtain defensible results. Based on experience at various Corps projects, it is recommended that the following steps be used when eliciting expert judgment. Details, of course, should be tailored to special needs; consequently, one or more of these steps may be eliminated.

a. Prepare background data and select issues. The initiator of the risk assessment should perform the following tasks in advance of the expert elicitation panel.

(1) Assemble and review all relevant site-specific data; visit the site; review related generic case histories.

(2) Develop and screen potential failure mechanisms for the site.

(3) Construct a preliminary event or fault tree for the site that includes all relevant modes of failure. Construct an additional event or fault tree for each potential remediation alternative for which judgmental probabilities are needed.

An event tree is a drawing that lays out the possible chains of events that might lead to adverse performance. The tree starts at the left with some initiating event, and then considers all possible chains of events that might lead from that first event (Figure E-1). Some of these chains lead to adverse outcomes; some do not. For each event in the tree, a conditional probability is assessed, presuming the occurrence of all the preceding events. The probability of a chain of events is obtained from the product of the probabilities of the events composing that chain.

A fault tree is a drawing that lays out possible sets of flaws in an engineered system that might lead to adverse performance. The tree starts at the right with some performance condition, and then considers the sets of faults (flaws) in the system that could have caused the adverse performance (Figure E-2). Most of these faults could only occur if earlier faults had occurred, and thus the tree is extended backward. Conditional probabilities for each fault are assessed as for an event tree, but the probability of a set of faults is calculated by starting at the adverse performance and moving backward.

Event trees are often easier for experts to conceive, but may become too complex. Fault trees, which focus only on adverse performance, may fail to uncover important combinations of events. Event and fault trees require a strict structuring of a problem into sequences. This allows probabilities to be decomposed into manageable pieces, and provides the accounting scheme by which those probabilities are put back together. In the process of decomposing a problem, it is sometimes helpful to construct an influence diagram that shows the inter-relationships of events, processes, and uncertainties. This diagram can be readily transformed into an event or fault tree.

Event and fault trees disaggregate failure sequences into the smallest pieces that can realistically be defined and analyzed, and can only be used for failure modes that are reasonably well understood. Failure modes, such as piping, for which the failure mechanism is poorly defined, cannot be further decomposed. Where the failure mechanism is well understood, it is usually good practice to disaggregate a problem such that component probabilities fall with the range [0.01 - 0.99], or better still, [0.1 - 0.9].

(4) Ensure a complete understanding of how the results of the expert elicitation will be

used by others involved in the risk assessment process.

(5) Select issues and uncertainties relative to the event trees that need to be assessed during expert elicitation. Form the issues into specific questions for the expert panel.

The questions should be carefully selected to represent the issues of concern and to achieve the desired objectives.

The initiator is now ready to use the expert elicitation process to formulate judgmental probabilities for all relevant modes of failure for each potential remediation alternative.

b. Select a balanced panel of experts. The choice of experts is the most important step in determining success or failure. Individuals selected must have an open mind and be willing to objectively judge different hypotheses and opinions that are not their own. Depending on personality and experience, experts may be individuals with special knowledge or individuals with a strongly argued point of view.

The panel must have a facilitator who is an individual versed in the issues who manages and encourages panel activities. The facilitator should be unbiased with respect to the outcome of the expert elicitation process and the facilitator must take care so that the expert panel's unbiased opinion is solicited, aggregated, and documented. Experts can be solicited both from within and from outside the initiator's organization. Appropriate USACE guidance on the use of technical experts should be followed. It is important that all panel experts be willing to be objective, commit time, and interact with others in a professional manner. The elicitation process has been shown to be most successful with between four and seven participants. A support team may be present to address panel questions regarding the site or specific events being investigated.

c. Refine the issues with the panel, and decide on the specific uncertainties. This phase sets up the problem, identifies specific uncertainties to be addressed, and defines the structure among those uncertainties. The goals are clear definitions of the uncertainties to be assessed, making unstated assumptions explicit, and dividing the technical problem into components with which experts can readily deal. For time-dependent requirements, it is best to request cumulative probabilities at different points in time from the panel. A minimum of three different dates should be requested so that a hazard function can be calculated form a data fit of the cumulative values provided by the panel.

A review package should be distributed to the experts well in advance. This package may include critical assumptions, interpretation of the foundation and other features, selected design parameters, analyses conducted, graphs and tables comparing principle issues, related generic case histories, proposed remedial measures, and the documentation for the construction of the event or fault tree. Time should be allocated at the initial meeting of the expert panel to review this information, ask clarifying questions, hold discussions with personnel knowledgeable about the site, and to visit the site if practicable.

All involved in the expert elicitation process must fully understand how the probability values and decisions made by the expert panel will be used and mathematically manipulated by other elements involved in the risk or reliability analysis. Economic accounting procedures may apply the numbers generated by the elicitation process and arrive at conclusions and recommendations far different than the experts envisioned.

d. Train the experts and eliminate error in eliciting judgmental probability. The training phase develops rapport with the experts, explains why and how judgmental probabilities are elicited, and how results will be used. Experts may be reluctant to participate unless assured about the intended use of the outcomes. During this phase, the philosophy of judgmental probability is reviewed, and an attempt is made to bring motivational biases out into the open.

The initiators of a risk assessment using expert elicitation must make every attempt to avoid the introduction of errors and bias into the result. Experts are known to display the following patterns when quantifying judgmental probability (Kahneman, Slovic, and Tversky, 1982).

(1) When asked to estimate the probability of an event, experts tend to assess larger values for the occurrence of those events that come readily to mind.

(2) When asked to estimate a numerical value, experts tend to fix on an initial estimate and then adjust for uncertainty by moving only slightly away from this first number.

(3) When asked to estimate the probability that an event 'A' originates from some process 'B', experts tend to base their estimate on the extent to which A resembles B rather than on statistical reasoning.

(4) An expert who perceives that he has had control over the collection or analysis of data tends to assign more credibility to the results than does an expert who only reviews the results.

The above patterns may lead to several errors including overconfidence, insensitivity to base rate probabilities, insensitivity to sample size, misconceptions of chance and neglect of regression effects.

The simplest manifestation of overconfidence occurs when people are asked to estimate the numerical value of some unknown quantity, and then to assess probability bounds on that estimate. For example, a person might be asked to estimate the undrained shear strength of a foundation clay, and then asked to assess the 10 and 90 percent bounds on that estimate.

People typically respond with probability bounds that are much narrower than empirical results suggest they should be.

Another display of overconfidence occurs when people are asked to estimate the numerical value of either small (<0.1) or large (>0.9) probabilities. People consistently underestimate low probabilities (unusually low shear strength) and overestimate high probabilities (continued satisfactory performance of a structure). Empirical results verify this effect (Lichenstein, Fischoff, and Phillips, 1982). With training people can learn to calibrate their estimates of probabilities between 0.1 and 0.9 (Winkler and Murphy, 1977). However, when required to estimate probabilities outside this interval, people error due to overconfidence. Research also suggests that the harder the estimation task the greater the overconfidence.

The simplest manifestation of insensitivity to base-rate occurs when people focus on recent information while ignoring background rates. For example, regional rates of seismic events provide important information about risk; yet this background information may be discounted if site reconnaissance fails to uncover direct evidence of a seismic hazard – even though the reconnaissance may be geologically inconclusive.

Insensitivity to sample size occurs when people presume that the attributes (averages, standard deviations) of small samples are close to the attributes of the populations from which the samples were taken. People tend to overemphasize the results of a small suite of samples even though the fluctuations in the attributes from one small suite to the next can be great.

Misconceptions of chance are familiar in the "gambler's fallacy" that events average out. People expect that the essential attributes of a globally random process will be reflected locally. Local variations of soil properties about some spatial average are not corrected as more measurements are taken; they are just diluted with more data.

Neglect of regression effects occurs when people overlook the fact that in predicting one variable from another (e.g. dry density from compactive effort), the dependent variable will deviate less from its mean than will the independent variable. Exceptionally high compactive effort produces, on average, high – but not exceptionally high – densities; and the converse for exceptionally low compactive effort. Representativeness leads people to erroneously overlook this regression toward the mean.

Beyond these statistical errors, an additional source for error is motivational biases. These are factors, conscious or not, that lead to inaccurate or incomplete assessments. The desire to appear knowledgeable, and thus under report uncertainty or the desire to advance a special cause, and thus refuse to credit alternate points of view are typical examples.

The training phase explains the how people can quantify judgmental uncertainties, and how well judgmental probabilities compare to the real world. The goal is to encourage the experts to think critically about how they quantify judgment, and to avoid common sources of statistical errors and biases discussed above. The training phase might involve having experts explain how they think about uncertainty and how they use data in modifying uncertainties. A few warm-up exercises can expose systematic biases in the experts' responses. "Thought experiments," which have experts explain retrospectively how unanticipated outcomes of an engineering project might have occurred, serve to open up the range of considerations experts entertain.

e. Elicit the judgmental probabilities of individual experts in quantified degrees of belief. This phase develops numerical probabilities for the component events or faults identified in the structuring phase. The goal is to obtain coherent, well-calibrated numerical representations of judgmental probability for individual experts on the panel, and to aggregate these into the probabilities for the entire panel. This is accomplished by presenting comparative assessments of uncertainty to the panel members and interactively working toward probability distributions.

(1) Associate probabilities with descriptive statements. In the early phases of expert elicitation, people find verbal descriptions more intuitive than they do numbers. Such descriptions are sought for the branches of an event or fault tree. Empirical translations are then used to approximate probabilities (Table E-1). This technique has been shown to improve consistency in estimating probabilities among experts. However, the range of responses is large, and the probabilities that an expert associates with verbal descriptions often changes with context.

(2) Avoid intuitive or direct assignment of probabilities. It is common for experts who have become comfortable using verbal descriptions to wish to directly assign numerical values to those probabilities. This should be discouraged, at least initially. The opportunity for systematic error or bias in directly assigning numerical probabilities is great. More experience with the process on the part of the experts should be allowed to occur before directly assigning numbers. At this initial point, no more than order of magnitude bounds on the elicited numerical degrees of belief is a realistic goal.

Table E-1. Empirical Translations of Verbal Descriptions of Uncertainty

Verbal Description	Probability Equivalent	Low	High
Virtually impossible	0.01	0.00	0.05
Very unlikely	0.10	0.02	0.15
Unlikely	0.15	0.04	0.45
Fairly unlikely	0.25	0.02	0.75
Fair chance, even chance	0.50	0.25	0.85
Usually, likely	0.75	0.25	0.95
Probable	0.80	.030	0.99
Very probably	0.90	0.75	0.99
Virtually certain	0.99	0.90	1.00

Source: Vick (1997), and Lichtenstein and Newman (1967).

(3) Quantify probabilities of discrete events. The theory of judgmental probability is based on the concept that numerical probabilities are not intuitive. This means that the most accurate judgmental probabilities are obtained by having an expert compare the uncertainty of the discrete event in question with other, standard uncertainties as if he were faced with placing a bet.

For a practical example, consider a dam site at which a fault zone in the foundation is suspected. Some exploration has been carried out, but the results are not definitive. If the expert would prefer to bet on the toss of a coin rather than on the existence of the fault, the judgmental probability of the fault existing must be less than half (0.5). Should he prefer to bet on the existence of the fault over the roll of a six-sided die, then the judgmental probability of the fault existing would be greater than one-sixth (0.17), and so forth. Changing the payoff odds on the gambles is another way of bounding the assessment.

Research on expert elicitation has addressed a number of issues regarding whether questions should be expressed in terms of probabilities, percentages, odds ratios, or log-odds ratios. In dealing with relatively probable events, probabilities or percentages are often intuitively familiar to experts. However, with rare events, odds ratios (such as, "100 to 1") may be easier because they avoid very small numbers. Definitive results for the use of aids such as probability wheels are lacking; and in the end, facilitators and experts must choose a protocol that is comfortable to the individuals involved.

(4) Quantify probability distributions. Not all uncertain quantities involve simple probabilities of discrete events. Many are defined over a scale, and the issue is to assess a judgmental probability distribution over that scale. For example, the friction between a concrete mass and its rock foundation should have a value between 0 and 90^{o}. A probability distribution summarizes the relative uncertainty about the parameter's value lying within specific intervals of the scale. In expert elicitation it is convenient to represent probability distributions as cumulative functions, which graph the scale of the parameter along the horizontal axis, and the judgmental probability that the realized value is less than specific values along the vertical axis.

The process starts by asking the experts to suggest extreme values for the uncertainty quantity. It is useful to have the expert describe ways that values outside these extremes might occur. Then, the experts are asked to assess probabilities that values outside the extremes occur. Starting with extreme values rather than best estimates is important in guarding against overconfidence and anchoring. Asking the experts to conceive extreme scenarios makes those scenarios 'available,' and allows one to think about the extremes more readily.

As numerical values are elicited, the facilitator should begin plotting these on graph paper; however, at this point the plot should not be shown to the experts, because it may bias future responses to conform to the previous ones. As ever more assessments are made, they are plotted on the graph to begin establishing bounds and to point out inconsistencies.

In checking for consistency, it is useful to compare numerical results elicited as values to those elicited as probabilities. In the *fixed probability* approach, the expert is given a probability, and asked for a corresponding value of the uncertain quantity; or given a probability interval, and asked for corresponding ranges of the uncertain quantity. For example, "What value of the friction angle do you think has a 1/3 chance of being exceeded?" "What values of the friction angle do you think have a 50:50 chance of bounding the true value?" In the *fixed value* approach, the expert is given a value of the uncertain quantity and asked the probability that the true value is less than that value, or the expert is given a range of values and asked the probability that the true value lies within that range. For example, "Would you be more inclined to bet on the chance of the friction angle being within the range 25 to 35 degrees or on drawing a diamond from this deck of cards?" Limited research suggests that fixed value procedures produce probability distributions that are more diffuse and better calibrated than do fixed probability or interval procedures.

(5) Use of normalized or base-rate frequency to estimate probabilities. The normalized frequency approach for assessing judgmental probabilities starts with an observed, empirical frequency of similar events in dam inventories and allows the experts to adjust the rates to reflect local conditions. The approach is appealing in that it begins with empirical frequencies. On the other hand, the procedure increases anchoring bias and a number of issues make the procedure difficult to use in practice. These include, identifying a relevant subcategory of events in the dam inventories with which to compare the present project, the fact that dam incidents are seldom simple cause and effect, and the complex procedures and calculations involved in adjusting base-rate frequencies. This method should be used only with caution.

(6) Use of reliability analysis to assess probabilities. For some component events, engineering models are available for predicting behavior. In these cases, reliability analysis can be used to assess probabilities associated with the components. Reliability analysis propagates uncertainty in input parameters to uncertainties in predictions of performance. The assessment problem is changed from having experts estimating probabilities of adverse performance to estimating probability distributions for input parameters. Once probabilities for the input parameters are assessed, a variety of mathematical techniques can be used to calculate probabilities associated with performance. Among these are, first-order second-moment approximations, advance second-moment techniques, point-estimate calculations, or Monte Carlo simulation. Sometimes, experts elect to assess an additional component of uncertainty in the reliability analysis to account for model error. While there are many ways to do this, the most common is to assign a simple, unit-mean multiplier to the model output, having a standard deviation estimated by the experts to reflect model uncertainty.

f. Revise and combine individual probabilities into a consensus. Once the judgmental probabilities of individuals have been elicited, attention turns to aggregating those probabilities into a consensus of the panel. Consensus distributions often outperform

individual experts in forecasting because errors average out (Rowe 1992). Both mathematical and behavioral procedures can be used to form consensus distributions.

After the initial round of elicitation, the value assessments may be plotted on a graph to establish bounds and to point out inconsistencies to the expert panel. The facilitator may wish to discuss "outlier" values with the individual expert(s) to ensure that the questions were understood. The experts should be given the opportunity to revise their opinions. Additional rounds of elicitation may be required until the panel is satisfied with the results.

Mathematical procedures typically use some form of weighted sum or average to aggregate individual probabilities. The weights, if not taken equal, are based on experts' self-weightings, on peer-weights, or on third-party weightings (e.g., by the 'evaluator'). Caution must be exercised if the experts group into "schools of thought," and thus do not give statistically independent answers (Ferrell, 1985).

Behavioral procedures involve an unstructured process in which experts discuss issues among themselves in order to arrive at a consensus judgment. The concept is straightforward. The potential information available to a group is at least as great as the sum of the information held by the individuals. It is presumed that errors are unmasked, and that the group discussion resolves ambiguities and conflict. Empirical evidence supports this contention, but strong involvement of a facilitator is the key to success of the expert elicitation process.

E-3. <u>Verifying and Documenting Judgmental Probabilities</u>. Once a set of probabilities has been elicited, it is important to ensure that the numerical probabilities obtained are consistent with probability theory. This can be done by making sure that simple things are true, such as the probabilities of complementary events adding up to 1.0. It is also good practice to restructure questions in logically equivalent ways to see if the answers change, or to ask redundant questions of the expert panel. The implications of the elicited probabilities for risk estimates and for the ordering of one set of risks against other sets is also useful feedback to the experts.

For credibility and defensibility, the process and results of an expert elicitation should be well documented, reproducible, subject to peer review, and neutral. The results of the process should also pass a “reality check” by the initiator’s organization. The process should be documented such that it is possible, in principal, to reproduce all the calculations involved and to arrive at the same answers. Calculation models should be fully specified. All questions asked and the responses of the experts should be tabulated. The source of all data and estimates in the study should be traceable to a person or a report. This means that the names of the expert panel members should be listed and the responses associated with each expert should be explicit.

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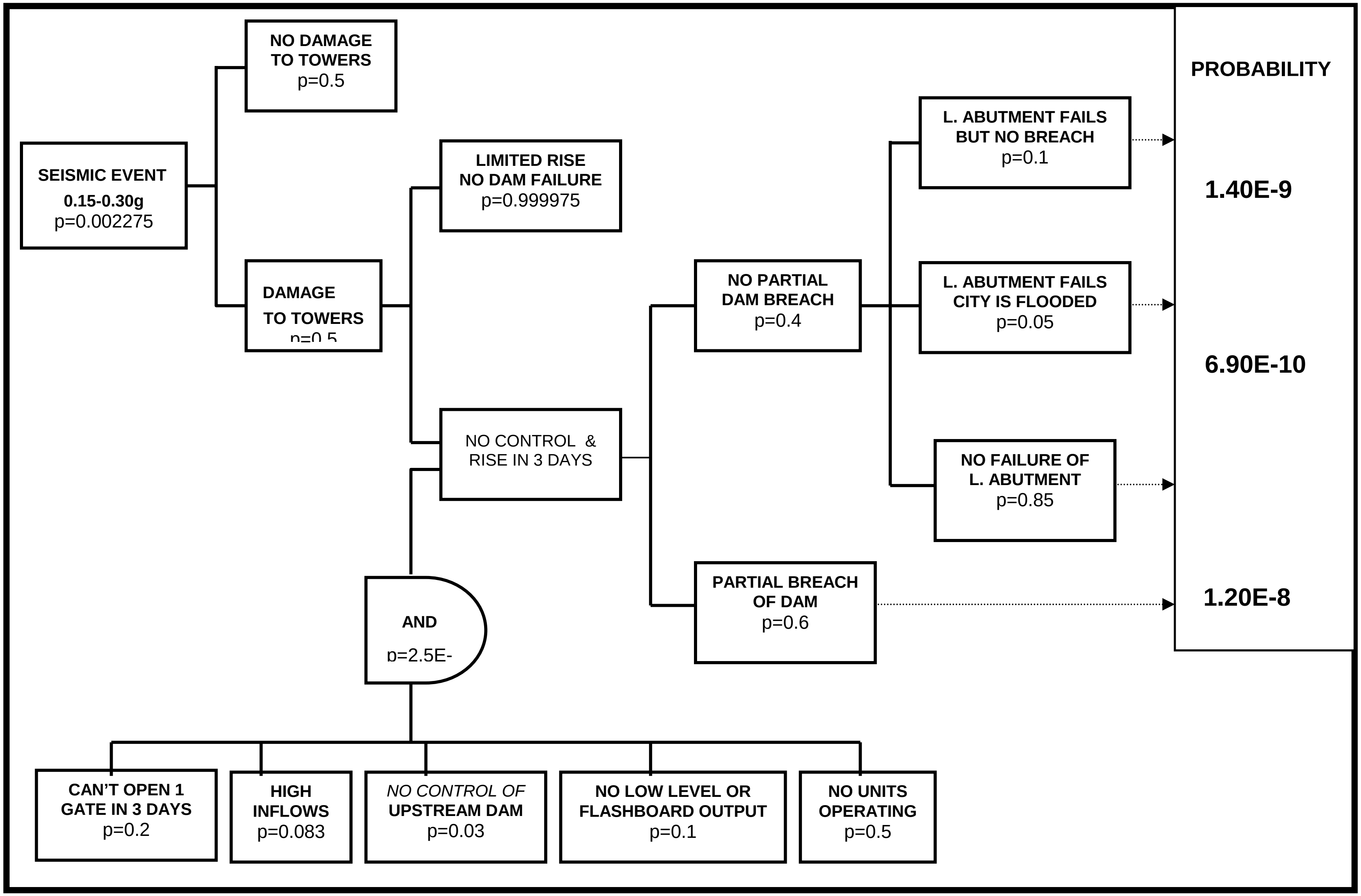
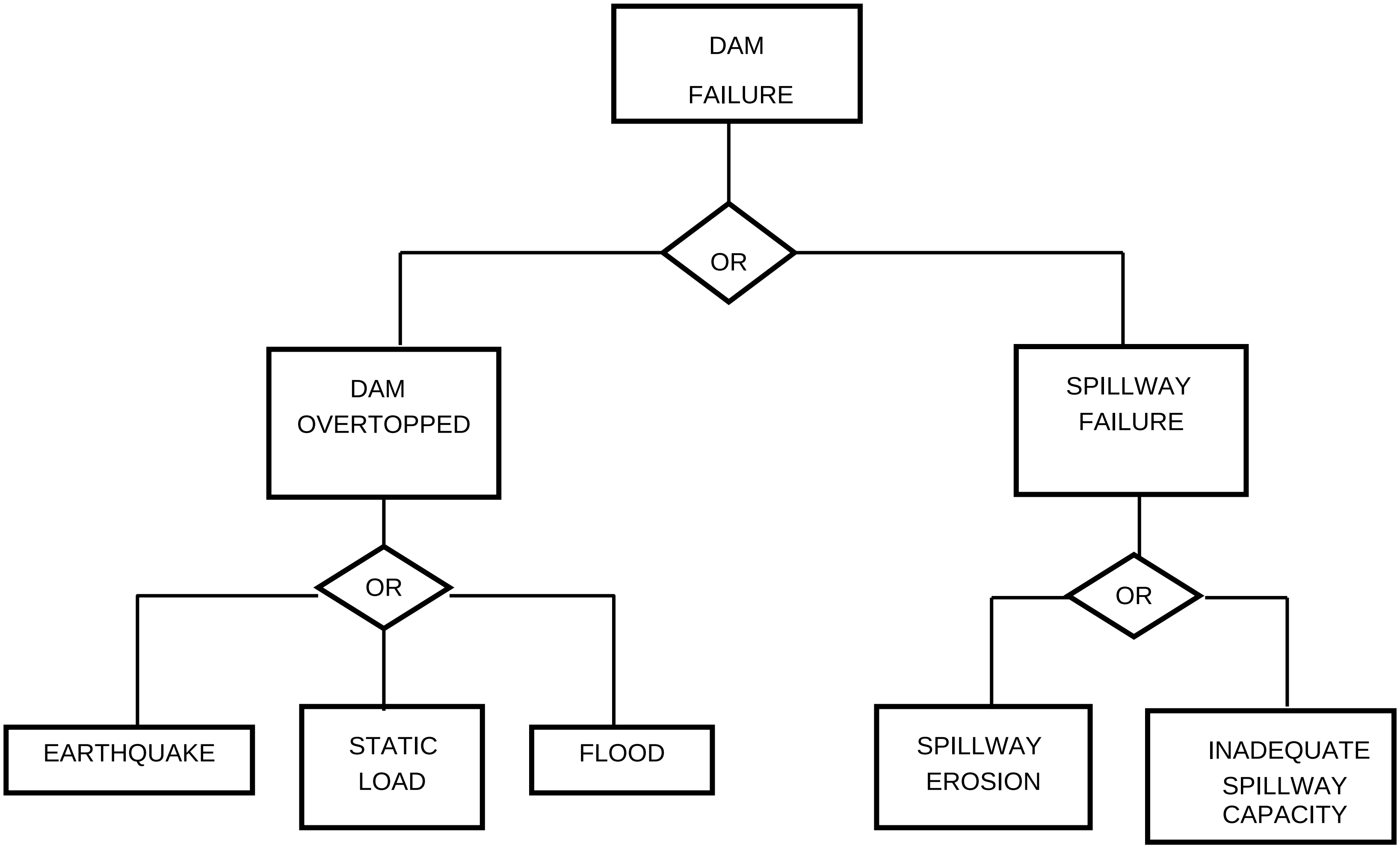


Event Tree: Earthquake loading of a dam.
Figure E-1.



FAULT TREE: Dam failure by overtopping or spillway failure.
Figure E-2

(D. MOSER)

APPENDIX F

RELIABILITY ANALYSIS FOR WALTER F. GEORGE LOCK AND DAM

FACT SHEET
WALTER F. GEORGE LOCK, DAM AND POWERHOUSE
MAJOR REHABILITATION EVALUATION REPORT
PREVENTION OF STRUCTURAL FAILURE

LOCATION: Located on the Chattahoochee River, approximately 1 mile north of Fort Gaines, Georgia, and approximately 1.6 miles upstream from the Georgia State Highway 37 bridge.

AUTHORITY: 1945 River and Harbor Act (House Document 342, 76th Congress, 1st Session), modified by the 1946 River and Harbor Act (House Document 300, 80th (Congress, 1st Session).

PURPOSE: To provide navigation, flood control, water supply, recreation and generate electric power on the Chattahoochee River.

TYPE OF DEFICIENCY:

The basic problem at Walter F. George involves seepage under the dam. This is a result of the deterioration of the limestone foundation material due to erosion and solutioning. Since this phenomenon cannot be accurately predicted, and distress in the structure does not become evident until the condition borders on imminent failure, this is a serious dam safety issue.

Seepage has been a problem at the Walter F. George Project since the beginning of the impoundment of the reservoir. The seepage problem can be attributed to three basic foundation conditions; (1) the presence of solution channels concentrated near the top of rock and cavernous horizons in the Earthy Limestone member of the Clayton Limestone Formation; (2) underseepage through the permeable section of the alluvial overburden; and (3) the high artesian heads in the Shell Limestone member.

HISTORY OF MAJOR SEEPAGE EVENTS:

1961: During stripping operations adjacent to the powerhouse non-overflow wall, two sinkholes developed near the dike centerline. A test excavation in the sinkhole area revealed extensive solution features. The decision was made to grout the earth dike sections on 5-foot centers to the top of the Shell Limestone.

1962: During filling of reservoir, numerous small boils developed in the drainage ditch at the downstream toe of the Alabama dike. To lessen the possibility of piping foundation material, relief wells were installed along the downstream toe of the floodplain portion of the dikes on 40-foot centers.

1968: Sinkhole activity increased on the left bank adjacent to the lock. Sinkholes occurred downstream of the dike, a spring developed adjacent to the lock wall, and a sinkhole was discovered in the lake. As a result of this sinkhole activity, remedial grouting was done along the Georgia dike, and a sand filter trench was constructed along the lock wall to intercept the seepage and prevent piping of material

1978: A supplemental design memorandum entitled “Study of Permanent Solutions to Underseepage Problems” was prepared which recommended the installation of a positive cutoff wall beneath the earth embankments to upgrade the dam to meet current standards.

A 24-inch thick concrete cutoff wall was chosen for installation to the bottom of the shell layer within the Earthy Limestone.

1981: Phase I of the cutoff wall was installed through a portion of the Georgia Dike.

1982: In between construction of the two phases of the cutoff wall, a boil was discovered immediately downstream of the powerhouse. The boil was a result of water flowing from a construction dewatering well (C-4) located at the downstream toe of the powerhouse. Construction piezometer (P-2) was found to be the source of the lake water flowing beneath the structure. The entrance point, dubbed “Hungry Hole”, was plugged with a tremie pipe. The Hungry Hole and the eroded channel beneath the powerhouse was filled with 175 cubic yards of termied concrete. Additional grouting was performed along the upstream face of the powerhouse, and the upstream western one third of the spillway.

1983-1985: Phase II of the concrete cutoff wall was installed through the remainder of the Georgia Dike and the Alabama Dike.

1992-1996: Walter F. George personnel reported an increase of flow rates in piezometer SP-5 and the powerhouse drains. In response to this flow increase, searches were started in 1993 for water entrance points in the lake bottom upstream of the structures. Simultaneous with the lake searches, grouting was conducted through holes drilled through the spillway monoliths. Sixty-four holes were drilled from the top of spillway into the Shell Limestone. In June 1996, foundation material was found in the draft tube floor, beneath drain D-12. During subsequent dives, it was confirmed that foundation material had piped, and was continuing to pipe through the drain. Based on this finding, the status of the project was downgraded to “a grave maintenance condition”.

FUNDS EXPENDED FOR MAJOR SEEPAGE REPAIRS:

1961: The total cost of the grouting program was $2.3 million.

1962: The total cost of the relief well system was $430,000.

1968: The total cost for the remedial work was $3.2 million.

1981-1985: The cost of installing the Phase I cutoff wall was $2.4 million, and the cost the Phase II cutoff wall was $9.1 million.

1982: The cost of plugging “Hungry Hole” plus additional grouting was $1.1 million.

1993-1996: Total cost for the lake searches and grouting from July 1993 through June 1996 was approximately $2.4 million.

IMPACTS OF FAILURE:

If no action is taken to correct the seepage problem at the Walter F. George Project, it will only be a matter of time before the concrete structures are completely undermined by the river. The erosion of the limestone foundation material is expected to occur slowly, reducing the risk of loss of life. However, if the situation is left unchecked, the erosion will eventually result in the complete destruction of the spillway and powerhouse structures.

PLAN OF IMPROVEMENT:

Construction of a positive concrete cutoff wall.

RELIABILITY ANALYSIS FOR WALTER F. GEORGE LOCK AND DAM

F-1. General.

a. A reliability analysis was performed for the overall project area within the limits of the project that will receive the recommended remedial work to be funded by the Major Rehabilitation program. The requirement for the proposed remedial measures is the result of the serious dam safety concerns for the project's integrity due to the high flows being measured from downstream sources of old piezometers, relief wells, drains, etc. Foundation materials of some size (1/2 to 1 inches) have been recovered from these discharges. Even as the known discharges are being remediated, many natural rock joints, core borings, construction piezometers, and dewatering wells located in the lake upstream of the structures and tailrace downstream of the structures may be in the process of developing into the next emergency.

b. Reliability analysis was conducted in accordance with the general guidance obtained in Memorandum from CECW-OM dated 29 SEP 1995, titled "Guidance for Major Rehabilitation Projects for Fiscal Year 1998" and ETL 1110-2-532, "Reliability Assessment of Navigation Structures". Additional guidance, methodologies and procedures were also obtained from the following references:

(1) Hodges Village Dam, Massachusetts, Major Rehabilitation Evaluation Report, June 1995, prepared by CENED.

(2) Wolff, Thomas F., 26 October 1996, Letter concerning Reliability Analysis at Walter F. George under contract DACWOl-97-P-0025 for CESAM.

(3) Wolff, Thomas F., 2 December 1996, Letter concerning review of assumed hazard function analysis under contract DACWO1-97-P-0025.

(4) Erwin, James W., 4 January 1997, Letter concerning Expert Elicitation and Cutoff Wall Design, under contract DACWOl-97-P-0l64

(5) Erwin, James W., 29 January 1997, Letter concerning Expert Elicitation and Cutoff Wall Design, under contract DACWO I -97-P-O 164

(6) Vick, Steven G. and Stewart, R.A., 1996 ASCE Conference on Uncertainty in Geotechnical Engineering, Risk Analysis in Dam Safety Practice., B.C. Hydro.

(7) Von Thun, J. Lawerence, 1996 ASCE Conference on Uncertainty in Geotechnical Engineering, Risk Assessment of Nambe Falls Dam, Bureau of Reclamation.

F-2. Event Tree.

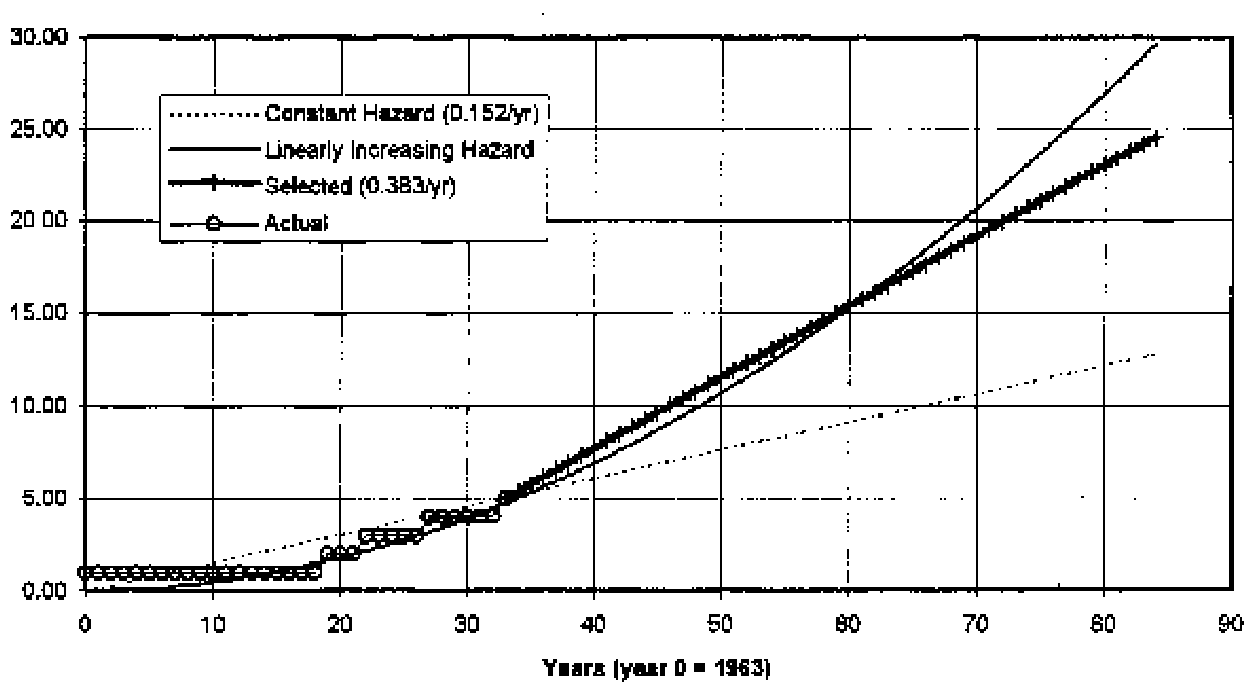
a. The reliability analysis is initiated with the preparation of an event tree with the main issue being the occurrence of significant seepage events over the course of the next fifty years. A significant seepage event is being defined as seepage from the reservoir which causes at least one of the following: a sinkhole, large increases in seepage flow, piping, or much higher than normal piezometric heads measured under and downstream of the dam. The development of a seepage event is not related to pool elevation since the greatest head differential is normal pool and tailwater conditions. The estimation of the frequency of significant seepage events 50 years into the future is based on the past history of events as they have been recorded over the life of the project and judgment as to the characteristic shape of the risk function over time. The risk of these seepage events is believed to have varied over time with an initial high risk period during and soon after construction and reservoir filling followed by a period of initially low but gradually increasing risk. The initial high risk stems from the fact that seepage problems are likely to show up upon initial exposure to high seepage gradients in any areas where existing subsurface conditions are conducive to seepage problems. The increase in risk over time after the initial high risk period is due to the progressive nature of seepage in limestone foundations, primarily due to progressive removal of fine-grained joint or cavity fill material in the limestone under high seepage gradients and to a lesser degree due to solutioning of the limestone. The deterioration of materials used for installation of relief wells and/or piezometers due to aging may also be a contributing factor. The increasing risk of seepage events over time is generally supported by the history of the seepage events at the project. Five significant seepage events have occurred over the life of the project from 1963 to date. The frequencies of the known significant seepage events, over the 33-year history of the project since reservoir filling in 1963, is shown in Table F-1. The frequencies have generally increased over time. The frequency since 1982 is significantly greater than the frequencies in prior years after the initial event in 1963. One of the events occurred immediately after reservoir filling in 1963 and the other four during the period 1982 to 1996.

Table F-1 - Time Intervals between Significant Seepage Events

Event No.	Event Start Date	Event End Date	Time Since Reservoir Filling, years	Years Between Event Starts	Cumulative Frequency	Incremental Frequency
1	1963	1963	0	NA	NA	NA
2	1982	1982	19	19	2/19=0.105	1/19=0.053
3	1985	1994	22	3	3/22=0.136	1/3=0.333
4	1990	1991	27	5	4/27=0.148	1/5=0.200
5	1996	1996	33	6	5/33=0.152	1/6=0.167

b. The average frequency for the 1963 to 1996 period was (5)/33=0.152 events per year. To estimate the frequency of significant seepage events 50 years into the future Figure F-1 was used. In Figure F-1 the actual seepage event are plotted as circles extended over the time period to the next seepage event. For the initial time period 1963 to 1996 a constant hazard function line of λ = 0.152 is plotted. The best fit to these initial seepage events is obtained using a Weibull distribution with a linearly increasing hazard function (see Figure F-1). However for computational ease it was decided to use a constant hazard function during the fifty year period from 1997 to 2047 instead of the linearly increasing hazard function. This constant hazard function line was determined by drawing a straight line for the time period of 34 to 84 years on the plot in Figure F-1, which closely approximates the curved linearly increasing hazard function line. This straight line is shown on Figure F-1 as the selected (0.383/yr) line. This straight line is a constant hazard function line with λ = 0.383. The value of λ which is the slope of the straight line was obtained as follows:

$$\lambda = \frac{Seepage Events}{Years} = \frac{(24.2-5)\, events}{(84-34)\, years} = 0.383$$



Number of Seepage Events
Figure F-1

c. The hazard function described above and presented in Figure F-1 is based upon the number of known significant seepage events in the past and provides an estimate of the expected number of known events in the future. The event tree includes some unknown events that would not have been counted as significant seepage events in the past if they have occurred. The appropriate frequency to be used with the initial event Al of the event tree should be the frequency of all events considered in the event tree including unknown seepage events. The frequency of unknown events is estimated based on back-analysis of the probabilities assigned by expert opinion to the events with "no problem" consequences most likely to be unknown. Specifically the probability of an event being unknown is estimated to be approximately the same as the probability of event C2 of the event tree. The probability of event C2 given that a significant event has occurred is the probability of the seepage being well connected to the reservoir only combined with the probability of the source being located far from structures. Therefore the probability of an event being unknown is estimated as 0.0880 based on the expert elicitation process and the probability of an event being known is estimated as the complement 0.9120. The frequency of unknown events is estimated as (0.0880/0.9120) x 0.383 = 0.037 events per year for the 1997-2047 period. The total frequency of known and unknown significant seepage events used for the initial event Al of the event tree is estimated as 0.383+0.037 = 0.420 events per year over the 1997-2047 period. The expected number of known and unknown events over the 1997-2047 period is estimated as 0.420x50 = 21.0 events for the same period.

d. Poisson Distribution on the Number of Possible Events in a Time Period t. The hazard function value h(t) = 0.420 events per year indicates that each year, there is a constant probability of p = 0.42 that an event will occur. Hence, over a long period of t years, the expected number of events N is

$$E[n] = tp = 0.42\ n$$

For a fifty-year period, the expected number of events is 21.

However, consider the fact that the number of events in a 50-year period will not always be exactly 21, as the process is random. In some periods there might be 23, 18, etc. Over a long time with n approaching infinity, the number of events would approach 0.42N.

When the hazard function is constant, events are assumed to occur at an average rate that is constant with time. In this case the probability distribution on the number of events N in the time period t is defined by the Poisson distribution, a discrete distribution given by

$$\Pr\,[(n = x) \mid t] = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where

$\Pr[(n = x) \mid t)$ is the probability that the number of events N in time period t will equal the value x

λ is the average number of events per time, which can be estimated from historical data, and

e is the base of the natural logarithms

Substituting the values x = 1, 2, … in the above equation, with $\lambda = 0.42$ and t = 50, the probability values in Table F-2 are obtained. These give the probability distribution on the number of seepage events in a 50-year period. Values less than 4 and above 41 have negligible probabilities.

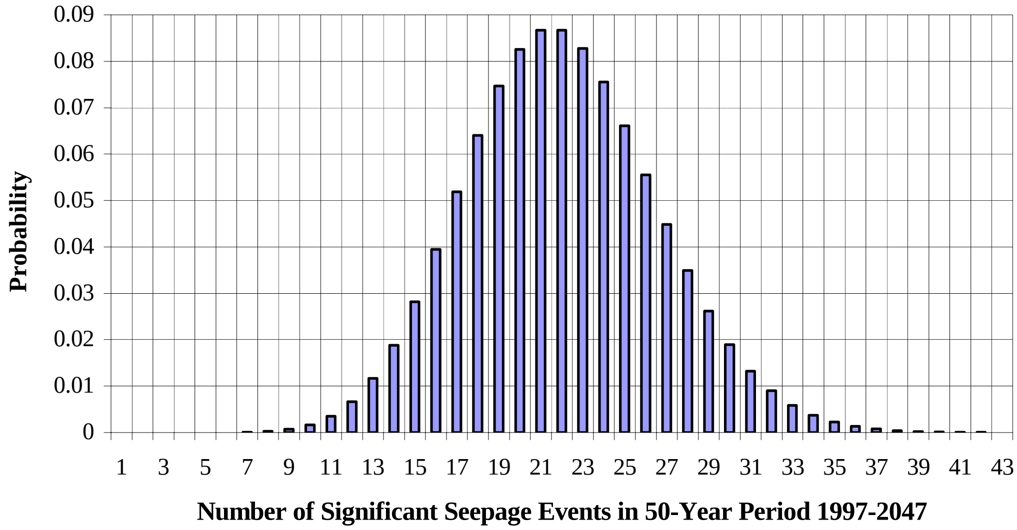
The probability that the number of events is between two values can be determined by summing the probabilities associated with the corresponding values. For example, the probability that the number of actual events in a 50-year interval is between 17 and 24 is

$$
\begin{aligned}
Pr\,(17 \leq n \leq 24) &= 0.06404 + 0.07472 + \ldots + 0.06613 \\
&= 0.6192
\end{aligned}
$$

which is more likely than not.

Table F-2. Probability of n Significant Seepage Events in 50-Year Period 1997 – 2047

No. of events, n	Pr		No. of events, n	Pr		No. of events, n	Pr
0	7.58E-10		15	0.0395		30	0.01327
1	1.59E-08		16	0.05184		31	0.00899
2	1.67E-07		17	0.06404		32	0.0059
3	1.17E-06		18	0.07472		33	0.00375
4	0.00001		19	0.08258		34	0.00232
5	0.00003		20	0.08671		35	0.00139
6	0.00009		21	0.08671		36	0.00081
7	0.00027		22	0.08277		37	0.00046
8	0.00071		23	0.07557		38	0.00025
9	0.00166		24	0.06613		39	0.00014
10	0.00349		25	0.05555		40	0.00007
11	0.00665		26	0.04486		41	0.00004
12	0.01164		27	0.03489		42	1.84E-05
13	0.01881		28	0.02617			
14	0.02821		29	0.01895			



Probability of n Significant Seepage Events in 50-Year Period 1997-2047
Figure F-2

e. The range of significant events to occur goes from 4 to 41 with probabilities occurring outside that range being so small they can be considered negligible. The number of events with the highest probability of occurrence is 21, as shown on Figure F-2. One should bear in mind that this number is the total number of events within the limits of the reservoir. The reservoir limit is deemed to be approximately 8,000 feet upstream of the existing dam and runs the entire width of the concrete monoliths in place (2000 feet±). Approximately 8,000 feet is the distance where the earthy limestone is discontinuous and the pool can enter into the Shelly Limestone directly.

F-3. Seepage Event Characteristics. Once a seepage event has occurred, then one must ask what characteristics of the event can manifest itself. The two main characteristics that will determine the consequences of the seepage event is its connection to headwater only, connection to both headwater and tailwater, or tailwater only and the distance that the source occurs from the structures.

a. A seepage connection to headwater only implies that there is no direct connection for flow to exit downstream of any structure but the full uplift pressure could be felt at the base of a monolith for most if not all of its base contact area. Seepage connected to both headwater and tailwater assumes that the reservoir head has a direct path from some area upstream of the dam, down to some strata below the bearing elevation of the structure, travel horizontally for its full length and exit some distance downstream. In classic seepage analysis, this exit point is next to the toe of the structure at the surface, but since the seepage media is channelized limestone, the exit could be some distance downstream without significant head loss. The final seepage connection would be to tailwater only.

This connection has no consequence for failure since is does not threaten the performance of the structures as defined herein.

b. The distance that the source occurs from the dam is defined by Near (0 to 300 feet) and Far (300 to 8000 feet). The 300 feet criteria was determined by the distance that barges could be placed against the dam and receive support for grouting during emergency repairs and the likelihood that a sinkhole would be noticed and the path length for seepage calculations. Far was considered for the 300 foot distance to the 8000 foot area in the reservoir where the impervious Earthy Limestone is discontinuous and the pool feeds directly into the Shelly Limestone.

F-4. Seepage Consequences. Once the characteristics of the seepage event have been determined, then some action called emergency repairs is required that will generate a cost or the event is unknown or located where repair action can not readily deal with it and the consequences will be evident later. These emergency repair actions are classified as successful, only partly successful, unsuccessful or not undertaken.

a. Successful is defined by repair action that fills the sinkhole and/or returns the observed flows and piezometric heads back to the status that was present prior to the event.

b. Partly successful will be defined as repair efforts that only remand a portion of the emergency situation back to its prior state. One example of this would be grouting of a sinkhole that only partially reduced measured flows or piezometric heads that remained above prior levels but the sinkhole was filled.

c. An unsuccessful repair is where the action has virtually no effect on the flow rates or the piezometric heads. An example of this would be grouting (chemical or cementitious) where the grout consistency was so thin and/or the seepage velocities so high that the grout merely washed out downstream or the grout set up so quickly that it plugged only the initial section of the opening.

d. Emergency repairs not taken means the seepage source was not discovered or surfaced too late for any repairs to be performed.

F-5. Emergency Repair Consequences. The direct result of the various unsuccessful emergency repair actions or lack of emergency repair action will be a major structure failure, a minor structure failure, or no problem for headwater only connected seepage events and excessive piping, excessive settlement beneath a monolith or no problems are observed for a seepage event connected to headwater and tailwater.

a. Major structural failure is defined as sufficient movement of a monolith to prevent it from performing its intended function. This movement could take the form of sliding or overturning from high uplift pressures to the degree that pool could not be maintained or power could not be generated or the lock could not be operated.

b. Minor structural failure is defined as relatively small movements of a monolith after which the functionality of the structure is retained but some repair is likely. An example of this would be a binding of a spillway gate.

c. Excessive piping is defined as flow measured from downstream sources that cannot be controlled or can only be reduced to a level, which still produces the transport of foundation materials from erosion. The path may be small or well below the foundation subgrade of the monolith and a loss of bearing area is not critical.

d. Excessive structural settlement or rotation will result from a cavity of sufficient size on the downstream end to cause loss of contact area back to the resultant. Based on the structural analysis, the monolith will remain stable until that happens, thereby producing a major structural failure without warning.

F-6. Results of Unsatisfactory Performance. All of these consequences result in varying degrees of unsatisfactory performance, which induce some amount of cost. These unsatisfactory performance events include a catastrophic dam failure, repairs to structure with a controlled drawdown of pool, or repairs to the structures without a pool drawdown being required.

a. Catastrophic dam failure is defined as uncontrolled flows resulting in a drawdown of the reservoir in less than 11 days. This criteria stems from the hydraulic analysis where for a given inflow, the complete drawdown of the pool must occur through the turbine draft tubes and the higher discharges will severely damage the turbines and some high erosional velocities will be felt on downstream structures.

b. The other events consist of repairs that require the pool to be drawn down but can wait until after 11 days or the pool is lost by a failure but the rate of loss is slow.

c. The third performance event is where repairs can be made without requiring the pool to be drawn down. These could be repairs similar to the emergency repairs in the pool or drilling from the top of dam.

F-7. Economic Consequences of an Unsatisfactory Performance Event. The following repair actions for economic analysis assume that the structure in question is a spillway monolith, which should provide an average level of effort between repairs to a lock monolith and repairs to a powerhouse monolith. This is not conservative since it is likely that more than one monolith could be involved and the costs would be increased accordingly.

a. Major Structure Failure - Catastrophic Dam Failure. This situation assumes that the monolith has moved, cracked and /or the gate which is in the opened position cannot be operated. Pool is being lost at a rate greater than 11 days. There will be turbine damage if the pool is intentionally drawn down at this rate. In order to effect repairs, a design must be prepared and plans and specifications produced for bid. This should take about three months to complete assuming O&M is used for funding. For construction, a cofferdam must be constructed around the upstream and downstream ends. With a 50-foot monolith width, assume three cells are required upstream and downstream to effectively dewater the work area. A total of six cells, assuming one month per cell to install, will take six months. Each cell costs some $200,000 to install with some $200,000 needed for mobilization and template construction. The existing monolith must be chipped out and the cavity in the foundation must be treated. Assume this takes two months to accomplish and costs $150,000. The 130-foot monolith must be placed in 5-foot lifts. This means 26 lifts are required, and at one week per lift, will take six months to perform. The total concrete volume is 18,648 cubic yards. Assuming $150 per cubic yard for placement costs, this portion of the work will cost some $3,000,000. The new gate must be set and the new driveway bridge erected and poured. This should take an additional three months to complete and cost about $600,000. The turbine blades will be reworked due to cavitation damage. This work will take six months and cost $200,000. Assuming that money is immediately available, a total of about 20 months is required to bring the structure back to pre-event condition. The total cost to accomplish this work is about $5,200,000. Approximately five months will be needed to refill the reservoir to pre-loss conditions based on normal flows. The pool could start to be refilled after the cofferdam is in place.

b. Major Structure Failure- Repair Structure with Controlled Pool Drawdown**.** This situation will require the same time line as that above except that due to the slow release of pool, the turbines will not be damaged and no damaging velocities will be felt downstream. The repair procedure and time should be the same as well as the cost less that accounted for the turbine repairs. This should add up to $5,000,000.

c. Minor Structure Failure- Repair with Pool Drawdown. This condition implies that the failure did not result in the immediate loss of pool and that the damage to the monolith is not severe enough to require replacement. The damage is severe enough to require chemical grouting and the velocities flowing through the cracks are too great for success. A controlled pool drawdown is required to effect repairs but the level is sufficiently low to prevent navigation, power generation, flood control and some recreation. Due to the numerous obstructions within the monolith, the grouting will take six months to accomplish using one crew full time and using an on site grout plant. The cost should add to about $750,000.00 based on out recent historical grouting efforts.

d. Minor Structure Failure- Repair without Pool Drawdown**.** This condition will be similar to the emergency repair efforts in terms of overall time but the costs should be somewhat less due to less problems with the grout mix design. Flood control may be affected, depending on the time of year of the event. We are assuming that one drill crew will be dedicated full time for five to six months. The overall costs for this work should be $500,000.

e. Emergency Repairs Prior to a Failure. This condition includes the remedial repairs to a discovered sinkhole found in the reservoir or known cavities beneath the structure that can be reached by drilling through the structure from the top driveway or from a barge anchored immediately upstream. The pool is not drawn down but the weather must not produce a severe fetch. We are assuming the average repair will take six months to perform using one five-person drill crew, the use of the government owned barge and support boat. The cost should run about $500,000.

f. No Problem. This condition is described as a result of the emergency repairs being only partly successful or unsuccessful and no immediate consequence is observed. This may be an intermediate condition where the emergency repairs are reinstated at a later date, perhaps after a potential upstream source is identified from soundings in the reservoir or observations from a diver.

F-8. Expert Elicitation.

a. General. The reliability of a structure shall be stated in terms of the probability of unsatisfactory performance of the feature. Unsatisfactory performance of a component may be indicated at various levels of performance, depending on the consequences of that performance level, from minor deflections to complete collapse of a structure. The guidance mandated that the probabilities of unsatisfactory performance be calculated using one of four methods: (I) Reliability Indices; (2) Hazard Functions; (3) Historical Frequency of Occurrence Analyses and (4) Expert Elicitation. Some of the above methods rely on a frequency approach requiring a significant database of historical events to judge the performance of a structure in the future. This data can take the form of the number of dam failures, the application of flood elevations to a reduced factor of safety, such as slope stability or conventional seepage through an earth embankment. In these cases, where classical failure mechanisms (sliding, overturning, structural deterioration, etc.) can be applied with confidence, then statistical analysis is warranted. Geotechnical engineering in general and karst bearing formations in particular do not lend themselves to the above. There are very few events to base any statistical analysis and the performance of the formations over time are very site specific, especially where normal operating conditions are the driving forces for degradation. Dam safety risk assessment can not be performed without liberal application of engineering judgment, which is subjective using expert opinion with a great deal of experience with the performance of the formation in question or similar materials elsewhere. The method is highly dependent on the experience and skill of the panel of experts selected and the procedures used to avoid biases in the probabilities. In consultation with CECW-ED and Dr. Wolff of Michigan State University, this office established the following procedures.

b. Board of Consultants. A Board of Consultants was convened on 21-22 May 1996 to review the underseepage problem experienced at Walter F. George Reservoir and to recommend potential solutions to the problem. The Board Members had many years of experience in geology and geotechnical engineering issues of major civil type structures in the United States as well as abroad, with representation from academia, engineering design, construction, and government. After much “brainstorming” of the issues, the group listed the main causes of seepage as: (1) inherent discontinuities in the Earthy Limestone formation from joints, natural weakness, sinkholes, etc; (2) manmade penetrations, such as core borings, monitoring and dewatering wells, sheetpile penetrations, and excavations for foundations; (3) erosion and solutioning of the formation due to seepage velocities and the chemical determination of the water.

The Board reinstituted the brainstorming session for potential solutions to the identified seepage causes. The group listed five possible courses of action that the District could undertake for the future of Walter F. George: (1) Do Nothing; (2) Continue current activities with grouting; (3) Install an impervious blanket upstream of the dam; (4) Perform innovative grouting; and (5) Install a concrete cutoff wall with a concrete apron immediately upstream of the dam structures. The only alternative that is considered capable of actually cutting off the seepage from upstream to downstream without a high risk of voids is the concrete cutoff wall. It is the District's position that the concrete cutoff wall is the only remedial measure that will accomplish the task without actually inducing other mechanisms for failure. Note: The NED plan must be an alternative that is feasible and reliable. Alternatives that are not feasible and reliable should be screened out early in the process and not carried to a conclusion. A brief write up discussing alternatives that are not considered feasible and reliable will suffice.

c. Expert Panel Member Selection. The District was then tasked to prepare a Major Rehabilitation Evaluation Report for Walter F. George Dam, using the recommendations of the Board of Consultants as guidance for remediation techniques. The first task of the report team was to prepare an event tree that describes the problems and resulting consequences. This was accomplished with input from geotechnical, geological, structural, hydraulic and hydrologic and economic disciplines. The event tree was created based on the "without project" or base condition and duplicated for the "with project (Base of wall at El. 20)" and the "with project (Base at El. –3.0 to –5.0)" condition. The use of statistical methods based on historical events could only be performed on the initial tree event. Probabilities for the subsequent events of the tree were based on engineering judgment using expert elicitation techniques since this was deemed to be the most applicable to the issue of seepage through limestone formations. The District assembled a team of geologists with many years of experience with geologic formations in general and with the behavior of the formations beneath Walter F. George in particular. This procedure was discussed with the technical counterparts in HQ and approval was given for this approach. It was suggested and agreed that an independent source outside of Mobile District be used in addition to the in-house personnel to give additional guidance and credence to the process. It was the District's belief that a renowned geologist in the private sector would give the impression of "less bias" but would be disadvantaged in predicting the behavior of formations in which he/she had no direct experience. The District desired a source of national stature that had not worked directly for Mobile District but who had some experience with the seepage events at this location. Dr. Jim Erwin, retired Division Geologist from the South Atlantic Division, fit all these requirements and agreed to serve in this capacity.

d. Procedures for Expert Elicitation. Expert elicitation is the use of highly experienced technical judgment to establish subjective probabilities to measure an individual's degree of belief concerning the likelihood of the occurrence of an event. Subjective probabilities are generally used whenever there is insufficient data to develop the probability of an event from historical frequency of occurrence or to conduct an analytical assessment of the probability. The method is highly dependent upon the experience and skill of the panel of experts selected and the procedures used to avoid biases in the probabilities. The procedure is primarily used for events that have a probability of occurrence between 0.001 and 0.999 since rare events with very low probabilities are more vulnerable to bias. Consistent methods must be used to reduce this bias.

(1) After each panel member was briefed on the project and the objectives of the Major Rehabilitation Report, they were asked to identify the most likely probability of a sub-event to occur as they proceeded through the event tree. They went back through the tree and established the range with upper and lower limits of probability for each given sub-event. The main activity of the event tree is a seepage event, which is defined as seepage from the reservoir which causes one or more of the following: a sinkhole; a large increase in seepage flow; piping of foundation material; or significantly higher than normal piezometric heads. These probabilities will have upper and lower limits of 0.999 to 0.001 and with all sub-events within that portion of the tree adding up to 1.0. The vote was given to the moderator by closed ballot for review to see if all panel members understand the event. If any analysis looked to be severely out of order, the moderator asked the respondent to reconsider the probability of the sub-event and ask any clarifying questions. Once all reasonable probabilities were returned, the probabilities for each sub event were averaged between the panel members and reported as the average most likely for that sub-event. The panel's high and low range of probabilities for that sub-event was averaged also. The final results of these probabilities were used in the calculation of risk assessment and economic consequence of that risk. This procedure was repeated through the entire event tree for the without project, with project (Base at El. 20 feet, NGVD) and the recommended plan, with project (Base at El. –3.0 to –5.0 feet, NGVD) conditions.

(2) The product of the most likely probabilities for each sub-event as it progressed throughout the event tree was calculated and these results have been incorporated into the economic analysis calculations.

e. With Project Probability of a Seepage Event. Since there is no historical data concerning the future number of likely events with the wall in place, the group was asked to predict the number of events likely to occur over the next 50 years given the reduction in pool area with possible joints and penetrations to act as sources of seepage. Estimates were given for the wall based at El. 20 and repeated for the wall extending down to El. –3.0 to -5.0 feet, NGVD. These numbers were totaled and averaged and then divided by 50 to give the average number of events per year. See Table F-3. These average frequencies were used below to estimate the probability of a given number of significant events over the 50-year period.

Table F-3. Predicted Number of Future Seepage Events for next the 50 Years with the Walls in Place

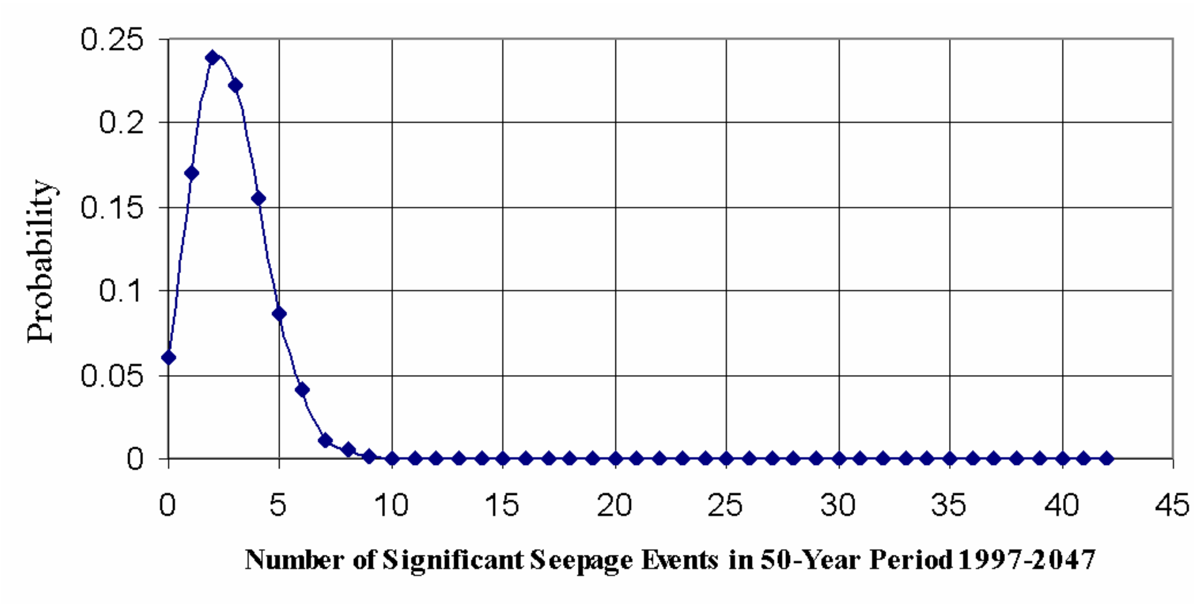
Plan El.	Dr. Erwin	John Baehr	Ross McCollum	James Sanders	Juan Payne	Average	Average per Annum
20	6	2	3	1	2	2.8	0.056
-3 to –5	0	0	1	1	0	0.4	0.008

The estimated probability of the total number of significant seepage events over the period 1997-2047 are shown in Table F-4 and illustrated in Figure F-3 for the wall based at El. 20. This probability is based on the Poisson equation:

$\mathrm{Pr} = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$ where, Pr is the probability of n events occurring in time interval t (50 years) and 2 is the frequency (0.056 events per year). The process was repeated with the bottom of the wall set 3 feet into the black clay stratum of the Providence Sand Formation (El. -3.0 to -5.0 feet, NGVD) and a frequency of 0.008 events per year was used to calculate the range of probabilities shown in Table F-5 and Figure F-4.

Table F-4. (Bottom of Wall at EL. 20) Probability of n Significant Seepage Events in 50-Year Period 1997-2047

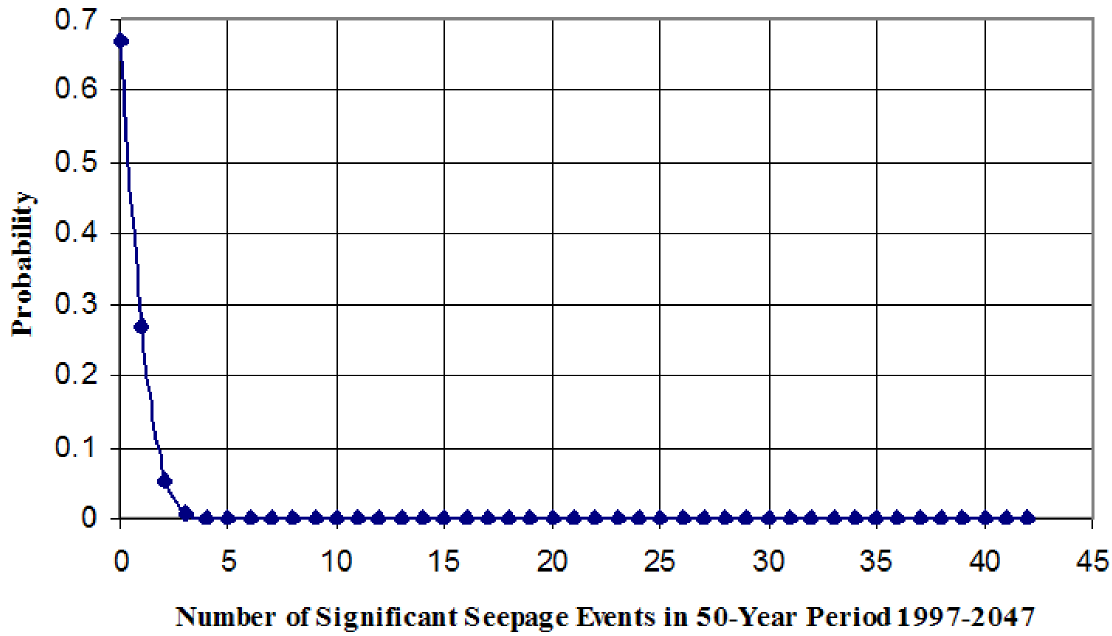
No. of events, n	Pr		No. of events, n	Pr		No. of events, n	Pr
0	0.06081		15	0.00000		30	0.00000
1	0.170268		16	0.00000		31	0.00000
2	0.238375		17	0.00000		32	0.00000
3	0.222484		18	0.00000		33	0.00000
4	0.15574		19	0.00000		34	0.00000
5	0.08721		20	0.00000		35	0.00000
6	0.0407		21	0.00000		36	0.00000
7	0.01628		22	0.00000		37	0.00000
8	0.0057		23	0.00000		38	0.00000
9	0.00177		24	0.00000		39	0.00000
10	0.0005		25	0.00000		40	0.00000
11	0.00013		26	0.00000		41	0.00000
12	0.00003		27	0.00000		42	2.61E-34
13	0.00001		28	0.00000			
14	0.00000		29	0.00000			



Probability of n Significant Seepage Events in Period 1997-2047 (Bottom of Wall at EL. 20)
Figure F-3

Table F-5. (Bottom of Wall at El. -3.0 to -5.0) Probability of n Significant Seepage Events in 50 Year Period 1997-2047

No. of events, n	Pr		No. of events, n	Pr		No. of events, n	Pr
0	0.67032		15	0.00000		30	0.00000
1	0.268128		16	0.00000		31	0.00000
2	0.053626		17	0.00000		32	0.00000
3	0.00715		18	0.00000		33	0.00000
4	0.00072		19	0.00000		34	0.00000
5	0.00006		20	0.00000		35	0.00000
6	0.000001		21	0.00000		36	0.00000
7	0.00000		22	0.00000		37	0.00000
8	0.00000		23	0.00000		38	0.00000
9	0.00000		24	0.00000		39	0.00000
10	0.00000		25	0.00000		40	0.00000
11	0.00000		26	0.00000		41	0.00000
12	0.00000		27	0.00000		42	9.23E-69
13	0.00000		28	0.00000			
14	0.00000		29	0.00000			



Probability of n Significant Seepage Events in 50-Year Period 1997-2047 (Bottom of Wall at El. -3.0 to -5.0)
Figure F-4.

F-9. Conclusions. This report has summarized the approach and results of the reliability analysis to assess the adequacy of the concrete structures against potentially erosive seepage flows from an increasing number of seepage incidents similar to those which have already occurred in significant numbers. These limestone foundations are the issue of the Evaluation and will be rehabilitated by the Recommended Plan. Pool levels are not an issue since normal pool creates the most differential head for seepage. The efficiency of the structure is not an issue since the monoliths have been designed for full uplift head for sliding and overturning and the recommended plan does not touch the structures. Therefore, all evaluations and rehabilitations have considered the limestone formations alone. Geotechnical features are rather unique for evaluation since there are few historical events and performance is very site specific. Significant engineering judgment from many years of experience has always been the hallmark of geotechnical engineering and probably always will be. It is our firm belief that use of highly experienced technical judgment by expert elicitation is the only way to analyze risk of future performance for the specific geotechnical events under consideration, i.e. erosive seepage in limestone, and the ability to locate and repair the seeps. We have known that Headquarters realized this when the criteria allowed the use of this technique and it is Mobile District's firm belief that it's guidance used by the experts produced the most technically feasible results with the least bias. This belief was facilitated by the development of a comprehensive event tree, which chronicled each cause and effect relationship to its logical conclusion and established costs for each action. These methods have been discussed and approved by technical disciplines in Headquarters and we trust what we have done here can be used and improved for the next Major Rehabilitation Report with similar issues.

APPENDIX G

RELIABILITY ANALYSIS FOR SEEPAGE AND PIPING FAILURE, WHITTIER NARROWS DAM

G-1. Introduction.

a. Whittier Narrows Dam is a 16,900-foot long embankment dam located 10 miles east of downtown Los Angeles. Condominiums and single-family residences line the downstream toe, less than 150 feet from the embankment. The dam is primarily used for flood control and groundwater recharge. Incidents of seepage sand boils and ponding on the downstream toe of the 4,000-foot long west embankment during relatively low pools led to a reevaluation of the existing seepage control system. The amount of seepage is directly related to pool elevation and the data shows the relationship between piezometric levels, seepage amount and pool elevation is constant over time.

b. The dam is founded on Recent Holocene Alluvium, which ranges in thickness from zero to about 120 feet. The upper foundation materials consist of loose to medium dense silty sand. The material becomes increasingly coarse and dense with depth. At depths greater than 30 feet, the material is typically very dense and most commonly classifies as poor to well graded sand with gravel.

c. The groundwater elevation downstream of the embankment ranges between the surface and 30 feet below surface and is very sensitive to seasonal conditions, groundwater recharge operations, and pool elevation behind the embankment. During flood control operations the groundwater rises very rapidly.

G-2. Computational Model.

a. A finite element seepage program was used to determine the flow lines and head drops in the vicinity of the toe. The escape gradient was then determined and the factor of safety against flotation was calculated. The conditional probability of failure was calculated for a pool elevation at elevation 229 feet, the approximate 100-year event. (For a total annualized risk assessment a range of pool levels would be analyzed to determine the total annualized consequences. This is a case where the analysis of pool elevations above the failure threshold is not required and only few pool elevations below the failure threshold elevation are required to determine consequences and related risks.)

b. The soils at the top of the foundation, which tend to be finer-grained and less permeable, were represented as a 10-ft thick semi-pervious top layer. The thickness of the deeper alluvium, which is coarser-grained and more permeable, varies from about 40 ft to about 120 ft along the length of the embankment. Density tests taken in the upper soils provide a range of unit weights, the average being 117.1 pcf.

c. Permeabilities for the foundation material were estimated based on data from pump tests, large-scale insitu tests and familiarity with similar materials. Due to the low permeability of the embankment relative to the foundation, flows through the embankment are insignificant. For all materials, the ratio of the horizontal to vertical permeabilities is assumed to be 4:1.

d. Sixty-two relief wells, spaced on 50-foot centers are located just downstream of the west embankment. Though approximately one-third of the wells no longer functioned, however the analysis assumed that the entire system functioned as designed.

e. In the finite element analyses, the relief wells were represented by a column of elements five feet wide and 50 ft high, at the location of the relief wells. Values of permeability were assigned to the elements in this column such that the same amount of head loss would occur through the column of elements as through a relief well. Head losses in the relief wells were based on the results of the pump tests.

G-3. Probabilistic Analyses.

a. The variation in the soil conditions along the dam made it difficult to select a single set of conditions for deterministic analyses. And while parametric analysis is useful for evaluating the relative importance of a variable, it provides no relationship between the value and its likelihood of occurrence. Probabilistic analyses (reliability analyses) are better able to cope with variable conditions because the combined effects of variations in all of the parameters can be evaluated on a logical theoretical basis.

b. A reliability analysis spreadsheet was developed which uses the Taylor series method to evaluate the coefficient of variation of the factor of safety, and computes the log normal reliability index (β_{LN}).

c. The reliability index (β_{LN}) is directly and uniquely related to "probability of unsatisfactory performance." "Probability of unsatisfactory performance" merely indicates the likelihood that some adverse event or condition, in this case a factor of safety less than 1.0, occurs. It is important to make clear that what is being computed in these reliability analyses is the probability that the factor of safety against the beginning of internal erosion and piping may be less than 1.0. This would be an undesirable condition, but it would not automatically result in failure of the dam. Depending on subsequent events, it might or might not result in serious consequences.

d. The Taylor series method involves these steps, as explained in the U. S. Army Corps of Engineers ETL 1110-2-556:

(1) For each random variable, the expected value and the standard deviation are determined. In this case there were five random variables: the permeability of the top stratum, the permeability of the lower stratum, the thickness of the lower stratum, the well flow losses in the relief wells, and the unit weight of the top stratum.

(2) The values of standard deviation were estimated using the "Three Sigma Rule" (Dai and Wang, 1992). The Three-Sigma Rule says that the lowest conceivable value is about

three standard deviations (three sigma) below the expected value, and the highest conceivable value is about three standard deviations above the expected value. The standard deviation is determined by estimating the highest and lowest conceivable values based on all available data and judgement.

(3) The values of standard deviation are computed using the following formula:

$$\sigma = \frac{HCV - LCV}{6}$$

where HCV = the highest conceivable value of the variable, and LCV = the lowest conceivable value of the variable.

(4) Seepage analyses are performed, and factors of safety against erosion at the downstream toe of the dam due to upward seepage are calculated.

(5) Expected values and standard deviations were determined and are presented in Table G-1.

Table G-1. Expected Value and Standard Deviation for the Probability Analysis

	T_{LOWER}, ft	k_{UPPER}, fpd	k_{LOWER}, fpd	k_{WELL}, fpd	γ_{SAT}, pcf
Expected Value	80	40	500	3200	117.1
Standard Deviation	40	15	150	800	9.1

G-4. <u>Results</u>.

a. The results of the reliability analysis calculations based on the finite element analyses are shown in Table G-2 below.

b. The results of this reliability study indicated that there was a 30 percent conditional probability that the factor of safety against the beginning of erosion and piping may be less than 1.0 for a pool elevation equal to the 100 year event. In addition to the analysis performed using the finite element method, a second analysis using a spreadsheet based on relief well equations in EM 1110-2-1901 indicated a 34 percent conditional probability of unsatisfactory performance for the 100-year pool elevation.

c. Table A-1 (page B-133) of Engineering Technical Letter 1110-2-556, categorizes reliability indexes and probabilities of unsatisfactory performance. By this classification, the condition at the West dam is ‘Hazardous.” Therefore, the condition at the west embankment represents an unacceptably high chance that erosion and piping will begin if the reservoir reaches its design pool elevation, 229 feet. As a result, the District remediated the problem by constructing a berm with a gravel drain on the downstream toe.

Table G-2. Taylor Series Reliability Calculations Based on Finite Element Analysis Results.

Analysis No.	T_{LOWER} (ft)	k_{UPPER} (fpd)	k_{LOWER} (fpd)	k_{WELL} (fpd)	γ_{SAT} (pcf)	i	FS	ΔFS	$(\Delta FS/2)^2$
1	80	40	500	3200	117.1	0.75	1.17		
2	40	40	500	3200	117.1	0.62	1.41		
3	120	40	500	3200	117.1	0.80	1.10	0.318	0.025
4	80	55	500	3200	117.1	0.70	1.25		
5	80	25	500	3200	117.1	0.80	1.10	0.157	0.006
6	80	40	650	3200	117.1	0.82	1.07		
7	80	40	350	3200	117.1	0.67	1.31	0.239	0.014
8	80	40	500	4000	117.1	0.72	1.22		
9	80	40	500	2400	117.1	0.78	1.12	0.094	0.002
10	80	40	500	3200	126.2	0.75	1.36		
11	80	40	500	3200	108.0	0.75	0.97	0.389	0.038
								sum =	0.086
								V_{FS} =	0.251
								β_{LN} =	0.509

APPENDIX H

HISTORICAL FREQUENCY OF OCCURRENCE ASSESSMENT OF EMBANKMENT FAILURE DUE TO PIPING

H-1. Introduction. Currently there are two different piping conditions identified that require different methods of analysis: 1) piping directly related to pool elevation which can be correlated to piezometric or seepage flow data, and 2) piping related to a complex seepage condition within the dam or its foundation that cannot be evaluated solely by analytical methods. Each of these piping conditions could be related to the development of higher pore pressures within the embankment resulting in a loss of shear strength and a resulting slope stability failure. However, the following discussion will be limited to seepage conditions leading to incidents or failure as a result of piping.

H-2. Piping Failure. An extensive analysis of piping has been conducted in the report *Analysis of Embankment Dam Incidents*, M. A. Foster, R. Fell, and M. Spanagle, UNICV Report No. R-374, September 1998. Following is the abstract from the report:

This report details the results of a statistical analysis of failures and accidents of embankment dams, specifically concentrating on incidents involving piping and slope stability. The aim of the study was to extend the existing compilations of dam incidents to include more details on the characteristics of the dams including dam zoning; filters, core soil types; compaction; foundation cutoff; and foundation geology. An assessment of the characteristics of the world population of dams was performed to assess the relative influence particular factors have on the likelihood of piping and slope instability.

Using the results of this analysis, a method was developed for estimating the probability of failure of embankment dams by piping taking into account the characteristics of the dam, age of the dam, dam performance and level of monitoring and surveillance. The so called “UNSW method” is intended to be used for preliminary risk assessments.

H-3. Procedure. Use of the procedures presented in the referenced report will generate an average annual probability of failure by piping which can then be used directly in analysis of risk associated with damages or loss of life. The procedure consists of the following steps:

(1) Identify the embankment zoning category shown on Figure H-1 (Figure 3.1 from UNSW report) that most closely matches the zoning of the dam to be evaluated.

(2) Determine the average annual probabilities of failure using Table H-1 (Table 11.1from UNSW report) for each of three modes of failure:

- -Piping through the embankment
- -Piping through the foundation
- -Piping from the embankment into the foundation

Select value corresponding to the age of the dam (i.e. less than or greater than 5 years).

(3) Calculate the weighting factors accounting for the characteristics of the dam for each of the three failure modes using Tables H-2 through H-4(Tables 11.2 through 11.4 from the UNSW report). The weighting factor for each mode of failure is obtained by multiplying the individual weighting factors for each characteristic (i.e. embankment filters, core compaction, foundation treatment, etc.)

(4) Calculate the overall probability of failure by piping by summing the probabilities for each of the three modes.

H-4. Use of Results.

a. As previously stated, the results of the above calculation will produce an average annual probability of failure by piping that can be used in further risk assessment calculations. The results may also be used in an assessment of the probability of piping failure compared to other dams of similar size, zoning, composition, geologic setting, operational history, etc. In this evaluation, the absolute value of probability is less important than the relative comparison of probability values. The method is intended to identify a dam whose probability of piping failure clearly stands out from other comparable dams. Such an analysis was recently (August 2000) conducted by the Baltimore District on Waterbury Dam. The results of the UNSW analysis for Waterbury were compared to several comparable Baltimore District dams located at sites with similar glacial geologic conditions. The analysis indicated that the probability of failure of Waterbury by piping through the embankment was more than 4000 times the probability of failure of comparable Baltimore District dams. Following is a tabulation of the results of that analysis:

Table H-5. Probability of Failure by Piping Through the Embankment.

WEIGHTING FACTORS (with typical range of values)	ASSIGNED WEIGHTING FACTOR VALUE						
	Waterbury (current condition)	**Waterbury (after repairs)**	Tioga-Hammond	Whitney Point	Cowanesque	Stillwater	Average of four Baltimore Dams
Zoning for dams after 5 years of operation (see Table 11.1)	24×10^{-6}	24×10^{-6}	25×10^{-6}	25×10^{-6}	25×10^{-6}	160×10^{-6}	5.875×10^{-5}
Embankment Filters (0.02 to 2.0)	2	0.02	0.02	0.2	0.02	0.02	0.065
Core Geological Origin (0.5 to 1.5)	1.5	1.5	1.0	1.0	0.5	1.0	0.875
Core Soil Type (0.3 to 5.0)	2.5	2.5	0.8	0.8	0.8	0.8	0.8
Compaction (0.5 to 5.0)	0.7	0.7	0.5	0.5	0.5	0.5	0.5
Conduits (0.5 to 5.0)	2.5	0.8	0.5	0.5	0.8	0.5	0.575
Foundation Treatment (0.9 to 2.0)	**5** to 10 **	0.9	0.9	0.9	0.9	0.9	0.9
Observations of Seepage (0.5 to 10.0)	**2** to 10	1.0	0.7	1.0	0.7	0.7	0.775
Monitoring and Surveillance (0.5 to 2.0)	0.5	0.5	0.8	0.8	0.8	0.8	0.8
W_E (total weighting factor)	6.56×10^{1}	1.9×10^{-2}	2.02×10^{-3}	2.88×10^{-2}	1.61×10^{-3}	2.02×10^{-3}	7.30×10^{-3}
P_eW_E (weighted probability)	$\mathbf{1.58 \times 10^{-3}}$	$\mathbf{4.54 \times 10^{-7}}$	5.04×10^{-8}	7.20×10^{-7}	4.03×10^{-8}	3.23×10^{-7}	$\mathbf{4.29 \times 10^{-7}}$

**Although it is greater than the maximum recommended value of 2, a foundation treatment weighting factor value of 5 was used based on the extreme foundation condition within the gorge of Waterbury Dam. Even this value may underestimate the potential negative influence of the gorge conditions compared with other dams that have failed by piping.

b. The analysis method was also applied to potential remedial alternatives to assess the relative benefits of the alternatives. The results of that analysis are reflected in the third column of the above tabulation showing the probability of failure of the dam after application of the proposed remedial repairs. The UNSW methodology may be used to provide a reasonable assessment of potential structural repairs (i.e. reconstruction of all or part of the dam or a cutoff wall), but cannot be used to assess the risk reduction associated with non-structural solutions (i.e. permanent reservoir drawdown or increased discharge capacity).

Assessment of Embankment Failure Due to Piping

The following procedure for determining the probability of failure by piping is based on the work by Foster, Fell, and Spanagle in the report published by the University of New South Wales (UNSW). References to tables or figures are from that report.

1. Identify the cross section on Fig H-1 (UNSW Figure 3.1) that most closely matches the section of the dam.

2. Select the base probability of failure of the dam by piping from Table H-1 (UNWS method Table 11.1) taking into account both the dam section and the age of the dam.

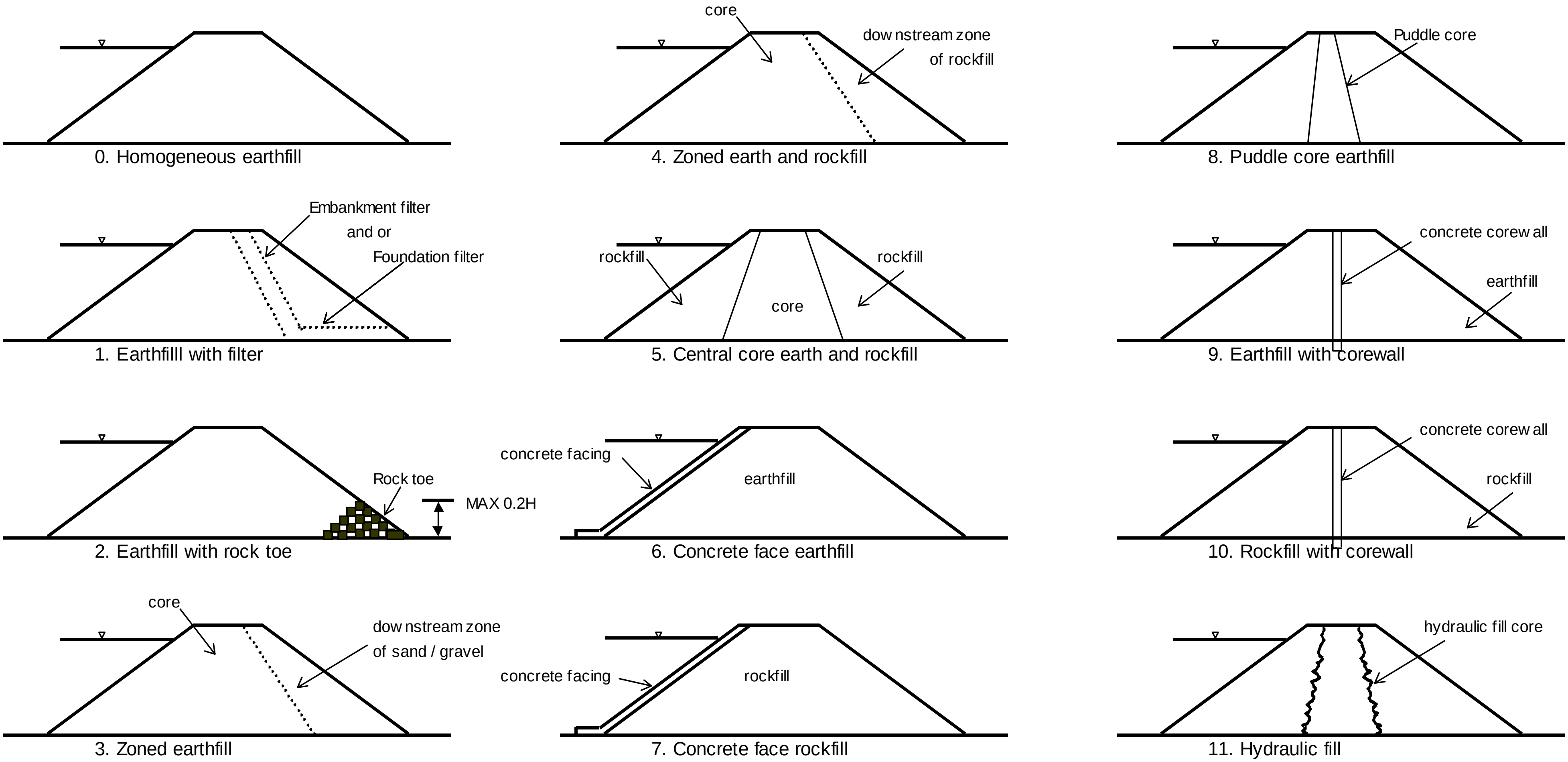
3. Determine the appropriate weighting factor from the list in Tables H-2 through H-4 (UNSW method Tables 11.2 through 11.4) for each of the 3 potential modes of piping failure (i.e. piping through the foundation, piping through the embankment, or piping from the embankment into the foundation).

4. Compute the probability of failure for each of the 3 potential failure modes.

5. Sum the 3 probability values to find the overall probability of failure by piping from all potential modes.

6. Perform the same analysis on other comparable dams considering size, zoning, site geology, construction methodology, and operation. Compare these results to that computed for the dam in question.

7. Comparison of probabilities can be used directly to assess the relative probability of failure, or computed probabilities can be used as input to further computations related to economic or environmental damage, or loss of life. Suggest use of a pool elevation that reflects the upper elevations of average annual range of pool elevations to determine consequences.



Dam zoning categories (UNSW Figure 3.1:)

Figure H-1

Reference : **Foster, Spanagle and Fell 1998**

Table H-1
(Table 11.1 UNSW): Average Probability of Failure of Embankment Dams by Mode of Piping and Dam Zoning.

ZONING CATEGORY	EMBANKMENT			FOUNDATION			EMBANKMENT INTO FOUNDATION		
	Average P_{Te} $(x10^{-3})$	Average Annual P_e $(x10^{-6})$		Average P_{Tf} $(x10^{-3})$	Average Annual P_f $(x10^{-6})$		Average P_{Tef} $(x10^{-3})$	Average Annual P_{ef} $(x10^{-6})$	
		First 5 Years Operation	After 5 Years Operation		First 5 Years Operation	After 5 Years Operation		First 5 Years Operation	After 5 Years Operation
Homogenous earthfill	16	2080	190						
Earthfill with filter	1.5	190	37						
Earthfill with rock toe	8.9	1160	160						
Zoned earthfill	1.2	160	25						
Zoned earth and rockfill	1.2	150	24						
Central core earth and rockfill	(<1.1)	(<140)	(<34)	1.7	255	19	0.18	19	4
Concrete face earthfill	5.3	690	75						
Concrete face rockfill	(<1)	(<130)	(<17)						
Puddle core earthfill	9.3	1200	38						
Earthfill with corewall	(<1)	(<130)	(<8)						
Rockfill with corewall	(<1)	(<130)	(<13)						
Hydraulic fill	(<1)	(<130)	(<5)						
ALL DAMS	3.5	450	56	1.7	255	19	0.18	19	4

Notes: (1) P_{Te}, P_{Tf}, and P_{Tef} are the average probabilities of failure over the life of the dam.
(2) P_e, P_f and P_{ef} are the average annual probabilities of failure.
Ref: Foster, Fell, & Spanagle 1998.

Table H-2
(Table 11.2 UNSW): Summary of the Weighting Factors for Piping Through the Embankment Mode of Failure.

FACTOR	GENERAL FACTORS INFLUENCING LIKELIHOOD OF FAILURE				
	MUCH MORE LIKELY	MORE LIKELY	NEUTRAL	LESS LIKELY	MUCH LESS LIKELY
ZONING	Refer to Table 11.1 for the average annual probabilities of failure by piping through the embankment depending on zoning type				
EMBANKMENT FILTERS $W_{E(filt)}$		No embankment filter (for dams which usually have filters (refer to text) (2)	Other dam types (1)	Embankment filter present - poor quality (0.2)	Embankment filter present - well designed and constructed (0.02)
CORE GEOLOGICAL ORIGIN $W_{E(cgo)}$	Alluvial (1.5)	Aeolian, Colluvial (1.25)	Residual, Lacustrine, Marine, Volcanic (1.0)		Glacial (0.5)
CORE SOIL TYPE $W_{E(cst)}$	Dispersive clays (5) Low plasticity silts (ML) (2.5) Poorly and well graded sands (SP, SW) (2)	Clayey and silty sands (SC, SM) (1.2)	Well graded and poorly graded gravels (GW, GP) (1.0) High plasticity silts (MH) (1.0)	Clayey and silty gravels (GC< GM) (0.8) Low plasticity clays (CL) (0.8)	High plasticity clays (CH) (0.3)
COMPACTION $W_{E(cc)}$	No formal compaction (5)	Rolled, modest control (1.2)	Puddle, Hydraulic fill (1.0)		Rolled, good control (0.5)
CONDUITS $W_{E\ (con)}$	Conduit through the embankment – many poor details (5)	Conduit through the embankment - some poor details (2)	Conduit through embankment - typical USBR practice (1.0)	Conduit through embankment - including downstream filters (0.8)	No conduit through the embankment (0.5)
FOUNDATION TREATMENT $W_{E(FT)}$	Untreated vertical faces or overhangs in core foundation (2)	Irregularities in foundation or abutment, Steep abutments (1.2)		Careful slope modification by cutting, filling with concrete (0.9)	
OBSERVATIONS OF SEEPAGE $W_{E(obs)}$	Muddy leakage Sudden increases in leakage (Up to 10)	Leakage gradually increasing, clear, Sinkholes, Seepage emerging on downstream slope (2)	Leakage steady, clear or not observed (1.0)	Minor leakage (0.7)	Leakage measures none or very small (0.5)
MONITORING AND SURVEILLANCE $W_{E(mon)}$	Inspections annually (2)	Inspections monthly (1.2)	Irregular seepage observations, inspections weekly (1.0)	Weekly - monthly seepage monitoring, weekly inspections (0.8)	Daily monitoring of seepage, daily inspections (0.5)

Ref : Foster, Fell, & Spanagle 1998.

Table H-3
(Table 11.3 UNSW): Summary of Weighting Factors for Piping Through the Foundation Mode of Failure.

FACTOR	GENERAL FACTORS INFLUENCING LIKELIHOOD OF FAILURE				
	MUCH MORE LIKELY	MORE LIKELY	NEUTRAL	LESS LIKELY	MUCH LESS LIKELY
ZONING	Refer to Table 11.1 for the average annual probability of failure by piping through the foundation				
FILTERS $W_{F(filt)}$		No foundation filter present when required (1.2)	No foundation filter (1.0)	Foundation filter(s) present (0.8)	
FOUNDATION TYPE (below cutoff) $W_{F(fnd)}$	Soil foundation (5)		Rock – clay infilled or open fractures and/or erodible rock substance (1.0)	Better rock quality	Rock - closed fractures and non-erodible substance (0.05)
CUTOFF TYPE (Soil foundation) $W_{F(cts)}$ OR CUTOFF TYPE (Rockfill foundation) $W_{F(ctr)}$	Sheetpile wall Poorly constructed diaphragm wall (3)	Shallow or no cutoff trench (1.2) Well constructed diaphragm wall (1.5)	Partially penetrating sheetpile wall or poorly constructed slurry trench wall (1.0) Average cutoff trench (1.0)	Upstream blanket, Partially penetrating well constructed slurry trench wall (0.8) Well constr4ucted cutoff trench (0.9)	Partially penetrating deep cutoff trench (0.7)
SOIL GEOLOGY TYPES (below cutoff) $W_{F(sg)}$ OR ROCK GEOLOGY TYPES (below cutoff) $W_{F(rg)}$	Dispersive soils (5) Volcanic ash (5) Limestone (5) Dolomite (3) Saline (gypsum) (5) Basalt (3)	Residual (1.2) Tuff (1.5) Rhyolite (2) Marble (2) Quartzite (2)	Aeolian, Colluvial, Lacustrine, Marine (1.0)	Alluvial (0.9) Sandstone, Shale, Siltstone, Claystone, Mudstone, Hornfels (0.7) Agglomerate, Volc. Breccia (0.8)	Glacial (0.5) Conglomerate (0.5) Andesite, Gabbro (0.5) Granite, Gneiss (0.2) Schist, Phyllite, Slate (0.5)
OBSERVATIONS OF SEEPAGE $W_{F(obs)}$ OR OBSERVATIONS OF PORE PRESSURES $W_{F(obp)}$	Muddy leakage Sudden increases in leakage (up to 10) Sudden increases in pressures (up to 10)	Leakage gradually increasing, clear, Sinkholes, Sandboils (2) Gradually increasing pressures in foundation (2)	Leakage steady, clear or not observed (1.0) High pressures measured in foundation (1.0)	Minor leakage (0.7)	Leakage measures none or very small (0.5) Low pore pressures in foundation (0.8)
MONITORING AND SURVEILLANCE $W_{F(mon)}$	Inspections annually (2)	Inspections monthly (1.2)	Irregular seepage observations, inspections weekly (1.0)	Weekly - monthly seepage monitoring, weekly inspections (0.8)	Daily monitoring of seepage, daily inspections (0.5)

Ref: Foster, Fell, & Spanagle 1998.

Table H-4
(Table 11.4 UNSW): Summary of Weighting Factors for Piping from the Embankment into the Foundation - Accidents and Failures.
SHEET 1 of 2

FACTOR	GENERAL FACTORS INFLUENCING LIKELIHOOD OF INITIATION OF PIPING - ACCIDENTS AND FAILURES				
	MUCH MORE LIKELY	MORE LIKELY	NEUTRAL	LESS LIKELY	MUCH LESS LIKELY
ZONING	Refer to Table 11.1 for the average annual probability of failure by piping from embankment into foundation				
FILTERS $W_{EF(filt)}$	Appears to be independent of presence/absence of embankment or foundation filters (1.0)				
FOUNDATION CUTOFF TRENCH $W_{EF(cot)}$	Deep and narrow cutoff trench (1.5)		Average cutoff trench width and depth (1.0)	Shallow or no cutoff trench (0.8)	
FOUNDATION TYPE $W_{EF\ (fnd)}$		Founding on or partly on rock foundations (1.5)			Founding on or partly on soil foundations (0.5)
EROSION CONTROL MEASURES OF CORE FOUNDATION $W_{EF\ (ecm)}$	No erosion control measures, open jointed bedrock or open work gravels (up to 5)	No erosion control measures, average foundation conditions (1.2)	No erosion control measures, good foundation conditions (1.0)	Erosion control measures present, poor foundations (0.5)	Good to very good erosion control measures present and good foundation (0.3 - 0.1)
GROUTING OF FOUNDATIONS $W_{EF(gr)}$		No grouting on rock foundations (1.3)	Soil foundation only - not applicable (1.0)	Rock foundations grouted (0.8)	
SOIL GEOLOGY TYPES $W_{EF(sg)}$, OR ROCK GEOLOGY TYPES $W_{EF(rg)}$	Colluvial (5) Sandstone interbedded with shale or limestone (3) Limestone, gypsum (2.5)	Glacial (2) Dolomite, Tuff, Quartzite (1.5) Rhyolite, Basalt, Marble (1.2)	Agglomerate, Volcanic breccia Granite, Andesite, Gabbro, Gneiss (1.0)	Residual (0.8) Sandstone, Conglomerate (0.8) Schist, Phyllite, Slate, Hornfels (0.6)	Alluvial, Aeolian, Lacustrine, Marine, Volcanic (0.5) Shale, Siltstone, Mudstone, Claystone (0.2)

Ref: Foster, Fell, & Spanagle 1998.

Table H-4 (Continued)
(Table 11.4 UNSW) (continued): Summary of Weighting Factors for Piping from the Embankment into the Foundation - Accidents and Failures.
SHEET 2 of 2

FACTOR	GENERAL FACTORS INFLUENCING LIKELIHOOD OF INITIATION OF PIPING - ACCIDENTS AND FAILURES				
	MUCH MORE LIKELY	MORE LIKELY	NEUTRAL	LESS LIKELY	MUCH LESS LIKELY
CORE GEOLOGICAL ORIGIN $W_{EF(cgo)}$	Alluvial (1.5)	Acolian, Colluvial (1.25)	Residual, Lacustrine, Marine, Volcanic (1.0)		Glacial (0.5)
CORE SOI L TYPE $W_{EF\ (cst)}$	Dispersive clays (5) Low plasticity silts (ML) (2.5) Poorly and well graded sands (SP, SW) (2)	Clayey and silty sands (SC, SM) (1.2)	Well graded and poorly graded gravels (CW, CP) (1.0) High plasticity silts (MH) (1.0)	Clayey and silty gravels (GC, GM) (0.8) Low plasticity clays (CL) (0.8)	High plasticity clays (CH) (0.3)
CORE COMPACTION $W_{EF\ (cc)}$	Appears to be independent of compaction – all compaction types (1.0)				
FOUNDATION TREATMENT $W_{EF(ft)}$	Untreated vertical faces or overhangs in core foundation (1.5)	Irregularities in foundation or abutment, Steep abutments (1.1)		Careful slope modification by cutting, filling with concrete (0.9)	
OBSERVATIONS OF SEEPAGE $W_{EF(obs)}$	Muddy leakage, Sudden increases in leakage (up to 10)	Leakage gradually increasing, clear, Sinkholes (2)	Leakage steady, clear or not monitored (1.0)	Minor leakage (0.7)	Leakage measured none or very small (0.5)
MONITORING AND SURVEILLANCE $W_{EF(mon)}$	Inspections annually (2)	Inspections monthly (1.2)	Irregular seepage observations, inspections weekly (1.0)	Weekly - monthly seepage monitoring, weekly inspections (0.8)	Daily monitoring of seepage, daily inspections (0.5)

Ref: Foster, Fell, & Spanagle 1998.

APPENDIX I

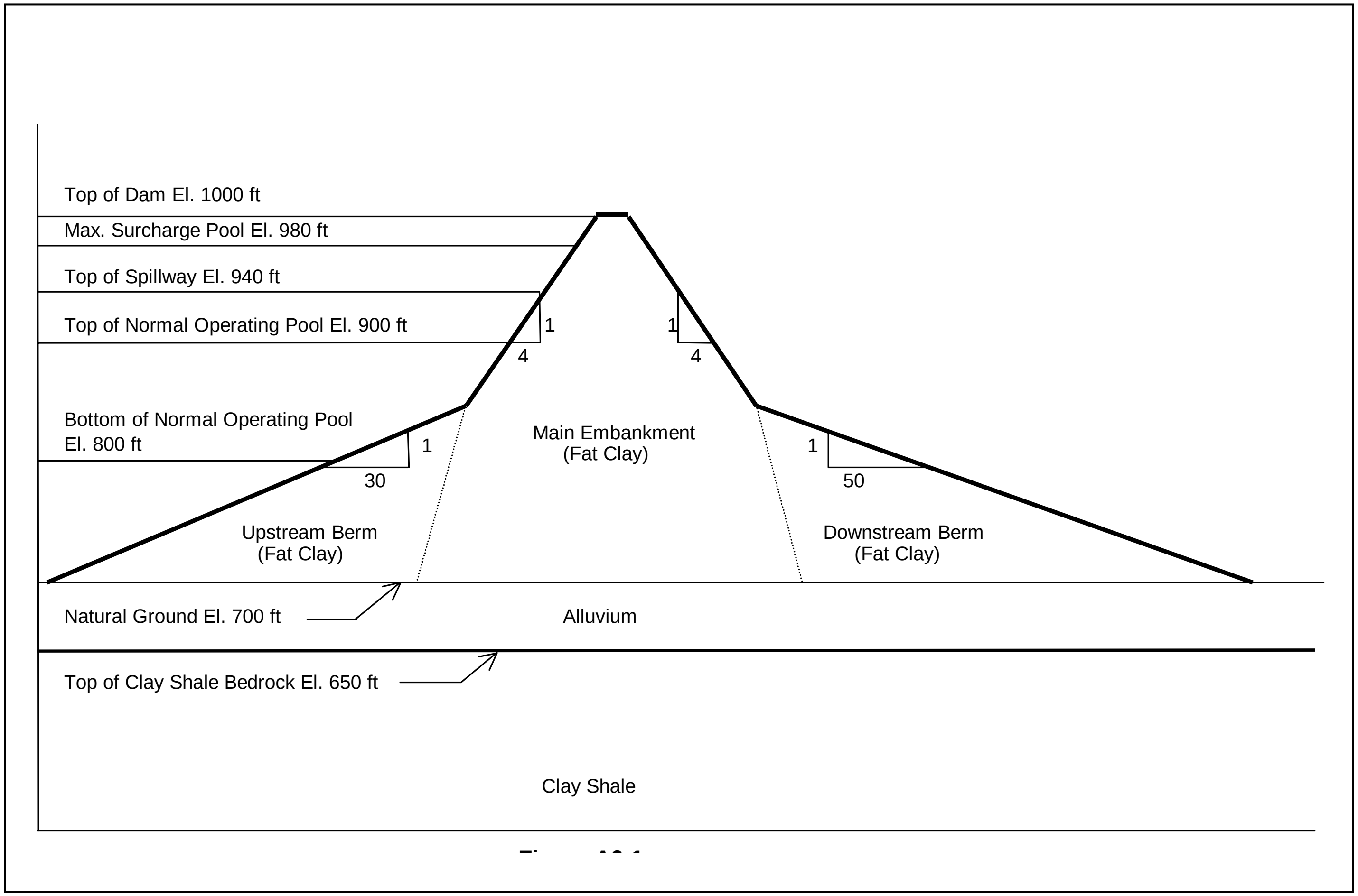
EXAMPLE PROBLEM FOR STEADY STATE SEEPAGE, SLOPE STABILITY RELIABILITY ANALYSIS

I-1. Introduction. Construction on Dam X started in 1950 and was completed in 1960. The project is a multi-purpose project for flood control, hydropower, environmental restoration and recreation. The dam is a rolled earth embankment and is approximately 300 feet high and approximately 2 miles long. The material used for the embankment came from required excavations and consists of fat clays and is fairly homogeneous, see Figure I-1. Dam X is a high hazard dam with several large cities located downstream within the flood plain.

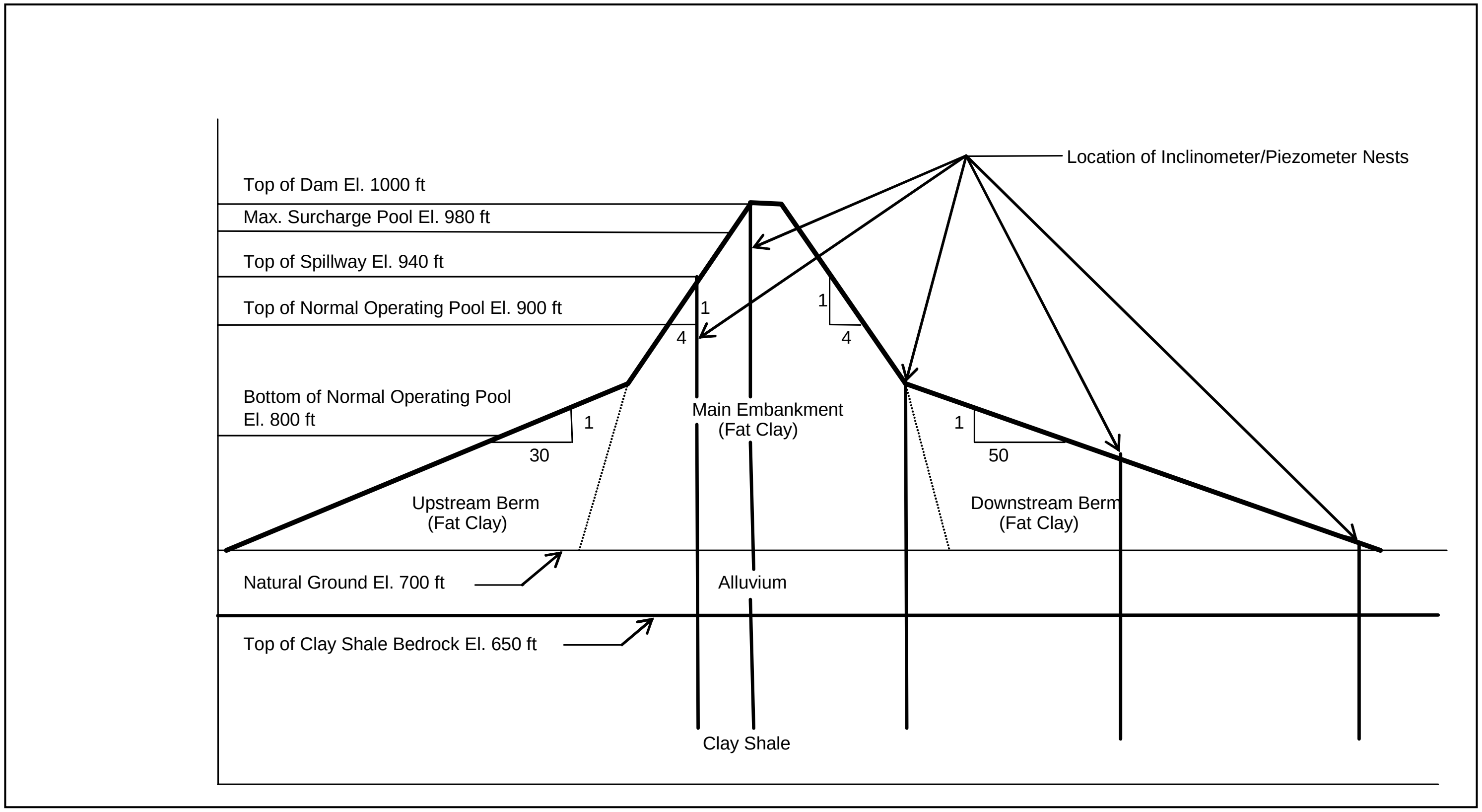
I-2. Current Project Conditions.

a. The original designers were concerned with the foundation conditions in an area approximately 1,000 feet long near the left abutment. The concern centered around a calcarious marine clay shale bedrock formation which is highly fractured and indicative of a high degree of movement during the river valley development. The original designers were concerned about how the foundation would behave after it was loaded with the embankment. Therefore, this area of the project was highly instrumented with settlement gauges, inclinometers and piezometers to monitor movement both during and after construction. Piezometers and inclinometers were placed in nests approximately 250 feet upstream of the crest of the embankment, at the upstream edge of the crest of the embankment and approximately 500 feet, 1000 feet and 1,500 feet downstream of the crest of the embankment, see Figure I-2. Settlement gauges were installed in the crest of the embankment. The normal operating pool ranges from elevation 800 feet to 900 feet. The exclusive flood control zone is between elevation 900 feet and 940 feet. The maximum surcharge pool or the maximum design pool elevation is 980 feet. Except for the historic record pool, the pool has fluctuated between elevation 800 and 900 feet. Some of the inclinometer data show movement in the downstream direction during and after construction within the clay shale bedrock. The data shows the rate of movement, in general, slowing down with time.

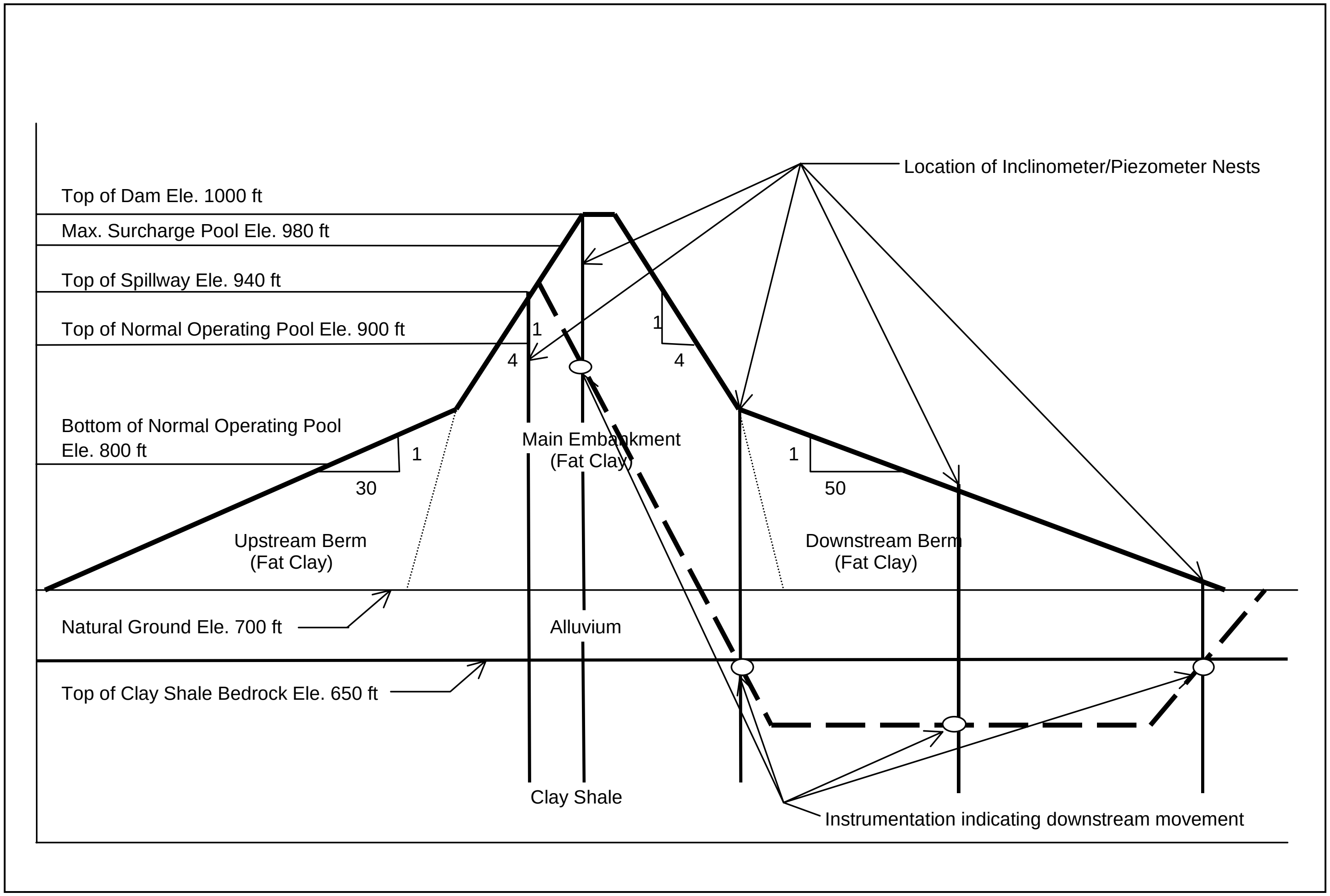
b. A recent flood event caused the pool to reach its record pool elevation of 940 feet (approximately 40 feet above the previous record pool). Shortly after the flood event, a physical inspection of the embankment showed signs of active movement. Cracks along the crest of the embankment where identified within the 1,000 foot reach describe above. The cracks also exhibited a vertical offset of approximately ½ of an inch when measured from the upstream side of the crack to the downstream side of the crack. A review of the instrumentation data revealed that the movements appeared to accelerate in response to the high reservoir pool. This situation is a significant concern since the maximum design pool is approximately 40 feet higher than the historic record pool. The review of instrumentation data also revealed that the movement appeared to represent a classic slope stability wedge failure, see Figure I-3.



Cross Section of Dam X
Figure I-1



Instrumentation Locations
Figure I-2



Movement Surface
Figure I-3

I-3. Problem Definition.

Additional sampling and testing of the bedrock shale material, within the movement zone, was performed after the signs of movement were identified. The testing revealed substantially lower shear strengths than those used in the slope stability analysis that was conducted in the original design phase of the project. The geologic history suggests that because of the movement experienced during valley formation, that the residual shear strength of the bedrock material had been mobilized and the new test results showed the shale material to be at residual shear strength. After reviewing the historic piezometeric data and comparing that to the assumptions made in the original design analysis, the design assumptions appear to be reasonable.

I-4. Project Site Characterization.

Table I-1 lists the representative soil properties used for the original design stability analysis. Table I-1 also gives the expected values for the same soil properties. In general, the expected values are different than the values used for design purposes. Within the Corps of Engineers, the 1/3 2/3 rule (the strength envelope used for design is drawn generally such that 1/3 of the test values are below the envelope and 2/3 of the test values are above) is generally used. The expected value is the value the designer thinks the material will exhibit in the field and can be substantially different than the values used for design purposes. The expected value for the weak shale is based on sampling and testing (that was conducted after the pool of record flood event) of the material located in movement zone.

Table I-1. Material Properties

Material Type	Unit Weight		Shear Strength Cohesion Φ			
	Design Value	Expected values	Design Value		Expected values	
			C	Φ	C	Φ
Embankment	115 pcf	125 pcf	0	23.0^{o}	0	27.1^{o}
Alluvial	100 pcf	105 pcf	0	15.0^{o}	0	18.0^{o}
Firm Shale	120 pcf	128 pcf	0	17.0^{o}	0	20.8^{o}*
Weak Shale	NA	102 pcf	NA	NA	0	8.0^{o}**

* This strength represents the cross bed shear strength in the shale.
** Residual shear strength of the shale.

I-5. Modes of Failure.

a. Looking at the physical distress and movement data from the inclinometers it appears that the problems experienced at Dam X are slope stability related. Therefore, a re-analysis of the stability of the dam was conducted at pool elevation 940 and 980 feet, in order to verify the conclusions of the instrumentation data. Using the expected values and the new shear strength data for the shale bedrock material along the apparent movement zone, a stability analysis was performed for a pool elevation of 940 and 980 feet.

b. The results of the analysis are shown below in Table I-2.

Table I-2. Results of Static Stability Analysis

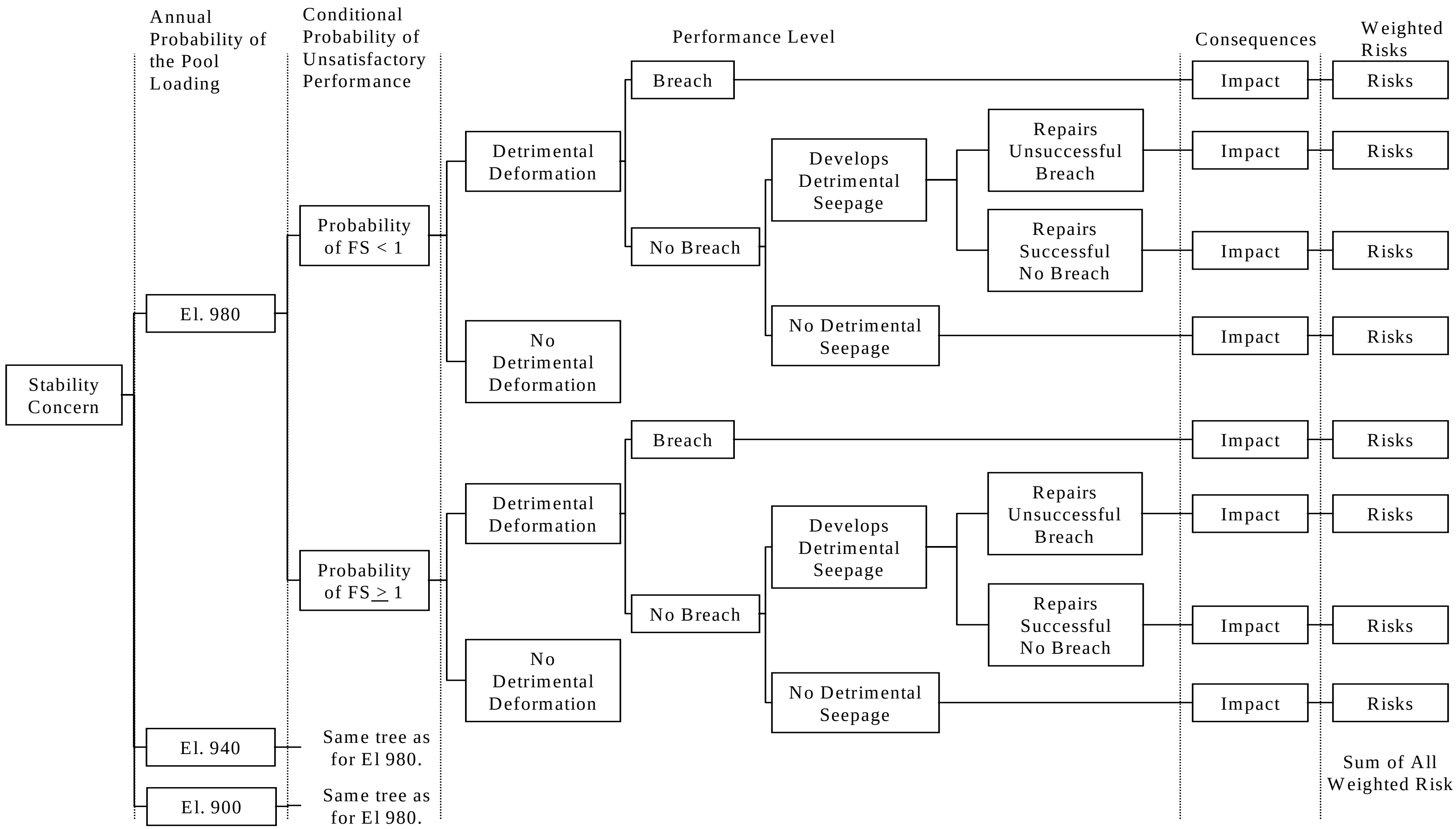
Reservoir Elevation	Factor of Safety	
	Original Design	Re-Analysis
940 feet msl	1.500	1.100
980 feet msl	1.200	0.997

c. The re-analysis shows that there is a significant concern associated with the stability of Dam X and a Major Rehabilitation Evaluation Report is warranted. The first thing that must be done after a decision is made to conduct a Major Rehabilitation Evaluation Report is to develop an event tree.

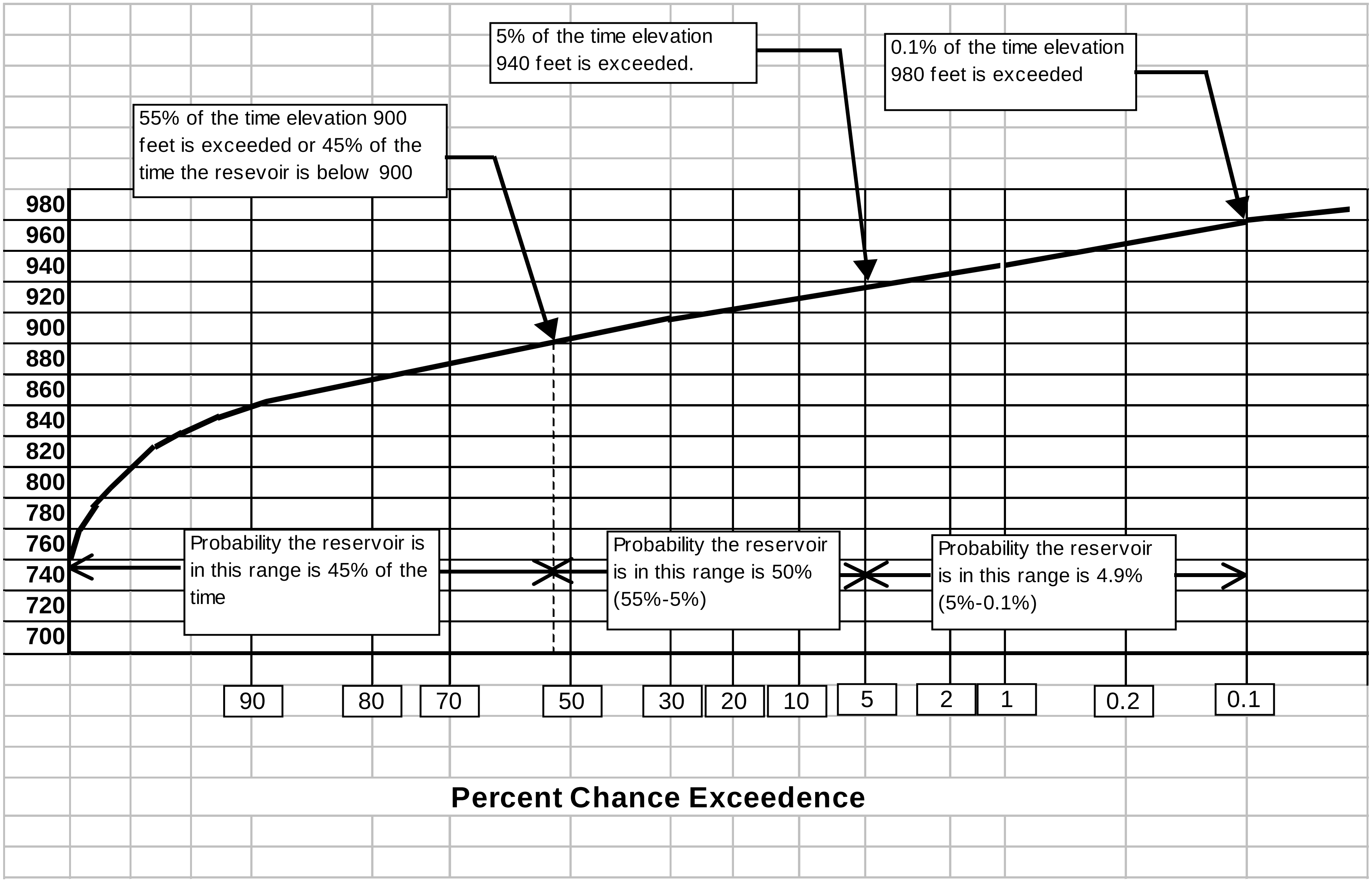
I-6. Event Tree.

a. An event tree is used to describe the possible events and the outcomes of those events given the current conditions of the project. For this example, an event tree was developed and is shown in Figure I-4. The event tree contains five major categories. The categories consist of events that are related to the frequency of loading (in this example the probability of the pool being at various elevations), the conditional probability of unsatisfactory performance, the performance level, consequences, and weighted damages (risk), that could occur for a particular set of events. In this case to determine the frequency of loading, a pool probability curve is needed, see Figure I-5. To select the number of cases to examine, the threshold condition must be determined. The threshold condition is simply the pool elevation at which below this condition there is not a problem and above this level problems start to occur. For Dam X the threshold elevation was selected at reservoir elevation 900 feet. This elevation was selected because there are no historic problems in either the instrumentation data or physical conditions in response to a pool elevation of 900 feet or below. The reservoir elevation has been there several times in the past and for extended periods of time with no adverse conditions identified.

b. Three reservoir pool elevations were selected for the event tree. Elevation 900 feet (threshold value), Elevation 940 feet (elevation at which problems were identified) and Elevation 980 feet (which is the maximum surcharge pool). The probability of loading for these elevations are calculated by using Figure I-5.



Event Tree
Figure I-4



Pool Probability Curve for Dam X
Figure I-5

I-7. Conditional Probability of Unsatisfactory Performance. For the purpose of this example problem, it is assumed that a factor of safety of 1.100 is considered unsatisfactory performance because of the physical movement that occurred at a pool elevation of 940 feet. To determine the conditional probability of unsatisfactory performance, the Taylor Series Method was used as described in Appendix D and ETL 1110-2-547.

I-8. Steps in the Reliability Analysis.

Step 1 - Determine the expected value (EV) of all the input parameters that are used to determine the factor of safety, see Table I-3.

Step 2 - Estimate the standard deviation of the input parameters. For the purpose of this example problem the standard deviation for the input parameters are contained in Table I-3.

Step 3 – Compute the factor of safety using the expected values for each parameter. Compute the factor of safety for each parameter increased and decreased by one standard deviation from the expected value while holding all others at the expected value, for each of the pool loading conditions, see Tables I-4 through I-6.

Step 4 - Compute the change in the factor of safety for each parameter. This is done by simply subtracting the factor of safety for the EV + one standard deviation from the factor of safety for the EV – one standard deviation, see Tables I-4 through I-6.

Table I-3. Expected Values and Standard Deviation of the Input Parameters

Variable	Expected Value	Standard Deviation
Shear Strength		
Embankment	27.1^{o}	2.56^{o}
Alluvium	18.0^{o}	3.92^{o}
Firm Shale	20.8^{o}	5.57o
Weak Shale	8.0^{o}	3.5^{o}
Unit Weight		
Embankment	125 pcf	5.67 pcf
Alluvium	105 pcf	2.33 pcf
Firm Shale	128 pcf	4.33 pcf
Weak Shale	102 pcf	6.67 pcf

Table I-4. Reliability Calculation for Pool Elevation 900 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = 1.500

VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66°	1.525	
Embankment	27.1°	2.56°			0.057
			24.54°	1.468	
			21.92°	1.515	
Alluvium	18.0°	3.92°			0.083
			14.08°	1.432	
			26.37°	1.496	
Firm Shale	20.8°	5.57°			0.008
			15.23°	1.488	
			11.50°	1.492	
Weak Shale	8.0°	3.50°			0.193
			4.50°	1.299	
Unit Weight					
			130.67	1.376	
Embankment	125	5.67			-0.043
			119.33	1.419	
			107.33	1.510	
Alluvium	105	2.33			0.028
			102.67	1.482	
			132.33	1.500	
Firm Shale	128	4.33			0.007
			123.67	1.493	
			108.67	1.502	
Weak Shale	102	6.67			0.011
			95.33	1.491	

Table I-5. Reliability Calculation for Pool Elevation 940 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = 1.100

VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66°	1.127	
Embankment	27.1°	2.56°			0.063
			24.54°	1.064	
			21.92°	1.115	
Alluvium	18.0°	3.92°			0.080
			14.08°	1.035	
			26.37°	1.096	
Firm Shale	20.8°	5.57°			0.008
			15.23°	1.088	
			11.50°	1.192	
Weak Shale	8.0°	3.50°			0.190
			4.50°	1.002	
Unit Weight					
			130.67	1.066	
Embankment	125	5.67			-0.063
			119.33	1.129	
			107.33	1.110	
Alluvium	105	2.33			0.101
			102.67	1.009	
			132.33	1.100	
Firm Shale	128	4.33			0.005
			123.67	1.095	
			108.67	1.105	
Weak Shale	102	6.67			0.010
			95.33	1.095	

Table I-6. Reliability Calculation for Pool Elevation 980 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = .997					
VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66°	1.051	
Embankment	27.1°	2.56°			0.060
			24.54°	0.991	
			21.92°	1.061	
Alluvium	18.0°	3.92°			0.078
			14.08°	0.983	
			26.37°	1.001	
Firm Shale	20.8°	5.57°			0.006
			15.23°	0.995	
			11.50°	0.998	
Weak Shale	8.0°	3.50°			0.191
			4.50°	0.807	
Unit Weight					
			130.67	1.021	
Embankment	125	5.67			-0.060
			119.33	1.081	
			107.33	1.042	
Alluvium	105	2.33			0.101
			102.67	0.941	
			132.33	1.031	
Firm Shale	128	4.33			0.006
			123.67	1.025	
			108.67	1.011	
Weak Shale	102	6.67			0.011
			95.33	1.000	

Step 5 - Compute the standard deviation of the factor of safety. The formula for the standard deviation of the factor of safety is computed using the following equation.

$$\sigma_F = \sqrt{(\Delta F_1/2)^2 + (\Delta F_2/2)^2 + (\Delta F_3/2)^2 + \ldots}$$

For this example problem, $\sigma_{F\text{-}900}$ is calculated as follows:

$$\sigma_{F\text{-}900} = \sqrt{(0.057/2)^2 + (0.083/2)^2 + (0.008/2)^2 + (0.193/2)^2 + (-0.043/2)^2 + (0.028/2)^2 + (0.007/2)^2 + (0.011/2)^2}$$

$$\sigma_{F\text{-}900} = 0.1121$$

For this example problem, $\sigma_{F\text{-}940}$ is calculated as follows:

$$\sigma_{F\text{-}940} = \sqrt{(0.063/2)^2 + (0.080/2)^2 + (0.008/2)^2 + (0.190/2)^2 + (-0.063/2)^2 + (0.101/2)^2 + (0.005/2)^2 + 0.010/2)^2}$$

$\sigma_{F\text{-}940} = 0.1233$

For this example problem, $\sigma_{F\text{-}980}$ is calculated as follows:

$$\sigma_{F\text{-}980} = \sqrt{(0.060/2)^2 + (0.078/2)^2 + (0.006/2)^2 + (0.191/2)^2 + (-0.060/2)^2 + (0.101/2)^2 + (0.006/2)^2 + (0.011/2)^2}$$

$\sigma_{F\text{-}980} = 0.1226$

Step 6 **-** Compute the Coefficient of Variation. The formula to compute the Coefficient of Variation (COV) is as follows:

$$COV = \sigma_F/F_{EV}$$

Where F_{EV} is the factor of safety computed using the expected values for all the input parameters.

For this example problem, the COV is:

$$COV_{900} = 0.1121/1.5 \text{ x } 100\% = 7.47\%$$

$$COV_{940} = 0.1233/1.1 \text{ x } 100\% = 11.21\%$$

$$COV_{980} = 0.1226/0.997 \text{ x } 100\% = 12.27\%$$

Step 7 **-** Compute the probability that the factor of safety is less than one.

Using the value of F_{EV} and the value of COV, compute the value of the log normal reliability index (β_{LN}) as follows:

$$\beta_{LN} = LN[F_{EV}/ (1+(COV)^{2})^{1/2}] / [LN (1 + (COV)^{2})]^{1/2}$$

Using Table D-1 of Appendix D or other published statistical tables find the standard cumulative normal distribution function of $\beta_{LN.}$ This value is the reliability. The probability of failure (P_f) is 1 minus this value.

For Example:
If $\beta_{LN} = 2.32$, then from Table D-1, the normal distribution function of 2.32 equals 0.9898 (the reliability). The probability of failure (P_f) is 1-0.9898 = 0.0102.

Using the mathematical procedure presented above gives the following results for this example problem:

A probability of the factor of safety is less than 1 for a pool elevation of 900 feet is approximately 0%. This tells us that at a reservoir elevation of 900 feet we are fairly confident that the dam is stable at elevation 900 feet.

A probability of the factor of safety is less than 1 for a pool elevation of 940 feet is approximately 21%.

A probability of the factor of safety is less than 1 for a pool elevation of 980 feet is approximately 50%.

These numbers are then added to the event tree. For example at elevation 980 feet, the event entitled “Probability of a FS < 1” would be assigned a 50% probability based on the calculations performed above.

I-9. Performance Level. The performance levels are developed as part of the event tree. These events are simply the events that the team feels are likely out comes given unsatisfactory performance. A probability of occurrence is assigned to each of these events. If an analytical tool is not available to compute the probability of occurrence, then expert elicitation is one method to perform this task. In the case of Dam X, the performance levels will be determined by use of expert elicitation and engineering judgement. These probabilities are then added to the event tree as well. Once all the probabilities have been assigned to each event, the probabilities are multiplied across the branch and then multiplied by the consequences to determine the annual weighted damages (risk).

APPENDIX J

HISTORICAL FREQUENCY OF OCCURRENCE ASSESSMENT OF PIPES THROUGH AN EMBANKMENT

J-1. Introduction. A reliability analysis model can be developed using historic data collected at a specific project or similar projects. The historic data would consist of known unsatisfactory performance events and the time of occurrence of the events. The historic frequency of occurrence model can be developed using the Weibull, Exponential, or Poisson distributions or a hazard function model. Examples of developing a historic frequency of occurrence model using the Weibull and Exponential distributions for pipes through an embankment are presented as follows.

J-2. Weibull Distribution.

a. The Weibull distribution is the most general method for developing a historic frequency of occurrence model and is used predominantly in industrial engineering. This method of reliability analysis is used to determine the probability of failure, reliability, or hazard function of a component with time using unsatisfactory performance data. A probability density function is fitted to the failure data versus time. This method of reliability engineering is based solely on unsatisfactory performance data without using any knowledge of the engineering properties or condition of the component. This method of reliability analysis is presented in the textbook "Reliability in Engineering Design" by K. C. Kapur and L. R. Lamberson.

b. The probability density function f(t) for the Weibull distribution is:

$$f(t) = \frac{b}{\alpha}\left[\frac{t}{\alpha}\right]^{b-1} \exp\left[-\left[\frac{t}{\alpha}\right]^{b}\right] \tag{1}$$

where

b is the shape parameter.

α is the characteristic life.

t is time.

F(t) is the cumulative distribution function, the probability that the system will fail by the time t or the probability of failure. F(t) is given as follows:

$$F(t) = 1 - \exp\left[-\left[\frac{t}{\alpha}\right]^{b}\right] \tag{2}$$

R(t) is the reliability function, the probability that the system will not fail by time t.

$$R(t) = \exp\left[-\left[\frac{t}{\alpha}\right]^{b}\right] \tag{3}$$

As can be seen from an examination of Equations 2 and 3, the reliability and probability of failure are related by the following equation:

$$R(t) = 1 - F(t) \tag{4}$$

The hazard function h(t) is the rate of failure at time t. The hazard function is the probability of failure in a given time period, usually one year, given that that item has not already failed. Simply stated, if there are ten items and one item fails in year one and another item fails in year two, the hazard function for year two is the number of items that failed in year two divided by the number of items existing at the beginning of year two. For this simple example the hazard function for year two would be:

$$h(t) = h(2) = \frac{1}{10-1} = 0.11$$

The hazard function for the Weibull distribution is given as

$$h(t) = \frac{b}{\alpha}\left[\frac{t}{\alpha}\right]^{b-1} \tag{5}$$

The Weibull distribution has the following characteristics: For a shape parameter of b = 1, the Weibull distribution becomes the exponential distribution, which gives a constant hazard function with an equal rate of failure in any year. For b = 2, the Weibull distribution becomes the Rayleigh distribution, which gives a linearly increasing hazard function. For b < 1, the hazard function decreases with time, giving a decreasing rate of failure with time. For b > 1, the hazard function increases with time, giving an increasing rate of failure with time. A b value of 1 would be representative of the occurrence of a random event, such as scour occurring adjacent to a structure, erosion, an earthquake, or an accident. Deterioration of sheetpiling could be represented by a b value between 1 and 2. For any Weibull distribution, there is a 63.2 percent probability that failure will occur before the characteristic life and a 37.8 percent probability that failure will occur after the characteristic life. Put another way, 63.2 percent of the components will fail by the characteristic life and 37.8 percent will not fail.

c. The term “failure” leads one to think in terms of a catastrophic event such as the failure of a large dam resulting in grave consequences. However, when addressing failures of components of a dam, failure could mean something as insignificant as an electric motor not starting when needed. So, to avoid the confusion that could result from the use of the term “failure”, the term “unsatisfactory performance” is used.

(1) From past known unsatisfactory performance events a table can be compiled of the unsatisfactory performance events that have occurred and the time of occurrence of that event. Each unsatisfactory performance event needs to be listed in the table according to the time of occurrence. The table needs to include the date the item was put into operation, the date of the unsatisfactory event, the time in years to the unsatisfactory event, and the corrected percentage of the components that have experienced unsatisfactory performance. The corrected percentage of components that have experienced unsatisfactory performance is a median rank correction to the data. The median rank correction adjusts the data such that the probability that the unsatisfactory performance event occurs before time t is 50%. The median rank correction is made using the following equation:

$$Median\ Rank\ Correction\,(\%) = \left[\frac{j-0.3}{N+0.4}\right]*100 \qquad (6)$$

where

j is the number of the item that had unsatisfactory performance, i.e., first item, second item, ……jth item

N is the sample size that is, the total of the items having unsatisfactory performance plus the items having satisfactory performance. To use the Weibull distribution to develop a historical frequency of occurrence model, the number of items that have had unsatisfactory performance and those that did not have unsatisfactory performance must be known.

(2) The data from the table (time and corrected percentage of components that have experienced unsatisfactory performance) is plotted on Weibull graph paper. A straight line is passed through the data points to determine b and α, where b is the slope of the straight line and α is the time at which 63.2 percent of the items have experienced unsatisfactory performance. The straight line is located by sight. While a least-squares fit or some other numerical method could be used, the sight method should be used based on the following quote from "Reliability in Engineering Design" by Kapur and Lamberson: "A least-square fitting procedure could be used; however, this somewhat defeats the purpose, namely, the ease of graphical estimation." The sight method of fitting the straight line to the data requires the data to be plotted and allows engineering judgment to be applied to the straight line fit. A mathematic method of fitting the straight line to the data, while being mathematically more accurate, can lead to serious errors, especially if the data is not plotted.

(3) An error in the historic database would be that it does not include all unsatisfactory performance events due to inadequate records. Thus, the historic database will tend to underestimate the actual number of events that have occurred and thus those that will occur in the future.

J-3. Weibull Distribution Example. In a particular levee system, there are 23 large diameter corrugated metal pipes (CMP) going through the levee. These pipes are connected to gravity drain structures. Three of these CMP, have failed during flood events causing water to flow into the protected area. Two CMP, failed during the 1973 flood and one failed during the 1993 flood. Refer to the Table J-1 to see when the pipes were installed. Using the Weibull distribution, calculate the hazard function in the year 2003 given that there is a flood. Calculate the hazard function for the years 2013 and 2023 given that there is a flood in those years.

Table J-1. Selected CMP Installation and Failure Information

	Year Installed	**Year of Failure**	**Time to Failure**	**Corrected % failure**
1st Failure	1950	1973	23	3.0
2nd Failure	1950	1973	23	7.3
3rd Failure	1950	1993	43	11.5

Solution:

First, use Equation (6) to find the corrected percent failure of the pipes:

For 1st pipe failure in 1973: Corrected % failure $= \dfrac{1-0.3}{23+0.4} \times 100 = 3$

For 2nd pipe failure in 1973: Corrected % failure $= \dfrac{2-0.3}{23+0.4} \times 100 = 7.3$

For 3rd pipe failure in 1993: Corrected % failure $= \dfrac{3-0.3}{23+0.4} \times 100 = 11.5$

On the Weibull graph paper, plot on the x-axis the time to failure. On the y-axis, plot corrected percent failure. Plot the points: (23,3), (23,7.3) and (43,11.5) on the Weibull graph paper (See Figure J-1).

Draw a best-fit line through the data points. The slope of the best-fit line is b = 1.4 (the slope b is obtained by matching the slope of the best fit line to the Weibull Slope indicator in the upper left hand corner of the Weibull distribution plot). The best fit line crosses 63.2% at 230 years so $\alpha = 230$

Calculate the hazard function for year 2003 using Equation (5): for t = 53 (2003 – 1950)

$$h(t) = \frac{1.4}{230}\left[\frac{53}{230}\right]^{(1.4-1)} = 0.0034$$

For the year 2013, t = 2013 – 1950 = 63 years. Using Equation (5):

$$h(t) = \frac{1.4}{230}\left[\frac{63}{230}\right]^{(1.4-1)} = 0.0036$$

For the year 2023, t = 73 years:

$$h(t) = \frac{1.4}{230}\left[\frac{73}{230}\right]^{(1.4-1)} = 0.0038$$

Over the twenty-year time period from 2003 to 2023 the hazard function increased from 0.0034 to 0.0038. This is the hazard function for a single corrugated metal pipe. Since there are 23 of the corrugated metal pipes in the levee system, a system hazard function must be developed. Guidance on calculating system reliability is given in ETL 1110-2-547.

J-4. Exponential Distribution. The Exponential distribution is a continuous probability density function. The probability density function (pdf) for this distribution is:

$$f(t) = \lambda e^{-\lambda t} \tag{7}$$

where

λ is the expected number of events in a time period

t is the time period

The exponential distribution can be used to model any process that is considered to be a random event such as floods, earthquakes, or scour. An important application for the exponential distribution is to model the time to the first occurrence of a Poisson event and the time between Poisson events. Poisson events are events that occur randomly in time or space and have an equally likely probability of occurrence in any unit time or space increment Δt. A Poisson distribution is used when the probability of a certain number of events in a given time period is desired.

In order to determine the probability that an event has occurred by time t, or within the interval (0,t) the cumulative distribution function (cdf) is used. The cdf for the exponential distribution is obtained by integrating the probability density function:

$$F(t) = \int_{o}^{t} \lambda e^{-\lambda t} = 1 - e^{-\lambda t} \tag{8}$$

Equation (8) provides the probability that the first event (or failure) occurs by time t. So F(t) is the probability of failure. To determine the probability that a failure will occur in the interval t_1 to t_2, the following equation can be used:

$$\Pr(\text{failure in interval } t_1 \text{ to } t_2) = F(t_2) - F(t_1) \tag{9}$$

The probability that no failure (or event) occurs by time t is represented by the reliability function R(t). R(t) is the complement of the cumulative distributive distribution and has the following equation:

$$R(t) = 1 - F(t) = e^{-\lambda t} \tag{10}$$

The hazard function h(t) for the exponential distribution can be obtained by dividing the probability density function by the reliability function. The hazard function is the conditional probability of failure in the next unit time increment given that the system has survived to time t. The hazard function is given as the following equation:

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \tag{11}$$

Since h (t) = λ, it can be seen that the exponential distribution has a constant hazard rate. Historical data can be used to determine λ. The information required to calculate λ is a data set of items that have been in service with and without failures. Of the items that failed, it is necessary to know the times to failure and times in service. λ can be calculated using the following equation:

$$\lambda = \frac{F}{T} \tag{12}$$

where

F = number of failures; and

T = the total time in service for failed and unfailed items.

J-5. Exponential Example. In a particular levee system, there are 23 large corrugated metal pipe (CMP) gravity drains. Three of these CMPs have failed during a flood. Two CMP failed during the 1973 flood and one failed during the 1993 flood. Refer to Table J-2 to see when all of the CMPs were installed. Using the exponential distribution, calculate the hazard function.

Solution:

The total time in service (T) for all of the pipes is $\sum_{i=1}^{23}(\textit{Life}\ \text{of Pipe}) = 1086$ pipe-years

Use Equation (12) to calculate λ:

$$\lambda = \frac{3\,failures}{1086\,pipe\ years} = 0.0028$$

The hazard function for a single pipe is calculated as

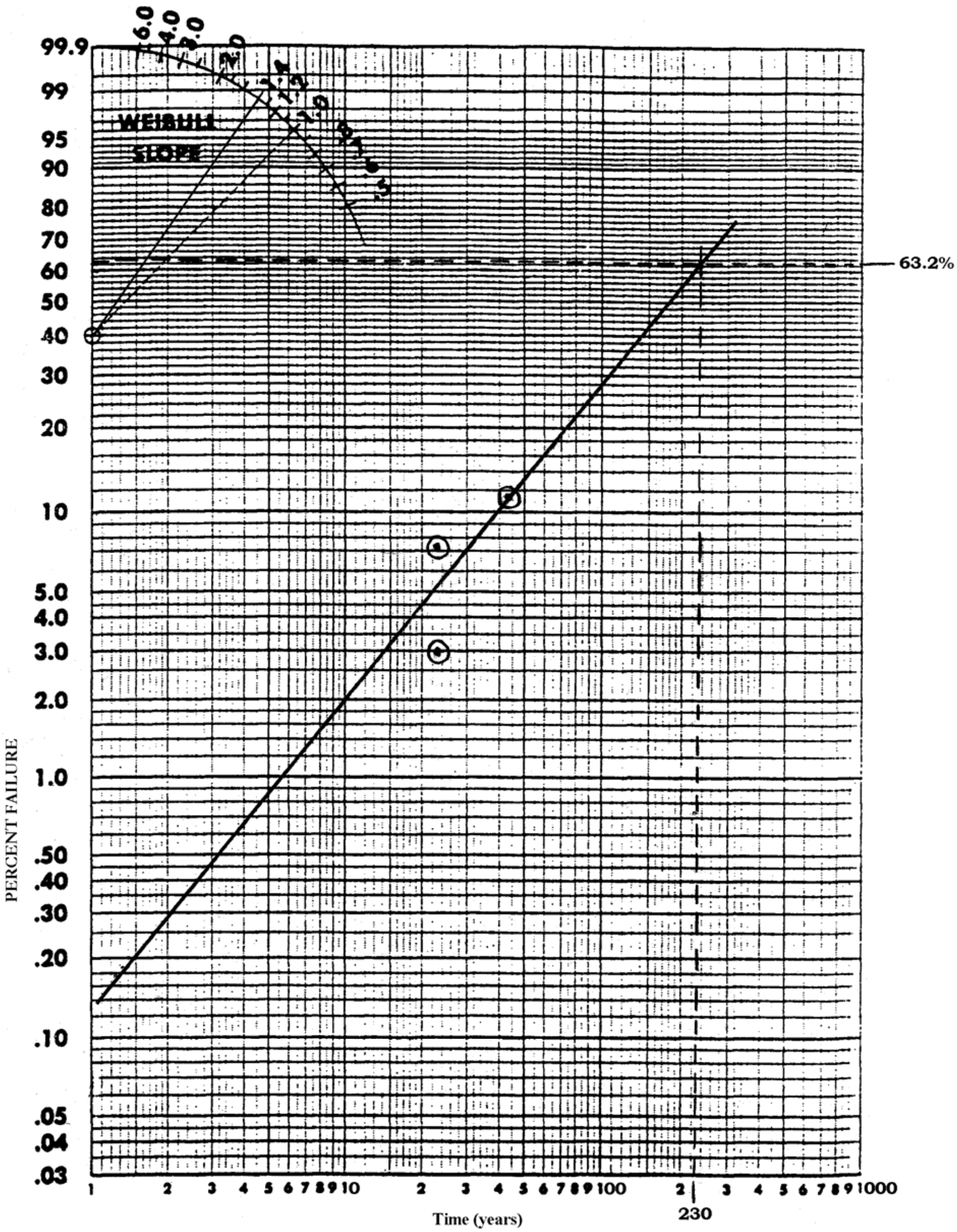
$h(t) = \lambda = 0.0028$

Using the Exponential distribution, the hazard rate is 0.0028 and is constant with time.

J-6. Comparison Between the Weibull Distribution and the Exponential Distribution. The Weibull distribution example and the exponential distribution example are the same problem, yet they yield different hazard rates. The difference between the two examples is that the Weibull example has an increasing hazard function while the exponential example has a constant hazard function. The Weibull distribution's hazard rate increases from 0.0034 in year 2003 to 0.0036 in the year 2013. The exponential example has a constant hazard rate for every year of 0.0028. If the b value of the Weibull example had equaled 1, the Weibull example and the exponential example would have yielded the same result. When b = 1 for a Weibull distribution, the Weibull distribution becomes the Exponential distribution. A b value of 1 would represent the occurrence of a random event. Since pipe failure is dependent on deterioration of the pipe (which leads to failure) it is legitimate to assume that the value of b would be between 1 and 2. If pipe failure were only dependent on a flood, it would be reasonable to assume that pipe failure follows an exponential distribution.

Table J-2. Total CMP Installation and Failure Information

Pipe #	Date Installed	Date Failed	Life of Pipe (to year 2003 or failure)
1	1950	1973	23
2	1950	1973	23
3	1948		55
4	1950		53
5	1950		53
6	1950		53
7	1950		53
8	1950		53
9	1950		53
10	1950		53
11	1960		43
12	1952		51
13	1952		51
14	1950	1993	43
15	1953		50
16	1956		47
17	1956		47
18	1956		47
19	1956		47
20	1956		47
21	1956		47
22	1956		47
23	1956		47
Sumation (Total time in service,T)			1086



Weibull Distribution Plot
Figure J-1

APPENDIX K

MONTE CARLO SIMULATION

K-1. Introduction. Monte Carlo simulation is a method of reliability analysis that should be used only when the system to be analyzed becomes too complex for use of simpler methods of reliability analysis, such as, the reliability index method. In Monte Carlo simulations, each random variable is represented by a probability density function and repeated conventional analyses are made (iterations) by changing the values of the random variables using a random number generator. To obtain an accurate Monte Carlo simulation solution, many thousands of these conventional analyses must be performed. The simpler methods of reliability analysis should be used whenever possible as the Monte Carlo simulation is much more costly and time consuming.

K-2. Probability Density Functions. When performing Monte Carlo simulations each random variable must be represented by a probability density function. In Geotechnical Engineering there are only four commonly used probability density functions: uniform distribution, triangular distribution, normal distribution, and lognormal distribution. Other probability density functions would only be used if there were test data that matched those functions. Table K-1 lists parameters that are used in geotechnical engineering and the probability density functions that typically best represent those parameters. The four probability density functions typically used in geotechnical engineering are discussed below.

a. Uniform Distribution. The uniform distribution (see Figure K-1) is the simplest of all distributions. All that is needed is the high and the low value. The uniform distribution gives the probability that observation will occur within a particular interval when the probability of occurrence within that interval is directly proportional to the interval length. If there is no available information, the Principle of Insufficient Reason says "the uniform distribution should be used". The uniform distribution is the distribution used to generate random numbers. It is used to highlight or exaggerate the fact that little is known about the parameter. It is used to model circular variables (like the direction of the wind coming from 0 to 360 degrees).

b. Triangular Distribution. A triangular distribution (see Figure K-2) is used when the smallest value (most pessimistic outcome), the largest value (most optimistic outcome), and the most likely value are known. The triangular distribution is the most commonly used distribution for modeling expert opinion.

c. Normal Distribution. The normal distribution (see Figure K-3) is the basic distribution of statistics. The Central Limits Theorem states "the sum of many variables tend to be normally distributed". Consequently the normal distribution is an appropriate model for many but not all physical phenomena. Most things in nature tend to be normally distributed. This distribution represents physical measurements of living organisms, intelligence test scores, product dimensions, average temperatures, etc. The famous bell shape curve that students are graded on in school is the normal distribution. This distribution is easy to use because of the many standardized tables available. This is a reasonable distribution for many things. If the normal distribution is used it is hard for someone to say that it is not the correct distribution.

d. Lognormal Distribution. The lognormal distribution (see Figure K-4) is the logarithm of the normal distribution. As such it best represents processes with are the multiplication of many variables. This distribution represents sizes from a breakage process, income size, inheritances, bank deposits, biological phenomena, life of transistor, etc. It is the distribution used for calculation of Corps of Engineers beta values. This distribution is used when the value of the variable cannot be less than zero. The extreme values of a normal distribution go below zero.

K-3. Methodology. Monte Carlo simulations, uses the same analysis method as conventional analysis does. It just runs each analysis many times. The random variables are represented as probability density functions. The deterministic variables are represented as constants. Monte Carlo simulation uses a random number generator to select the value of each random variable using the probability density function specified for that random variable. For each iteration (analysis) a factor of safety is calculated. Each factor of safety resulting from an iteration is counted to develop a probability density function for the factor of safety and to determine the number of unsatisfactory performance events. The iterations are repeated thousands of times until the process converges. From the probability density function for the factor of safety or from the number of factors of safety counted that are less than one divided by the number of iterations the percent of the factor of safeties less than one is calculated to determine the probability of unsatisfactory performance. To accomplish the many thousands of iterations a computer program is needed. If the problem being analyzed is simple enough, then the conventional analysis can be accomplished in an Excel spreadsheet and the Monte Carlo simulation can be accomplished using the add-on program @RISK. @RISK attaches a designated probability density function to each random variable, generates the random numbers needed to determine the value of each random variable, performs the thousands of iterations, and collects each value of the factor of safety to determine the probability density function of the factor of safety and the number of factors of safety less than one to determine the probability of unsatisfactory performance. If the problem being analyzed is too complex to be analyzed in an Excel spreadsheet, then special probabilistic programs that perform Monte Carlo simulations must be used. Some of these programs are SLOPE/W, a slope stability computer program, and RCSLIDE, a sliding of gravity structures computer program.

K-4. Monte Carlo Simulation Example. The example problem given in Appendix D using the reliability index method to calculate the probability of unsatisfactory performance of a simple slope will be repeated here using the Monte Carlo method of analysis. The problem (see Figure D-3) discussed in Appendix D had two random variables ϕ and b. The Taylor Series method of analysis used in Appendix D showed that the random variable b had little effect on the analysis and that the random variable ϕ controlled the analysis. Therefore, to further simplify this Monte Carlo simulation, b will be taken as a deterministic variable of 1.5.

a. The Monte Carlo simulation will be performed using the infinite slope equation given by Equation 1.

$$FS = b \tan \phi \tag{1}$$

The following values used in Appendix D will be used for the analysis:

b = 1.5

E[ϕ] = 38°

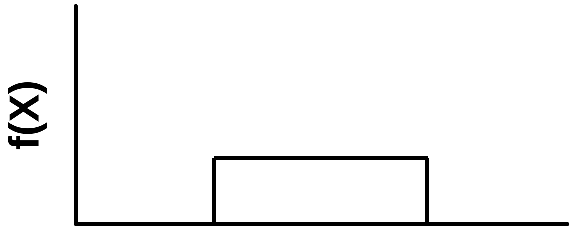
σ_ϕ = 3.8°

Based on the information given in Table K-1 the friction angle of sand is best represented by a normal probability density function. The normal probability density function for the expected value and standard deviation of the friction angle of sand given above is shown in Figure K-5. The normal probability density function is converted into a cumulative distribution function in the @RISK computer program, which is shown in Figure K-6. The @RISK computer program (version 3.5 was used for this example) generates random numbers using the uniform probability density function shown in Figure K-7. The Monte Carlo simulation works as follows. A random number is generated by @RISK between zero and one. For example, say that the random number is 0.25. The @RISK program goes to the cumulative distribution function with the random number of 0.25 and calculates a ϕ of 35° (see Figure K-6 for a graphical representation). Using Equation 1 with b = 1.5 and ϕ = 35°, @RISK calculates the value of the factor of safety to be 1.05 and stores that value. Say that the next random number is 0.75; @RISK determines ϕ = 41° and calculates a factor of safety of 2.35. And at a random number of 0.5, @RISK determines ϕ = 38° and calculates a factor of safety of 1.17. This process is repeated thousands of times until the process converges. Convergence can be determined by one of two methods. The first method is that @RISK has built in functions that determine when the analysis has converged. Secondly convergence occurs when two Monte Carlo simulations, say for 10,000 and 20,000 iterations, result in the same probability of unsatisfactory performance. If there is no convergence in this second method, then @RISK is run for 40,000 iterations. The number of iterations is doubled for each simulation until convergence occurs. The result of these Monte Carlo simulations is a probability density function for the factor of safety shown in Figure K-8. The probability of unsatisfactory performance is then calculated by @RISK as the area under the curve in Figure K-8 to the left of the values of the factor of safety of one. This gives a probability of unsatisfactory performance of 12.8%, which compares well with the probability of unsatisfactory performance calculated in Appendix D of 14% using the reliability index method. Alternatively, the probability of unsatisfactory performance can be calculated by the number of factors of safety count by @RISK as being less than one divided by the number of iterations. For this example @RISK counted 2560 values of the factor of safety less than one for 20,000 iterations. Thus the probability of unsatisfactory performance equals 2560 divided by 20,000 giving a value of 12.8%. When @RISK was set to use the built-in function to determine when the analysis had converged, the program gave a different probability of unsatisfactory performance. On the first run, the program said that the simulation converged after 1050 iterations had been run and yielded a probability of unsatisfactory performance of 10.86%. On a second attempt, the simulation converged after 600 iterations with a probability of unsatisfactory performance of 13.33%. Thus, it is recommended when using @RISK version 3.5 that the built in convergence criteria not be used; newer or updated versions may be acceptable, if tested. Table K-2 shows that convergence was achieved with 10,000 iterations but 20,000 iterations had to be run to determine that the Monte Carlo simulation had converged.

Table K-1
PROBABILITY DENSITY FUNCTIONS
FOR RANDOM VARIABLES IN GEOTECHNICAL ENGINEERING

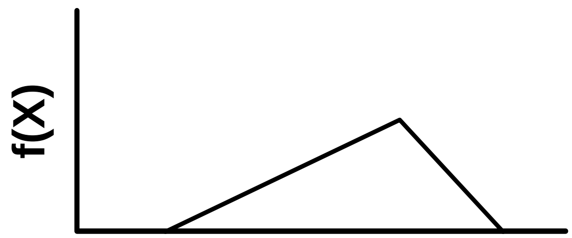
PARAMETER	PROBABILITY DENSITY FUNCTION	REFERENCE
Variables That Do Not Take Negative Values	LN	Lacasse & Nadim (1996)
Unit Weights	N	
Cone Resistance		
Sand	LN	Lacasse & Nadim (1996)
Clay	N/LN	Lacasse & Nadim (1996)
Undrained Shear Strength S_u		
Clay	LN	Lacasse & Nadim (1996)
Clayey Silt	N	Lacasse & Nadim (1996)
Ratio S_u/σ'_u		
Clay	N/LN	Lacasse & Nadim (1996)
Liquid and Plastic Limits	N	Lacasse & Nadim (1996)
Friction Angle		
Sand	N	Lacasse & Nadim (1996)
Void Ratio & Porosity	N	Lacasse & Nadim (1996)
Overconsolidation Ratio	N/LN	Lacasse & Nadim (1996)
Limited Information		
High & Low Only	U	Mosher
High, Low & Most Likely	T	Mosher
No Information	U	Hobbs
Construction Costs	N	Mosher
Distributions Resulting From		
Summation	N	
Multiplication	LN	

N: normal distribution
LN: lognormal distribution
N/LN: normal and lognormal distribution
U: uniform distribution
T: triangular distribution



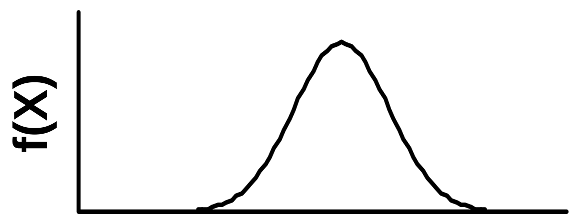
Uniform Distribution

Figure K-1



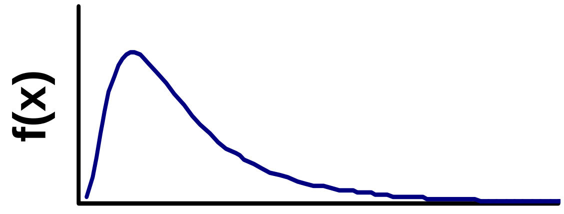
Triangular Distribution

Figure K-2



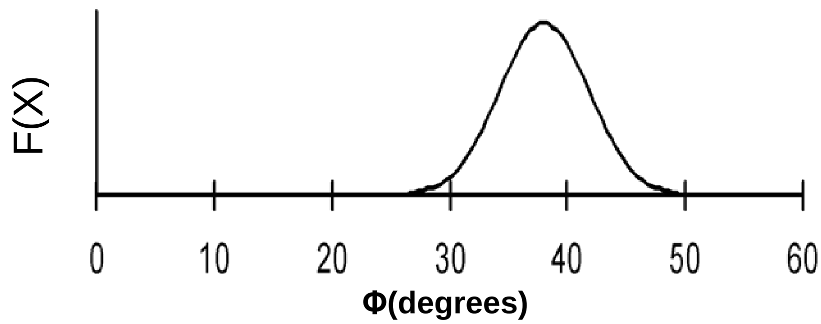
Normal Distribution

Figure K-3

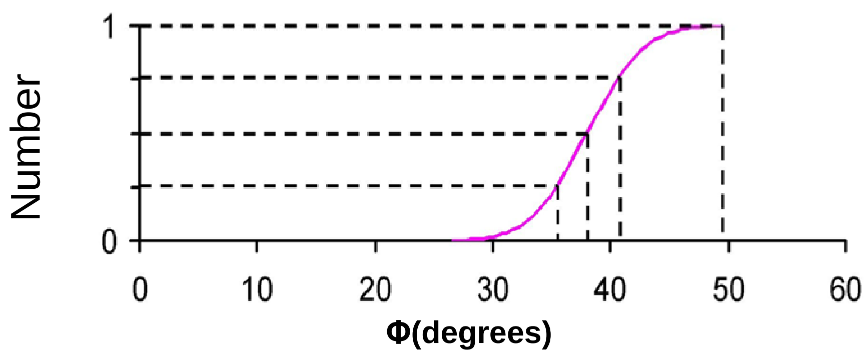


Lognormal Distribution

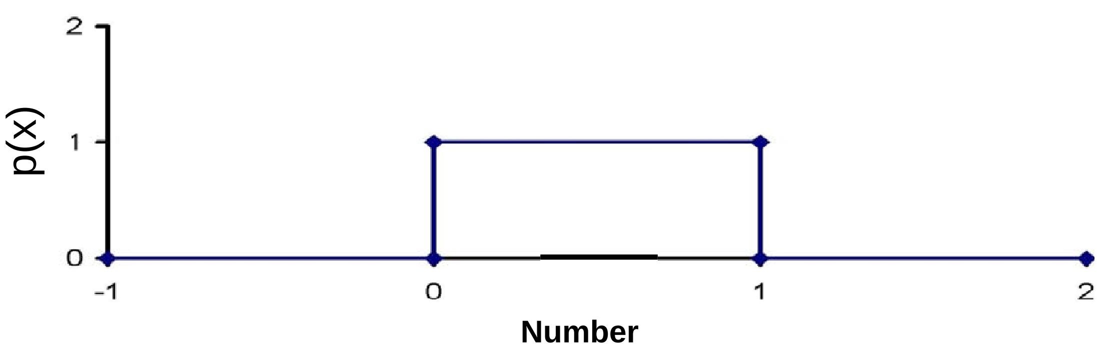
Figure K-4



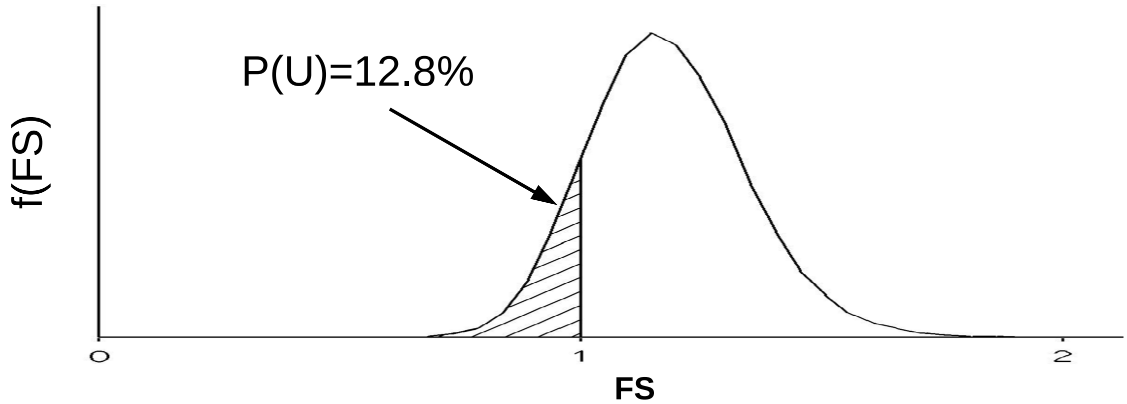
Normal Distribution
Figure K-5



Cumulative Distribution Function
Figure K-6



Randomly Generated Number
Figure K-7



Probability Density Function

Figure K-8

Table K-2. Determination of Convergence for Monte Carlo Simulation Example

Simulation	Total # of Iterations	# of Iterations with FS<1	Probability of Unsatisfactory Performance for FS<1	@RISK set to determine Convergence	Actual Convergence
1	600	80	13.33%	converge	does not converge
2	1050	114	10.86%	converge	does not converge
3	2500	325	13.00%		does not converge
4	5000	646	12.92%		does not converge
5	10,000	1284	12.84%		converge
6	20,000	2560	12.80%		converge
7	40,000	5102	12.76%		converge
8	100,000	12797	12.80%		converge