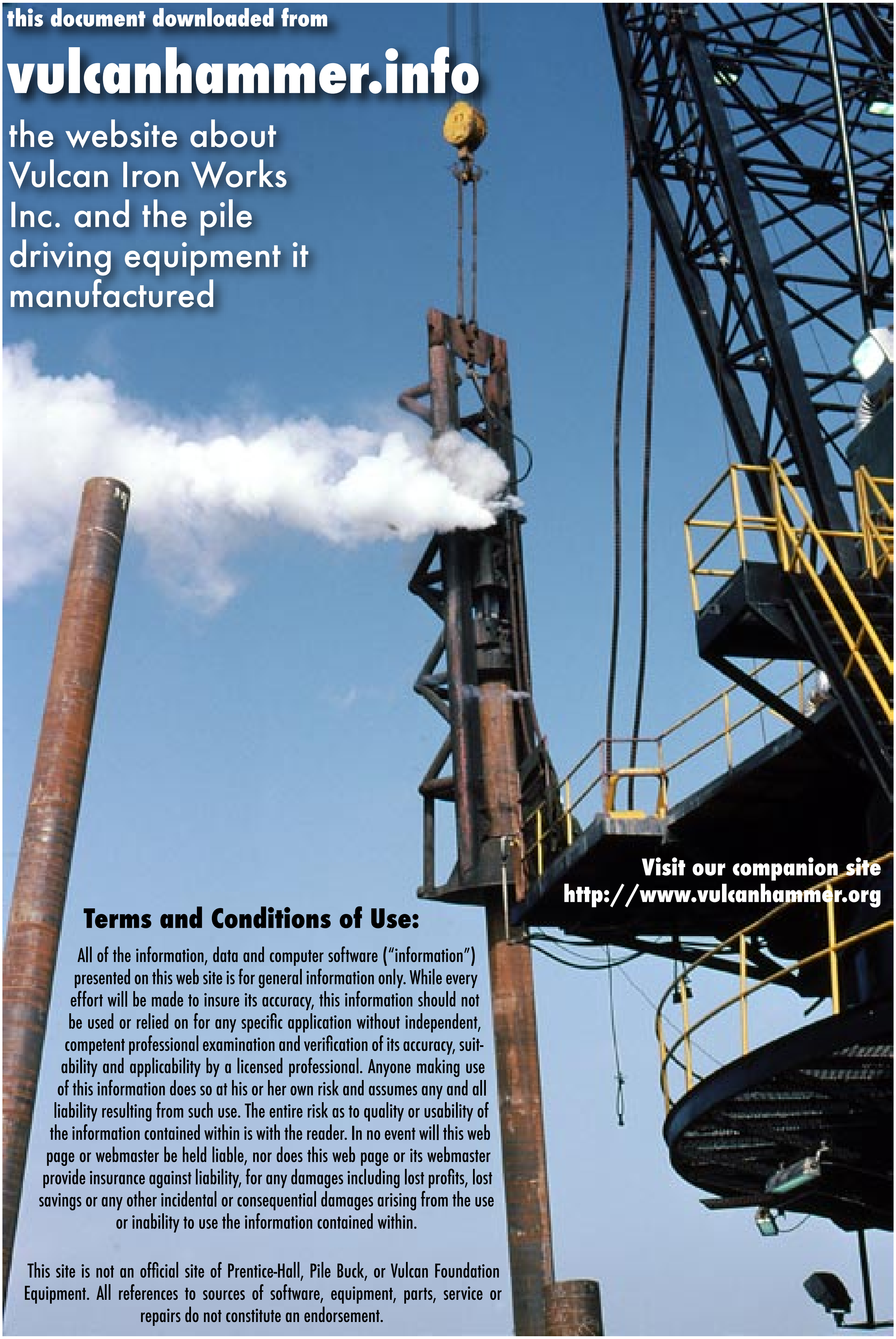

APPLICATION OF WAVE PROPAGATION THEORY IN PILE DRIVING ANALYSIS

AAE-646
WAVE PROPAGATION IN STRUCTURES

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1.0 INTRODUCTION

Pile foundations are commonly used for off-shore structures, bridge supports, in areas were the applied loads cannot be supported by the underlying soil and they have to be transferred to a more competent stratum. It is clear that the amount of load that each pile can support will depend on the resistance characteristics of the surrounding soil (shaft friction) as well as the soill beneath the pile (tip resistance). In order to evaluate much load each pile can support a number of models based on the one-dimensional wave equation have been presented (Smith, 1960; Rausche et al, 1972; Lee et al 1988). These models are used to predict the stresses on driven piles induced by impact loads as well as the permanent displacements associated with these impact loads. The soil dynamic resistance is modeled as a system of spring and dashpots concentrated at the nodes (Fig. A). The numerical procedures usually employed in the solution of these problems are finite differences and finite element. In the solution scheme, the soil resistance is assumed to be elasto-plastic, i.e. proportional to the soil displacement up to a given maximum value (Fig. D). The non-linear behavior of the soil resistance restrics the use of spectral analysis for this type of problems. Nonetheless, spectral analysis can be used for cases where the impact loadings are small enough that the soil response is within the elastic range.

2.0 THEORETICAL DEVELOPMENT

A pile embedded within a homogeneous soil can be modeled using the generalized rod theory considering external influences (Doyle, 1989). The soil surrounding the pile influence the dynamic response of the pile; it can be shown that the differential equation of motion considering forces proportional to displacement and velocity can be written as follows:

$$EA\frac{\partial^2 u}{\partial x^2} - K_s u - \eta_s \frac{\partial u}{\partial t} = \rho A \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

where:

K_s = Spring soil stiffness/unith length of pile shaft
η_s = Damping soil parameter/unit length of pile shaft
E = Young modulus of the Pile
A = Area of the pile cross section
ρ = Pile density
u = Pile displacement

assuming that the pile displacement u(x,t) can be expressed as:

$$u(x,t) = \Sigma\, \bar{u}(\omega_n)\, e^{-i(k_n x - \omega_n t)} \qquad (2a)$$

where

k_n = wavenumber
ω_n = angular frequency (radian/time)

it follows from Eq. (2a) that the particle velocity, v(x,t), and the particle acceleration, a(x,t), can be written as:

$$v(x,t) = \Sigma\, i\omega_n\, \bar{u}(\omega_n)\, e^{-i(k_n x - \omega_n t)} \qquad (2b)$$

$$a(x,t) = \Sigma\, -\omega_n^2\, \bar{u}(\omega_n)\, e^{-i(k_n x - \omega_n t)} \qquad (2c)$$

replacing Eq. (2) in (1) leads to:

$$EA\frac{\partial^2 \overline{u}}{\partial x^2} + \left[\omega^2 \rho A - K_s - i\,\omega \eta_s\right]\overline{u} = 0 \qquad (3)$$

notice that the subscript n for the variables k, ω, and u has been dropped out, but it should be understood that Eq (3) must be solved at each frequency and the following spectral relationship is for all values of n:

$$k = \pm\left[\omega^2 \frac{\rho A}{EA} - \frac{K_s}{EA} - i\,\omega\frac{\eta_s}{EA}\right]^{\frac{1}{2}} \qquad (4)$$

Eq. (4) indicates that the solution consists of two modes and the total solution is expressed as a linear combination of them:

$$\overline{u} = Ae^{-kx} + Be^{+kx} \qquad (5)$$

where A, B are the undetermined amplitudes coefficients to be determined when the boundary conditions are taken into account. These coefficients are also dependent on the frequency ω_n.

Diferentiating Eq. (5) respect to x, the force in the pile can then be written as:

$$\overline{F} = EA\frac{\partial \overline{u}}{\partial x} = iEA\left[-Ae^{-kx} + Be^{+kx}\right] \qquad (6)$$

Eqs. (5) and (6) are the general equations describing the displacement and force as a function of frequency. Specific solutions will be obtained when the boundary conditions are taken into account. The boundary conditions in pile driving problems are the usually specified as: (a) impact load at the top of the pile and (b) concentrated mass, spring and a dash-pot system at the tip (Fig. B). The concentrated mass has been included because it is very common to drive hollow steel piles with a steel cover-plate located at the tip.

Assuming the origin the coordinates to be at the pile top (Fig. B), the following relationships are established:

at x=0

$$\overline{F}(0) = -\overline{P} = EAik[-A + B] \tag{7}$$

at x = L (length of the pile)

$$\overline{F}(L) + K_t\overline{u}(L) + \eta_t \frac{\partial \overline{u}(L)}{\partial t} = m_t \frac{\partial^2 \overline{u}(L)}{\partial x^2} \tag{10}$$

where:

K_t = Spring soil stiffness/unit are of the pile tip
η_t = Damping soil/unit are of the pile tip
m_t = mass of the bottom plate

replacing the values of displacement, velocity and acceleration of the pile tip of the pile, the following expression is obtained:

$$EAik[-Ae^{-ikL} + Be^{-ikL}] = [-K_t - i\omega\eta_t - m_t\omega^2][Ae^{-ikL} + Be^{ikL}] \tag{9}$$

after a convenient grouping of terms the following expression is obtained:

$$\frac{B}{A} = \frac{[ik - \alpha_t]}{[ik + \alpha_t]} e^{-i2kL} \tag{10}$$

where

$$\alpha_t = \frac{[K_t + i\omega\eta_t + m_t\omega^2]}{EA}$$

combining Eq. (7) and Eq. (10) the magnitudes of the coefficients A and B can be determined yielding the following expressions for the particle displacement and force in the pile:

$$\overline{u}(x,\omega) = \frac{\overline{P}}{(1 - \lambda_t e^{i2kL})\, ikEA}\left[e^{-ikx} + \lambda_t e^{-ik(2L - x)}\right] \tag{11}$$

$$\overline{F}(x,\omega) = \frac{\overline{P}}{(1 - \lambda_t e^{i2kL})}\left[-e^{-ikx} + \lambda_t e^{-ik(2L - x)}\right] \tag{12}$$

where

$$\lambda_t = \frac{[ik - \alpha_t]}{[ik + \alpha_t]}$$

Eqs. (11) and (12) describe the displacement and Force spectrum of the pile. The program SHELLPIL (Appendix A), written in FORTRAN, contains this two expressions. The spectrum at any specified location of the pile can be obtained by means of this program. It is noted that to obtain the force and displacement-time history at any specified location a Fourier inverse transform must be applied. Analytical solution of the inverse transform, even for the simplest forcing function, is not possible to obtain; thus a numerical inversion has to be performed. This is done with help of the program CFFTCOMP (ikayex tools, 1991).

3.0 SOIL PARAMETERS

In order to evaluate the pile displacement, the soil spring and dash-pot coefficients for the shaft and tip must be assessed. Novak et al (1978) derived an expression for the resistance at the pile shaft using elasto-dynamic theory:

$$K_s = Sw_1(a_o)\ G_s \tag{14}$$

$$\eta_s = \frac{Sw_2(a_o)\ G_s\ r_o}{a_o\ V_s} \tag{15}$$

where G_s = soil shear modulus; $a_o = \omega r_o/V_s$, the dimensionless frequency ratio; r_o = outer pile radius; V_s = shear wave velocity in the soil; and Sw_1 and Sw_2 are related of the Bessel functions of first and second kind of order zero and one (Chow, 1985).

Notice that for values of the dimensionless frequency ratio (a_o) higher than 0.5 (Fig. C) the values of Sw_1 and Sw_2 can be approximated by the frequency independent constants 2.75 and $2\pi a_o$ respectively. Since impact load comprises mostly high frequency components the above values can be replaced in Eqs. (14) and (15), yielding the following expressions for the soil spring and damping coefficients:

$$K_s = 2.75\ G_s \tag{16}$$

$$\eta_s = \frac{2\pi\ G_s\ r_o}{V_s} \tag{17}$$

The soil resistance at the tip can be approximated by that of a vertically vibrating rigid disc on a elastic half-space. Lysmer and Richart (1966) derived the frequency-independent soil spring and damping coefficients that were generalized later Gazetas (Lee et al, 1988):

$$K_t = \frac{2 G_s r_o}{(1-\nu_s)\,\Omega(r_i/r_o)} \tag{18}$$

$$\eta_t = \frac{3.4\,(r_o^2 - r_i^2)\,\sqrt{\rho_s G_s}}{(1-\nu_s)} \quad (19)$$

where: $\Omega(r_i/r_o) =$ a function of the ratio of inner to outer radius of pile (Lee at al, 1988), it is equal to 0.5 for close-ended piled; ν_s = soil poisson ratio.

4.0 ANALYZED CASES

The proposed model is compared with measured data and with other models published earlier. Two piles were analyzed with different geometric and soil characteristics were analyzed. Both piles were hollow piles with 25 mm steel plate welded at the bottom of the pile to make it close-ended. This was considered in the model as a concentrated mass located at the bottom of the pile.

Case 1

The soil parameters and pile characteristics adopted from Simons and Randolph (1985) are the following:

Pile

r_o = 0.75 m
r_i = 0.68 m
L_p = 30.0 m
E_p = $210\text{x}10^6$ KN/m^2
ρ_p = 7.75 Ton/m^3

Soil

G_s = 10,000 KPa
ρ_s = 2.1 Ton/m^3
ν_s = 0.48

Thus:

A_p = 0.31 m^2
C_p = $(EA/\rho A)^{1/2}$ = 5,205 m/s
m_t = 0.342 Ton

C_s = $(G_s/\rho_s)^{1/2}$ = 69 m/s
K_s = 27,500 KN/m^2
η_s = 683 KN sec/m^2
K_t = 57,692 KN/m
η_t = 683 KN sec/m

The input load considered in the analysis was described by a exponential decaying function F = 37,493 exp(-1563 t) KN, t in seconds. Fig. 1 shows the variation with time of this function. This case was the only case found in the literature were the applied impact load was small enough so that no permanent displacements on the pile were imposed. Thus, comparisons with the spectral analysis model would be more accurate.

The input data for this case is shown in Table 1 in appendix A.

Case 2

This case utilizes similar pile and soil characteristics as in case 2 but a different impact load F = 75,000 exp(-1042t) KN (Fig. 1). This impact load induces permanent displacements on the pile. The input data for this case is shown in Table 2 in appendix A.

Case 3

The soil and geometric characteristics of this case were adopted form Lee et al. (1988). Although the applied force induces permanent displacements, the provided measured stresses at different points on the pile makes this case study a quite interesting one.

Pile

r_o = 0.457 m
r_i = 0.438 m
L_p = 12.0 m
E_p = 207×10^6 KN/m^2
ρ_p = 7.75 Ton/m^3

Soil

G_s = 16,000 KPa
ρ_s = 1.8 Ton/m^3
ν_s = 0.50

Thus:

A_p = 0.053 m^2
C_p = $(EA/\rho A)^{1/2}$ = 5,168 m/s
m_t = 0.127 Ton

C_s = $(G_s/\rho_s)^{1/2}$ = 94.3 m/s
K_s = 44,000 KN/m^2
η_s = 487 KN sec/m^2
K_t = 58,496 KN/m
η_t = 329 KN sec/m

The impact load was obtained from strain measurements at the top of the pile (Fig. 4). The input data for this case is shown in Table 3 in appendix A.

5.0 DISCUSSIONS OF THE RESULTS

The theoretical impact load described by an exponentially decaying function requires large frequency values to be describe in the frequency domain (Figs. 2 and 3). This values have been compared to those obtained using an exact fourier transform that were obtained for this particular case:

Function : $F(t) = A \exp(-bt)$

Fourier Transform : $\{F(w)\} = (b - i\omega)\ A/(b^2+\omega^2)$

For impact loads that are infinite in time (cases 1 and 2) the required values of frequencies to accurately describe the pulse in the frequency domain are quite large (Fig. 5).

The computed maximum displacement at the pile top (Fig. 6) can be compared to those obtained by the different models shown in Figs. E. Notice that the maximum displacement occurs at about 0.002 sec in all models shown. It is reminded that for this case, no permanent displacement was computed.

Fig. 7 shows the moving applied forces at the top, center and bottom of the pile. The continuous sign changes in the traveling pulse observe at the mid-pile indicates the nature of both boundary conditions.

Comparison between the tip response using spectral analysis (Fig. 8) and those shown in Fig. F is not as direct as in the previous case. This is due to the permanent displacement induce by the applied load. The maximum displacement (Fig. 8) is about 3 mm. and it occurs at approximately 0.008 sec. Computed displacements using the finite differences and finite element approach (Fig. F) at about 0.008 sec. shows displacements of about 7 mm. The spectral analysis furnishes a smaller value because the soil resistance does not have a maximum value as it is assumed in the other models. Nonetheless, if the permanent displacement of the pile is assumed to be the maximum value (3 mm) obtained using spectral analysis

(Fig. 8), this value can be compared to those displacement values obtained at 0.1 sec, (Fig. F). Fig. 9 shows the traveling force pulse trough the pile. A similar behavior as in case 1 is observed.

Fig. 10 shows the displacements at the top, center and tip of the pile. The maximum value at the pile top is approximately 4.5 mm. where as at the pile tip is 3 mm. No records of the pile displacements were reported. Fig. 11 shows the behavior of the moving force pulse along the pile. Notice that there are not reflected waves in the pile. Measured values (Fig. F) shows a different behavior of the force pulse. Wave reflections are observed and the force magnitude does not steadily decrease. Notice that the measured peak values of the first pulse at the tip are higher than those measured at the center pile.

Fig. 12 shows the wave behavior for varying values of K_s considering zero shaft damping coefficient. The remaining paramenters are the same as those used in case 3. The parameter K_s is varied from 0 to 176,000 KN/m. Using the soil parameters given in section 4 (Case 3), the calculated value is 44,000 KN/m. It is noted that a 400 % increase in the shaft friction results in a change of less than 20%.

6.0 CONCLUSIONS

The use of spectra analysis to predict the behavior of the pile during driving conditions can provide useful results when negligible permanent displacements have occurred. Case study shows that the displacement-time history can reasonable well predicted. Similar remark can be made regarding the stresses-time history.

For large permanent displacements, the model could be used to estimate the values of the expected permanent displacements assuming the maximum value of displacement obtained at the tip is equal to the permanent set. On the contrary, stresses in the pile may not be predicted reasonable well using this procedure.

The main advantage of using spectral analysis is the ease to visualize the influence of the different parameters affecting the pile behavior. Knowing the frequency components of the impact load, the behavior of the traveling pulse can be predicted.

REFERENCES

Chow Y.K. (1985). Dynamic response of axially loaded pile foundations. Proceedings, Fifth International conference in numerical methods in Geomechanics, Nagoya, Japan, 1457-1464.

Doyle, J. F. (1989). Wave propagation in structures: An FFT-Based spectral analysis Methodology. Springer-Verlag, N.Y.

Lee, S.L., Chow, Y.K., Karunaratne G.P., and Wong, K.Y. (1988). Rational wave equation model for pile driving analysis. J. Soil Mech. and Found. Div. ASCE 114(3), 306-325.

Lysmer, J., and Richart, F. E. (1966). Dynamic response of footings to vertical loading. J. Soil Mech. and Found. Div. ASCE 2(1), 61-91.

Novak, M., Nogami T., and Aboul-Ella, F. (1978). Dynamic soil reaction for plain strain case. J. of Engrg. Mech. Div. ASCE 104(4), 953-959.

Rausche, F., Moses, F., and Goble, G.G. (1972). Soil resistance prediction from pile dynamics. J. Soil Mech. and Found. Div. ASCE 98(9), 61-91.

Simons H.A. and Randolph M.F. (1985). A new approach to one dimensional pile driving analysis. Proceedings, Fifth International conference in numerical methods in Geomechanics, Nagoya, Japan, 1457-1464.

Smith, E.A.L. (1960). Pile driving analysis by the wave equation. J. Soil Mech. and Found. Div. ASCE 86(4), 35-61.

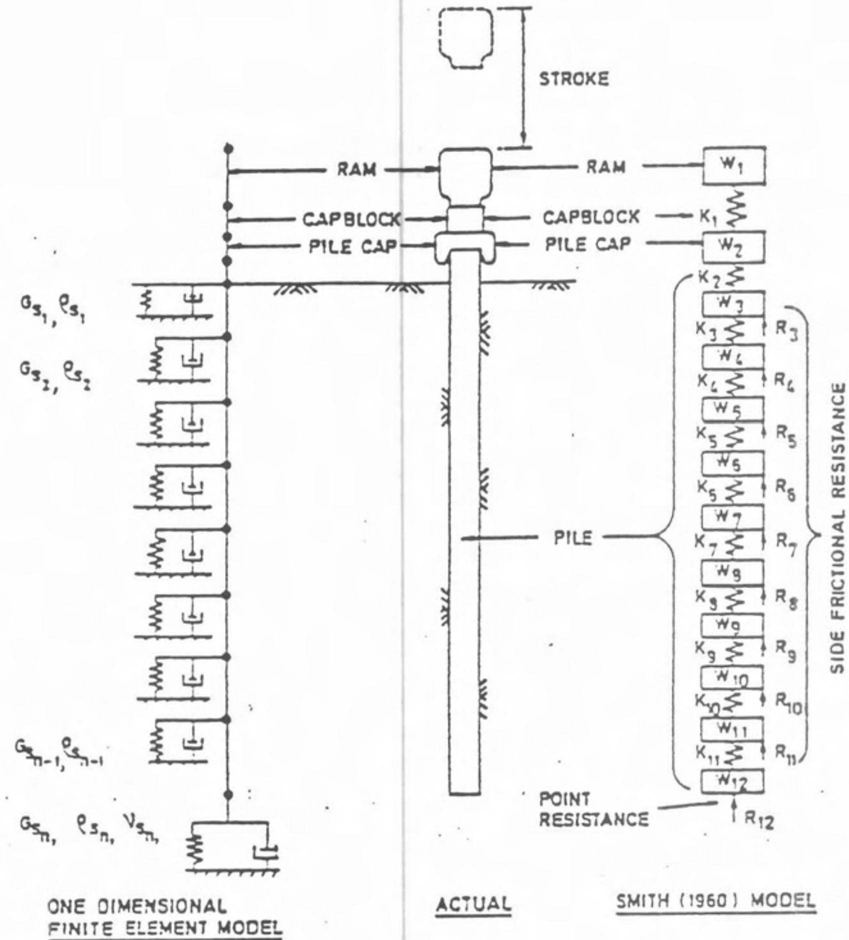


FIG. A. Wave Equation Models

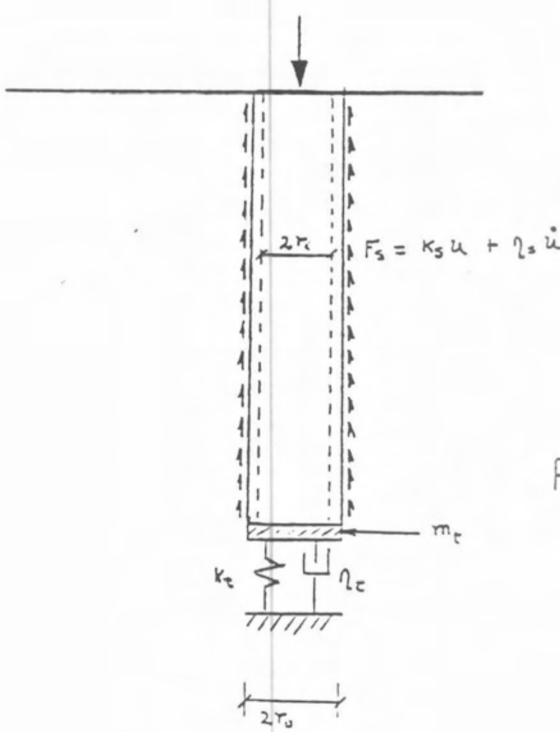


FIG. B. Pile Model

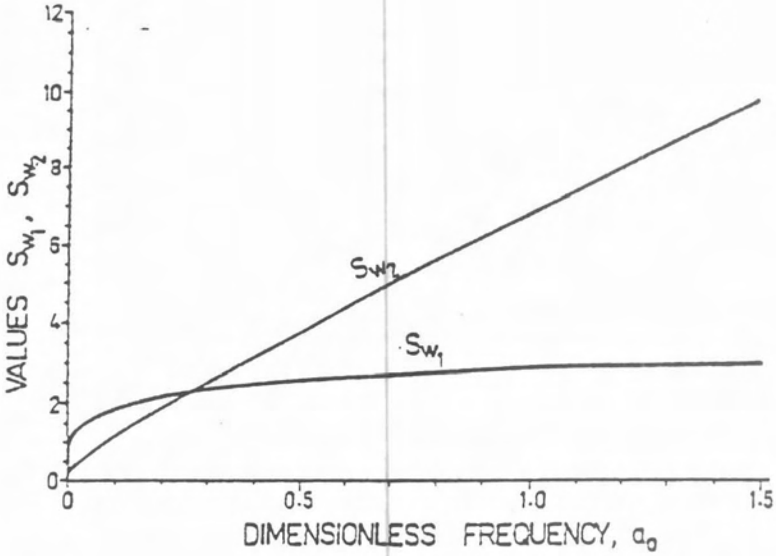


FIG. C Variations of Sw_1 and Sw_2 with Dimensionless Frequency Ratio a_0 (after Novak et al, 1978)

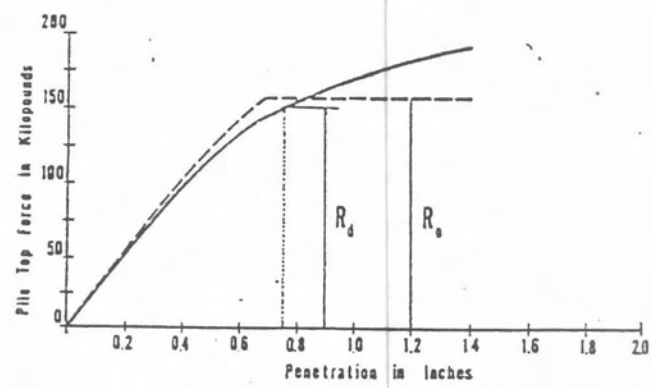
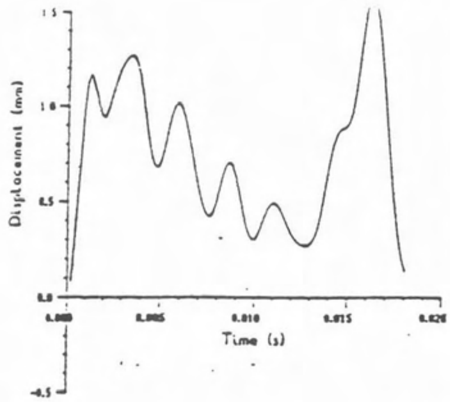
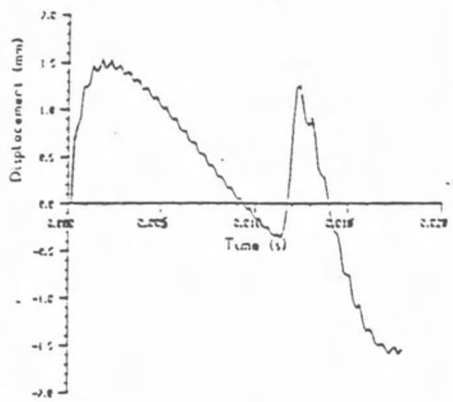


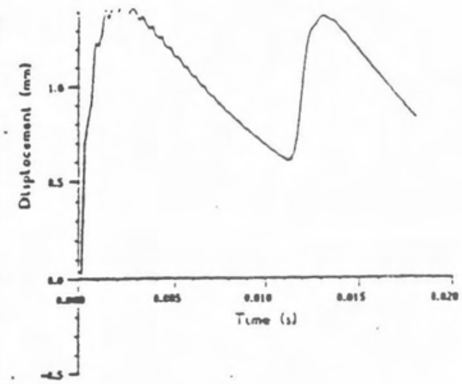
FiG. D



(a) Response using Finite Element method



(b) Response using Smith (1960) approach



(c) Response using new approach

Fig. 4 Comparison of elastic responses

FIG. E

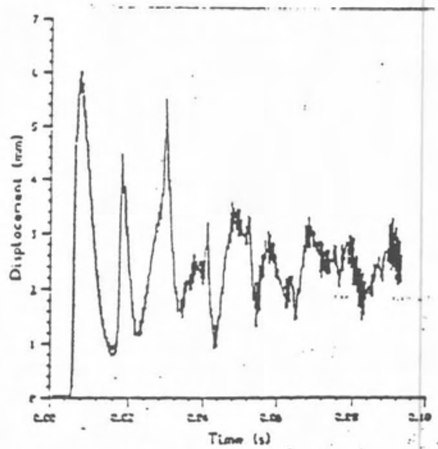
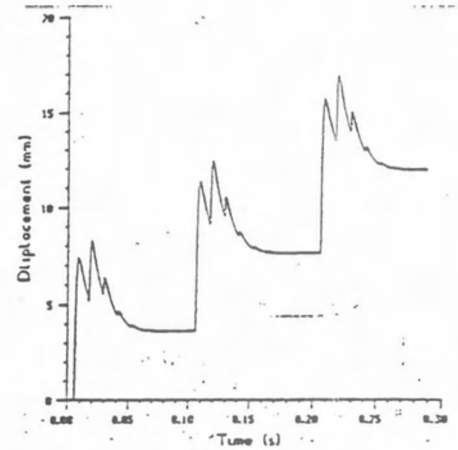


Fig. 5 Tip response using Smith (1960) method



(a) Tip response

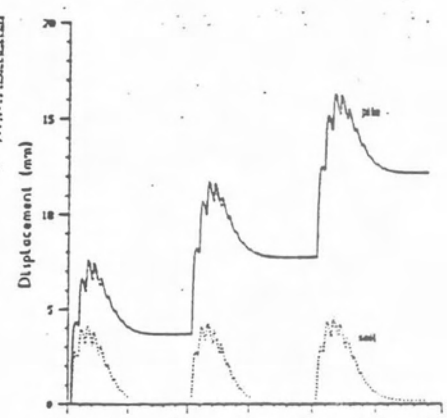


FIG. F

Fig.1 - Force (Time Domain)

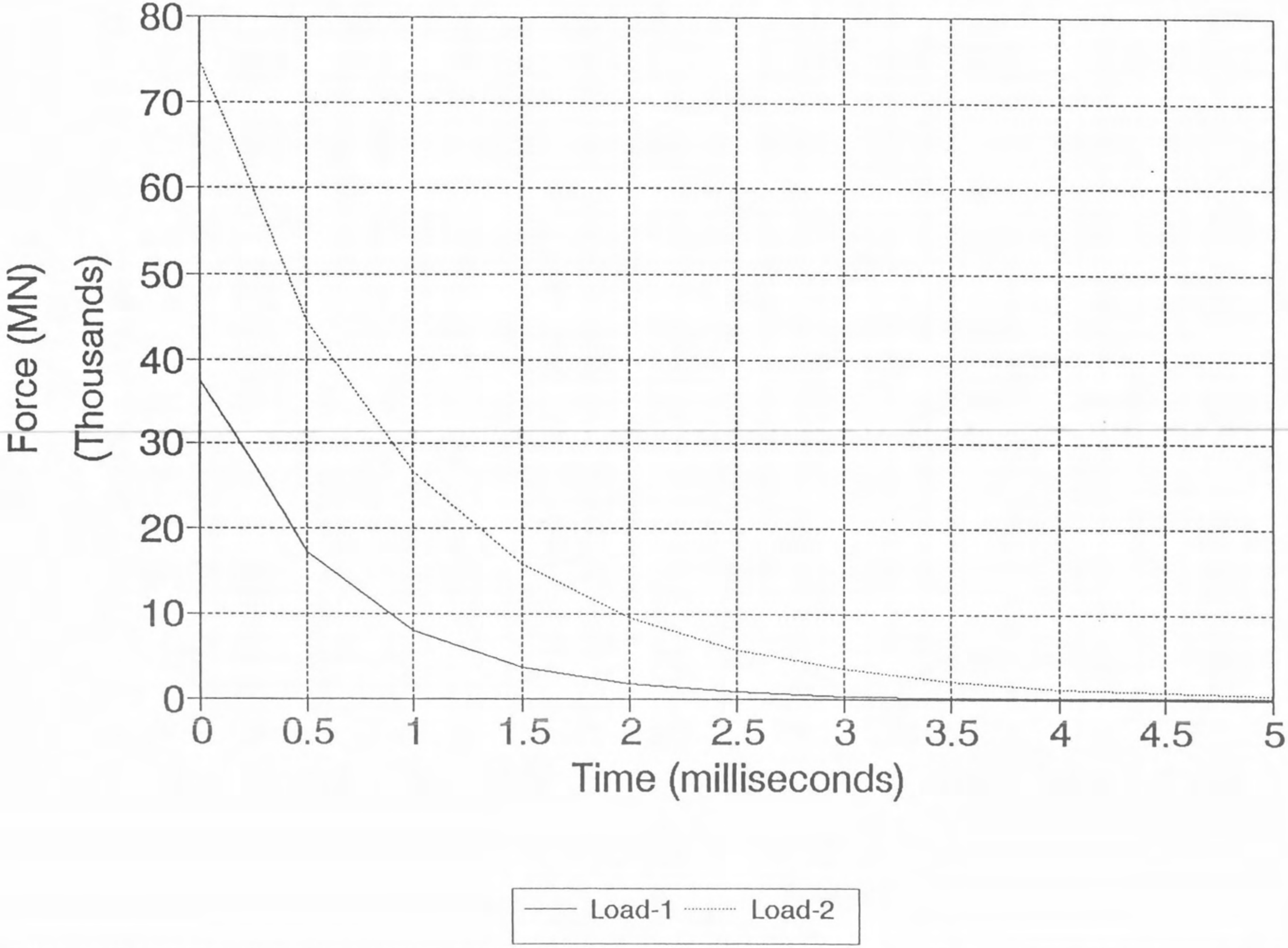


Fig. 2 - Load1 (Frequency Domain)

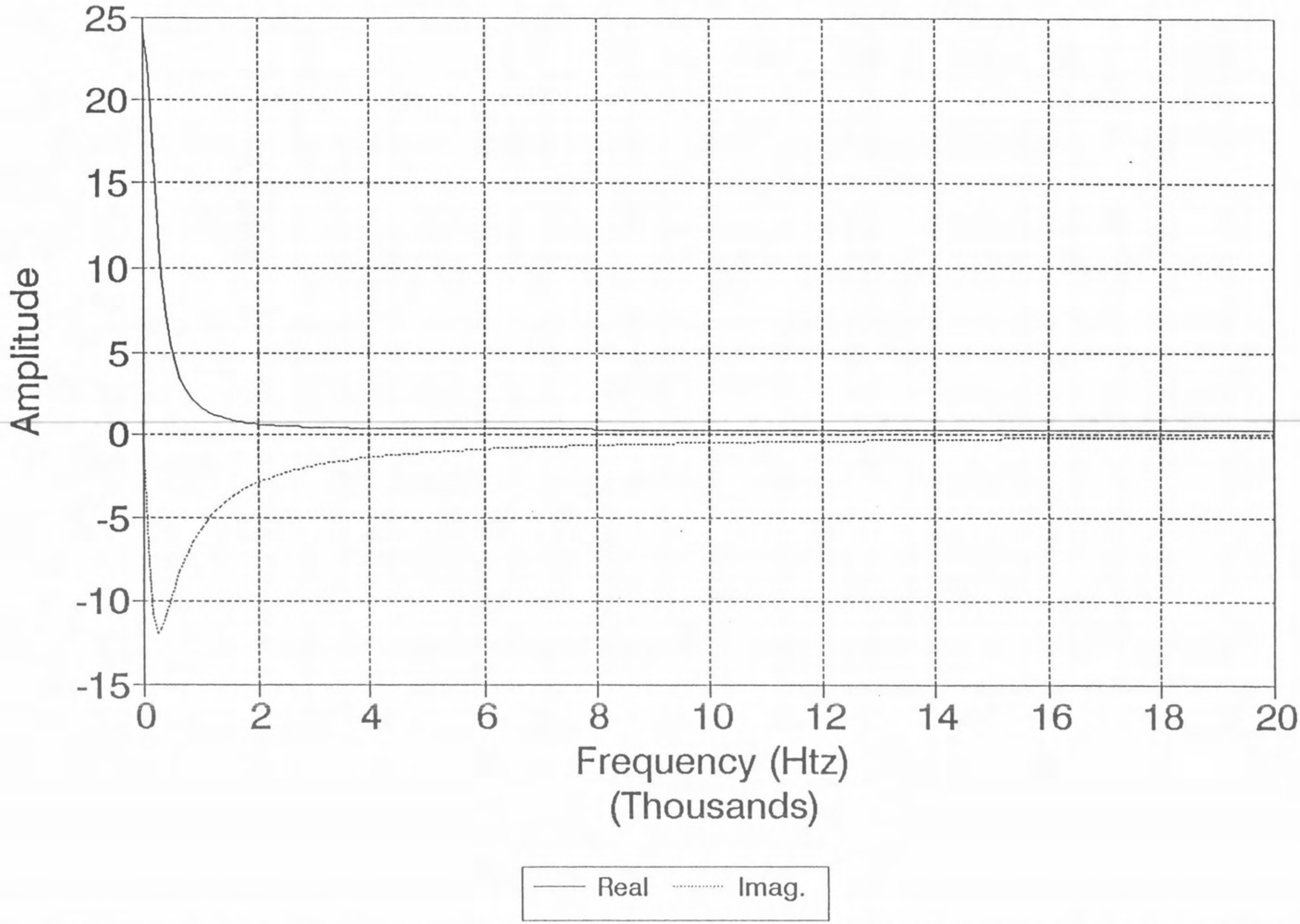


Fig 3 - Load2 (Frequency Domain)

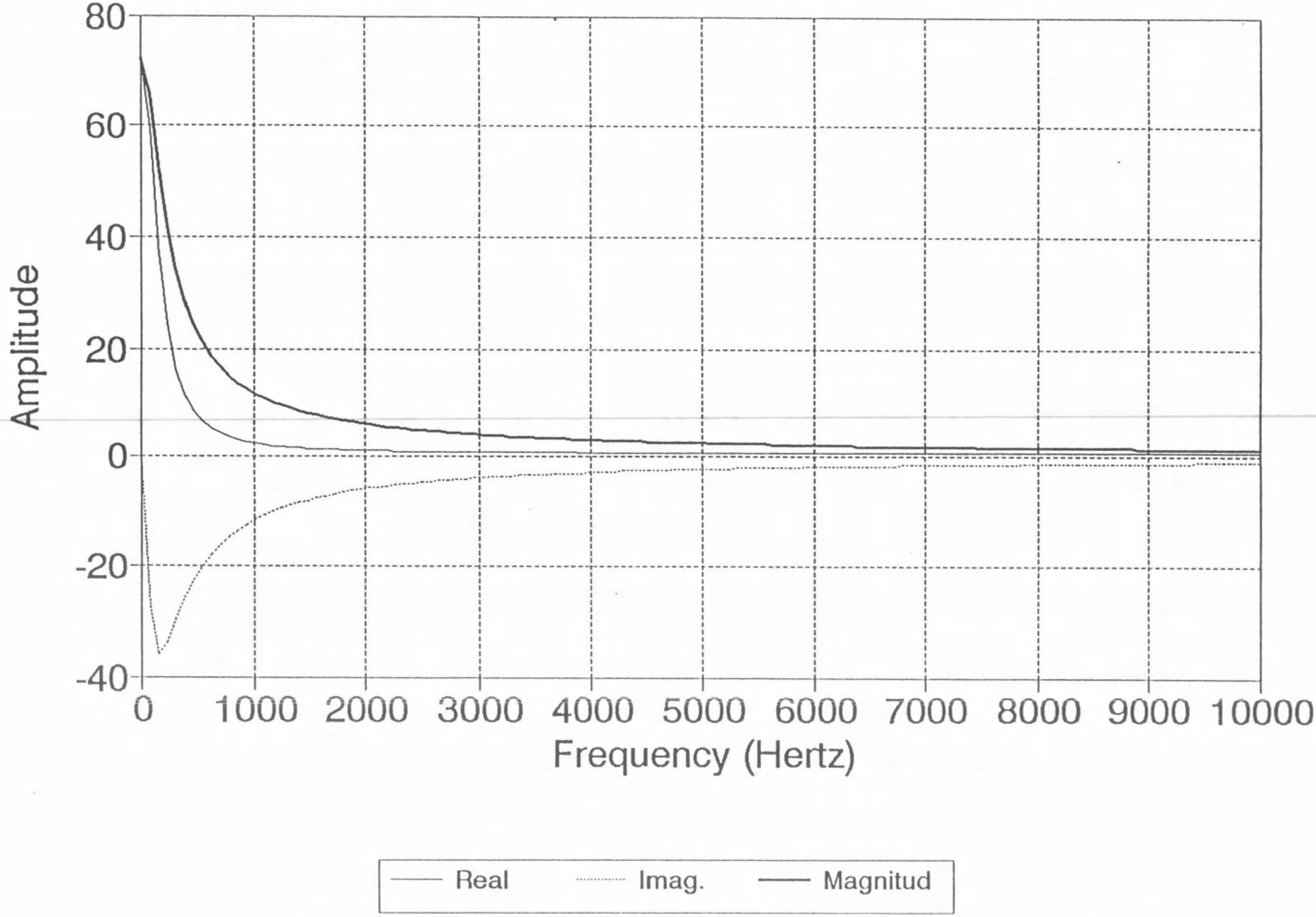


Fig.4 - Force (Time Domain)

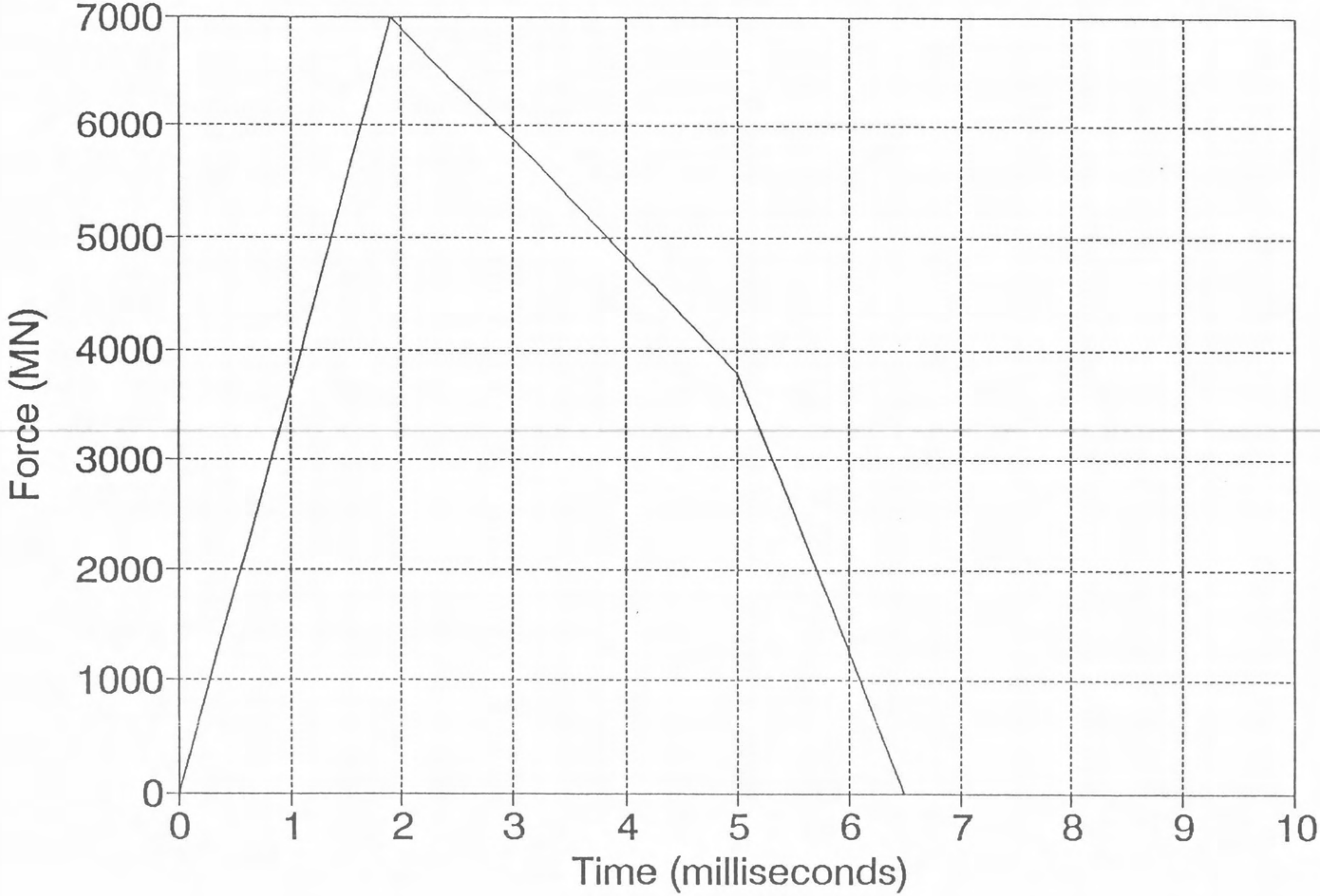


Fig. 5 - Load3 (Frequency Domain)

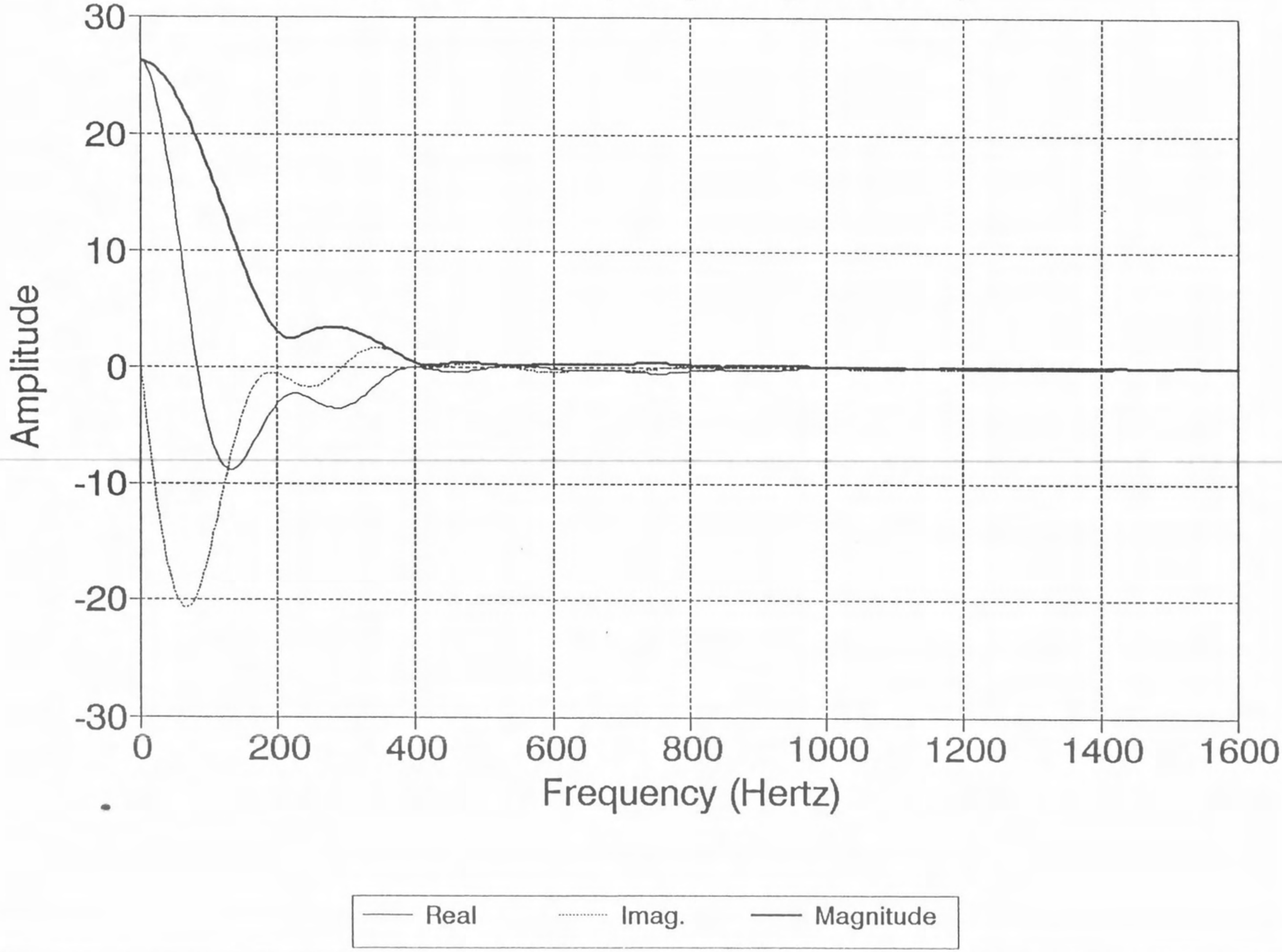


Fig. 6 - Displacement at Top-Middle-Tip

Aplied Force - Load1

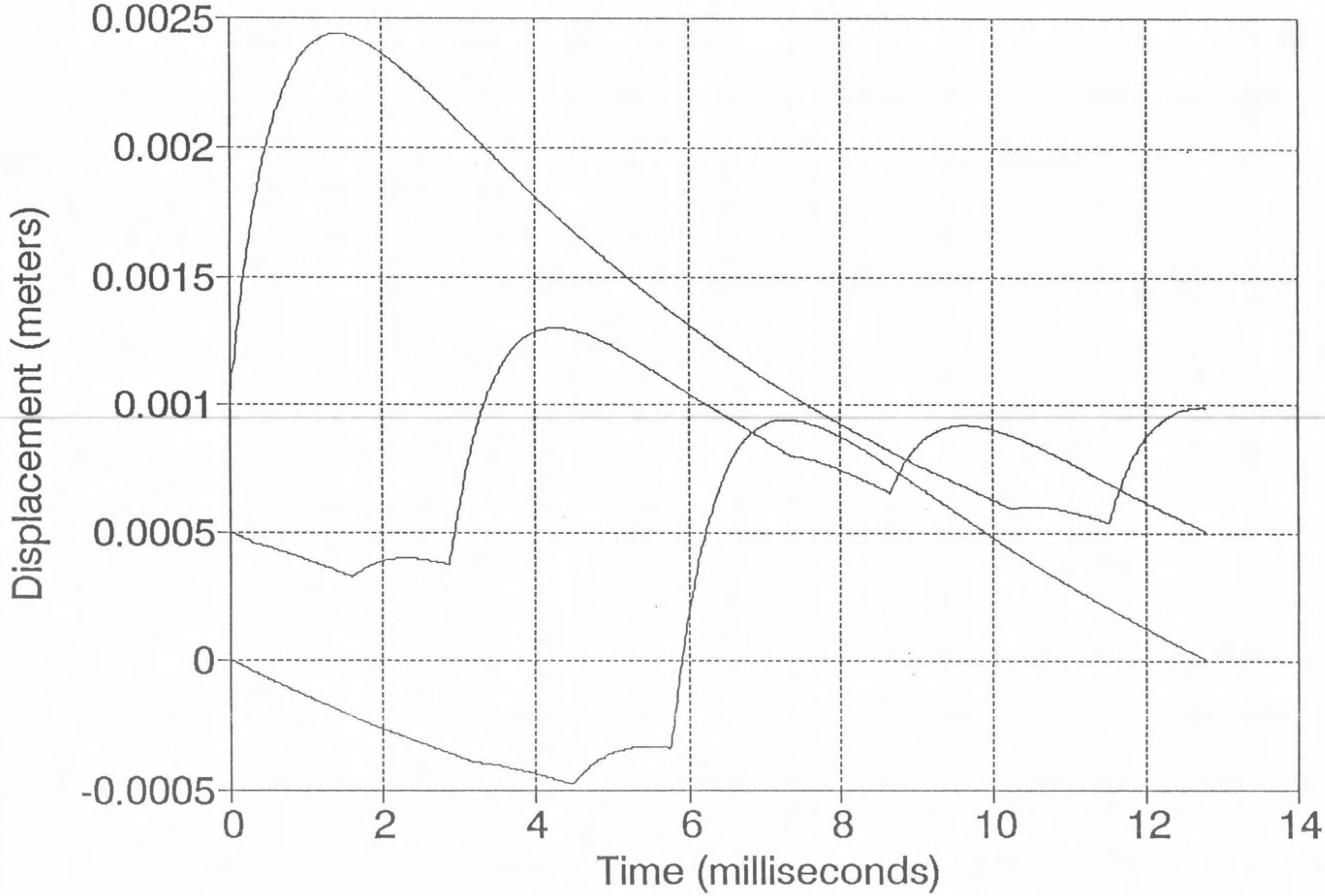
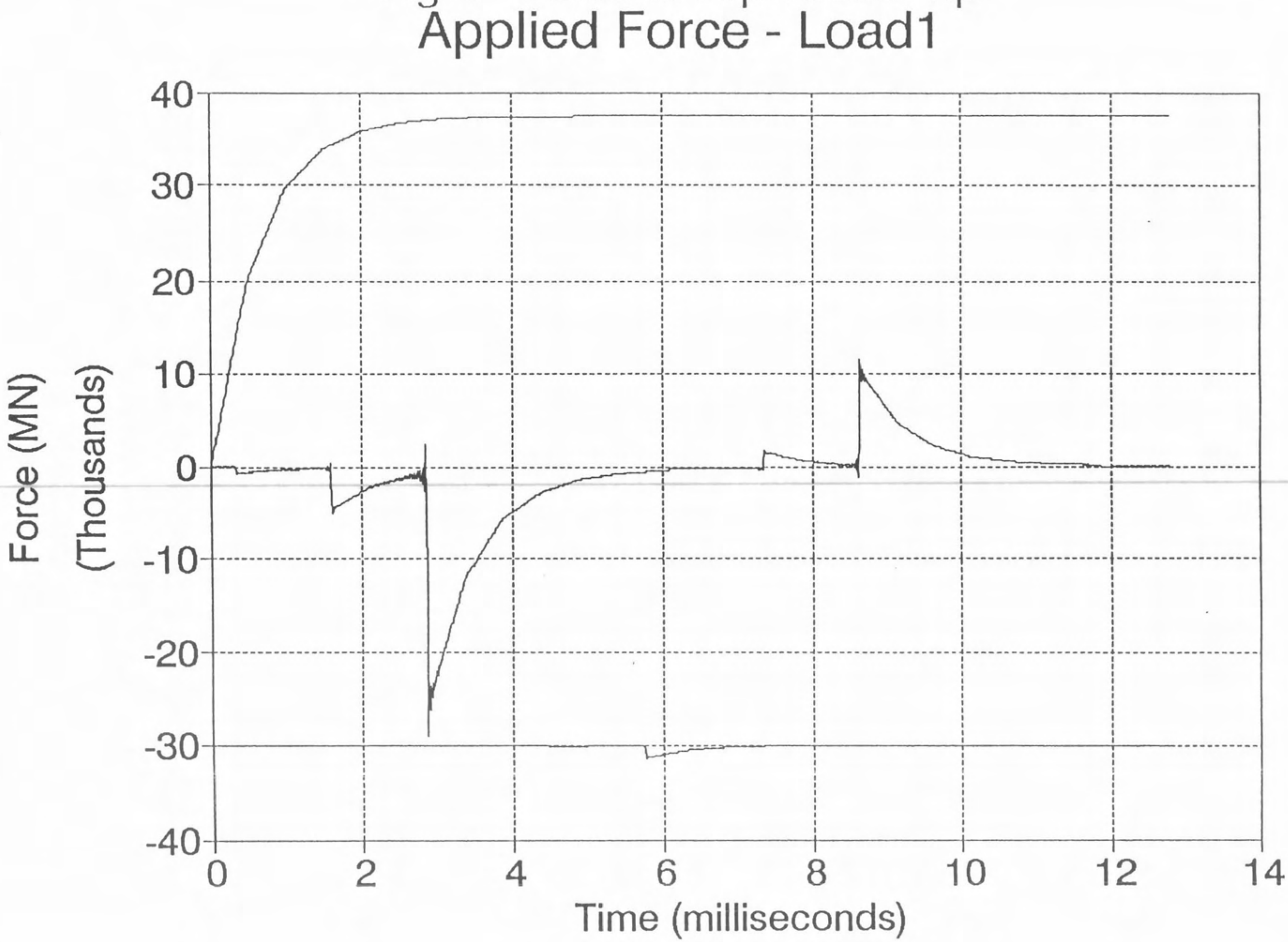


Fig. 7 - Force at Top-Middle-Tip



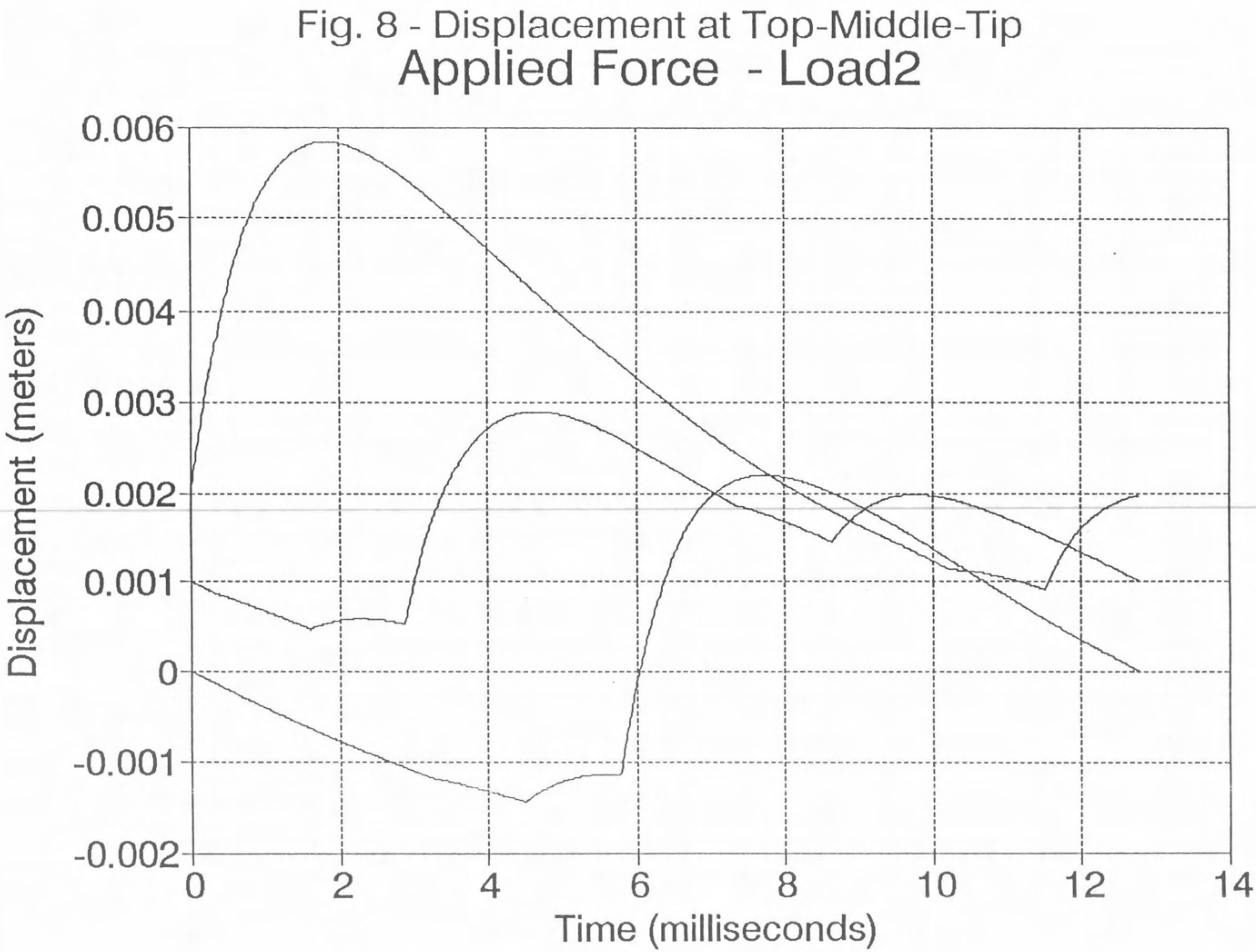


Fig. 8 - Displacement at Top-Middle-Tip
Applied Force - Load2
Displacement (meters)
0.006
0.005
0.004
0.003
0.002
0.001
0
-0.001
-0.002
0
2
4
6
8
10
12
14
Time (milliseconds)

Fig. 9 - Force at Top-Middle-Tip

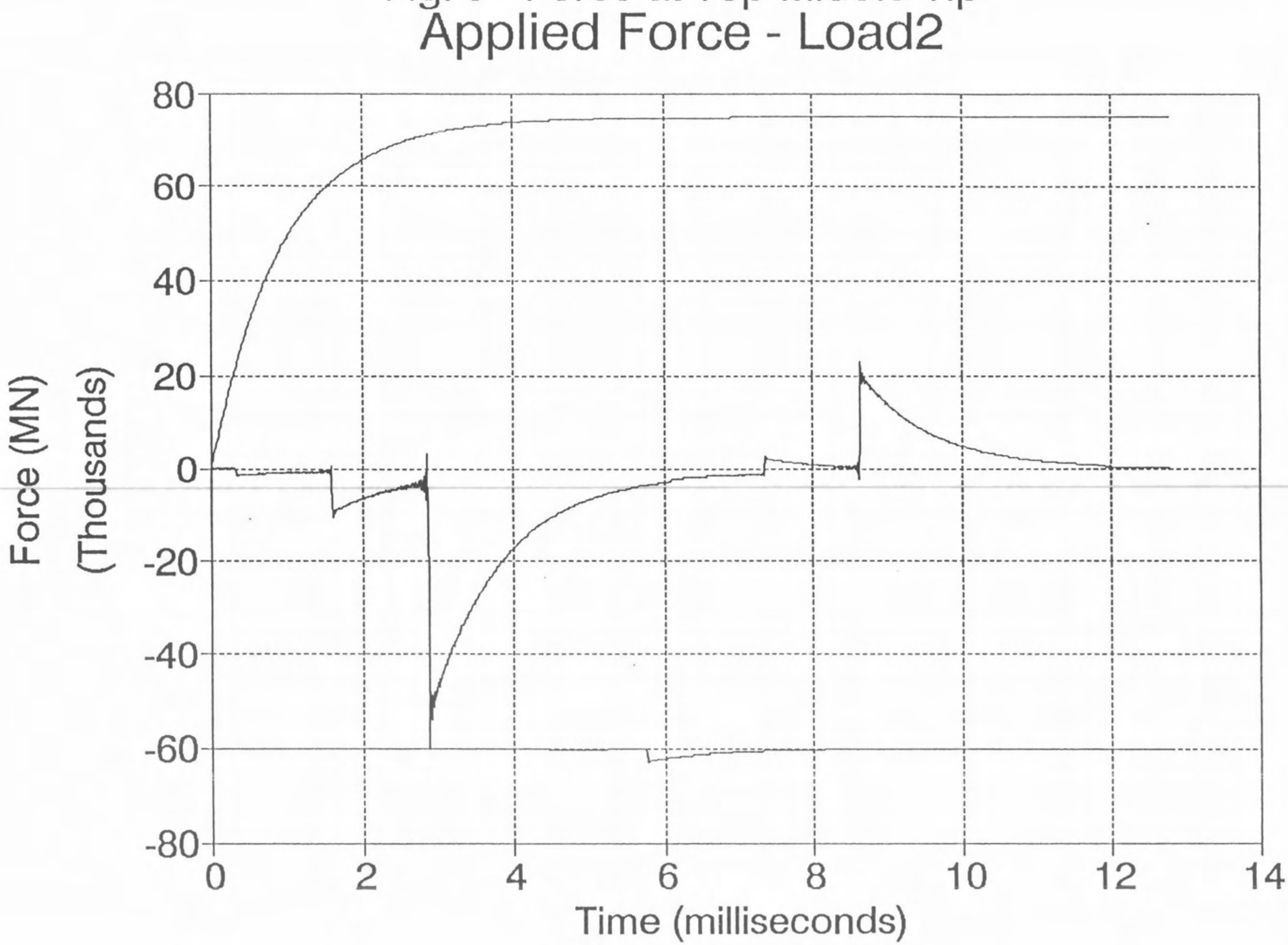


Fig. 10 -Displacement at Top-Middle-Tip

Applied Force - Load2

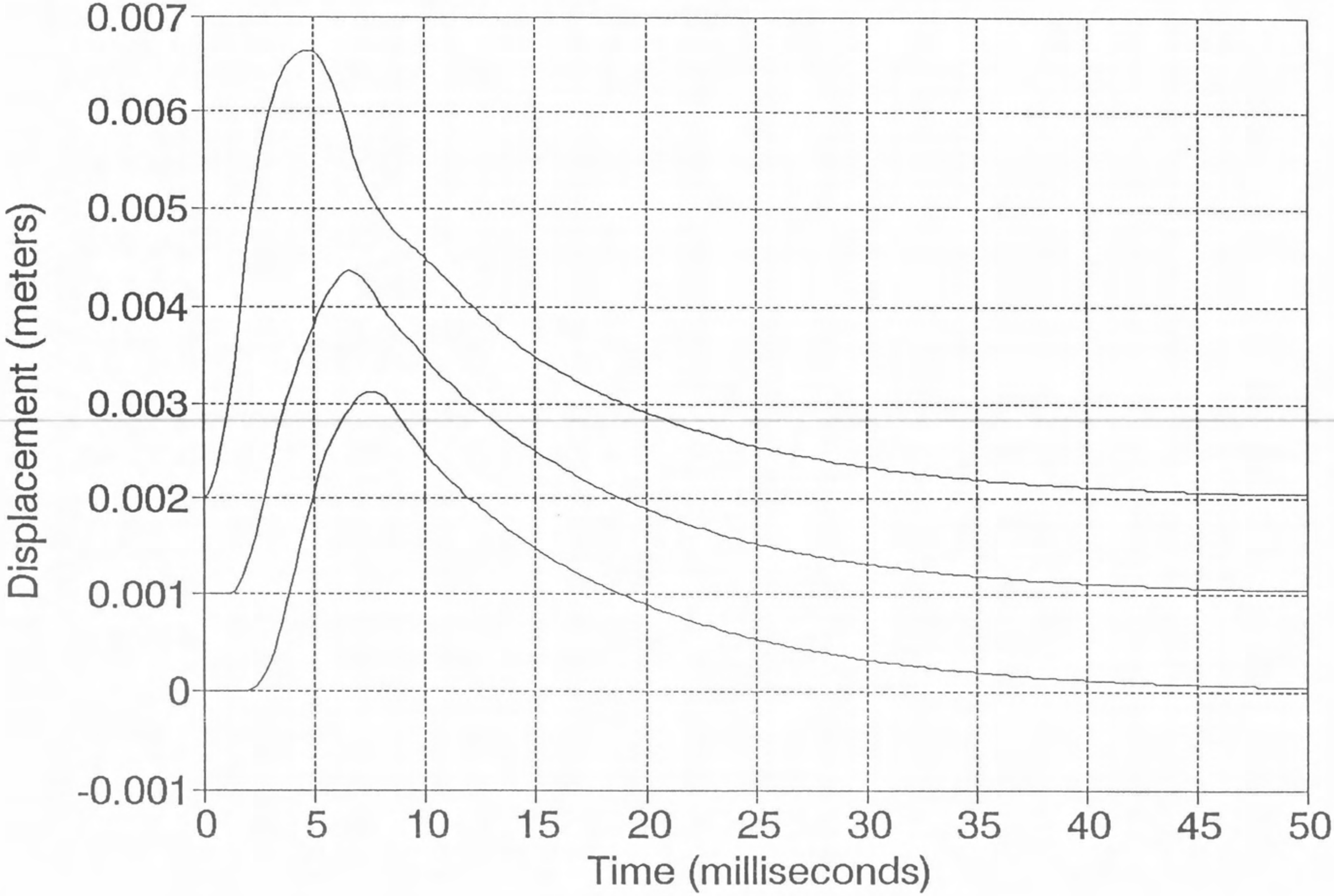


Fig. 11 - Force at Top-Middle-Tip

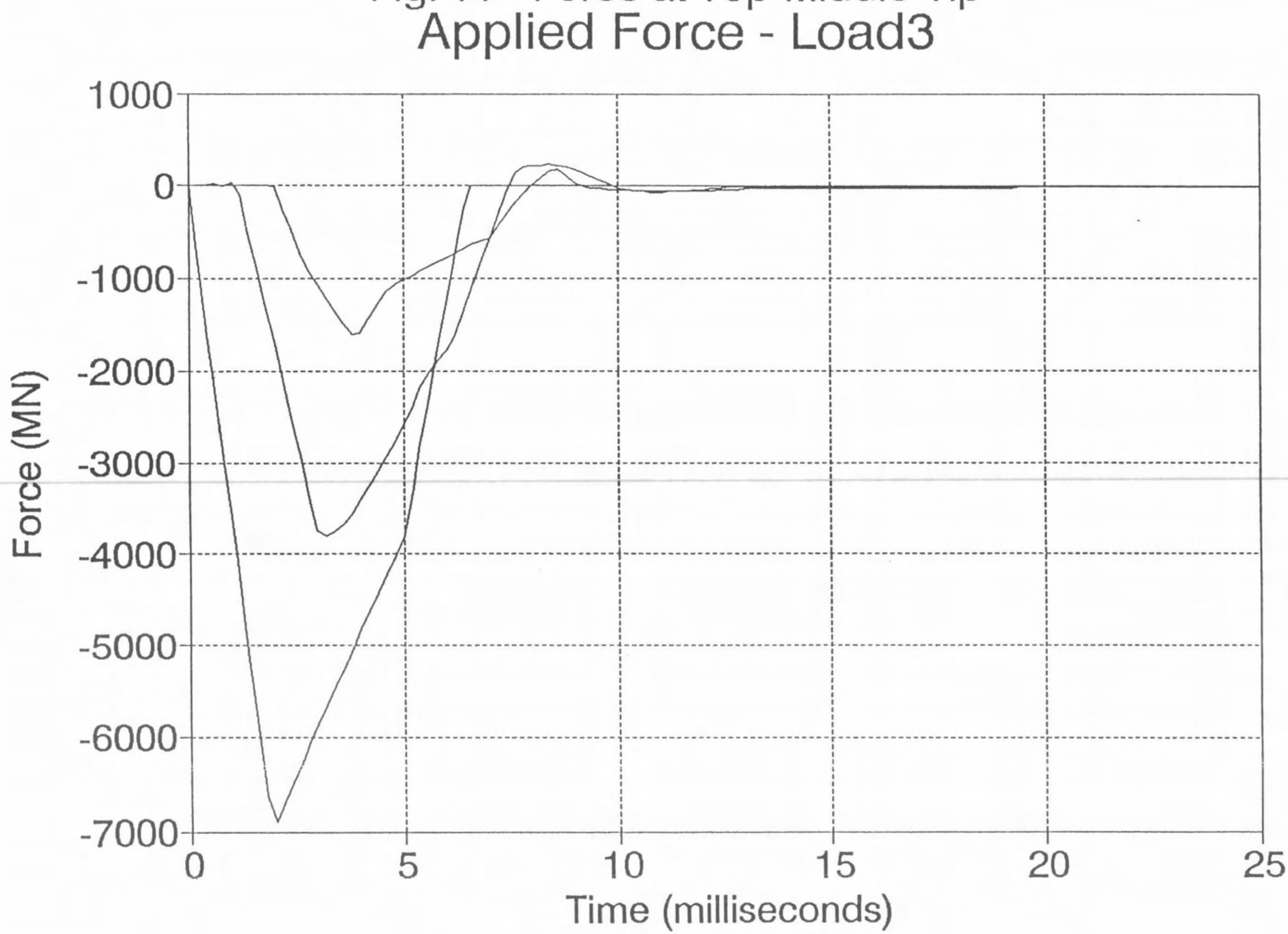
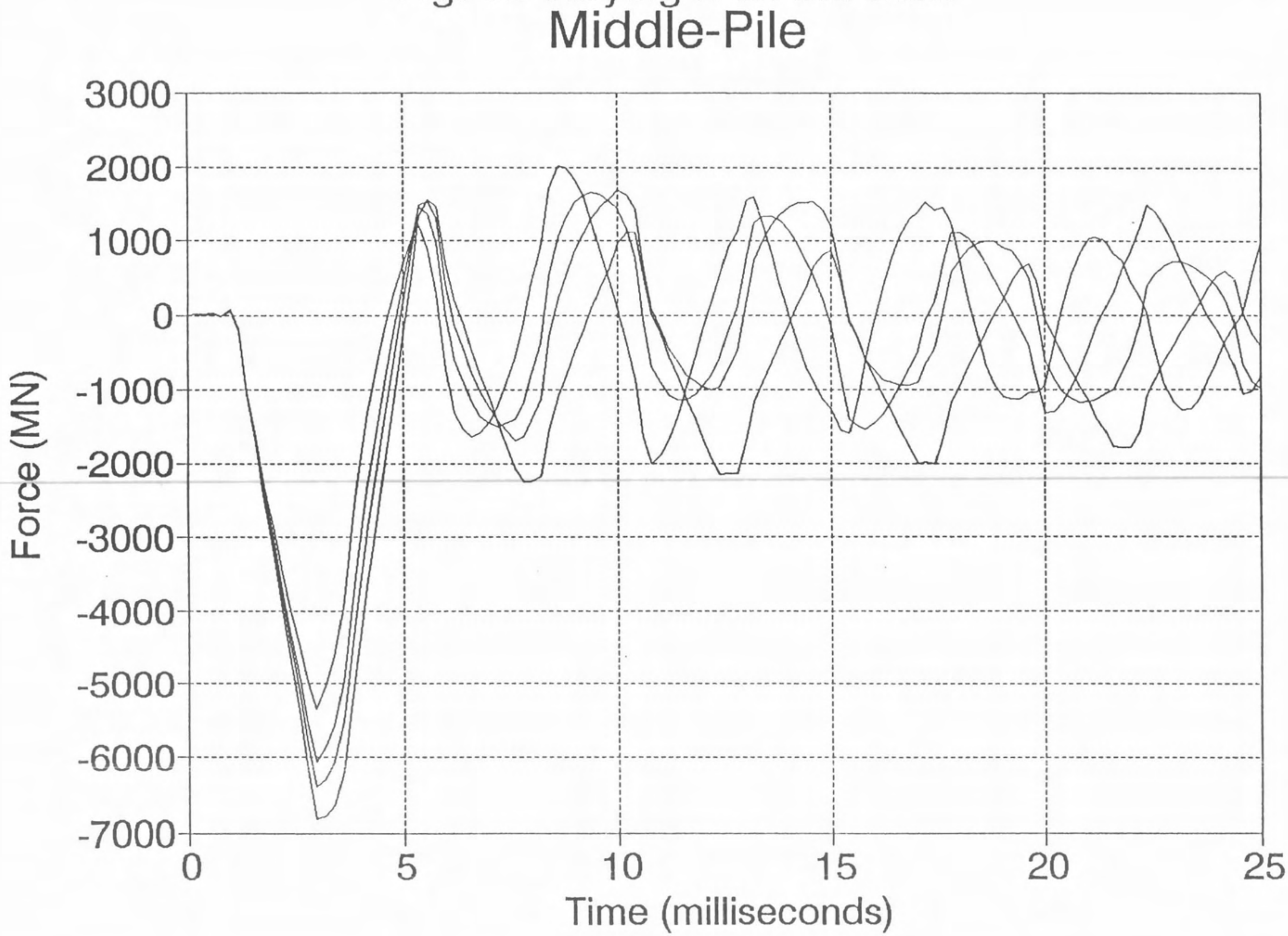


Fig. 12-Varying K on the shaft



Appendix A

```
c         SHELL.for
c         This is for the AE646 Project
          dimension tran(8400,3)
          complex ci,fcmplx,ocmplx,tf1,tf2,w,den
          complex xk,betac,eomega,omega,arg1,alfs,alft
          character*30 file1,file3
          real Le, alsr, alsi, altr, alti, ea, eat
c
          ci=cmplx(0,1)
          pi=4.0*atan(1.0)
c
          write(*,'(//)')
          write(*,*)'SHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELL'
          write(*,*)'                    version 1.02, August 1991 '
          write(*,*)'                       ikayex SOFTWARE TOOLS'
          write(*,*)'SHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELLSHELL'
          write(*,'(//)')
c
c         MATERIAL PROPERTIES
              e1=10.e6
              rho1=2.5e-4
              width1=1
              thick1=0.25
              area1 = thick1*width1
              le = 12
              write(*,*)'e1 etc',e1,rho1,width1,thick1
c
c         READ INPUT FILE
          write(*,'(a\)')' TYPE:  input filename--> '
          read(*,'(1a30)') file1
          write(*,*) file1
          write(*,*)' INPUT: eta,  # lines,  in-pos'n, out-posn  '
          read(*,*) e1, rho1, area1, masst, le
          write(*,*) e1, rho1, area1, Thick2, le
          read(*,*) alsr, alsi, altr, alti
          write(*,*) alsr, alsi, altr, alti
          read(*,*) ndata,nlocx
          write(*,*) ndata,nlocx
          write(*,*)'Ouput : Displacement (1), Force (2) Velocity (3)'
          nyqst=ndata/2+1
              zi=1/12.*width1*thick1**3
              ea=e1*area1
              eat = e1*areat
              c0=sqrt(e1/rho1)
              freq0=10000.
              omega0=freq0*2*pi
              xmass = rho1*Area1*Thick2
c
      do 35 ii = 1,nlocx
                 open(unit=3,file=file1)
                 write(*,*)'reading <ascii> file'
                 rewind 3
                 do 20 nn=1,ndata
                    read(3,*) tran(nn,1),tran(nn,2),tran(nn,3)
 20              continue
                 close (unit=3)
           read(*,*)xx1,Iopt
           write(*,*)xx1,Iopt
c
c
```

```
c         BIG DO_LOOP OVER FREQUENCIES only up to Nyquist
          do 30 nn=2,nyqst
             freq=tran(nn,1)
             omega = tran(nn,1)*2.0*pi
             eomega=e1
c
c            Generalized Rod transmission
                alfs = (alsr + ci*omega*alsi)/ea
                alft = (altr + ci*omega*alti + masst*omega**2)/ea
                xk = csqrt(omega**2*(rho1/e1) - alfs)
                betac = (ci*xk - alft)/(ci*xk + alft)
                den = (1 - betac*cexp(-2*ci*xk*Le))
                arg1=xk*xx1
                tf1=(cexp(-ci*arg1)+betac*cexp(-ci*(2*le*xk-arg1)))/Den
                tf1=tf1/(ci*xk*ea)
                tf2=(-cexp(-ci*arg1)+betac*cexp(-ci*(2*le*xk-arg1)))/Den
c
c               obtain output
                fcmplx = tran(nn,2)+ci*tran(nn,3)
c  Iopt = 1,2,3 (displacement, force, Velocity
                if (Iopt.eq.1) ocmplx = fcmplx*tf1
                if (Iopt.eq.2) ocmplx = fcmplx*tf2
c
c
c            FORM THE COMPLEX CONJUGATE
             tran(nn,2)          = real(ocmplx)
             tran(nn,3)          = aimag(ocmplx)
             tran(2+ndata-nn,2) = tran(nn,2)
             tran(2+ndata-nn,3) =-tran(nn,3)
 30       continue
c         ASSIGN ZERO TO INITIAL VALUE
          sum=0.0
          do 32 n=2,nyqst
             sum=sum+tran(n,2)
 32       continue
          tran(1,2)          = -2*sum
          tran(1,3)          = 0.0
          tran(2+ndata-1,2) = tran(1,2)
          tran(2+ndata-1,3) =-tran(1,3)
c
c         STORE RESULTS
 800      continue
          write(*,'(a\)')' TYPE:  output filename--> '
          read(5,'(1a30)') file3
          write(*,*) file3
            open(unit=2,file=file3)
            write(*,*)'storing <ascii> file'
            rewind 2
            do 820 nn=1,ndata
              if(Iopt.eq.1) write(2,*) tran(nn,1),tran(nn,2),tran(nn,3)
              if(Iopt.eq.2) write(2,*) tran(nn,1),tran(nn,2),tran(nn,3)
              w = 2*pi*tran(nn,1)
              VelR = -w*tran(nn,3)
              VelI =  w*tran(nn,2)
              if(Iopt.eq.3) write(2,*) tran(nn,1), VelR, VelI
 820        continue
            close(2)
  35   continue
c
```

```
      stop 'terminated ok'
      end
c
c
```

TABLE 1

```
c:\temp\Load1fr
210.0e6  7.75  0.31 0.342 30.0
27500.0  683.0  57692.0  533.0
1024   6
0.0  1
DisL1a
15.0 1
DisL1b
30.0 1
DisL1c
0.0  2
ForL1a
15.0 2
ForL1b
30.0 2
ForL1c
```

TABLE 2

```
c:\temp\Load2fr
210.0e6  7.75  0.31 0.342 30.0
27500.0  683.0  57692.0  533.0
1024   6
0.0  1
DisL2a
15.0 1
DisL2b
30.0 1
DisL2c
0.0  2
ForL2a
15.0 2
ForL2b
30.0 2
ForL2c
```

TABLE 3

```
c:\temp\Load3fr
207.0e6  7.75  0.05 0.05 12.0
44000.0  487.2  58496.0  329.3
1024   6
0.0  1
DisL3a
5.8  1
DisL3b
10.2 1
DisL3c
0.0  2
ForL3a
5.8  2
ForL3b
10.2 2
ForL3c
```

```
c:\temp\Load1fr
210.0e6  7.75  0.31 0.342 30.0
27500.0  683.0  57692.0  533.0
```