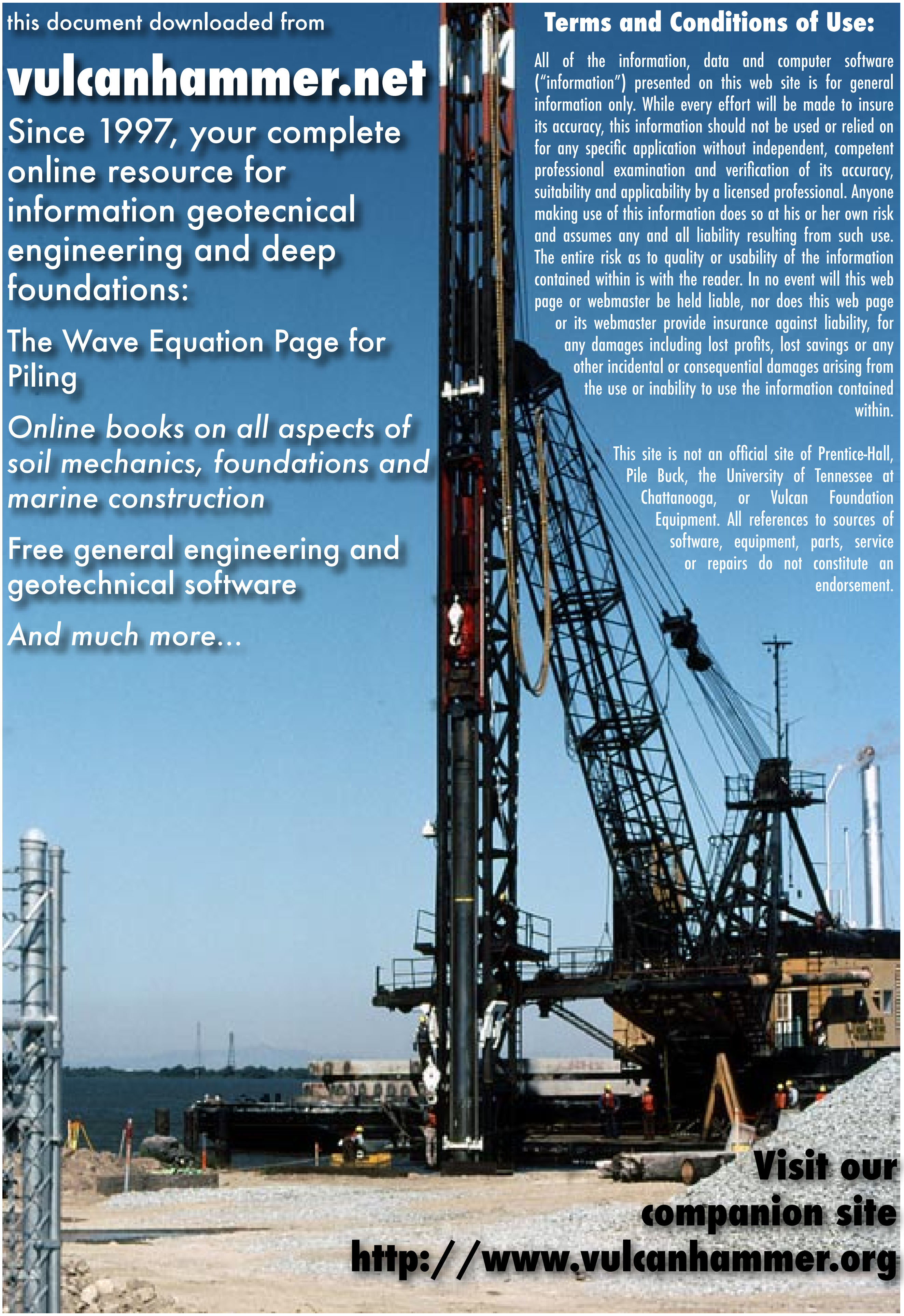

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
February 1991

3. REPORT TYPE AND DATES COVERED
Final Report

4. TITLE AND SUBTITLE
An Introduction to Wave Propagation in Pavements and Soils--Theory and Practice

5. FUNDING NUMBERS

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Alaska Dept. of Transportation & Public Facilities
Statewide Research
2301 Peger Road
Fairbanks AK 99709-5316

8. PERFORMING ORGANIZATION REPORT NUMBER
AK-RD-91-02

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Same as Above

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AK-RD-91-02

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This paper introduces the physics and analysis of wave propagation in pavement and soils. The study of wave propagation in soils can yield useful results to engineers concerned with the resilient characteristics of a particular site, dynamic soil-structure interaction (e.g., pavements or soils), and earthquake analysis. Types of waves considered here are those resulting from forced impulses and ground penetrating radar. Background of the development in measuring waves in soils beginning in the late 1930's to the present is given. Practical and useful applications are presented along with equipment necessary to obtain results. Use of wave measuring equipment may expedite soil exploration in the future.

14. SUBJECT TERMS
Wave propagation, pavement analysis, soil dynamics, modulus, soils, pavement

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

AN INTRODUCTION TO WAVE PROPAGATION IN PAVEMENTS AND SOILS

Theory and Practice

Final Report

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February 1991

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2301 Peger Road
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Publication Cost: $5.00

ACKNOWLEDGEMENTS

Special thanks to my wife Nancy, for "watching the kids" and making the time available to do the work. Special thanks to Nicole R. Greene, Clerk Typist III, for word processing and putting up with my messes. Thanks to Steve Rice, student editor, for editing. Thanks to Leroy Hulsey, professor of the Earthquake Engineering class for which the original paper was written for. Many of the references used in compiling this report were obtained from the Alaska Department of Transportation Statewide Research Section library. Without the availability of these references the report quality would be severely compromised. Thank you to Billy Connor, Statewide Research Manager, for making this library available to me and anyone interested in increasing their knowledge of transportation engineering. Thanks to the University of Alaska Fairbanks library for making references available and obtaining inter-library books for those that weren't there.

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SUMMARY

This report introduces the physics and analysis of wave propagation in pavement and soils. The study of wave propagation in soils can yield useful results to engineers concerned with the resilient characteristics of a particular site, dynamic soil structure interaction (e.g., pavements on soils) and earthquake analysis. The types of waves considered result from forced impulses and ground-penetrating radar waves. Some applications of this analysis that are discussed include:

1. Determining pavement/soil layer elastic properties.
2. Predicting of loads and deflections resulting from a design event.
3. Estimating pavement/soil layer thicknesses and depth of a rigid layer.
4. Estimating moisture content of soil layers.
5. Determining when load restrictions are warranted.
6. Determining pavement life through dynamic analysis.
7. Determining the distance of an earthquake epicenter in distance from a point of measurement.

This paper presents the background of the development in measuring waves in soils for analysis beginning in the late 1930's to the present. Practical and useful applications are presented along with the equipment necessary to obtain results. Accuracy of results, ongoing research and recommended future research are given.

Background research for and writing of this report were done in the author's "off-work" hours. The original report was submitted and presented as part of the requirements for a University of Alaska Fairbanks Civil Engineering graduate course in Earthquake Engineering. This edited version is dedicated to those interested in learning more about the subject: Use this knowledge for the benefit of others.

INTRODUCTION

The field of wave propagation analysis in soils is relatively new. It has been and is limited by the accuracy of the measuring and data processing equipment. Developments in electronic equipment and computer analysis make this subject more viable. The Transportation Research Information Service conducted a literature search for all articles and books which mention wave propagation in soils and/or pavements. This paper summarizes the findings from the literature, emphasizing applications to airports and highway pavements.

It is felt that most civil engineers do not have a good understanding of wave propagation. Therefore, this paper was written as a practical introduction to the subject. For in-depth study of analysis and experimental techniques, references cited throughout the paper are given at the end.

The elastic plate wave equation was developed about one hundred years ago by Rayleigh [48] and Lamb [30]. The equation is a three-dimensional, time-dependent hyperbolic partial differential equation. Analytical solutions for this type of equation are only available for ideal (initial and boundary) conditions. Numerical solutions using computers give excellent results especially considering the range of soil properties. The idea is that soil properties affect the velocity and wavelengths of waves within and on the surface of the soil. Therefore, by measuring wavelengths and velocities, we can determine the soil properties.

This paper is written in three sections. The first discusses background and development, showing past research and study that have been done in the field. The second presents recent (within the last ten years) studies and proposed uses, showing applications to today's problems. The final section describes proposed research and recommends future research.

GENERAL THEORY AND INTRODUCTION TO WAVE MEASUREMENT

Vibrating or impact loads on the surface of an ideal, homogeneous, isotropic, linear, elastic material generate different types of waves:

1. Primary waves, often referred to as compression, dilatational or P-waves.
2. Shear waves, often referred to as secondary, distortional or S-waves.
3. Rayleigh waves, also termed surface waves or R-waves.
4. Love waves.

For each type, the velocity of wave propagation (V_p, V_s, V_r, V_l) is related to the properties of the material. Each wave-type also displays a different type of motion. These same types of waves travel through soils.

The Primary wave exhibits a push-pull motion and hence is referred to as a dilational wave. This dilational motion occurs in the radial direction from the impulse source. Primary waves travel at higher velocity than the other waves and occur first in a travel-time record of wave motions. The velocity of the P-wave is given by the following equation:

$$V_p = [(\lambda + 2G)/\rho]^{1/2} \qquad (1)$$

where λ (Lame's constant) $= \mu E/[(1+\mu)(1-2\mu)]$
G (Shear modulus) $= E/2(1+\mu)$

and E, μ, ρ are Young's modulus, Poisson's ratio and the mass density, respectively, of the elastic material. Sound waves travel through water approximately 4 times more rapidly than through air. Similarly, the velocity of P-waves depends greatly upon the degree of saturation and the void ratio of a porous medium such as soil.

The S-wave exhibits shearing motion perpendicular to the radial direction of wave propagation. Shear waves travel at significantly lower velocities than P-waves and as a result they appear later in a travel-time record, hence the name "Secondary" waves.

The velocity of Shear waves is generally given by the following equation:

$$V_s = \sqrt{\frac{G}{\rho}} \qquad (2)$$

Shear wave velocity is independent of the degree of saturation since fluid cannot transmit shearing motion.

An American scientist named M. Biot derived the wave equation for propagation through a porous medium and published the results in 1956 [7]. According to his theory, the Shear wave velocity in saturated porous media is:

$$V_S = \sqrt{\frac{G}{\bar{\rho} + \frac{\rho^* \rho_A}{\rho^* + \rho_A}}} \qquad (3)$$

where ρ = mass density of the soil particles
ρ^* = mass density of the fluid (water) given by the mass of fluid per unit volume of soil.
ρ_A = mass density of an additional apparent mass which relates to the coupling the fluid and the soil.

There is, again, no structural coupling between the soil and the water, but in the water there is a rotation which is in the same direction (with different magnitude) as that of the particles in the soil. The rotation is a function of ρ_A, which will vary with grain size and permeability. Determination of the ρ_A term is not well understood, so the equation is virtually always solved by assuming the total mass density of the soil mass for determining the Shear wave velocity as in the previous equation.

Biot found that both P-waves and S-waves are transmitted through a saturated medium, with the P-waves moving through the soil and the S-waves moving through the water. P-wave velocities decrease as the moisture content decreases. S-waves are unaffected by the moisture content.

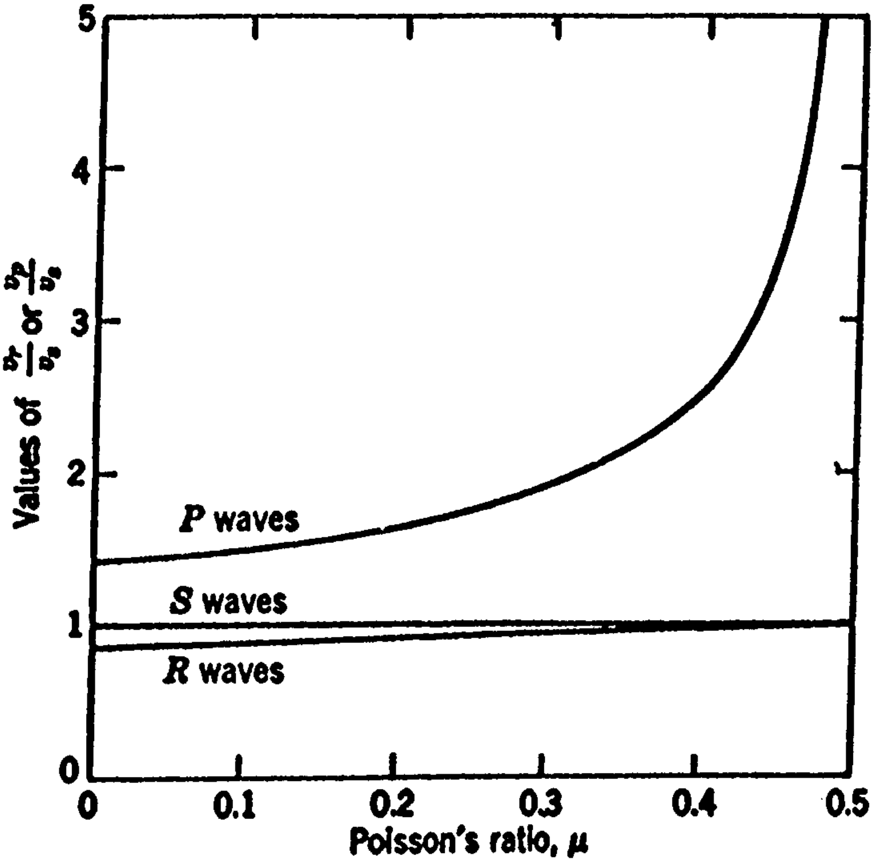
Rayleigh waves do not spread into the body of the elastic medium, but travel along the surface. The wave motions cause both horizontal and vertical particle displacements, which describe a vertically inclined ellipse at the surface [45]. The velocity of Rayleigh waves is nearly equal to Shear wave velocity, particularly for media with Poisson's ratio above about 0.25 (which are typical of soils).

Another surface wave form is the Love wave. It is a horizontally polarized Shear wave that results when the Shear modulus increases with depth [27,45]. At high frequencies, i.e., short wavelengths, the velocity of Love waves approaches the velocity of Shear waves in the medium of the surface layer. At low frequencies, the velocity of Love waves approaches the velocity of Shear waves in the stiffer subgrade.

Solutions are available for Rayleigh, Love, and other types of wave equations under a variety of stratification conditions [9]. In many such solutions the velocity of wave propagation is a function of the wave frequency. When this happens, unless we are dealing with sinusoidal, steady-state conditions, the shape of the disturbance changes as it travels along the medium in question. Sharp disturbances become trains of waves, each train containing oscillations of essentially equal frequency. The velocity of a group of waves under these conditions differs from the velocity of an individual wave. This type of dispersion does not necessarily combine in an additive manner with the dispersion due to internal damping [24]. This phenomenon accounts partly for the increase in duration of earthquake motions with focal distance.

Since pavement systems generally have decreasing modulus with depth, Love waves are not considered. Figure 1 shows the relationship between P and R-wave velocities to the S-wave velocities as a function of Poisson's ratio. Several important conclusions can be drawn from Figure 1. First, notice that V_s is independent of the Poisson's ratio (μ) of the soil, while V_p is highly dependent upon this value. While V_r is slightly dependent upon μ it can be seen that, for common μ values associated with soils (0.3-0.5), this effect is small. For most practical situations it is assumed that

$$\alpha V_s = V_r \qquad (4)$$



Effect of Poisson's ratio upon various wave velocities. (From Richart.)

FIGURE 1 [49]

Rayleigh waves, being surface waves, can be measured. Using the predicted Shear wave velocity, the underlying soil properties may be estimated using Equation (1). The depth Rayleigh waves reach is proportional to their wavelength. A proportionality constant, α, is used such that

$$\alpha = \frac{V_r}{V_s} = 0.875 + 0.16\mu \qquad (5)$$

With $\mu = 0.35$, for example, this gives $\alpha = 0.931$. So here $V_r = 0.931 V_s$.

Rayleigh wave velocity is independent of frequency in homogeneous material. Since an ideal elastic half-space has a unique R-wave velocity, each frequency has a corresponding wavelength according to the following relationship:

$$V_r = f \times L_r \qquad (6)$$

where f is the input frequency of excitation that generates a Rayleigh wave of wavelength L_r. Since Rayleigh waves travel through a homogeneous soil layer at a velocity independent of the frequency of excitation, homogeneous soil layers can be identified.

Figure 2 shows schematic results of two tests on an elastic isotropic medium. A steady-state vibrating load of frequency f is applied to the surface. The resulting wavelength can be obtained using an oscilloscope and electrical pickup devices. The number of waves, n, occurring at a given distance was determined by moving the pickups to find the zero amplitude points. Modern test set-ups can calculate phase angle at given sensor spacing, as will be described later. A plot of n versus x as shown in the figure yields the average wavelength as the slope inverse of the established relationships. Once the wavelength and frequency are known, the elastic properties of the material can be determined from the equation for Shear wave velocity.

However, for layered materials, the ideal case must be modified. Low frequencies generate long wavelengths corresponding to deep sampling of the site. Conversely, high frequencies generate short wavelengths corresponding to shallow sampling. In multilayered systems, the Rayleigh wave travels at a velocity that reflects the properties of the layer(s) that the wavelength samples. Each wavelength will have a corresponding phase velocity, depending on how much of each layer the wave samples.

An important assumption, reasonably valid for uniform and layered structures, is that the majority of the Rayleigh wave travels to a depth equal to L_r. Thus the average material property, E, is approximately typical of the material at a depth of one-half L_r [61]. More recent studies (1982) showed good correlation with the pavement profile when the effective sampling depth was taken to be $1/3L_r$ [21]. The newest techniques find solutions independent of these assumptions.

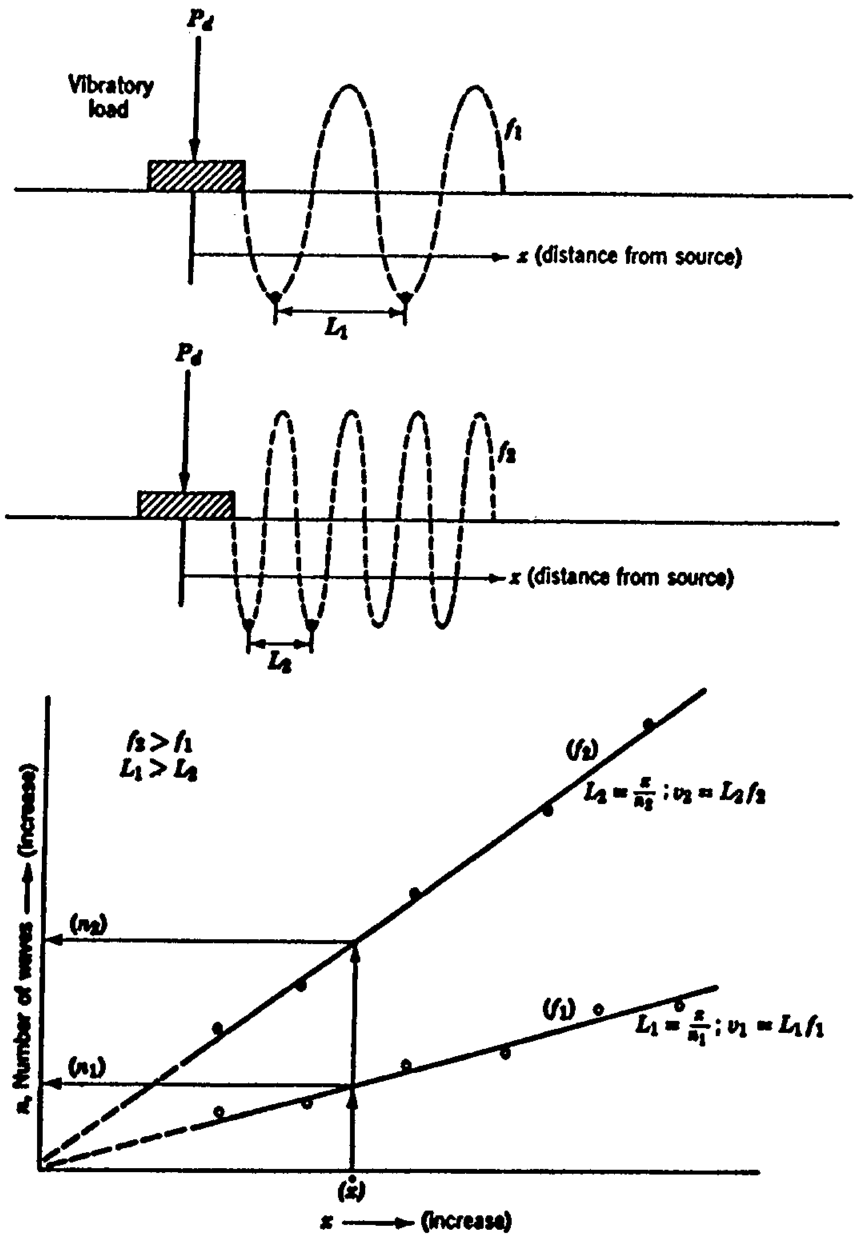


Figure 2 [61] Schematic illustration of surface wave propagation through ideal elastic media.

In order to effectively sample a location, the vibration equipment must be capable of producing a wide range of frequencies. The results are interpreted using a *dispersion curve* that relates wavelength to wave velocity. Figure 3 shows an actual vibratory response for a pavement structure. The upper part of the diagram is a plot of frequency versus wave velocity with the slope of any line being equal to the wavelength.

This diagram can be obtained from the results of Figure 2. Note that within given frequency ranges the resulting wave velocities are constant, bounded by given wavelengths. Using the assumption that the wave velocities (and therefore E values) are representative of material properties of one-half the wavelength. The dispersion curve is produced on the bottom portion of the diagram. The structure analyzed is also shown in Figure 3. The resulting V_r for each layer can be read directly from the diagram. The modulus for each layer can be readily obtained by solving the Shear wave velocity equation for E:

$$E = \frac{2(1+\mu)\, V_r^2\, (\gamma_m / g)}{\alpha^2} \tag{7}$$

where γ_m is the unit weight of layer m, and g is the acceleration of gravity.

In the above example, steady-state frequencies were applied to the surface and measured at radial offsets to obtain results. In the past this method required large, expensive vibrating equipment in order to generate a wide range of frequencies which were individually measured. Newer techniques which eliminate this burdensome equipment by analyzing multiple frequency inputs concurrently will be shown later.

An alternative to surface measurement techniques is cross-hole testing [60]. The source and receivers are placed in drilled holes so that direct arrivals of waves can be determined. Both P and S-wave velocities can be measured by this type of test. Layering and velocities are accurately determined. Proper spacing of the bore-holes minimizes problems caused by refracted waves. The constraint of a quick, nondestructive test precludes application of this method to pavement evaluation. However, cross-hole testing is sometimes used as a tool to validate surface measurements, or in other types of geological and seismic testing.

Another type of wave that travels and deflects in soils is ground-penetrating radar. Its use in analyzing airport, highway and bridge pavements is a relatively new field which looks promising for determining layer thicknesses and moisture content.

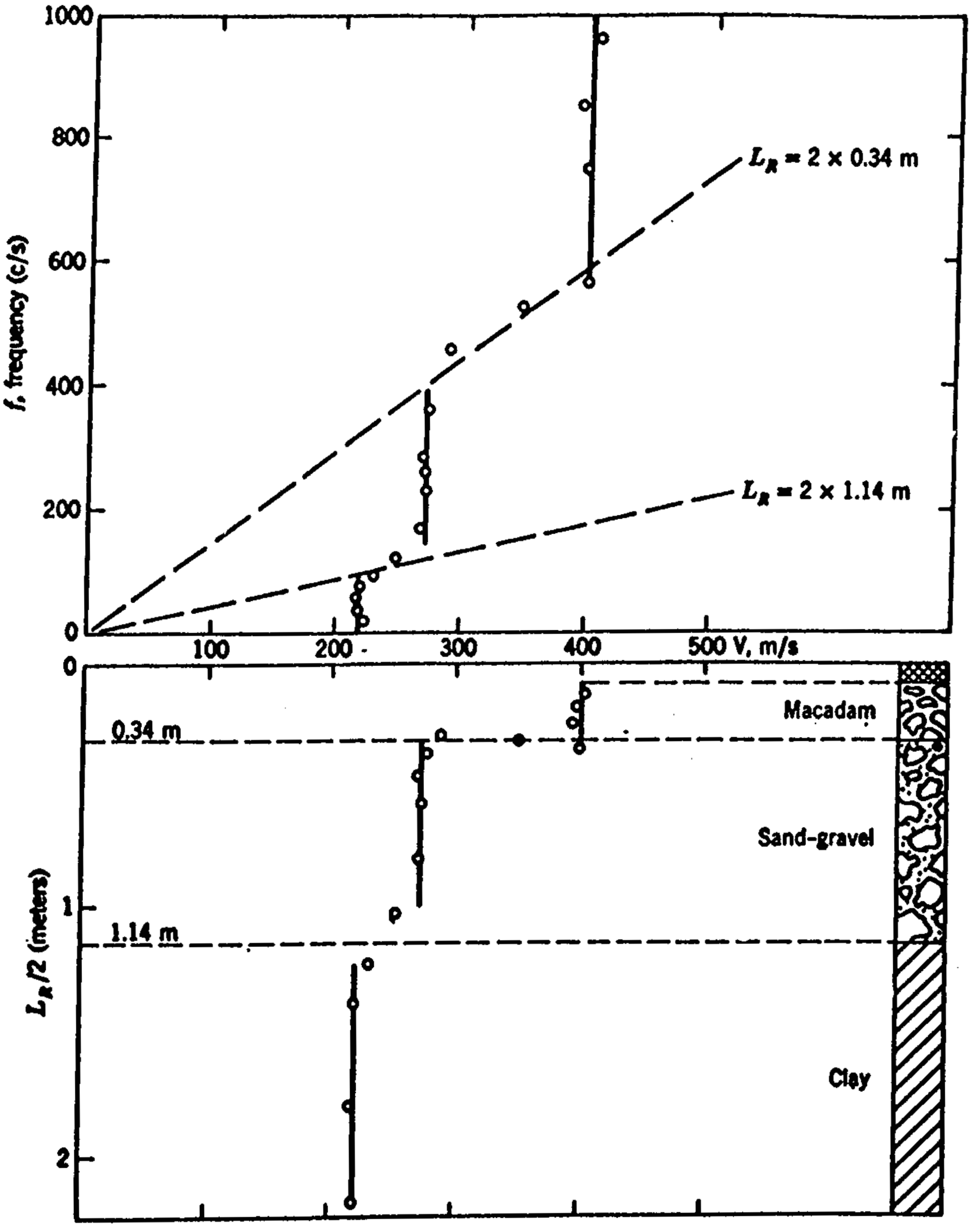


Figure 3 Wave propagation results on stratified pavement system. (From Heukelom and Foster.) [23,61]

Ground-penetrating radar is the electromagnetic analog to ultrasound. With radar, short pulses of electromagnetic energy are emitted from an antenna. These pulses penetrate materials which act as dielectrics (insulators), such as rock, soil, concrete and asphalt pavement. Dielectric discontinuities in these materials, for example at material interfaces and metal inclusions, produce echoes which are received at the antenna. A pattern of echoes, referred to as the wave form, is produced by successive arrivals of echoes from different interfaces. So the wave form contains information regarding the location of interfaces, the nature of the material contrast at the interface and the properties of the material layers. The usefulness of ground-penetrating radar will be further discussed in the applications section of this paper under the headings **Estimating Pavement/Soil Layer Thicknesses** and **Estimating Moisture Content of Soil Layers.**

EARLY DEVELOPMENTS

Early experiments in soil wave propagation were performed with crude equipment (by today's standards), but their findings have helped develop the field today (which will probably be crude by tomorrow's standards). Until the 1960's, equipment was not available that could measure frequencies to sample surface layers (>1000 cps).

The resonant column technique of laboratory analysis of soils was first used by the Japanese in the late 1930's [60]. With this method, a cylindrical soil sample is excited at the ends by waves whose frequency is varied until maximum deflection is reached (resonance). Soil properties are then determined using wave equations.

Propagation of vibrations in soil was studied in Germany in 1938 [14]. The German work showed that, on many sites, the velocity of propagation of the vibrations decreased as frequency increased. The variation was attributed to a depth-related change in soil properties. Attempts were made to fit the actual thickness and properties found at bore-holes. However, the type of wave being studied was not definitely established; some workers considered it a Rayleigh wave while others thought the waves were vertically polarized Love waves. In view of current knowledge it appears that the first opinion was correct. The German work also showed that the velocity of propagation was related to the strength of the soil.

Similar experiments were carried out in Sweden in 1946 [6], on a fairly uniform site, and there was no appreciable variation in the phase velocity at frequencies between 14 and 32 cps. Plate bearing tests were also made on the soil, using plates of different thickness and diameters to derive the relationship between the load and deformation under static conditions. The conclusion drawn from the results was that the dynamic method gave in this case a relatively correct idea of the behavior of the soil under the action of a distributed load if the maximum value of $\mu = 0.5$ was used. It was implied that, when calculating traffic stresses in the soil beneath a road [1,17], the elastic constants calculated from vibration experiments apply.

In Holland measurements were made at the surface of bituminous paved roads ([59] 1951; [45] 1953) at frequencies of 10 to 60 cps. The wave propagation was still governed primarily by the properties of the soil beneath the road. In these experiments the investigators observed two or more distinct velocities at certain frequencies: the wave of lowest frequency was usually present near the vibration source, and waves of higher velocity predominated at more distant points. It was assumed that the vibrations were Shear waves arising at different soil layers and traveling with the Shear wave velocity appropriate to the medium within the layer. In retrospect, considering Figure 3, it is seen that when frequencies and wavelengths are used that are near a boundary between different layers in the soil system, that several velocities could be measured at nearly the same frequency.

In the 1950's, Ronald Jones (of England) performed several dynamic wave field experiments to test for the soil's elastic properties and thickness of pavement layers. His tests were performed in the range of 35 to 400 cps frequency. This was considered very fast for the time, since previous experiments had been done at frequencies of less than 60 cps or so.

The object of Jones' 1958 paper [26] was to ascertain whether an experimental technique operating in the lower frequency range could form a reliable means of deriving the Shear modulus (G) of the soil and ultimately provide an estimate of its strength and ability to resist deformation.

A single seismic geophone was used to locate successive positions away from the vibration source on the soil to find where measured vibration was in phase with the source at distances of 40 to 80 feet. This allowed at least four waves to be measured to obtain an average value for the wavelength.

Rayleigh waves are considered to act in the principal mode [26], i.e., the solution of the wave equation giving the lowest velocity. Jones indicates that a matrix of order six is required for the characteristic solution to the wave equation (requiring the determinant). Computations for multi-layered systems in those days were limited by the number of hand computations required. Jones also poses solutions for the Love wave equation, which are valid only when lower layers of the soil have higher Shear moduli. He used the then recent findings [37,38] regarding wave propagation in a semi-infinite elastic medium, which are similar to today's.

Jones estimated that wave velocity peaks of greater than 1000 fps would be the phase velocity of gravel. Modern experiments show gravel phase (Rayleigh wave) velocities of over 500 fps and asphalt concrete phase velocities around 1500 fps.

Under small vibrating forces, soil is assumed to behave elastically and effective moduli do not change. The values of Young's modulus calculated from vibration experiments on undisturbed soil are often at least twice those derived from stress/strain measurements in triaxial compression under slowly applied loads. This is because the stress and strain induced by vibrations are much less than those caused by traditional soil tests, which often go beyond the elastic range, and the moduli tend to drop off non-linearly as a function of stress/strain.

Jones also attempted to measure of local Shear modulus by the Resonance method. He placed a small vibrator of known mass upon the soil. The frequency of the vibration was changed until maximum amplitude was measured. This frequency of maximum amplitude was considered to be the resonant frequency and similar relationships for field testing were derived to solve for the Shear modulus as earlier described for laboratory tests. Shear moduli deduced from measurement of resonant frequency at low amplitude vibration were in good agreement with moduli deduced from phase velocity measurements. In applying the resonant method, Jones assumed soil damping to be negligible.

In 1963 William Thompson [55] of the United States summarized results [58] of previous American studies [11,13,28] to solve the problem of viscously damped road vibrations where the structure is subjected to a concentrated vertical load moving longitudinally at constant velocity on an elastic pavement and subgrade. After making several substitutions and assumptions (1′ wide pavement infinitely long and steady-state vibrations), he came up with a fourth order differential equation in terms of vertical displacement with changing longitudinal coordinate. Although the study is highly theoretical, it is interesting since it explores the effects of damping on the pavement system.

Contrary to the conditions wherein the damping ratio $\zeta \Leftrightarrow 1$ characterizes under-damped, critically damped and overdamped motions, respectively, of a second order dynamic system, no simple, single value of the function of ζ separates the solution of the fourth order, road-vehicle system. The value of the discriminate (Δ) of the characteristic equation is indicative of the three regimes of the solution, but the separate solutions for the motion of the pavement in front of and behind the load obscure this distinction.

Irrespective of the value of Δ, the pavement vibrations in front of the load exhibit a typically underdamped behavior (i.e., oscillation). Behind the load an underdamped motion again appears when $\Delta > 0$, while for $\Delta \leq 0$, the vibrations decay with distance behind the load without ever achieving an oscillatory wave form.

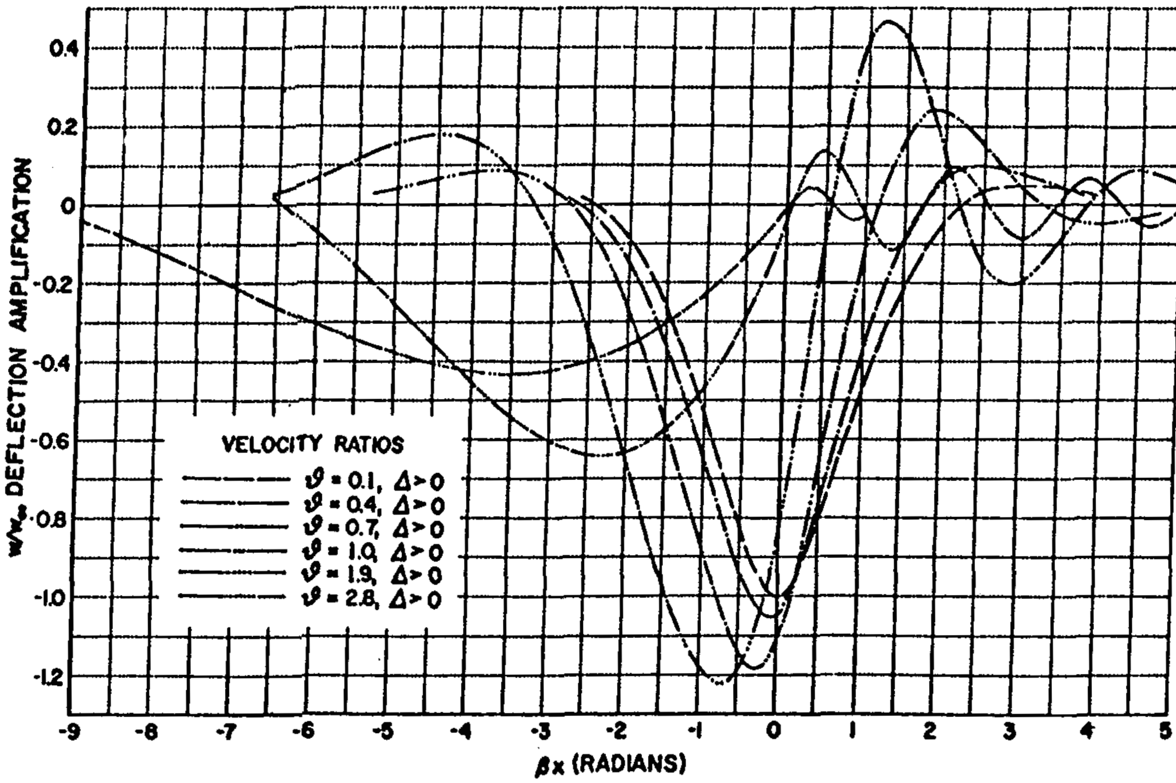


Figure 4 General nondimensional surface displacements; variation of surface displacement with velocity of the moving load for a case of light damping, ζ = 0.4. [55]

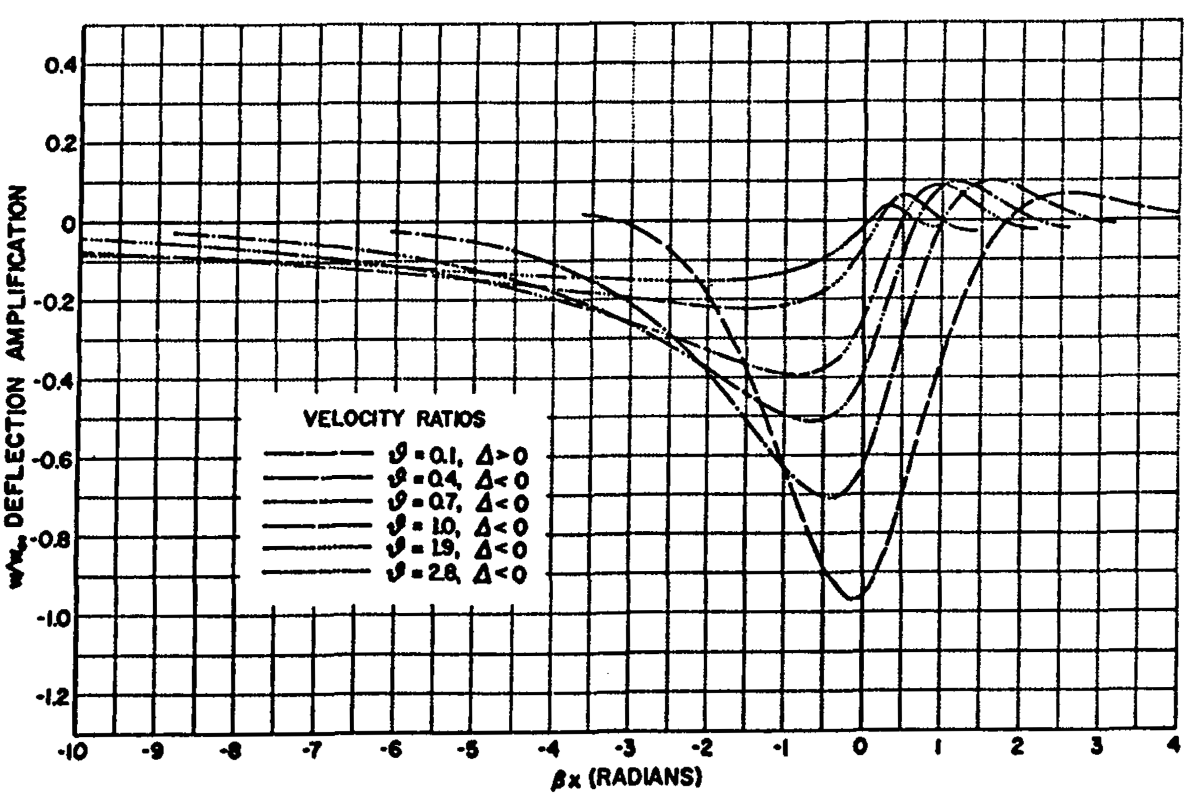


Figure 5 General nondimensional surface displacements; variation of surface displacement with velocity of the moving load for a case of heavy damping, ζ = 2.0. [55]

The pavement profile computations demonstrate that the position of maximum deflection of the pavement falls behind the load as the velocity increases. Thompson showed many graphs for normalized displacements versus normalized offsets from the load center of the moving load on the pavement (see Figures 4,5,6,7). These graphs are for various damping (ζ) and velocity (ϑ) ratios.

Damping ratios are the ratio of the system's damping to critical damping, which is the damping of a system just at the point where it transforms between oscillatory motion and logarithmic decay. Velocity ratios are the ratio of the wave velocity to critical velocity which is the propagation velocity of transverse displacement along a freely vibrating elastically supported plate.

Thompson's model does not include

1. energy losses due to pavement flexure,
2. transient terms associated with unsteady load velocity and magnitude,
3. inertia of the subgrade and
4. dynamics of the vehicle.

These problems were anticipated to be second order terms without great effect on the solution. Current studies show that dynamics of the vehicle, e.g., suspension type, play an important role in dynamic loading characteristics.

Lambe and Whitman [31] summarize that moduli determined by wave propagation techniques are usually 2-3 times greater than those determined by standard loading tests - for reasons mentioned earlier. However, the modulus determined under very small stresses or after many cycles of loading is about equal to the modulus calculated from wave velocity. This appears to be true regardless of the frequency of repeated loads. This conclusion makes the wave velocity technique more valid for determining the elastic properties of engineered embankments which are compacted and have been subjected to repeated loads for a certain amount of time.

Research [2,46,53] and common sense indicate that better predictions of pavement life can be afforded by proper dynamic analysis of the system. As our understanding of pavement and soil response to dynamic loads increases, undoubtedly pavement designs, will improve.

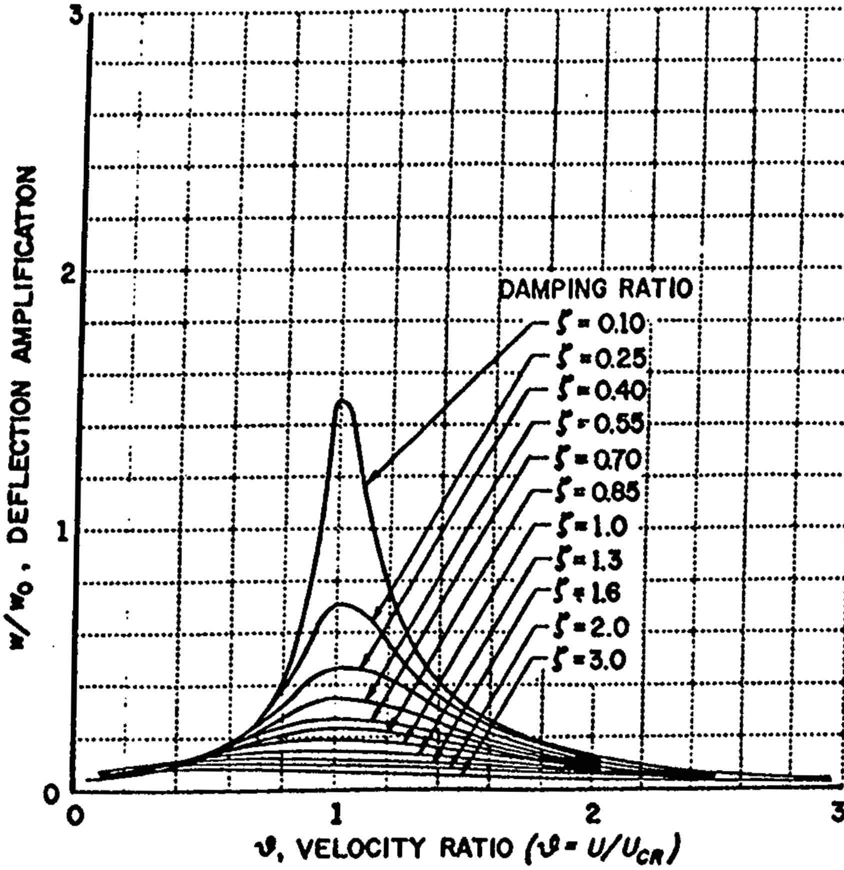


Figure 6 Stability diagram for surface displacements ahead of the load; maximum deflection amplification ahead of the load is normalized to the static deflection under the load. [55]

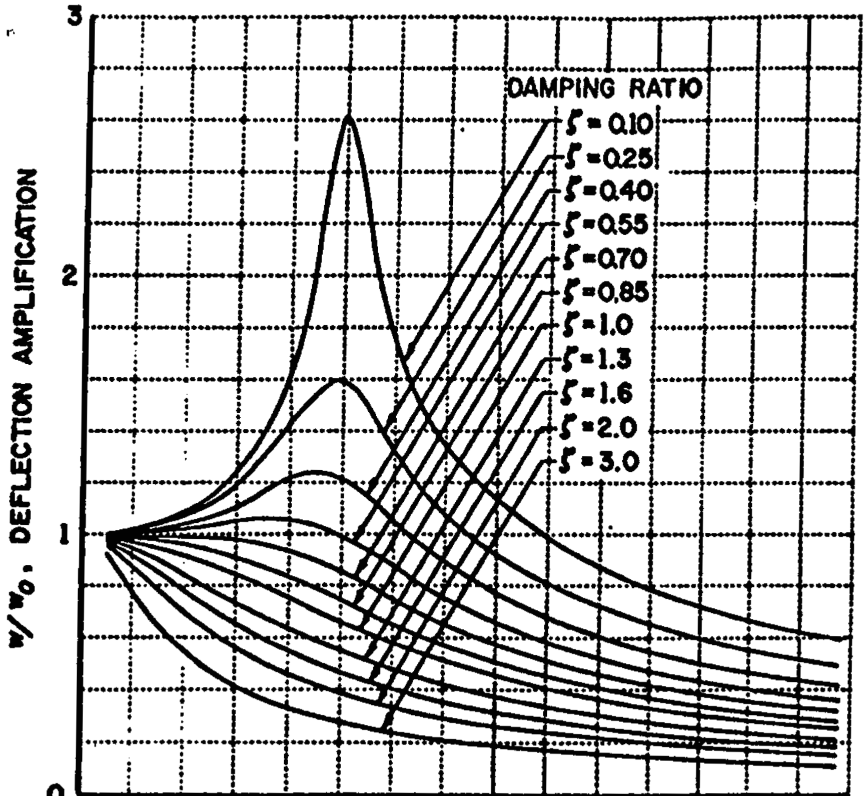


Figure 7 Stability diagram for surface displacements behind the load; maximum deflection amplification behind the load is normalized to the static deflection under the load. [55]

APPLICATIONS

1. Determining Pavement/Soil Layer Elastic Properties

Since, as has been shown, wave propagation velocities are dependent upon the medium's Shear modulus, Poisson's ratio and density, the background of this application has already been discussed. Here current developments in equipment and analysis of wave propagation are presented.

Spectral Analysis of Surface Waves

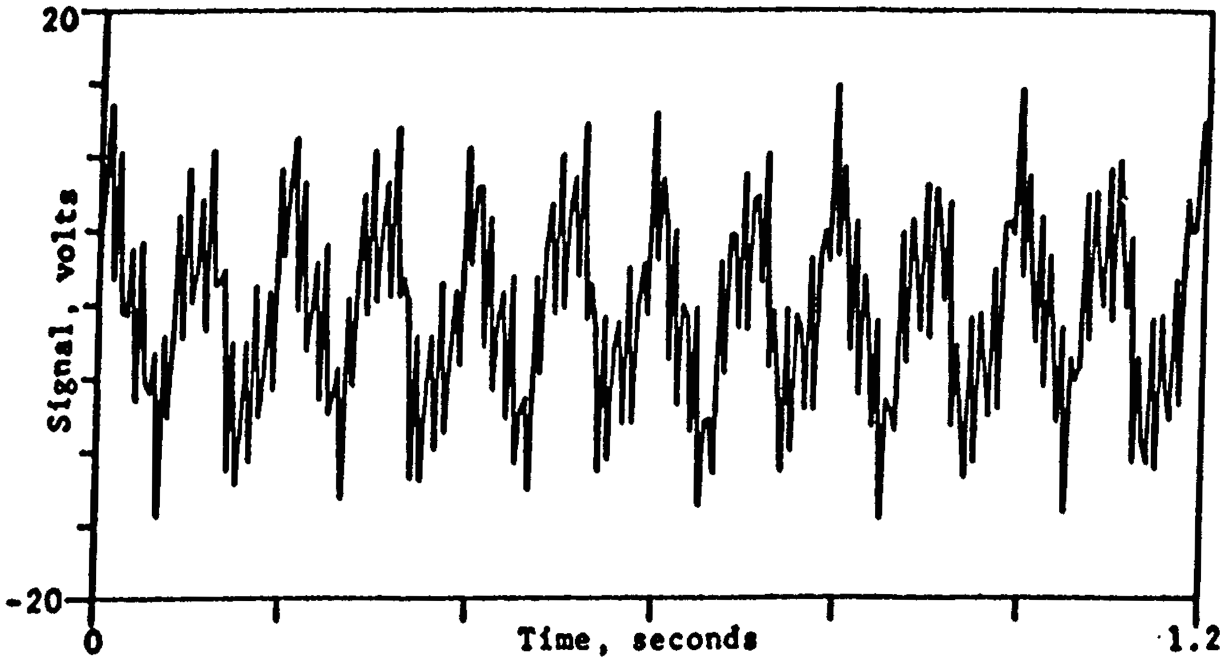
In the mid-seventies, a new technique incorporating an impulse source was developed to replace the slower steady-state vibration machines. Using a drop hammer, a wide range of frequencies (or wavelengths) can be input to the system as a transient source. The problem with excitation at a large number of frequencies at the same time is that time domain measurements are cluttered with data. The development of microprocessors and the Fast-Fourier-Transform (FFT) algorithm has greatly extended the capability to measure and analyze dynamic systems in the frequency domain. Instrumentation now exists that rapidly filters and converts an analog signal to a digitized signal, transforms the signal from its representation in the time domain into its frequency components, and analyzes the data in various formats. Consequently, frequency spectrum analysis provides a quick and feasible approach to evaluate the propagation of elastic waves through layered systems. The process is termed Spectral Analysis of Surface Waves (SASW).

The primary reason for utilizing SASW is that information can be extracted from data that is not apparent in the time domain (as measured). For example, the components of the signal in Figure 8a are indistinguishable in the time record, but each wave and its relative contribution to the overall wave form are easily observed in the frequency spectrum shown in Figure 8b. The amplitude and phase of each frequency component in the wave form can be determined. In addition, relationships between two signals can be identified.

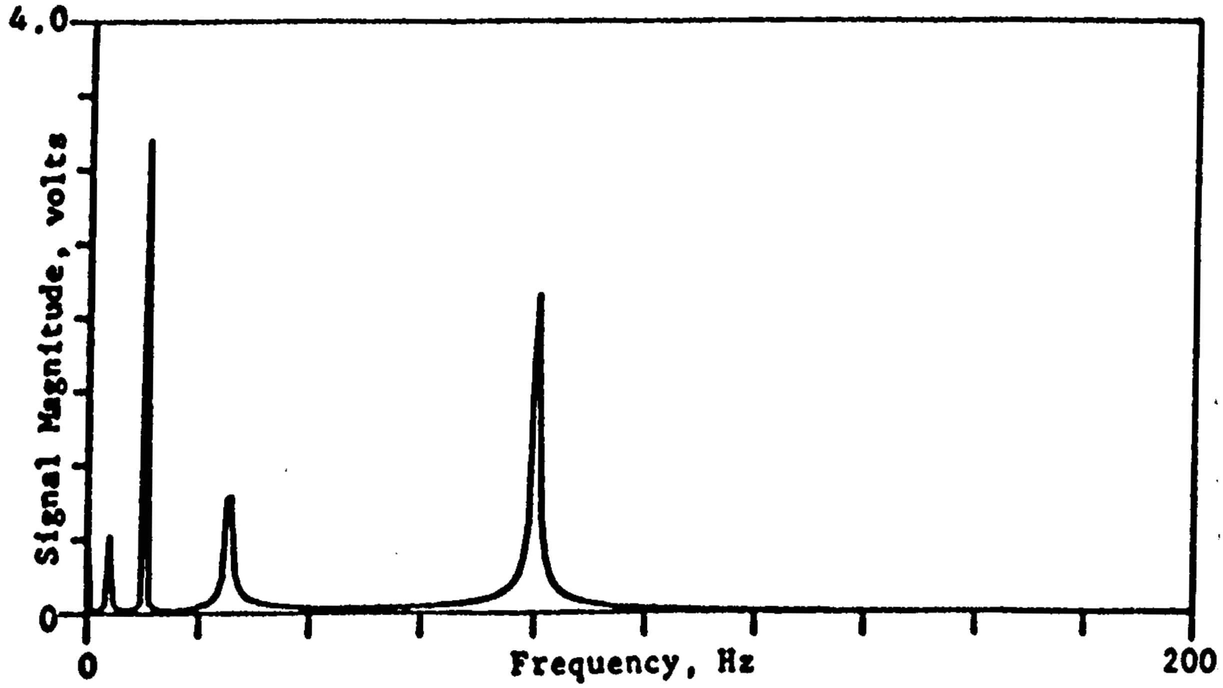
The test setup for the SASW method is shown in Figure 9. This setup has been primarily developed and presented by S. Nazarian and K.H. Stokoe II [39, 40, 41, 42, 43]. Their papers describe the development and various uses of the technique which has become one of national interest.

The main tools required for collecting data in the field are an impulse source, two or more receivers and a recording/analyzing device. One apparent advantage of such a system is its portability. The equipment could easily be transported to remote sites and used for testing. In the future this type of testing could replace soil borings currently used in engineering.

Figure 8 Representation of complex time signal by its frequency spectrum.



(a) Signal in the time domain.



(b) Signal in the frequency domain.

[21]

Impulse sources used in the SASW test should be able to generate frequencies from 10Hz to more than 50kHz when testing pavements. Ordinary hammers, such as hand-held or sledge hammers, can be used on typical pavement sites. Usually several hammers are needed. Small hammers are used to generate high frequency waves to sample the surface layer. Larger hammers, such as sledge hammers, are used to generate low-frequency waves so that properties of the subgrade can be determined. Recently, pulsating crystals have been successfully used to sample the upper few inches of a pavement surface layer.

Typically, accelerometers capture higher-frequency waves (above 500Hz). Geophones (velocity transducers) are recommended to capture lower-frequency waves of less than 500Hz.

A spectral analyzer records field data. Spectral analyzers can capture data in the time domain, digitize the data properly, perform an FFT on the signals and conduct spectral analysis.

Several receiver spacings are required to collect the necessary field data. Nazarian and Stokoe determined that the quality of the dispersion curve could be maximized by a common receiver midpoint (CRMP) array. Other experimenters have successfully tried different arrangements.

This array is depicted in Figure 10. An imaginary centerline is assumed and the receivers are moved symmetrically relative to the centerline for each test. Note that at each receiver spacing impulses are tested on each side, giving a self-check for each spacing.

This testing requires about 20 to 40 minutes per location. As such, the SASW tests are not nearly as economical as other methods of non-destructive testing. Current work is moving towards automation of the procedure.

The phase of cross power spectrum (CPS) is used to determine phase velocity and wavelength information. A typical CPS determined at a flexible pavement site is shown in Figure 11. The phase information simply represents the phase lag between the signals captured at the two receivers at various frequencies. The phase lag is due to the fact that the signal has to travel farther to get to the sensor located farther from the source. A phase lag of 360° (or 0° on the figure) corresponds to a time lag equal to the period of the signal. The period of the signal (T) is the reciprocal of the frequency (f). Therefore, the travel time (t) between the two receivers can be obtained from the phase of the CPS angle (ϕ) by

$$t = (\phi/360)T = (\phi/360)/f \quad (8)$$

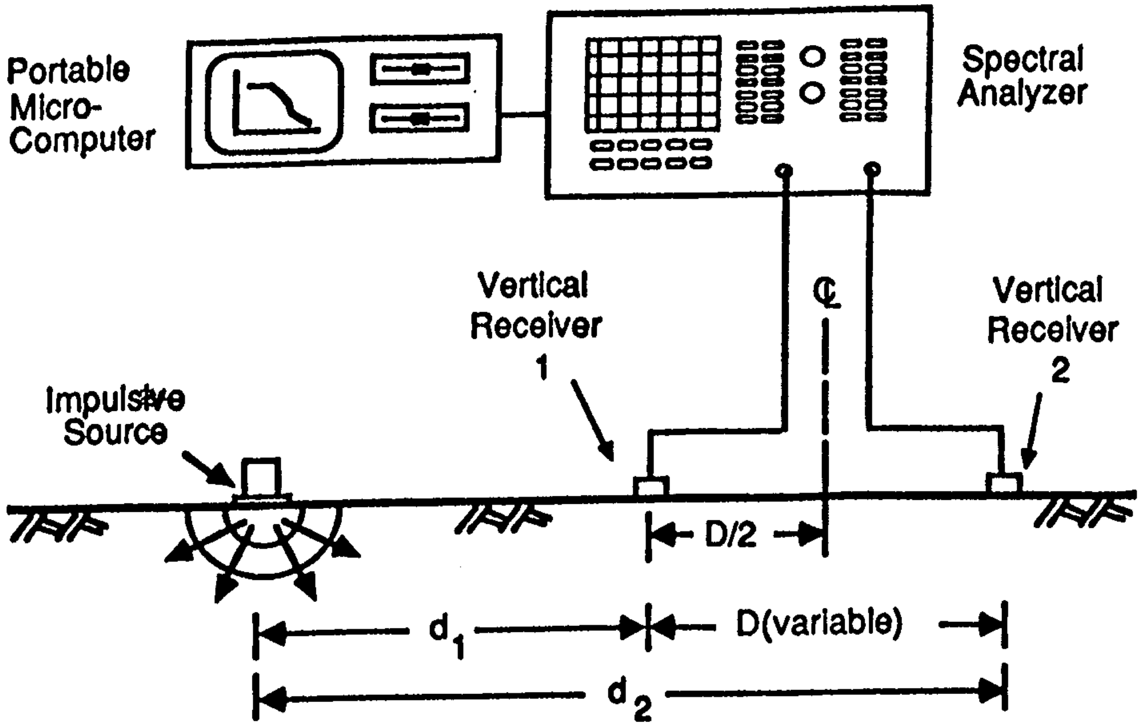


FIG.9 —*General configuration of SASW testing.* [43]

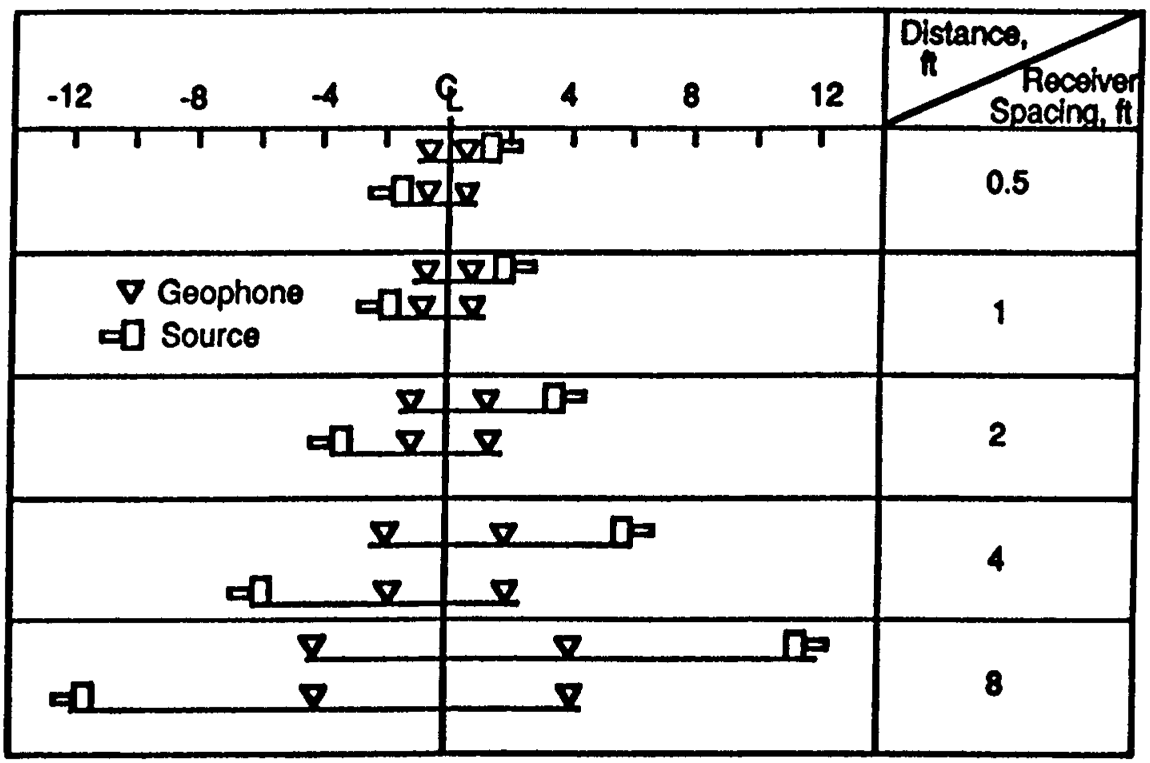


FIG10—*Schematic of experimental arrangement for SASW tests.* [43]

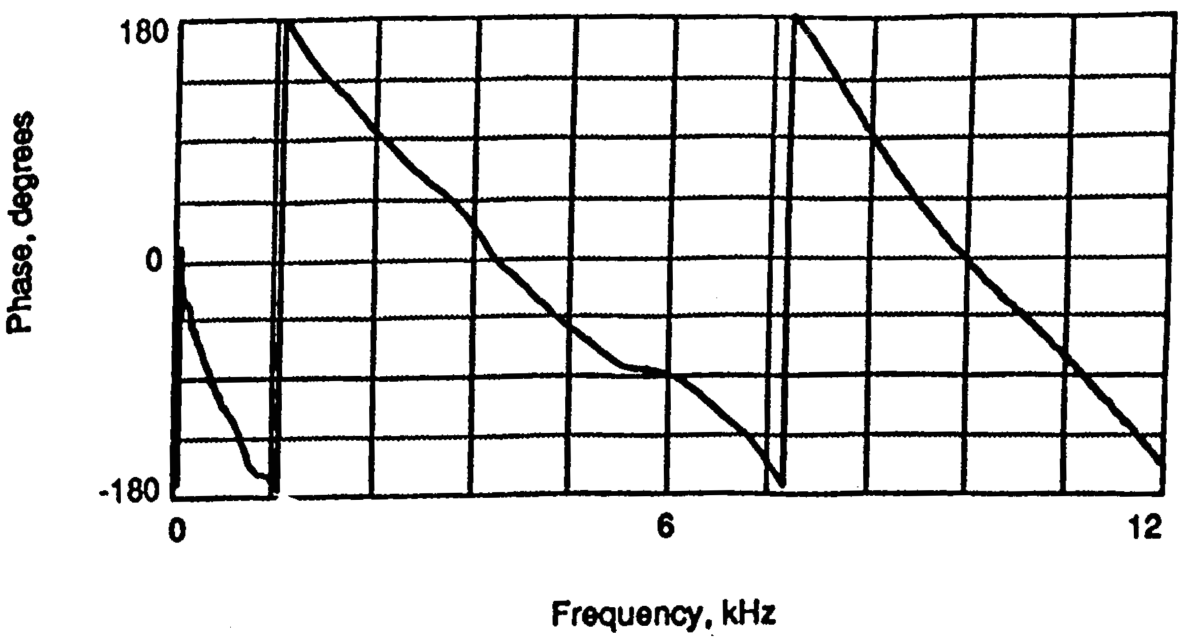


FIG.11—*Typical cross power spectrum measured on a flexible pavement (receiver spacing of 0.3 m).* [43]

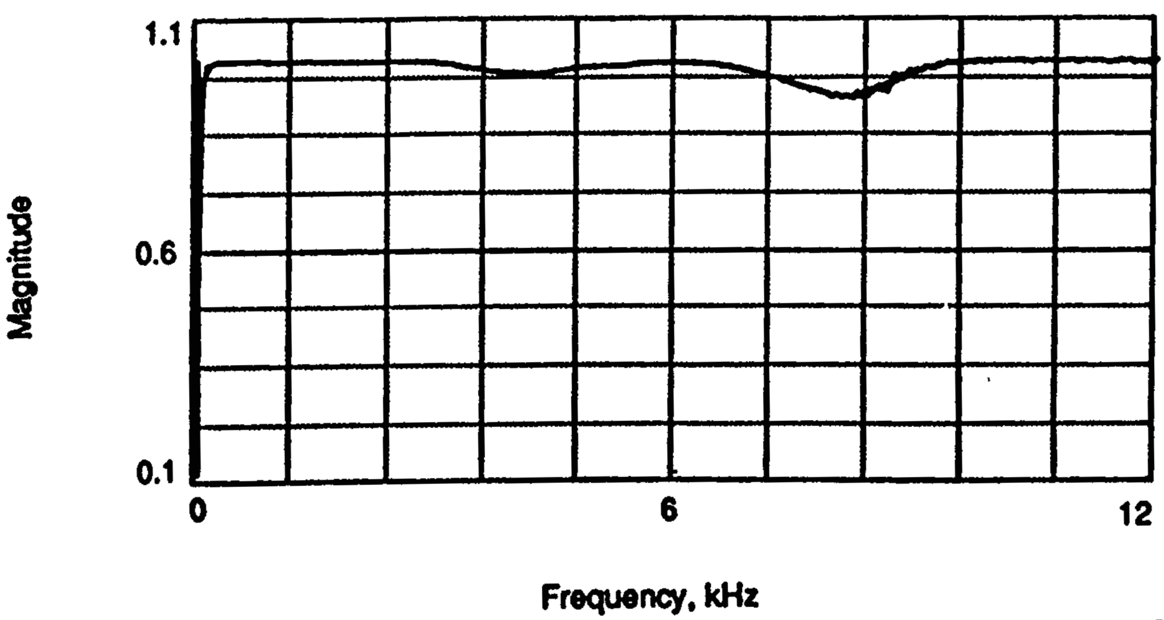


FIG.12—*Typical coherence function measured on a flexible pavement (receiver spacing of 0.3 m).* [43]

Since the distance between the receivers (D) is known, the phase velocity (V_{ph}) can be determined by $V_{ph} = D/t = Df/(\phi/360)$. From wave propagation theory (Equation 6), the wavelength (L) = $V_{ph}/f = D/(\phi/360)$.

A coherence function is used to determine the ranges of frequency over which the signal received by the two sensors is not coherent due to wave refraction or other equipment problems. A value of near unity for the coherence function at a given frequency corresponds to a low signal-to-noise ratio at that frequency. Data in the ranges of low coherence are deleted from use in the dispersion curve. A coherence function which was calculated simultaneously with the cross power spectrum record shown in Figure 11 is shown on Figure 12. Low coherence only exists in this record at frequencies below about 50Hz.

Repeating the computations for phase velocity and wavelength, dispersion curves of wavelength (ft) versus surface-wave phase velocity (fps) are obtained. Sheu, et al. have a program to automate the determination of dispersion curves [52]. It is claimed that a microcomputer with this program can be connected to a spectral analyzer and determine dispersion curves in the field, if desired. The most important step in performing SASW tests is the in-house data reduction. This step is called the inversion process [43].

Inversion is an iterative process where a Shear wave velocity profile is assumed and a theoretical dispersion curve is obtained. The theoretical dispersion curve is compared with the field dispersion curve. If the two curves compare within a small tolerance, the stiffness profile of the site is obtained, along with layer thicknesses. Otherwise, the assumed Shear wave velocity is adjusted and the comparison repeated. This process, whereby a best fit is obtained, eliminates the assumption of wavelength sampling used in earlier applications. One author [22] proposes a finite difference approach with optimization routines to let a computer perform the inversion process.

The theoretical dispersion curve is obtained by using matrix solution [21,54,57]. Values for Poisson's ratios and for the mass density must be assumed for each layer. It is said that these assumptions should not cause more than a 20% error in modulus values.

Advantages of the SASW method are

1. No assumptions with regard to thickness of the pavement layers need to be made. In fact, layer thicknesses can be determined using the method.
2. Variation in moduli within different layers can be determined.
3. Moduli of thin layers can be determined.
4. Subgrade moduli may be determined before, during, and after placement of the pavement system.
5. Decreasing moduli with depth is not a criterion as with some other methods.

Problems with the SASW method are

1. Computed moduli are generally higher than moduli predicted by deflection testing due to the lower strains imparted.
2. The method is not now automated for purchase of standard test equipment, although work is being done in this area.
3. If the actual surface waves do not follow sinusoidal wave forms, the FFT method may not be applicable. One author [15] proposes the use of Bessel functions to model the wave forms.
4. The analysis is complex, possibly more so than the Boussinesq elastic theory analysis required to understand deflection analysis. But for practical use, either method must be computerized, and memorization of the theory is not required.

Falling Weight Deflectometer

A Falling Weight Deflectometer (FWD), in conjunction with wave propagation analysis equipment, can be used as the load impulse to analyze lower layers of a pavement (>15"). But the FWD cannot excite high enough frequencies to generate the short wavelengths needed to sample the upper levels. Nearly all the energy of excitation from an FWD is within 100Hz and essentially no frequencies are above 250Hz. These results [21] indicate that the FWD does not sufficiently excite the necessary frequencies (up to 2-5 kHz) to evaluate the entire pavement by the wave propagation methods.

However, the FWD can analyze the entire pavement system using maximum deflections measured at radial distances from the load impulse (also measured). The Dynatest System of back-calculation of pavement layers utilizes the method of equivalent thicknesses and Boussinesq's equations to determine layer moduli according to higher stress levels [58] typical of traffic loading. The FWD method using the computer program ELMOD can determine the equivalent depth of a stiff layer and the elastic moduli of a maximum of four layers above this.

An FWD applies an impact to the pavement surface equivalent to the design vehicle. The peak load is measured and the peak deflections are determined at various radii from the load center by use of velocity-measuring transducers. An actual deflection basin composed of peak deflections never occurs during the test since there is a wave propagated from the load and phase lags across the transducers. See Figure 13. Nearly all models used for deflection basin analysis are based on a static deflection basin and ignore wave propagation effects. Besides phase lag, wave propagation could significantly influence the measured peak deflections when very stiff layers exist at shallow depths [2,15].

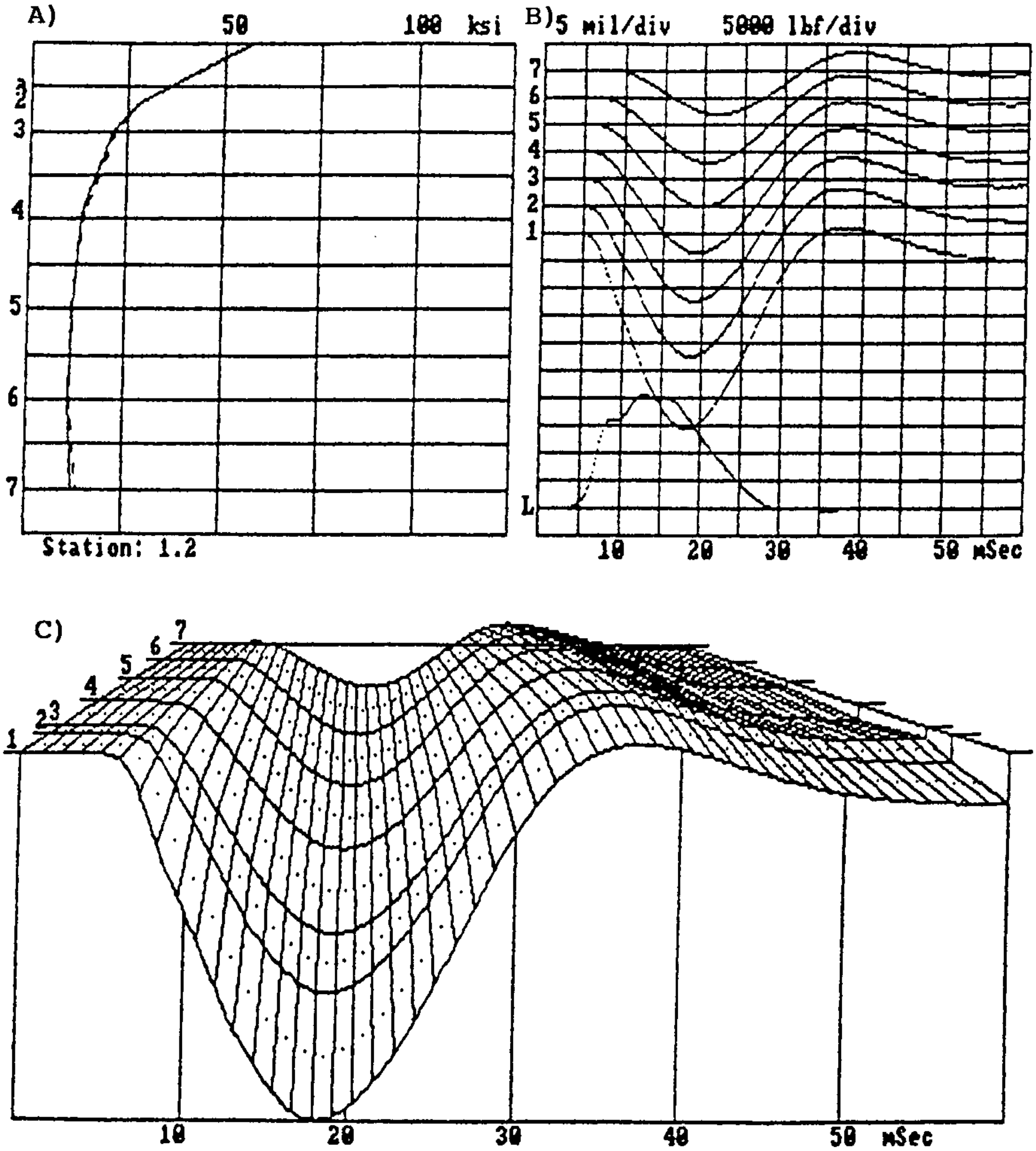


Figure 13. Typical Falling Weight Deflectometer Time History

A) Surface Stress vs. Distance from Load
B) Time vs. Load and Sensor Deflections
C) Time vs. Deflection vs. Distance from Load

NOTE: Sensor 1 is at the Load center, Sensor 7 is offset 47.2"
The others in between.

In deflection basin analysis, two to five layers and their thicknesses are assumed. ELMOD computes an equivalent depth to a stiff layer, which helps it match basins during modulus back-calculation. Each layer is assumed to be continuous and of uniform thickness in all horizontal directions, homogeneous, elastic and isotropic. Poisson's ratio values must be assumed. The properties within each layer are assumed to be constant. These assumptions are also true of the SASW method. Obviously, in any real pavement all of the engineering properties vary.

The procedure followed in most computer programs in determining layer moduli from deflection basin measurements is by first giving a "seed" modulus for each of the assumed layers. The peak applied dynamic load is input as a static load on the surface, and a static deflection basin is calculated for the model. The calculated deflection is compared with the measured deflection. Moduli estimates are adjusted, and through an iterative process a best possible match is obtained. The result is a set of moduli for the given layers which give a calculated static deflection basin close to the measured dynamic deflection basin. Users must be aware that the moduli are properties of the model used to simulate the real pavement system, rather than true values. Dynamic properties are supposed to be built into the design procedure by use of failure criteria based upon load repetitions of a design vehicle. These criteria are empirical and their range of accurate application is not well defined. The method does not account for inertia or damping of the system.

<u>Road Rater</u>

The Road Rater is a trailer-mounted vibratory vehicle. The Road Rater 2008 is capable of generating vibratory loads of variable frequency with magnitudes in the range of highway loadings. Time and frequency domain measurements are recorded at four geophones including magnitudes of load deflections and phase shifts, from a reference sensor. Peak, time, and frequency domain values may be used in analysis. The Fast-Fourier-Transform is used to convert time domain values to frequency domain. A cross spectrum function on the Road Rater analyzer may be used to record phase shifts.

The standard Road Rater System of back-calculation simply takes the measured peak deflections utilizing multi-layer linear elastic theory as in the other methods to back-calculate moduli. Several modifications to this program are available with the highest form utilizing real time basins input to a dynamic back-calculation algorithm. Dynamic response predictions are determined using Green's function solutions [2] to an eigen value problem developed for a frequency-dependent stiffness matrix. Phase information was found to be critical to accurate analysis of dynamic response.

2. Prediction of Loads and Deflections Resulting From a Design Event

Once the moduli for a soil stratum have been determined, the stresses, strains and deflections resulting from a design event can be estimated using elastic theory analysis. Many programs are available for this analysis, including ELSYM5, BISAR, CHEV5L and modified versions of SAPIV, to name a few.

The resilient modulus usually used in analysis and design of dynamic soil systems is dependent on stress and recoverable strain. This value is generally decreasing in subgrade soils for stress levels above 5 psi or so (see Figure 14) and strain levels above 0.01% (see Figure 15). In granular materials back-calculated moduli values are sometimes increasing with stress.

Wave propagation techniques do not impart stress or strain levels above these limits, yet the design event may. Therefore if you use moduli determined from wave propagation techniques in an elastic analysis of a design event of greater magnitude, your solutions for stress and strain may not be conservative. A safety factor is usually used in geotechnical engineering, and would be recommended here also.

3. Estimating Pavement/Soil Layer Thicknesses and Depth of Rigid Layer

Rigid bottom depth can be predicted using the theory of stress wave propagation in an elastic medium in conjunction with FWD tests [57]. For example, if the measured duration of an FWD load is 25 msec, the period (T) of a harmonic wave form can be approximated to 50 msec., and the frequency (f) is 1/T = 20Hz. The length of a compression body wave is given by

$$L_p = \frac{V_p}{f} = [(\lambda + 2G)/\rho]^{1/2}/f \tag{9}$$

where V_p, λ, G and ρ are P-wave velocity, Lame's constant, Shear modulus and mass density, respectively, as shown earlier. The Lame's constant and Shear modulus are functions of Young's modulus and the Poisson's ratio. Assuming typical mass density and Poisson's ratio, the modulus from the fifth FWD sensor can be computed using the Boussinesq equation:

$$E_o(x) = \frac{(1-\mu^2) * \sigma_{vo} * r^2}{(x * d(x))} \tag{10}$$

where $E_o(x)$ is the surface modulus at a distance of x from the load center, here at the fifth sensor.
σ_{vo} is the maximum contact stress under the FWD loading plate
r is the radius of the loading plate
d(x) is the measured deflection at x.

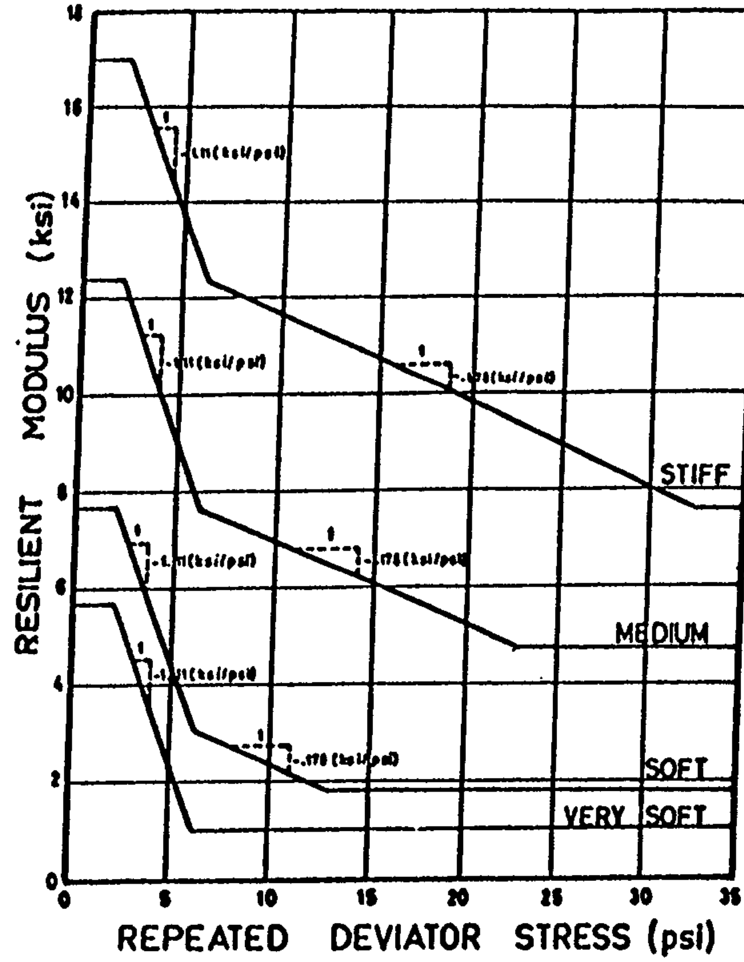


FIGURE 14 Resilient properties of subgrade [47]

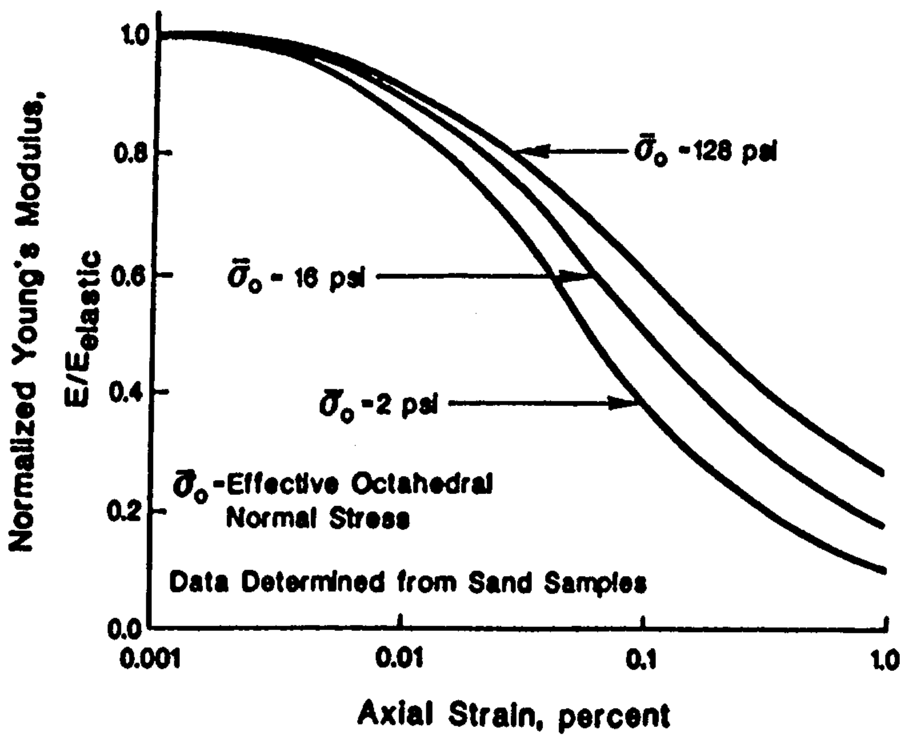


FIGURE 15 Nonlinear model used for determining equivalent linear moduli of geotechnical materials. [41]

Once the Young's modulus is found using Equation 10, the estimated length of the P-wave is found by Equation 9. The thickness of the subgrade then is assumed to be 1/2 L_p. This type of estimation is said to give better predictions of moduli and better basin matching using deflection-based methods [65].

Using the SASW method the pavement section is broken into many layers in the inversion process, and a modulus versus depth profile is the result (see Figure 16). Here layer moduli are shown, and even variation in layer moduli are given. Also, the SASW method can sample to much greater depths than FWD tests. The maximum depth of (accurate) FWD modulus determination is not much more than the offset distance of the last sensor, which is usually around 5 feet. The problem, then, with SASW moduli versus depth results is how to rationally break the section into a smaller amount of layers since most elastic analysis programs only allow input of 5 or less layers.

Ground-penetrating radar can be used to determine thicknesses of pavement layers. Radar has already been applied to the determination of thicknesses and subsurface properties of concrete pavements [54] and bridge decks [10,35]. The basic principal underlying its application to asphalt pavements is illustrated in Figure 17. Here it is seen that the principal components of the wave form are the reflections from the top of the asphalt pavement, from the asphalt pavement/base boundary and from the base/subgrade boundary. The intensity of these reflections will be proportional to the dielectric properties between the layers.

Asphalt pavement thickness can be computed from the time difference between points A and C (t_{ac}) shown in Figure 16 and the velocity of the electromagnetic wave in asphalt, V_a [36]. This velocity can be computed as

$$V_a = \frac{11.8}{\sqrt{\epsilon_a}} \quad inches/nanosecond \tag{11}$$

where ϵ_a is the dielectric permitivity of asphalt. The dielectric permitivity can be estimated using the flat plate reflection test [27] on the pavement of interest. Previous measurements give values of ϵ_a ranging from 5.0 to 6.0, which yield V_a ranging from 4.8 to 5.3 in/ns since t_{ac} is the round-trip travel time of the pulse in the asphalt layer. The thickness can be computed as

$$h_a = \frac{V_a t_{ac}}{2} \tag{12}$$

With a ground-penetrating radar device installed in a vehicle, pavement thickness determinations can be made for each linear foot of pavement, at speeds up to 30 mph.

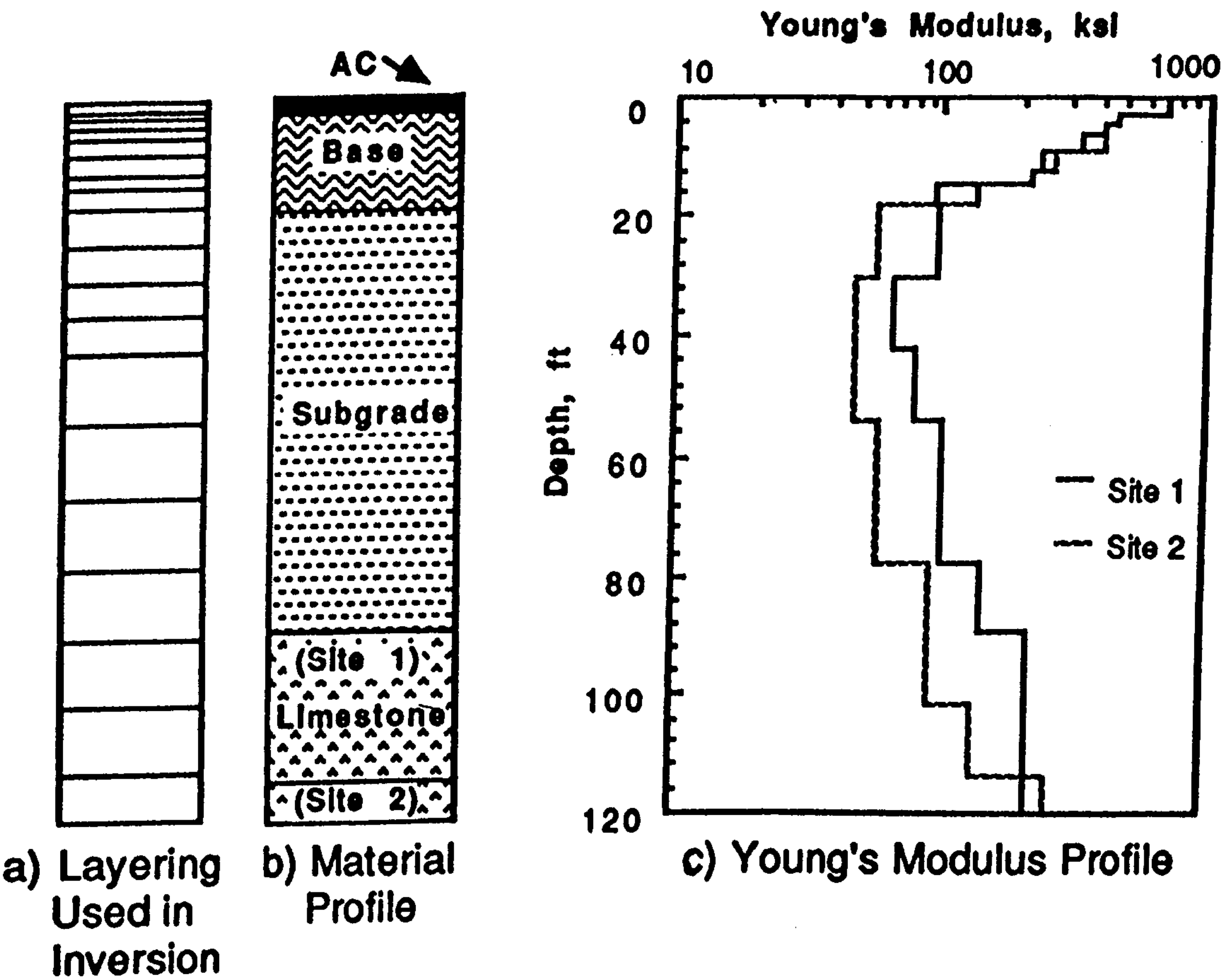


FIG.16 —*Composite profiles of Site 1 and Site 2 for Case Study* [43]

4. Estimating Moisture Content of Soil Layers

The response of radar to a layered medium can be predicted analytically if one knows the thickness, dielectric permitivity (ϵ) and conductivity (σ) for each layer. Such a model has been applied to pavements [32]. Littlefield has computed dielectric properties of pavement subgrade ϵ_s and σ_s as a function of moisture content and has studied radar response versus subgrade moisture content. Figure 18 shows a typical result of this work. The amplitude of the reflection from the top of the subgrade is plotted versus percent saturation of the subgrade. Note that the relationship is strong, suggesting that this portion of the radar wave form can be used to infer spacial variations with moisture content.

The relationship described above has been successfully used to detect moisture under asphalt overlays on reinforced concrete bridge decks (34). Its adaptation to asphalt pavements is straightforward, but variation in subgrade material properties must be considered.

5. Determining When Load Restrictions Should Be Applied

Unlike deflection measurements, wave propagation has seldom been used to determine when load restrictions should be applied. Since wave propagation techniques have the ability to determine depths of stiff layers (e.g., frozen ground), their use may be warranted in the future.

In 1966, wave propagation techniques were used to detect variations in the load carrying capacity of flexible pavements [25]. It was assumed load carrying capacity varied with seasonal variations in the elastic properties of the pavement layers and in the ultimate strengths of the pavement layers and subgrade.

A two-sensor velocity pickup system was used along with a Dynamic Displacement Transducer (DDT) near the load to test pavements subjected to a weight falling onto a striker plate. Tests were performed over a nine-month period (October to June) in New York.

Several parameters were analyzed, including DDT reading, rebound time, impulse duration and wave propagation time. Only the DDT reading was found to indicate seasonal variations. The wave propagation time was only analyzed for a three month period. There wasn't enough information to be conclusive, but the tests generally showed increased travel time on a poor road.

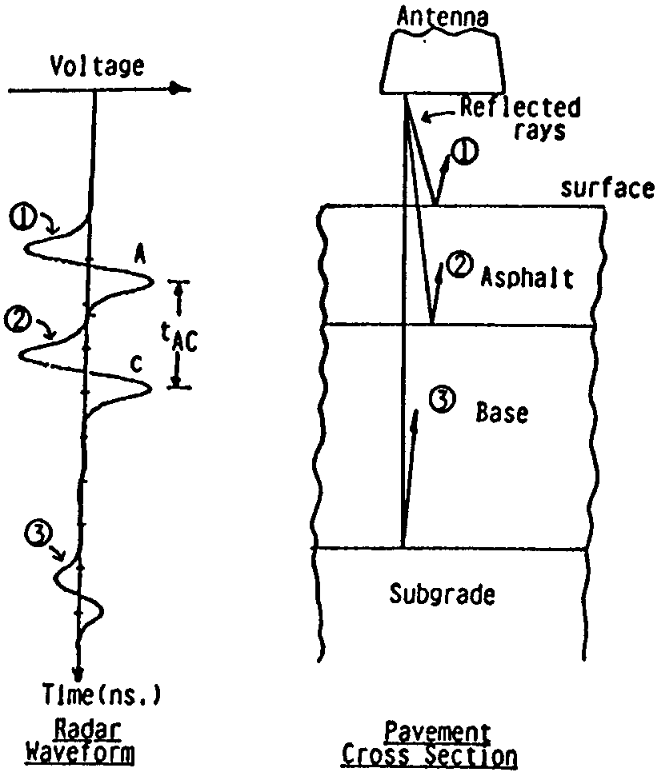


FIGURE 17 **Radar model for pavement.** [36]

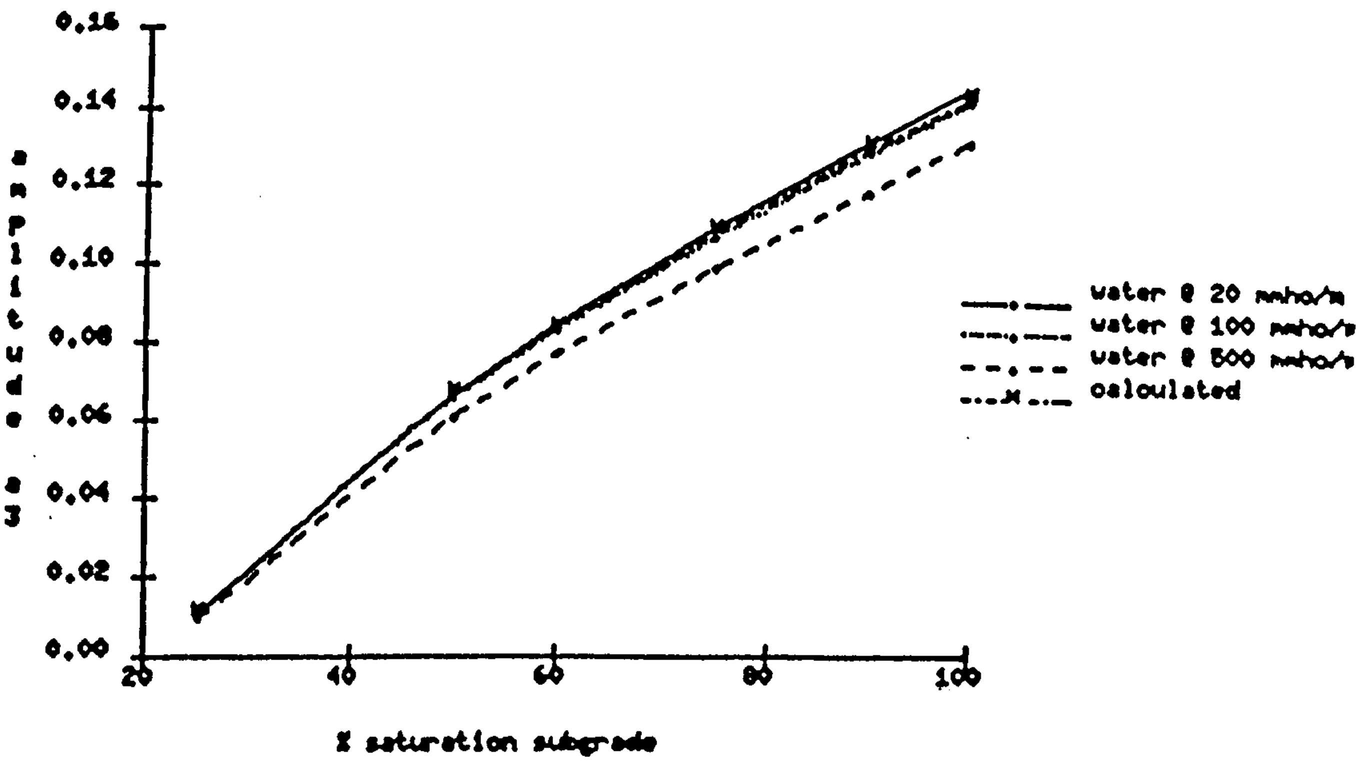


FIGURE 18 **Subgrade return (a_3) versus saturation of subgrade (base course 25%).** [36]

Tests performed in Texas using SASW before and after one week of intense rainfall showed a decrease in modulus of the subgrade [40]. Should this method be automated for accurately determining layer thicknesses and moduli, its application to this field could be exceedingly useful.

Most pavement engineers agree that the rate of deterioration of a pavement depends on critical stresses and/or strains in different layers of a pavement. These stresses and strains can be computed only if the layer moduli are known. It can be shown that deflection is not linearly proportional to strain [58]. Therefore, deflections must be converted to stresses and/or strains before being used to evaluate structural condition.

6. Determining Pavement Life Through Dynamic Analysis

The hypothesis is that proper consideration of dynamic loads improves the accuracy of pavement performance predictions.

Assessment of pavement response to dynamic loads requires the following information:

1. History and spatial distribution of loads that specific vehicles apply to the pavement as a function of vehicle speed and pavement profile.
2. Solutions for stresses, strains and deformations of representative pavement structures subjected to loads that duplicate those applied in-situ both in magnitude and as functions of time.
3. Measurements of the dynamic response of the materials constituting pavement systems over the range of foreseeable loading times, stress states and material states.

Dynamic response of vehicles is influenced by

1. Suspension type
2. Vehicle type (Load configuration)
3. Loading—magnitude and center of gravity
4. Vehicle speed
5. Pavement roughness

Current capabilities to perform dynamic analysis are highly developed. However, the computational effort required to solve a completely general three-dimensional dynamic problem representative of a pavement system subjected to a moving load is too large to be acceptable for practical pavement analysis. Therefore simplifying assumptions must be made.

Cole and Huth [12] found that maximum stresses that occur in a uniform elastic half-space subjected to a moving load (at constant velocity) are greater than the stresses produced by a similar static load. This increase can be closely approximated by the ratio:

$$Amplification\ ratio = \frac{maximum\ dynamic\ stress}{static\ stress} = \frac{1}{1-(V/V_s)^2} \tag{13}$$

where V = velocity of the moving load
V_s = shear wave velocity in the half-space

The above equation is not directly applicable to pavement structures due to their multilayered nature. But, if, for example, a Shear wave velocity of 1100 fps is assumed as an average for a system and the vehicle is traveling at 55 mph (81fps), the amplification ratio is only 1.005. Even for 150 mph (220fps), which is reached by some aircraft, the amplification ratio is 1.042, which is basically a 4% increase over static and likely well inside the accuracy attainable by computer aided stress analysis. One paper [53] showed that even after performing multilayered dynamic analysis, the amplification ratio would be less, rendering it negligible. However, the benefit of dynamic analysis is representation of changes in stress and strain with passing vehicles, thereby affording evaluation of fatigue.

Weight in motion (WIM) studies are performed at sites measuring peak dynamic loads of a vehicle passing over a set of sensors. These may be used to validate dynamic analyses at points but do not provide time histories of the complete load.

Methods of measuring dynamic loads using moving vehicles include

1. Tire pressure transducer
2. A combination of strain gages and accelerometers on the axles
3. Wheel force transducer mounted to the tire hub

One study [46] concluded that tire inflation pressure is not in phase with the dynamic load and therefore the tire inflation pressure transducer was considered unsuitable for measuring dynamic loads.

Once layer moduli and Poisson's ratio's are accurately accounted for, the stresses and strains can be computed for the design traffic and the pavement life predicted in terms of empirical equations. Govind and Walton analyzed surface wave propagation under moving load to assess pavement damage. For this study, a finite element program was implemented to perform stress analysis under moving wheel loads.

The program allows input of layer properties, number of axles, load per axle, spacing between axles, wheel path width, load type, and truck velocity - to name a few. The simulation depends on considering steady-state harmonic forces and displacements at a given frequency. Because of the program's large memory requirements, it can be executed only on mainframe computers with one or more gigabytes of memory. A CRAY-XMP was used. A "damage transform" was developed and defined as a system of mapping from a load (or stress) domain to a linear damage domain.

With respect to fatigue, it is not only the magnitude of the force that determines the extent of fatigue damage but also the rate at which it is applied and withdrawn. For cyclic loading, this is equivalent to assuming that damage would be a function of both amplitude and frequency of the load pattern. It is hypothesized that the rate of change of force, stress, or energy might correspond better to fatigue damage, rather than just maximum strain and modulus.

The results of the study compare well with the AASHO experiment of the 1950's and could be used to assess damage due to other axle groupings and structural systems. The model is overly cumbersome for general engineering design or analysis applications, but considering how technology progresses, it is easily conceivable that this form of analysis may be more accessible in the future.

7. Determining the Distance of an Earthquake Epicenter from a Point of Measurement

When the first P and S-waves that arrive at a station follow the same path, which is almost always true [44], the time elapsed between their arrivals is:

$$\Delta t = \frac{x}{V_s} - \frac{x}{V_p} \tag{14}$$

where x = the focal distance of the station

Therefore the focal point of a disturbance can be identified from two or more seismological stations which measure Δt, V_p and V_s by the intersection of computed focal distances from the stations using

$$x = \left(\frac{\Delta t}{\frac{1}{V_s} - \frac{1}{V_p}} \right)$$

PROPOSED AND FUTURE RESEARCH

The Strategic Highway Research Program (SHRP) has sent out requests for proposed wave propagation technology and Ground-penetrating Radar (GPR) to test selected sites. The proposed outcome of their research will be detailed specifications, cost estimates, user manuals and training guides for equipment utilizing GPR and wave propagation technology.

The pavement conditions to be measured by prototype equipment include

1. Moisture in the base, subbase and subgrade beneath flexible pavements (asphalt concrete)
2. Moisture under rigid pavement (Portland cement concrete) joints
3. Moisture within an asphalt layer
4. Fine cracking
5. Subsurface problems or discontinuities
6. Voids or loss of support under rigid pavement joints
7. Overlay delamination
8. Asphalt aging

The author has submitted proposals for two test sites in Alaska with requests that the scope be expanded to include determination of thicknesses and elastic properties of pavement structural system layers. These proposals were received with interest.

The US Air Force has funded development of a trailer/van which can perform (1) vibratory load deflection tests, (2) resonant frequency tests, (3) deflection-basin tests, (4) wave propagation tests and (5) analysis in one unit [5]. This equipment is bulky and expensive, so present research is directed towards development of more mobile, air-transportable testing systems.

Future research must include further investigations into dynamic response of pavements and soils so that ultimately the analysis can be performed on personal computers. Specific items that appear to need development are

1. Instrumentation for measuring pavement response under actual traffic loading. The weight-in-motion installations are a step in the right direction, but ultimately we must develop a means of measuring dynamic loading effects over an entire influence area with respect to load, time, displacement, and pore-water pressure.

2. Sensitivity analysis of the density terms assumed in back-calculation of elastic properties using Shear wave velocities. Biot's theory shows a more complex density term which accounts for additional apparent mass relating to the coupling of the moisture and the soil. Current applications of wave propagation analysis use only an assumed bulk density to cover the term.

3. Dynamic analysis of horizontal loads that are imparted to a surface by vehicles braking and failure criteria for this type of loading in pavement layers. These are now apparently unexplored fields. For example, the Boeing 727 airplane characteristic s data available from the Boeing commercial airplane company shows that this type of airplane may impart more than 78,000 lbs. of horizontal force per strut at instantaneous braking. A Boeing 747, under the same conditions, is listed to give over 150,000 lbs. of horizontal force. Any consideration of this type of force in pavement design is from indirectly related empirical concerns.

4. More flexible and powerful instrumentation for vehicles. Future techniques may allow us to actually measure dynamic loads along sections to be analyzed. If a vehicle could also be instrumented to measure surface deflections in-transit, the possibilities would be limitless.

CONCLUSIONS

The field of wave propagation and dynamic analysis of soils and pavements is an interesting, yet not well-understood branch of civil engineering. A better understanding of this subject will undoubtedly improve engineering designs. Proper analysis of dynamic soil properties and forces can also reduce the need for factor-of-safety multipliers. This means that understanding soil dynamics will save dollars on civil engineering designs and maintenance.

Any new research that improves engineers' understanding and ability to analyze dynamic systems will pay for itself. Emphasis must be on accuracy, ease and simplicity of application.

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