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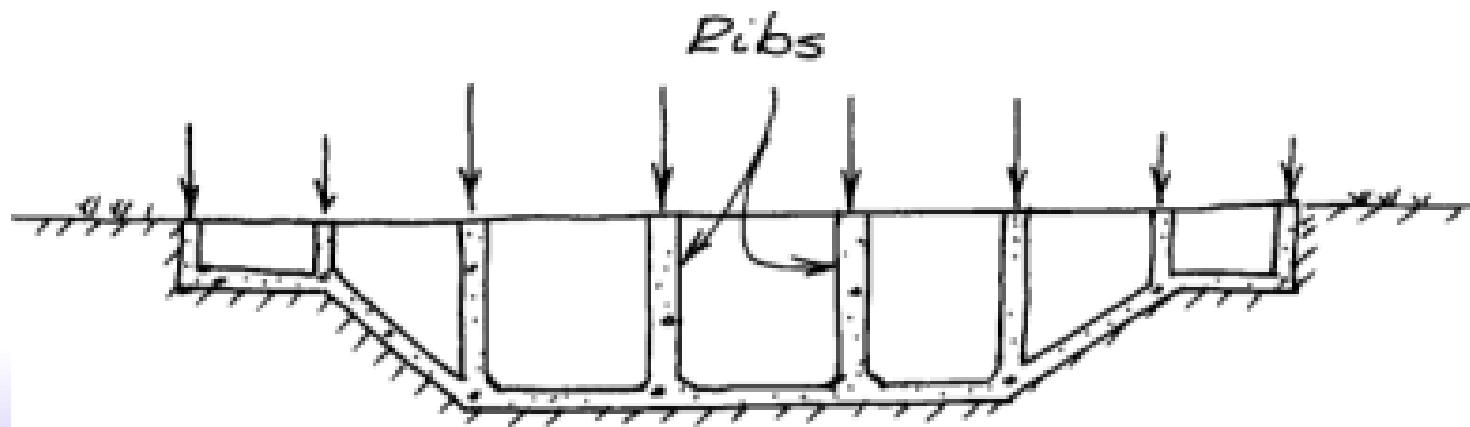
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# ENCE 461

## Foundation Analysis and Design

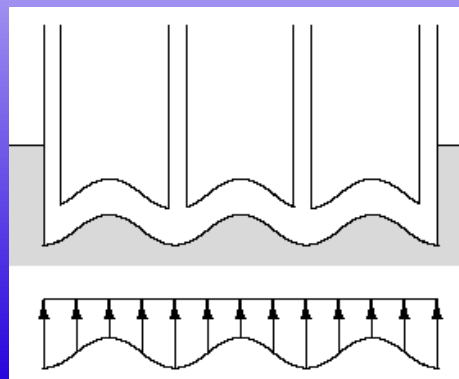


Waffle structure.  
(International Plaza)

Mat Foundations (Part II)

# Nonrigid Methods

- Nonrigid methods consider the deformation of the mat and their influence of bearing pressure distribution.
- These methods produce more accurate values of mat deformations and stresses
- These methods are more difficult to implement than rigid methods because of soil-structure interaction



# Nonrigid Methods

- Coefficient of Subgrade Reaction
- Winkler Methods
- Coupled Method
- Pseudo-Coupled Method
- Multiple-Parameter Method
- Finite Element Method

# Coefficient of Subgrade Reaction

- Nonrigid methods must take into account that both the soil and the foundation have deformation characteristics.
- These deformation characteristics can be either linear or non-linear (especially in the case of the soils)
- The deformation characteristics of the soil are quantified in the coefficient of subgrade reaction, or subgrade modulus, which is similar to the modulus of elasticity for unidirectional deformation

# Coefficient of Subgrade Reaction

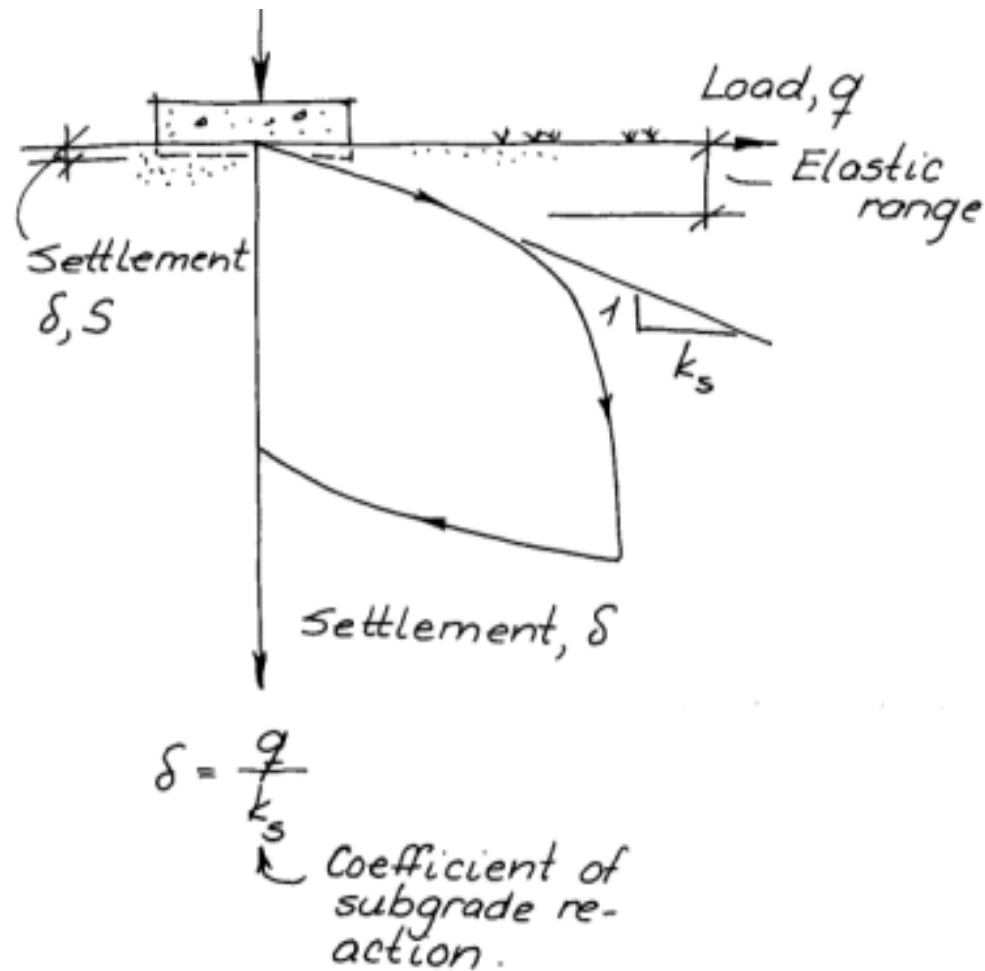
- Definition of Coefficient of Subgrade Reaction

$$k_s = \frac{q}{\delta}$$

- $k_s$  = coefficient of subgrade reaction, units of force/length<sup>3</sup> (not the same as unit weight!)
- $q$  = bearing pressure
- $\delta$  = settlement

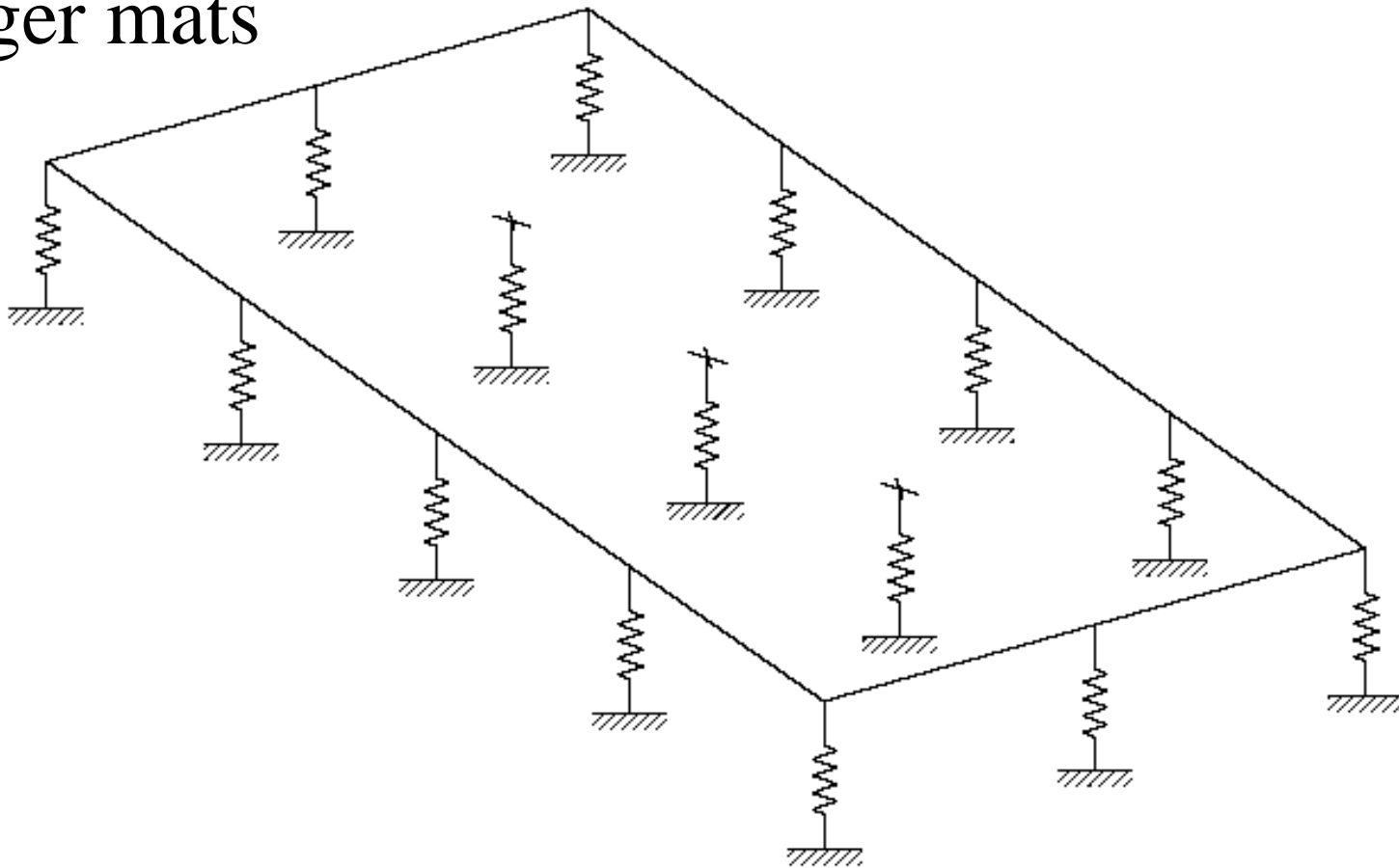
# Coefficient of Subgrade Reaction

- Plate load test for coefficient of subgrade reaction



# Coefficient of Subgrade Reaction

- Application of coefficient of subgrade reaction to larger mats



**Figure 10.6** The coefficient of subgrade reaction forms the basis for a “bed of springs” analogy to model the soil-structure interaction in mat foundations.

# Coefficient of Subgrade Reaction



Figure 10.6 The coefficient of subgrade reaction forms the basis for a "bed of springs" analogy to model the soil-structure interaction in mat foundations.

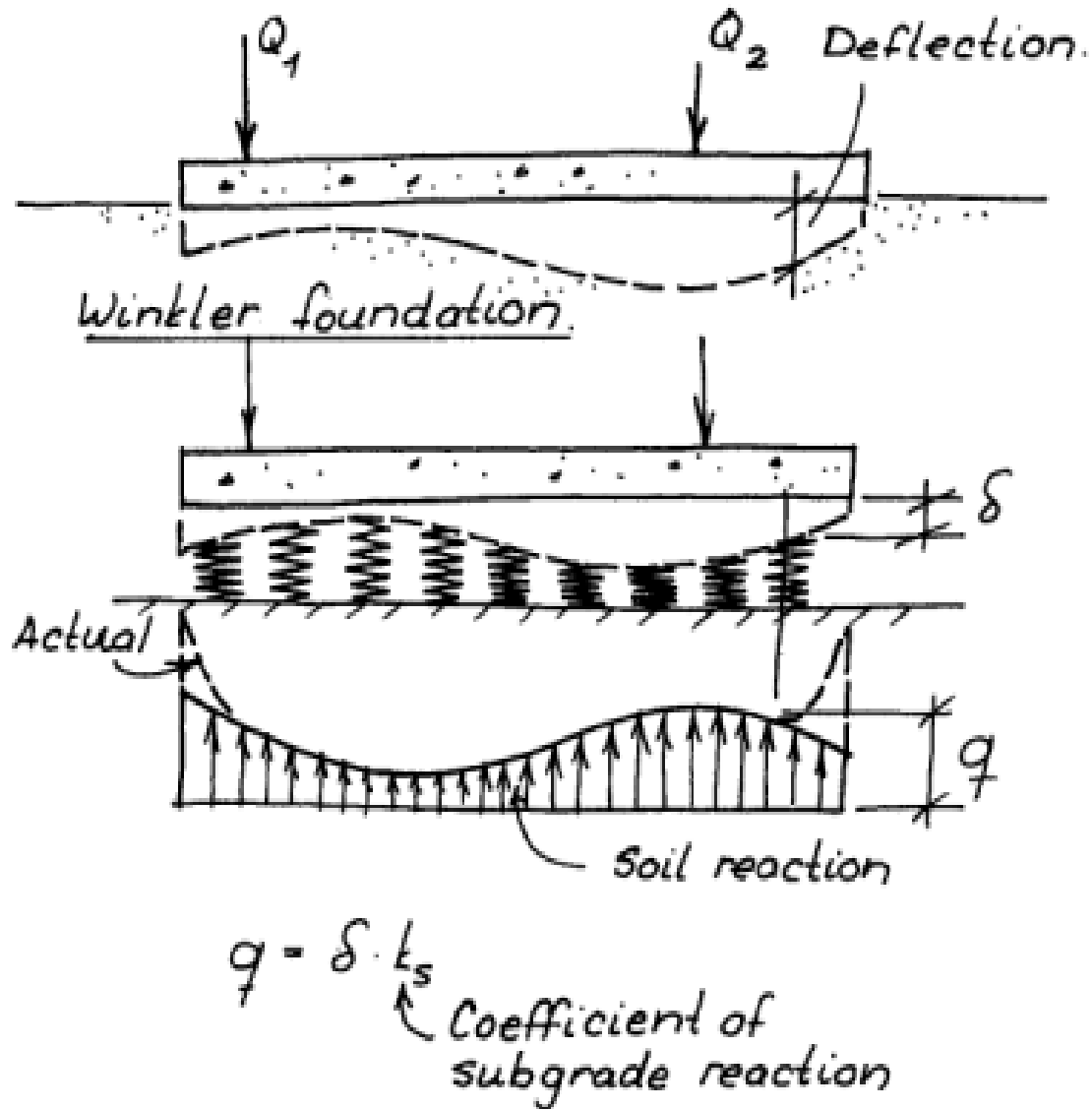
- Portions of the mat that experience more settlement produce more compression in the springs
- Sum of these springs must equal the applied structural loads plus the weight of the mat

$$\sum P + W_f - u_D = \int q dA = \int \delta k_s dA$$

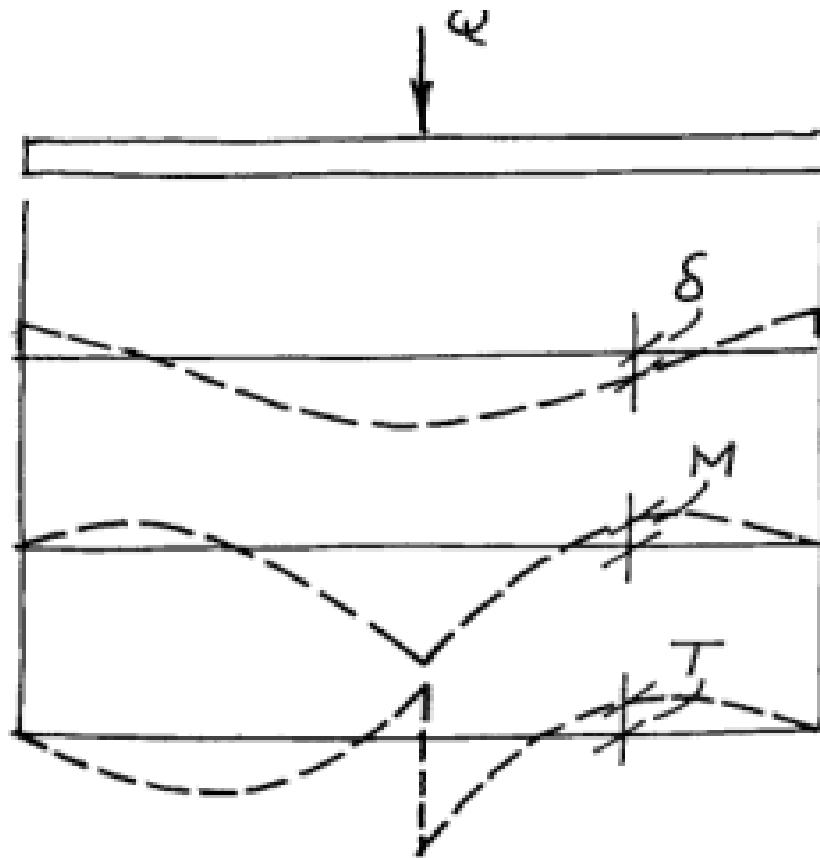
# Winkler Methods

- The earliest use of these "springs" to represent the interaction between soil and foundation was done by Winkler in 1867; the model is thus referred to as the Winkler method
- The one-dimensional representation of this is a "beam on elastic foundation," thus sometimes it is called the "beam on elastic foundation" method
- Mat foundations represent a two-dimensional application of the Winkler method

# Beams on Elastic Foundations



# Beams on Elastic Foundations



$\delta$ , deflection

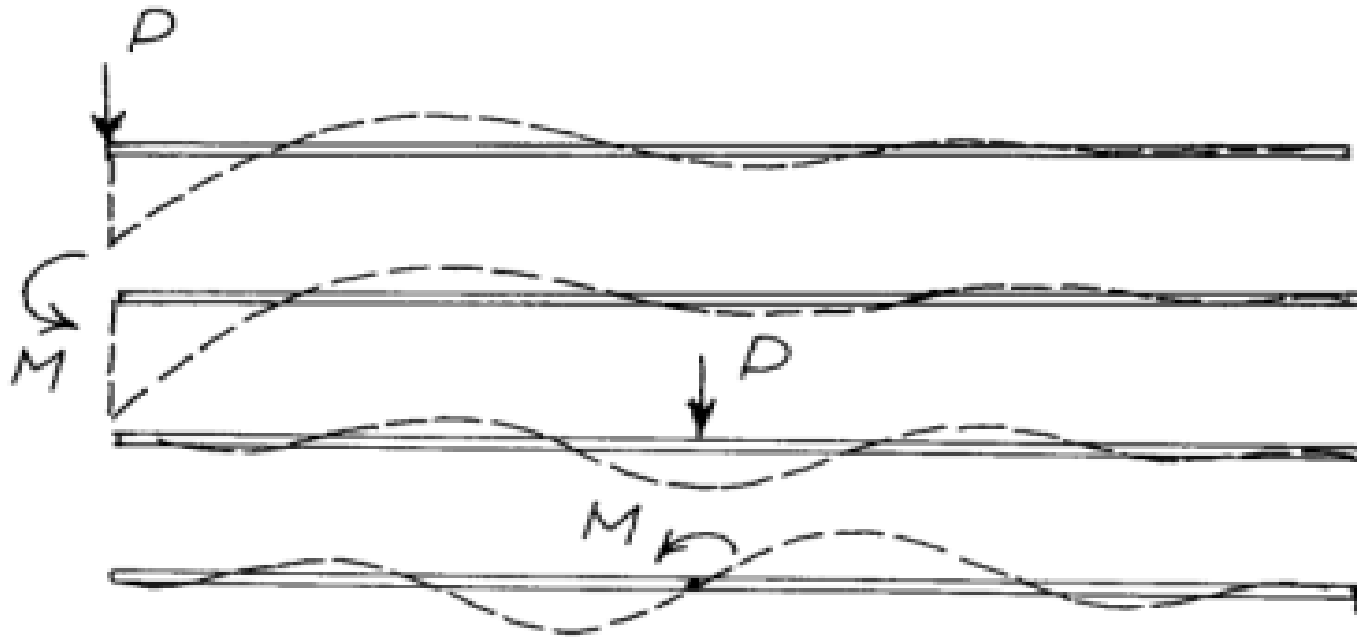
$M$ , moment

$T$ , shear force

$$EI \frac{d^4 y}{dx^4} = q - - k_s B y$$

$$L L = \sqrt{\frac{k_s B L^4}{4 EI}}$$

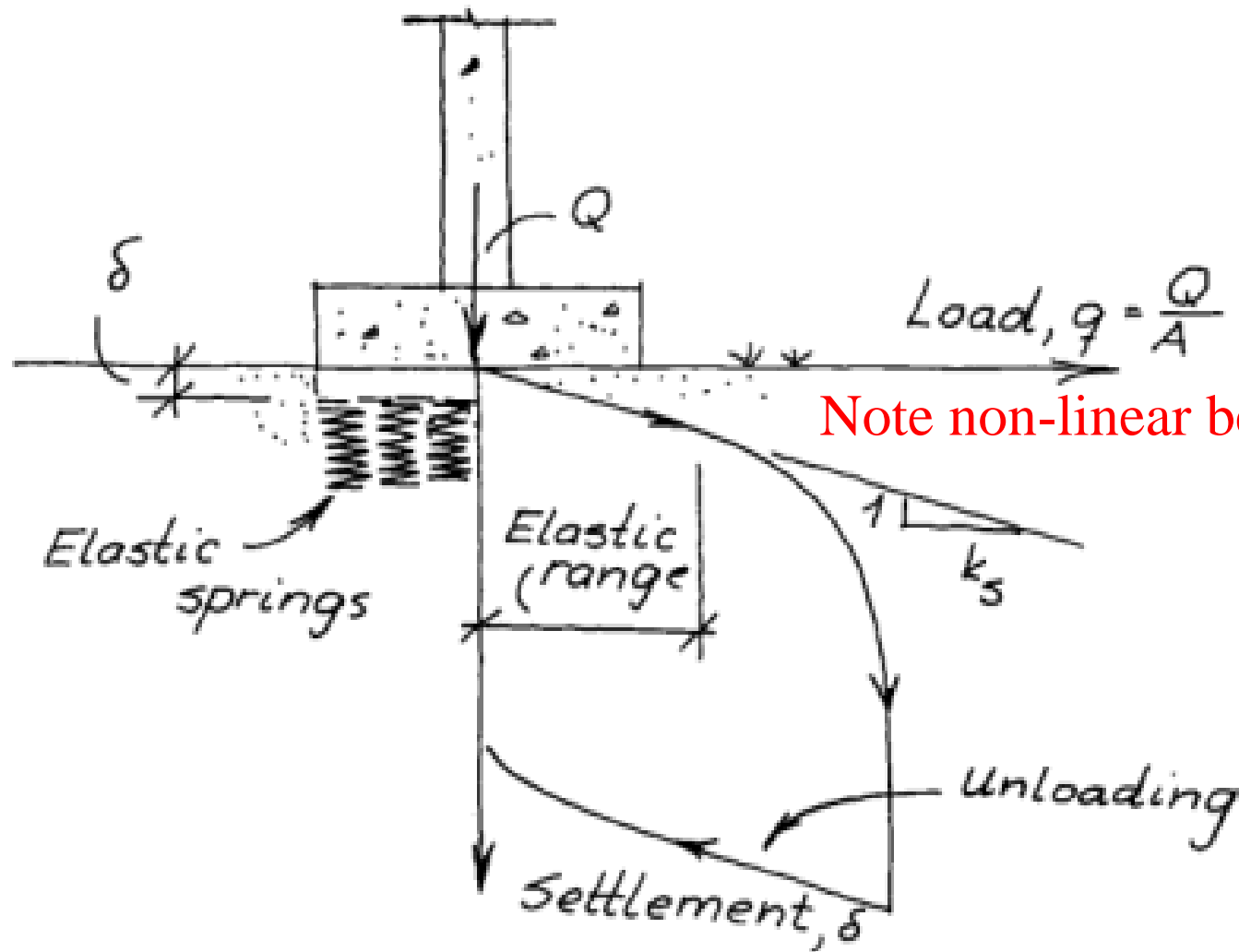
# Beams on Elastic Foundations



$$\Delta L = \frac{4}{\sqrt{\frac{k_3 B L^4}{4EI}}}$$

Shear force  
Moment  
Slope  
Deflection

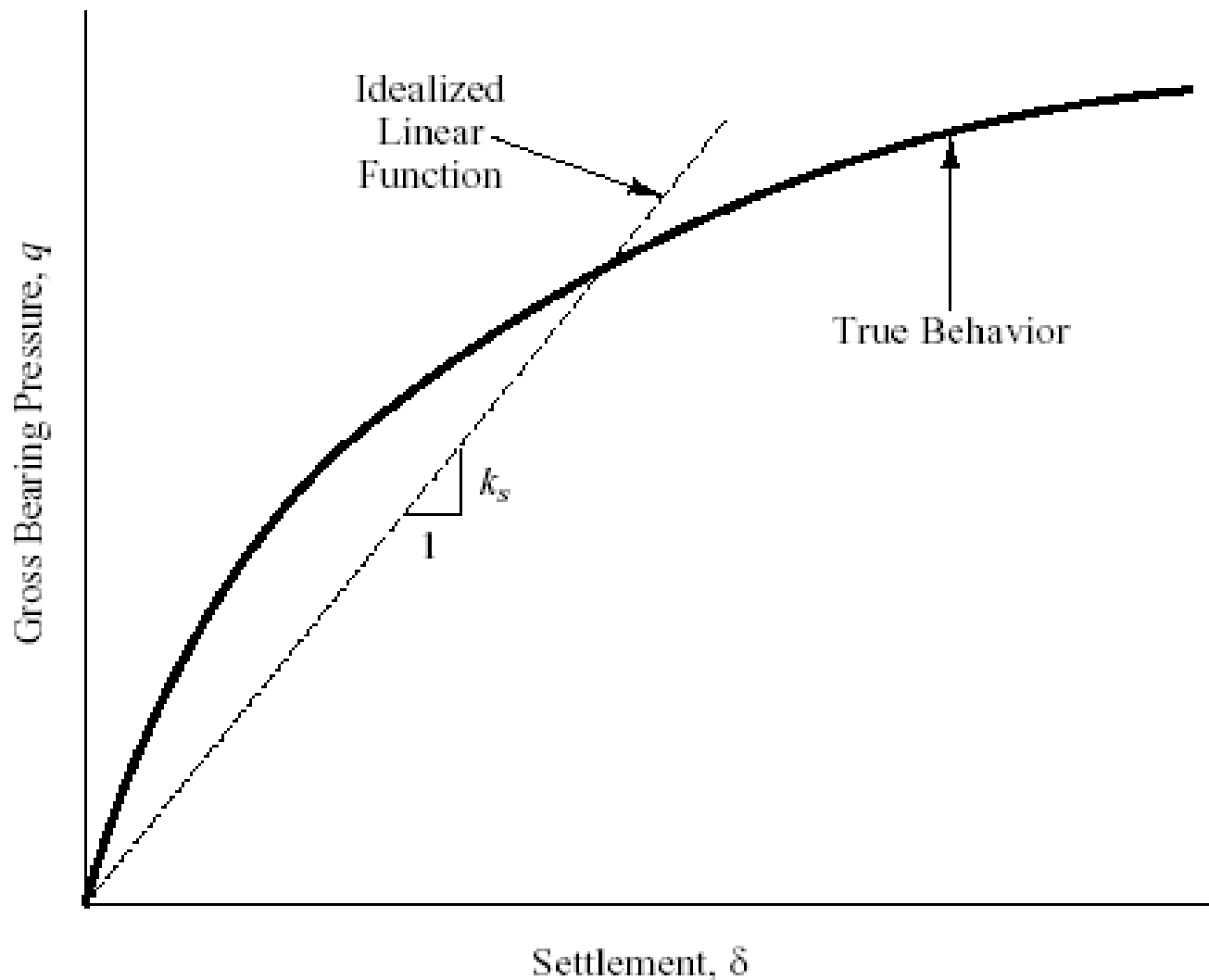
# Application to Spread Footings



Note non-linear behaviour

$$\text{Settlement, } \delta = q/k_s$$

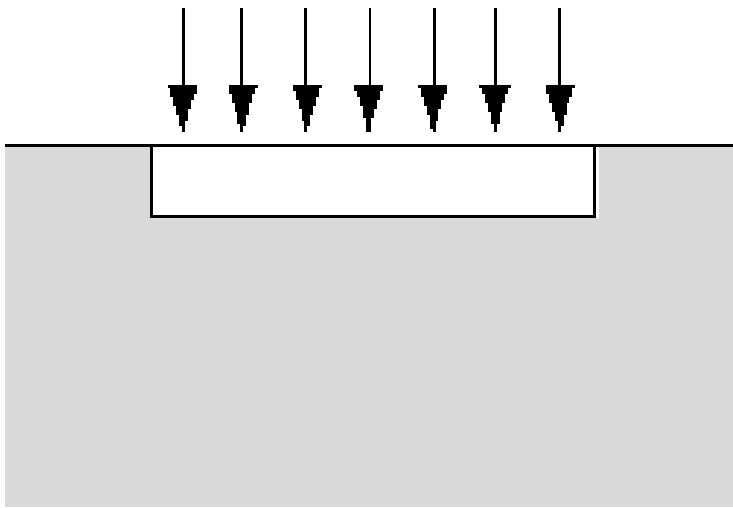
# Non-Linear Characteristics of Soil Deformation



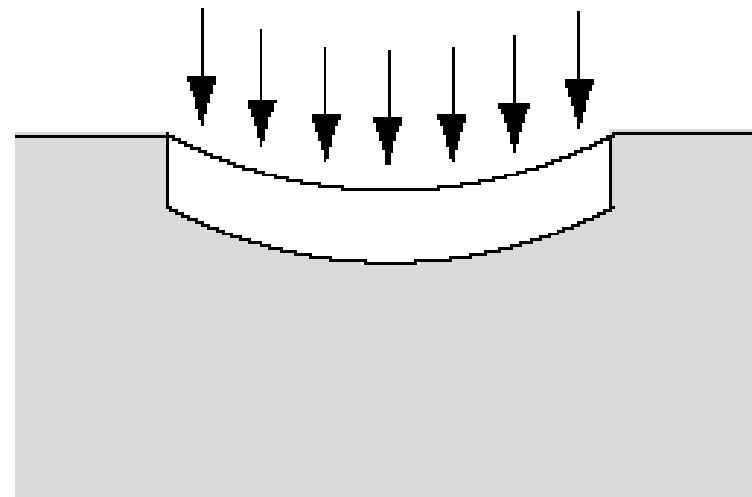
# Limitations of Winkler Method

- Load-settlement curves are not really linear; we must make a linear approximation to use the Winkler model
- Winkler model assumes that a uniformly loaded mat underlain by a perfectly uniform soil will uniformly settle into the soil.
  - Actual data show that such a mat-soil interaction will deflect in the centre more than the edges
  - This is one reason why we use other methods (such as Schmertmann's) to determine settlement

# Limitations of Winkler Method



Per Winkler



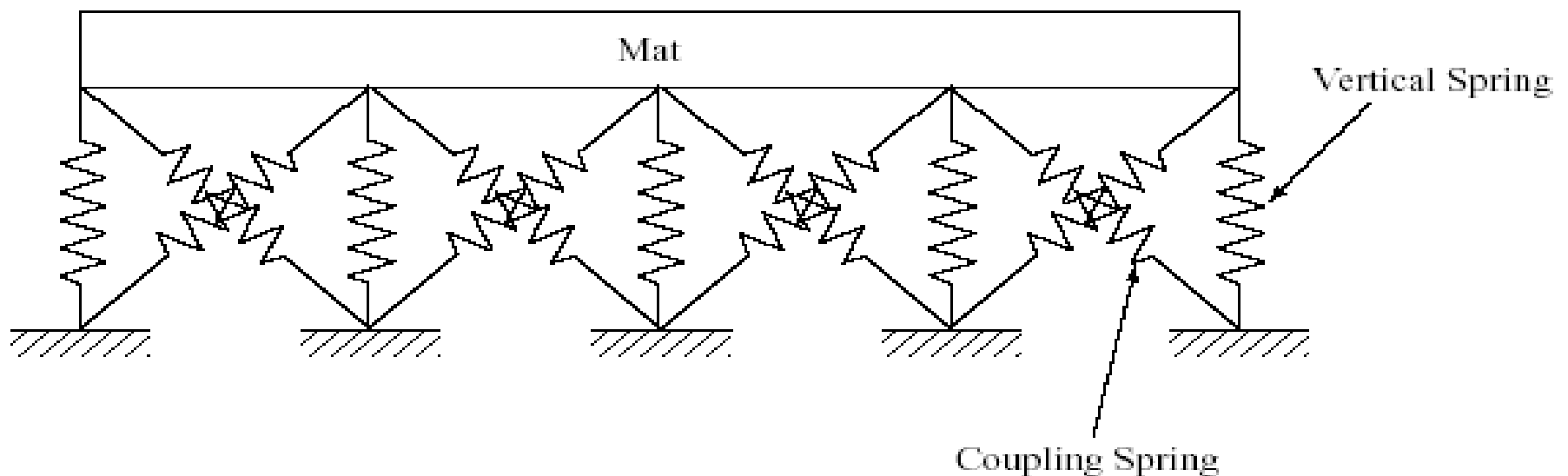
Actual

# Limitations of Winkler Method

- Soil springs do not act independently. Bearing pressure on one part of the mat influences both the "spring" under it and those surrounding it (due to lateral earth pressure)
- No single value of  $k_s$  truly represents the interaction between the soil and the mat
- The independent spring problem is in reality the largest problem with the Winkler model

# Coupled Method

- Ideally the coupled method, which uses additional springs as shown below, is more accurate than the Winkler method
- The problem with the coupled method comes in selecting the values of  $k_s$  for the coupling springs



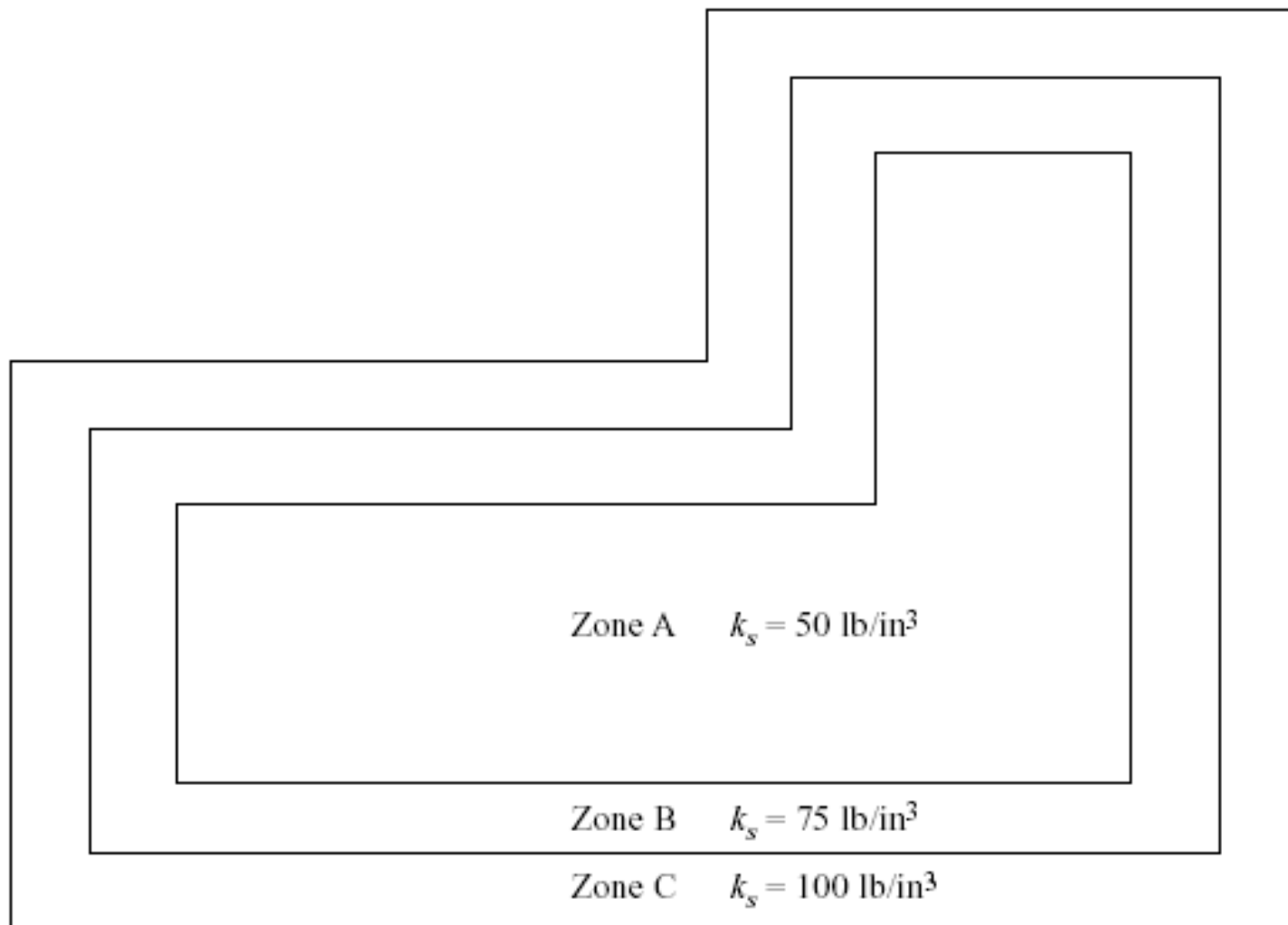
# Pseudo-Coupled Method

- An attempt to overcome both the lack of coupling in the Winkler method and the difficulties of the coupling springs
- Does so by using springs that act independently (like Winkler springs), but have different  $k_s$  values depending upon their location on the mat
- Most commercial mat design software uses the Winkler method; thus, pseudo-coupled methods can be used with these packages for more conservative and accurate results

# Pseudo-Coupled Method

- Implementation
  - Divide the mat into two or more concentric zones
    - The innermost zone should be about half as wide and half as long as the mat
  - Assign a  $k_s$  value to each zone
    - These should progressively increase from the centre
    - The outermost zone  $k_s$  should be about twice as large as the innermost zone
  - Evaluate the shears, moments and deformations using the Winkler method
  - Adjust mat thickness and reinforcement to satisfy strength and serviceability requirements

# Pseudo-Coupled Method



**Figure 10.10** A typical mat divided into zones for a pseudo-coupled analysis. The coefficient of subgrade reaction,  $k_s$ , progressively increases from the innermost zone to the outermost zone.

# Multiple-Parameter Method

- This method replaces the independently-acting linear springs of the Winkler method with springs and other mechanical elements
  - The additional elements define the coupling effects
- Method bypasses the guesswork involved in distributing the  $k_s$  values in the pseudo-coupled method; should be more accurate
- Method has not been implemented into software packages and thus is not routinely used on design projects

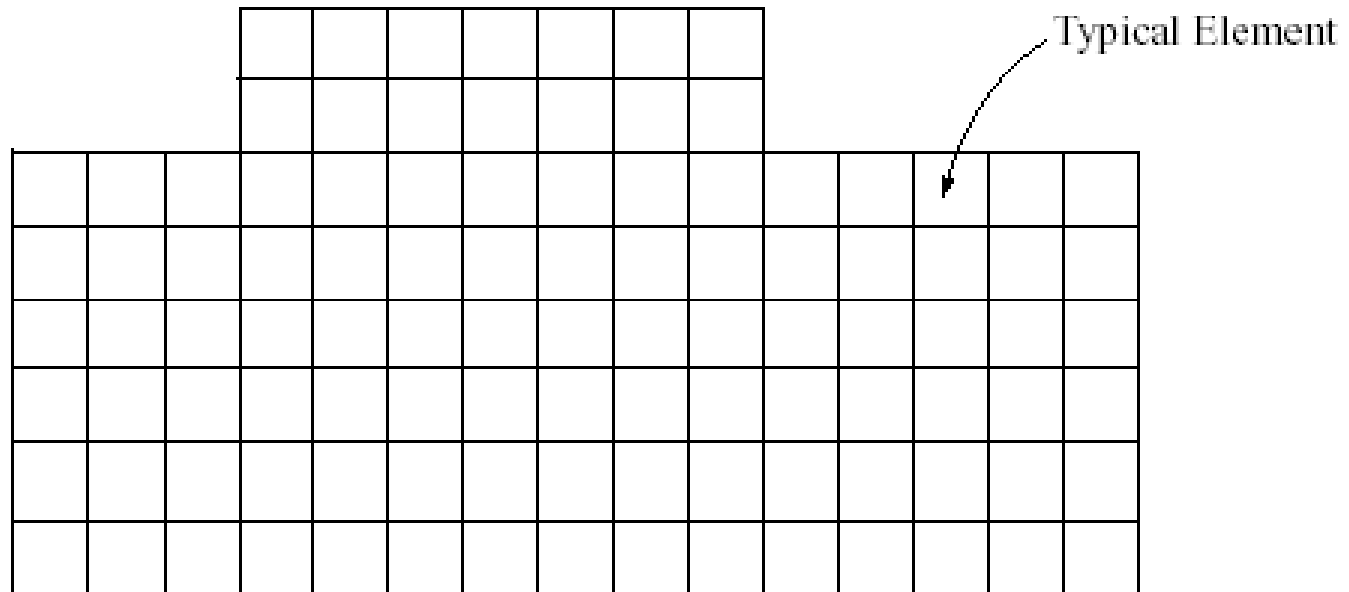
# Finite Element Method

- Models the entire soil-mat system in a three-dimensional way
- In theory, should be the most accurate method
- Method is not yet practical because
  - Requires large amount of computing power to perform
  - Difficult to determine soil properties in such a way as to justify the precision of the analysis, especially when soil parameters are highly variable
- Will become more in use as these problems are addressed

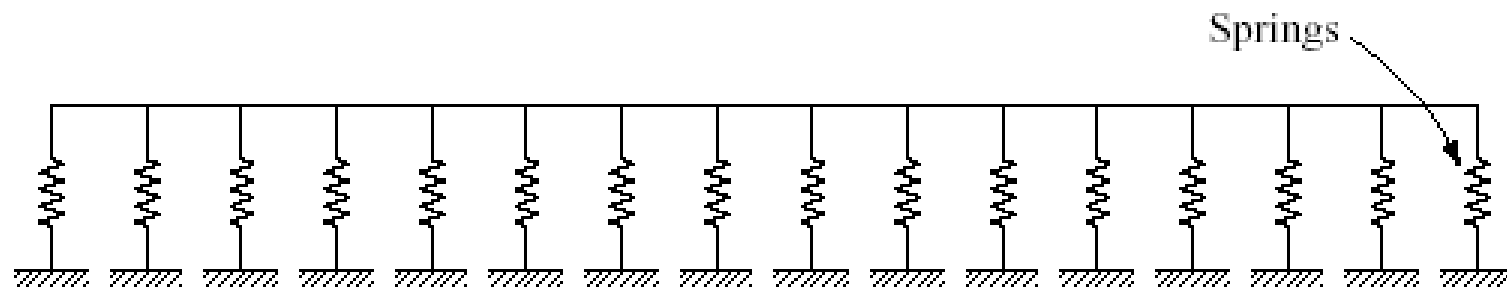
# Finite Element Method

- Finite element method is used for structural analysis
- Mat is modelled in a similar way to other plate structures with springs connected at the nodes of the elements
- Mat is loaded with column loads, applied line loads, applied area loads, and mat weight
- Usually superstructure stiffness is not considered (conservative)
  - Can be done but is rarely performed in practice

# Finite Element Method



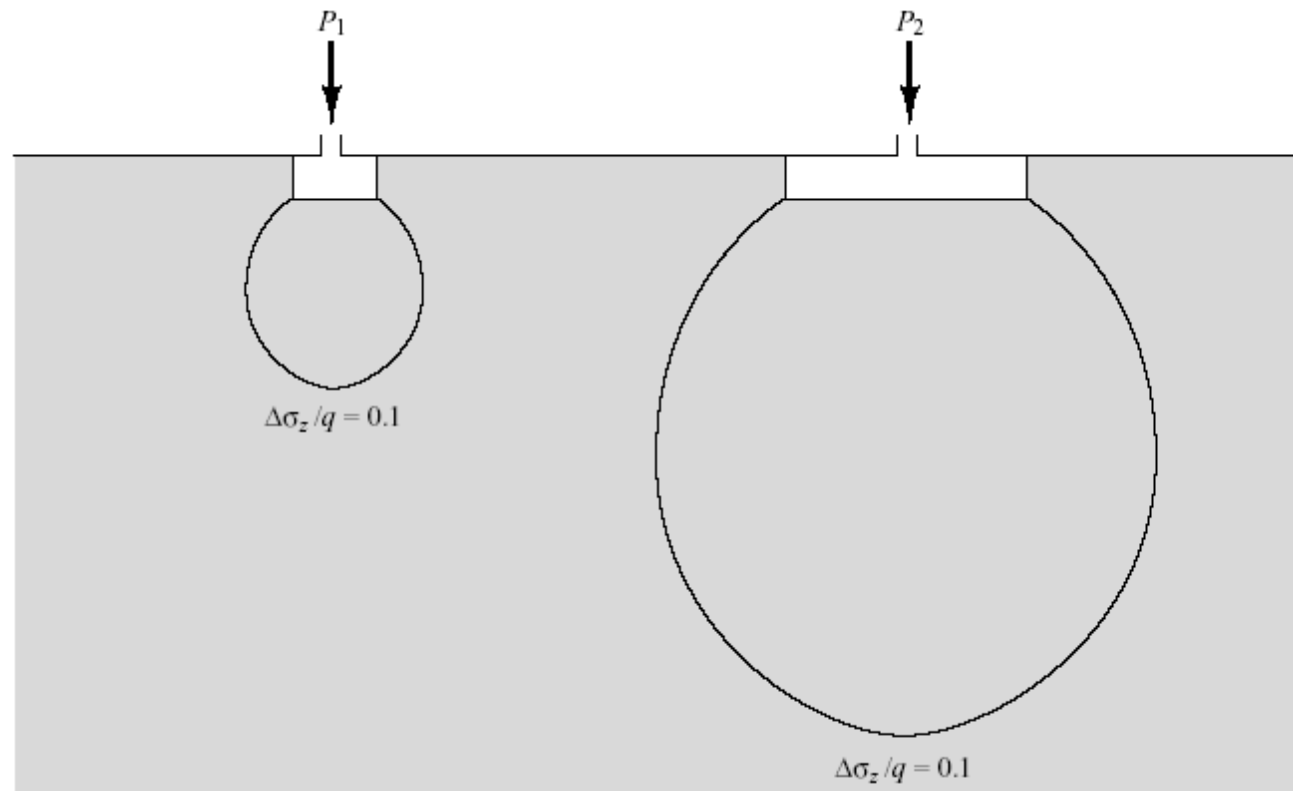
Plan



Profile

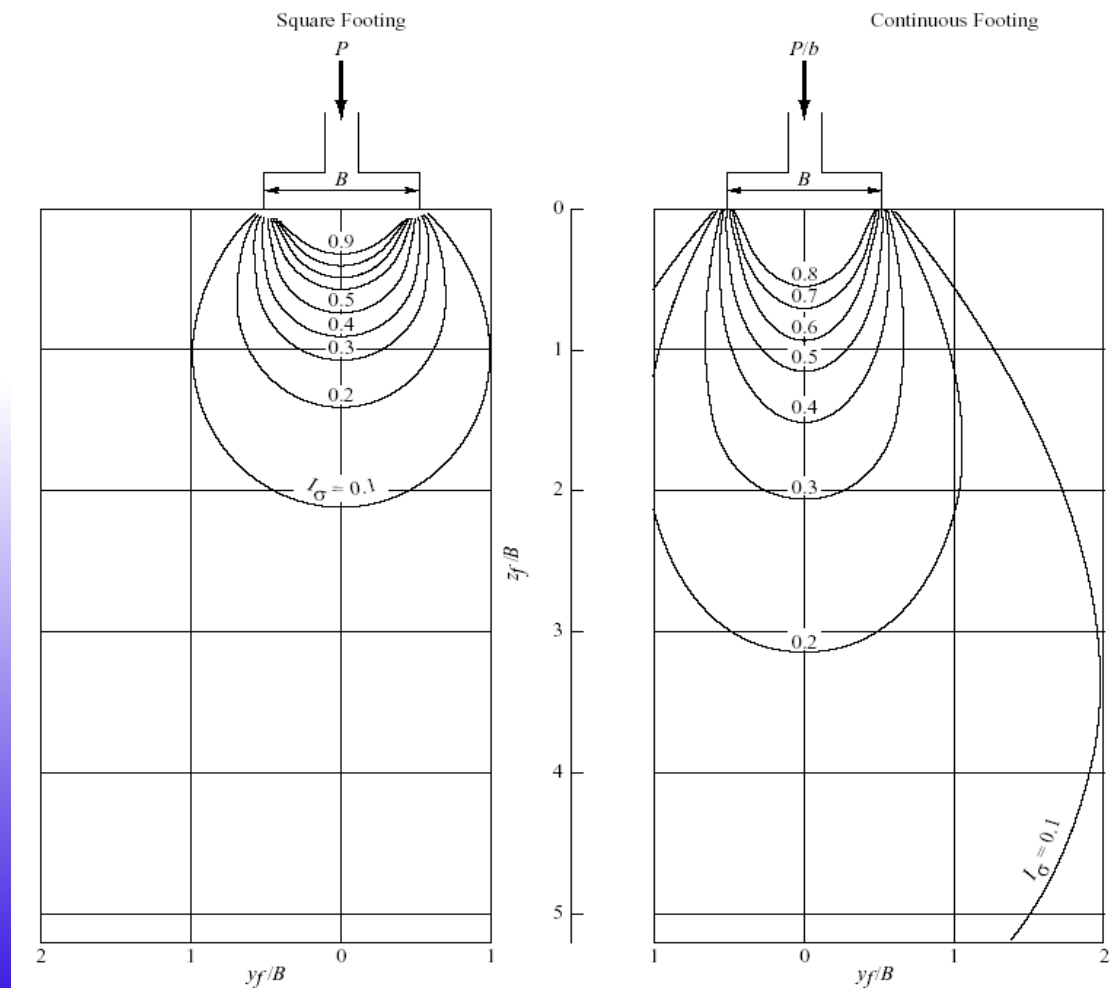
# Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - Width of the loaded area; wide mat will settle more than a narrow one because more soil is mobilised by a wide mat



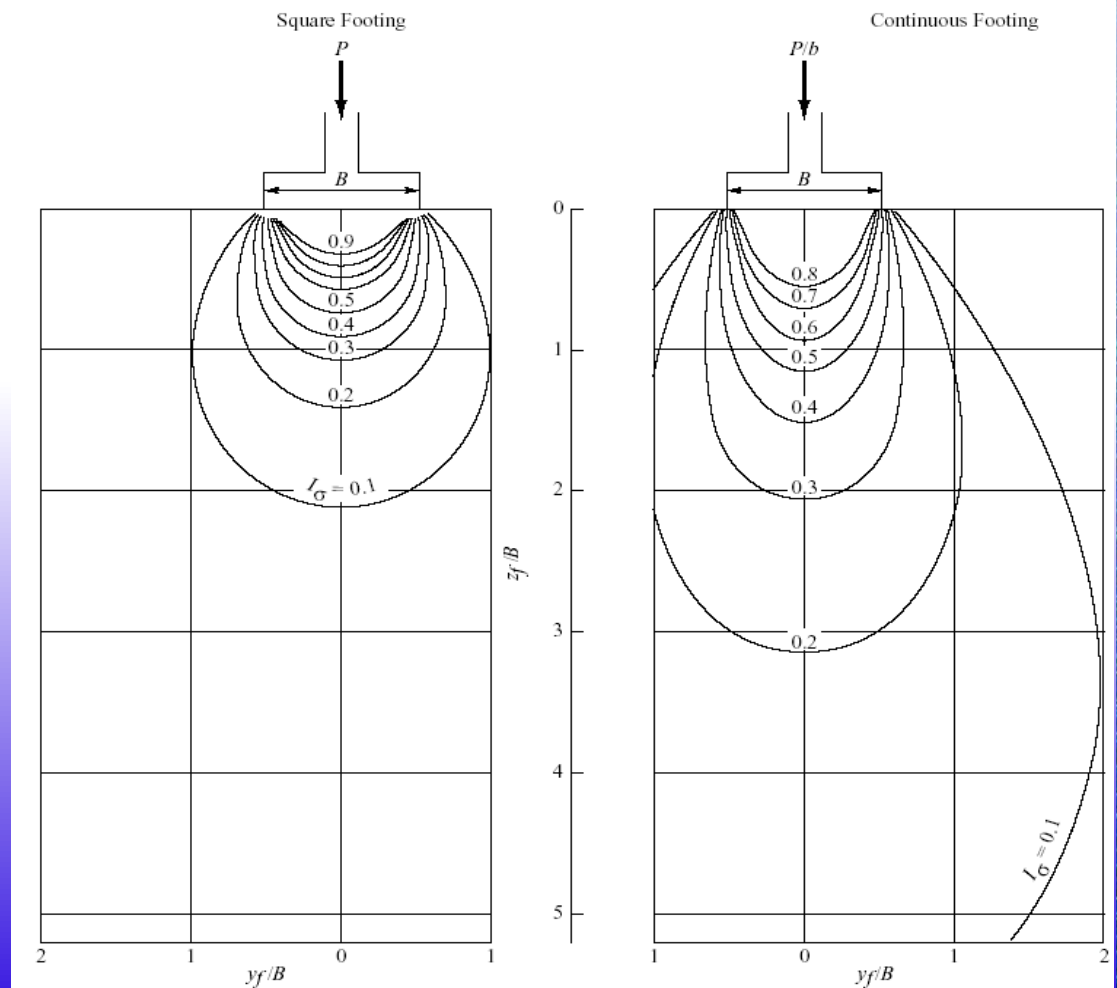
# Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - Shape of the loaded area: stresses beneath long, narrow loaded area is different from those below square loaded areas



# Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - Depth of the loaded area below the ground surface
  - Change in stress in the soil due to  $q$  is a smaller percentage of the initial stress at greater depths



# Determining the Coefficient of Subgrade Reaction

- Not a straightforward process due to:
  - The position of the mat
    - To model the soil accurately,  $k_s$  needs to be larger near the edges of the mat and smaller near the centre
  - Time
    - With compressible (and especially cohesive compressible soils) mat settlement is a process which may take several years
    - May be necessary to consider both short and long term cases
  - Non-linear nature of soil deformation makes unique value of  $k_s$  non-existent

# Determining the Coefficient of Subgrade Reaction

- Methods used to determine coefficient
  - Plate load tests
    - Test results must be adjusted between the shape of the loading plate and the actual shape of the foundation
    - Adjustment must also be made for the size of the plate vs. the size of the foundation, and the influence of size on the depth of soil stress
    - Attempts to make accurate adjustments have not been very successful to date
  - Derived relationships between  $k_s$  and  $E_s$ 
    - Relationships developed are too limited in their application possibilities

# Determining the Coefficient of Subgrade Reaction

- Methods used to determine coefficient
  - Use settlement techniques such as Terzaghi's consolidation theory, Schmertmann's method, etc., and express the results in a  $k_s$  value
    - If using a pseudo-coupled value, use values of  $k_s$  in the centre of the mat which are half those along the perimeter
    - This methodology has the potential of eliminating the problems described earlier while at the same time yielding values of  $k_s$  which then can be used in a structural analysis of the mat with some degree of confidence

# Example of Determining Coefficient of Subgrade Reaction

- Given
  - Structure to be supported on a 30 m wide by 50 m long mat foundation
  - Average bearing pressure is 120 kPa
  - Average settlement determined  $\delta = 30$  mm using settlement analysis method
- Find
  - Design values of  $k_s$  used in a pseudo-coupled analysis

# Example of Determining Coefficient of Subgrade Reaction

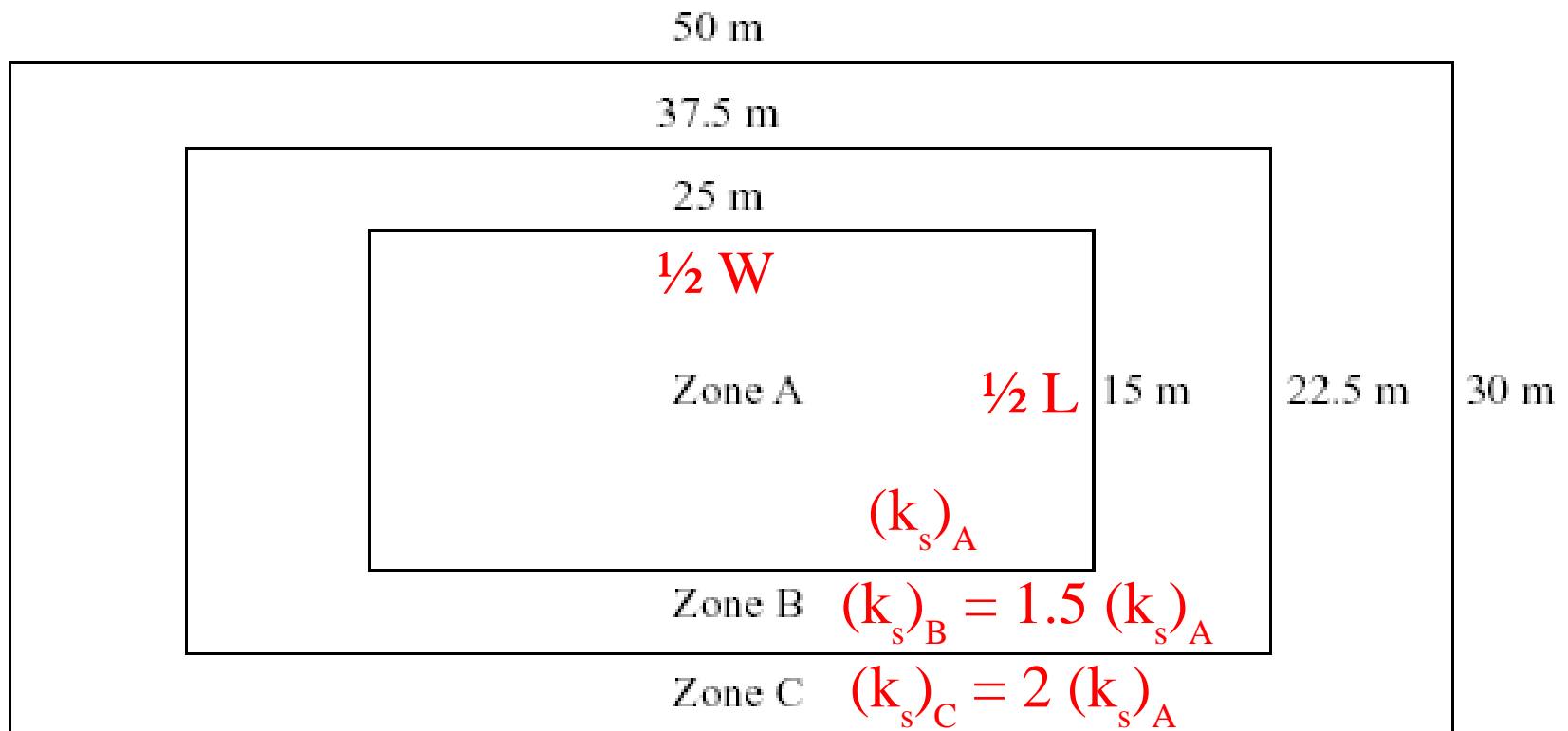
- Solution
  - Compute average  $k_s$  for entire mat

$$k_s = \frac{q}{\delta}$$

$$k_s = \frac{120 \text{ kPa}}{0.030 \text{ m}} = 4000 \text{ kN / m}^2$$

# Example of Determining Coefficient of Subgrade Reaction

- Solution
  - Divide mat into three zones as shown



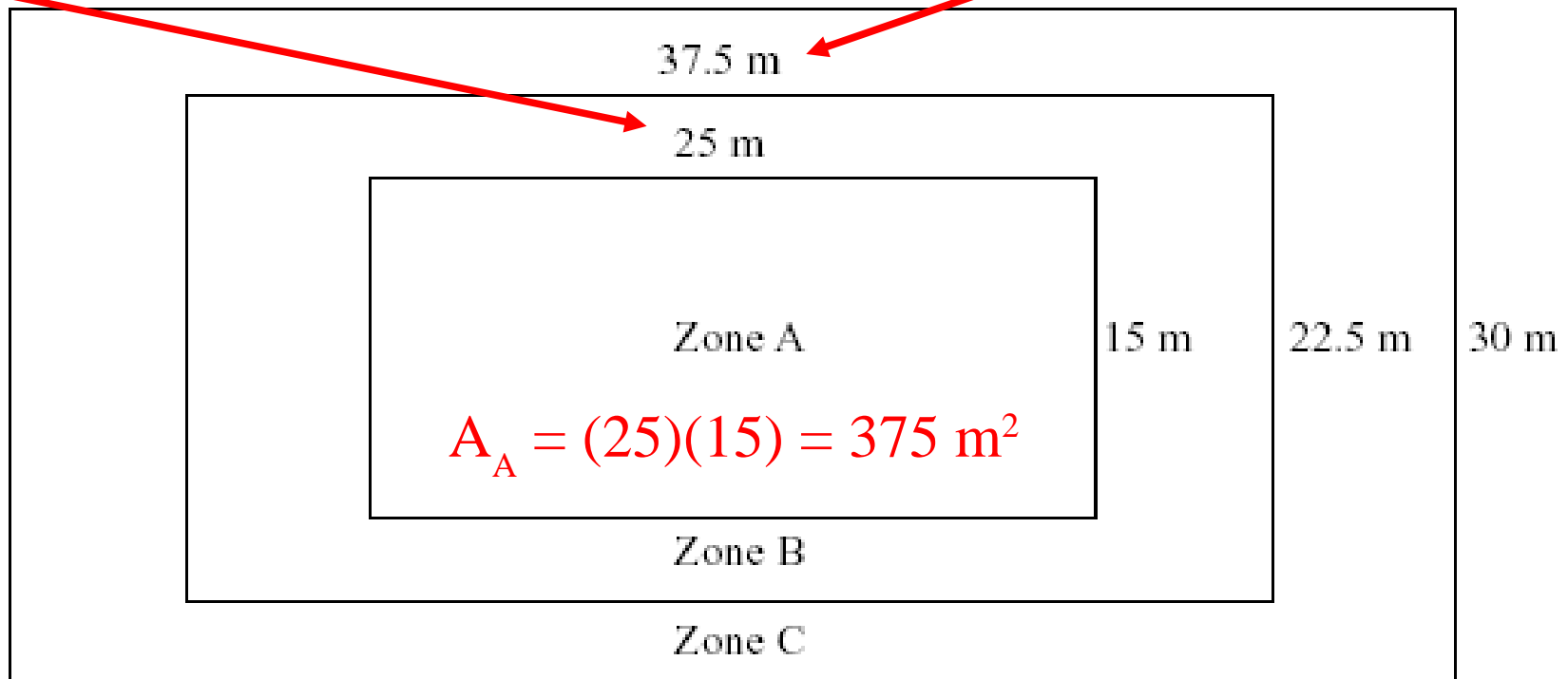
# Example of Determining Coefficient of Subgrade Reaction

- Solution

$$A_C = (50)(30) - 469 = 656 \text{ m}^2$$

- Compute the area of each zone

$$A_B = (37.5)(22.5) - 375 = 469 \text{ m}^2$$



# Example of Determining Coefficient of Subgrade Reaction

- Solution

- Compute the design  $k_s$  values

$$A_A(k_s)_A + A_B(k_s)_B + A_C(k_s)_C = (A_A + A_B + A_C)(k_s)_{avg}$$
$$375(k_s)_A + (469)(1.5)(k_s)_A + (656)(2)(k_s)_A = 1500(k_s)_{avg}$$
$$2390(k_s)_A = 1500(k_s)_{avg}$$

$$(k_s)_A = 0.627(k_s)_{avg} \quad (k_s)_A = (0.627)(4000) = 2510 \text{ kN/m}^2$$

$$(k_s)_B = (0.627)(1.5)(4000) = 3765 \text{ kN/m}^3$$

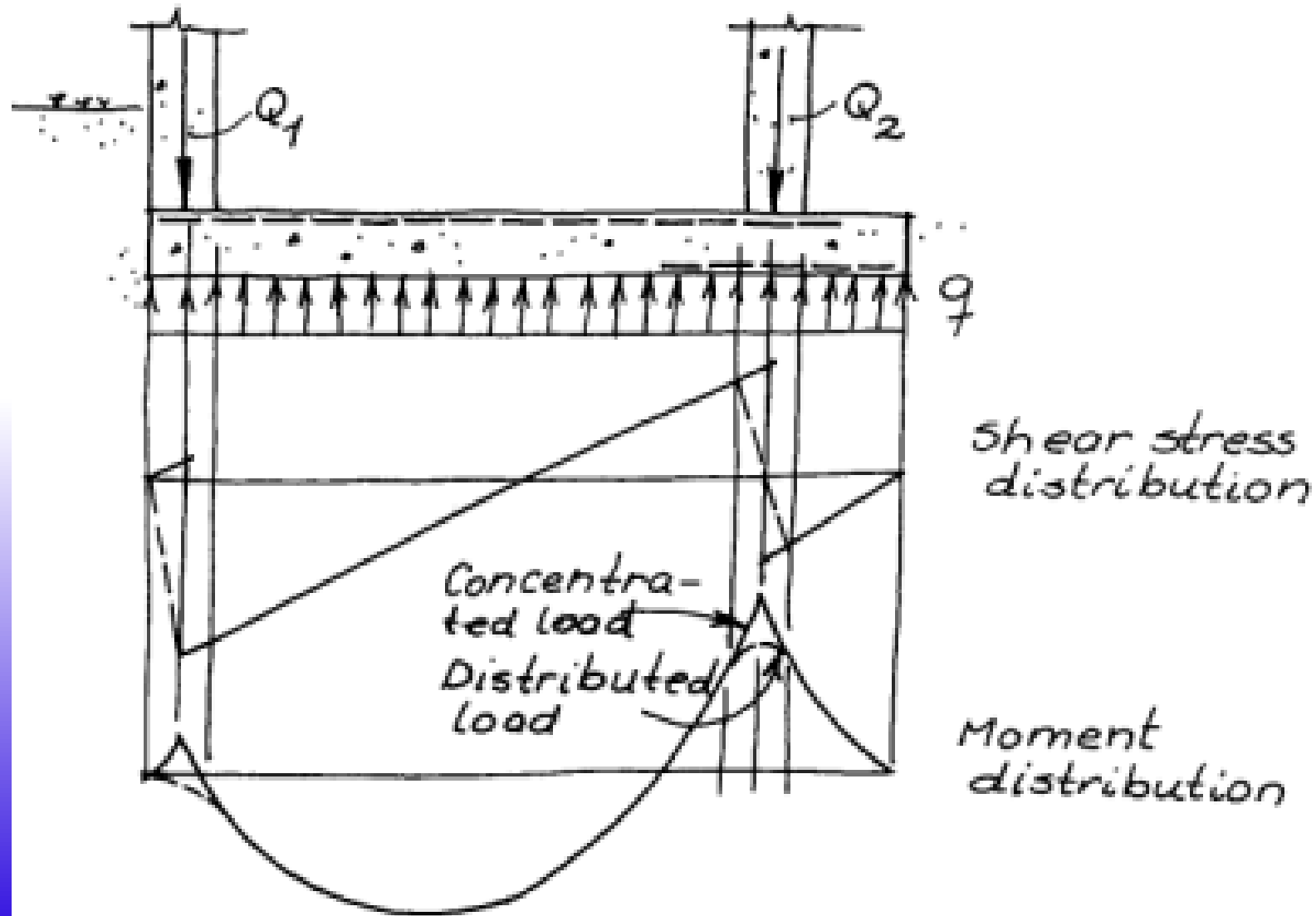
$$(k_s)_C = (0.627)(2)(4000) = 5020 \text{ kN/m}^3$$

- ACI suggests varying  $k_s$  from  $\frac{1}{2}$  its computed value to 5 or 10 times the computed value, then base the structural design on the worst condition

# Structural Design of Mats

- Structural design requires two analyses
  - Strength
    - Evaluate these requirements using factored loads and LRFD design methods
    - Mat must have sufficient thickness  $T$  and reinforcement to safely resist these loads
    - $T$  should be large enough so that no shear reinforcement is required
  - Servicability
    - Evaluate using unfactored loads for excessive deformation at places of concentrated loads, such as columns, soil non-uniformities, mat non-uniformities, etc.
    - This is the equivalent of a differential settlement analysis
    - Mat must be made thicker if this is a problem

# Structural Design of Mats



# Structural Design of Mats

Finite difference method.

$$\frac{\partial^4 w}{\partial x^4} + \frac{2 \partial^4 w}{\partial x^2 \partial x^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} + \frac{D}{D(\partial_x \partial_y)}$$

Computer programs are available

Uncertainties:  $E_{\text{concrete}}, I_{\text{raft}}, k_s, E_s$

↑  
Coefficient of sub-grade reaction.

# Structural Design of Mats

- Closed form solutions
  - Once popular; however, with the advent of computers, have fallen out of favour
    - For example see <http://www.vulcanhammer.net/download/piletoe.pdf>
- Finite difference methods
- Finite element methods
  - Spring values as computed in the example can then be used in finite element analysis
  - The stiffer springs at the edges will encourage the foundation to sag in the centre, which is what we actually see in foundations

# Other Considerations in Mat Foundations

- Total settlement
  - "Bed of springs" solution should not be used to compute total settlement; this should be done using other methods
- Bearing capacity
  - Mat foundations generally do not have bearing capacity problems
  - With undrained silts and clays, bearing capacity needs to be watched
  - Methods for spread footings can be used with mat foundations, including presumptive bearing capacities

# Questions

